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# Some unexpected thermal effects on squeal events observed on the experimental bench FIVE@ECL

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## Abstract

The proposed study investigates the potential links between thermal effects and structural modifications of a brake system which can lead to minor modifications of the nonlinear squeal signature at temperatures lower to 100 °C. Experiments are performed on the test bench Friction-Induced Vibration and noise at Ecole Centrale de Lyon (FIVE@ECL). The analyses are carried out on both the vibratory nonlinear responses and the acoustic noise occurring during transient and stationary squeal events.

The second main contribution is to propose a detailed analysis and an in-depth methodology to demonstrate experimentally the effects of temperature increase not only on the non-linear squeal phenomenon but also on the modal behavior of a specific part of the braking system which corresponds to the brake disc. It is shown that the characteristics of the squeal signature related to the frequencies decrease or amplitude fluctuation for specific frequencies contribution can be explained in part by a relation between temperature changes and associated structural changes in the brake system.

*Keywords:* friction-induced vibration, squeal noise, transient squeal events, experimental benchmark FIVE@ECL

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## 1. Introduction

Many studies have been conducted to investigate friction-induced vibration and noise of disc brake systems. One of the major and very ambitious challenges is the ability to provide a better understanding of the physical phenomena involved that cause squeal noise and its potential complex evolution during a braking test. Even if several mechanisms and physical factors have been proposed in the literature to explain friction-induced vibrations [1, 2, 3, 4] their impact on brake squeal are often studied and dealt with separately. Some industrial and academic experimental studies are more focused on the characterization of squeal phenomena based on the self-sustaining vibrations of the brake system [5, 6, 7, 8, 9, 10, 11, 12, 13, 14] while other researchers approach the subject from a purely tribological point of view [15, 16, 17].

Only few studies have been addressed to illustrate the fact that the problem of friction-induced vibrations is in reality the consequence of various physical mechanisms and potentially the combination of complex phenomena including both the dynamic behaviour of the brake system but also some additional tribological problem. Some researchers demonstrated that squealing events are correlated with the physical or chemical changes that induce modifications in the contact behaviour [18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29]. Others clearly state that some geometrical modifications of the friction such as grooved surfaces may significantly influence the noise performance of the brake system [30, 31, 32]. Similarly, brake materials can reach high temperatures during braking. Consequently the influence of temperature on both the occurrences and frequencies of the brake squeal is quite complex. In particular, studies have focused on temperature sensitivity in brake materials by considering the effects of thermo-physical properties such as specific heat, thermal conductivity and diffusivity on the contact pressure and surface morphology [33, 34, 35, 36, 37, 38]. Lazarri et al. [39] explained that an increase in temperature results in a higher acoustic energy generated by the contact due to an increase in the physio-chemical reactivity of the surface, resulting in higher friction noise. All these studies demonstrated that the pad structure and its physical characteristics at the contact surface play a significant role on the self-excited vibrations and squealing noise of brake systems. Although extensive experimental research has been conducted to examine the influence of thermo-physical properties on interfacial characteristics and consequently on the squeal propensity, little work has been developed to determine the effect of temperature on the overall braking system and its potential impact on squeal [10]. Recently, a study illustrated the effect of temperature distribution on the disc surface [40], but the proposed approach was more focus on the effect of friction block configuration on the brake squeal.

It is also nowadays very clearly established that the appearance of friction-induced vibrations is weighted relatively to the system configuration, operating and environmental conditions. Being able to have a global vision on these

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different origins and their role and interaction on the appearance of brake squeal is a very delicate task. Very few studies today address a transverse vision of the problem of squealing brakes by including complementary points of view of vibratory and tribological themes. Only a few upstream studies, carried out by researchers who are experts in the field (see for example [20, 11]), approach the subject of brake squeal from this point of view. One of the objectives of this study is to make a contribution on the need to consider this dual approach based on vibrational and tribological phenomena for a better understanding of squeal events observed on the experimental bench FIVE@ECL. More specifically, one of the original contributions will be to discuss the fact that certain tribological problems such as thermal effects may play a role in the evolution of the non-linear vibration signature of the brake squeal event at operating temperatures which are considered to remain low. Based on the previous remarks one of the contributions of the present study is to investigate the effect of temperature on the whole braking system and more particularly its influence on the characteristics of the brake disc. One of the originality will be to highlight that structural modifications of the disc can be induced by a temperature evolution which can lead to a modification of the squeal signature. Such observations, as yet unreported in the literature, will illustrate some unexpected thermal effects on squeal events that are directly correlated to the influence of temperature on the structural disc vibration behavior.

This work is in the continuity of the previous works [41, 42] that address the problem of transient and stationary friction-induced vibration and squeal noise based on the laboratory experimental bench Friction-Induced Vibration and noise at École Centrale de Lyon (FIVE@ECL).

This manuscript is organized as follows. A brief reminder of the general description and characteristics of the laboratory brake system Friction-Induced Vibration and noise at Ecole Centrale de Lyon (FIVE@ECL) is first discussed. Then the most relevant results on the squeal phenomena previously discussed in [41, 42] are recalled in order to position the objective and the originality of the proposed study. In the later sections, thermal characteristics during squeal events as well as thermal effects on modal analysis of the brake system are carried out to provide a deep and original contribution on a better understanding of the thermal effects on squeal events for braking tests below 100 °C performed on the experimental bench FIVE@ECL.

## 2. Preamble and positioning of the study

The main objective of this section is to give a brief reminder of the general description and characteristics of the laboratory brake system as well as the experimental set-up methodology that have been previously described in [41, 42]. For the interested reader all the raw data of the specific braking test are available as open-data [43].

### 2.1. Description of the experimental bench FIVE@ECL

Firstly, a brief description of the experimental bench FIVE@ECL and the vibrational and acoustic measurements in link with the proposed study are given. Figure 1(a) shows the brake system under study that is composed of a disc with a 0.16 m external diameter, 0.033 m internal diameter and a thickness of 0.002 m, two pads F03C and a caliper XT M785 from the Japanese manufacturer Shimano. The rotating and no-rotating elements of the brake system are mounted on two independent rigid supports to avoid a transmission of parasitic vibration during braking tests with a potential back loop on the squeal phenomenon (for more details see [41]). The braking system is activated by a hydraulic pump system inducing friction forces caused by the pressure of the brake pads against the disc. The disc is driven by an electric motor which allows braking tests to be carried out at a constant controlled speed. This ensures that potential evolutions of friction-induced vibration and squeal noise are not due to uncontrolled fluctuations of the rotational speed but to real additional physical phenomena involved during the braking process.

Considering more specifically the vibrational and acoustic measurements, the complete list of the experimental device used for the proposed study is as follows:

- five miniature triaxial Integrated Electronic Piezo-electric (IEPE) accelerometers: two for each pad and one placed on the brake caliper (ref. Bruël & Kjær Miniature Triaxial DeltaTron@Accelerometer - Type 4520);
- four proximitor sensors to perform non-contact measurement on the rotational disc (ref. Bently Nevada 3300 XL NSv). They will be used for several purposes: to measure the evolution of the static position and the dynamic vibration values during a braking test, to perform modal analysis for the non-rotating brake system at several temperatures, and to estimate the disc thickness variation due to thermal effect.
- two microphones close to the brake system to perform measurements in near-field (ref. Bruël & Kjær 20 kHz Prevision Array Microphone - Type 4958).

Two CompactDAQ devices from National Instrument are used to perform the dynamic signal acquisition with a sampling rate of 25600 S/s and a 24 bit resolution for each channel.

In addition to these vibrational and acoustic measurements, four operational parameters are monitored during experiments: the pressure, the rotating speed of the disc, the motor torque and the temperature close to the pad/disc brake system. The channel identification and position of each sensor are provided in [41, 42]. Figure 1(b) shows the position of the T-type thermocouple sensor close to the frictional interface where the origin of the temperature increase is located. Given the position of the temperature sensor, it allows to measure the temperature of the backplate near one of the contact surface.

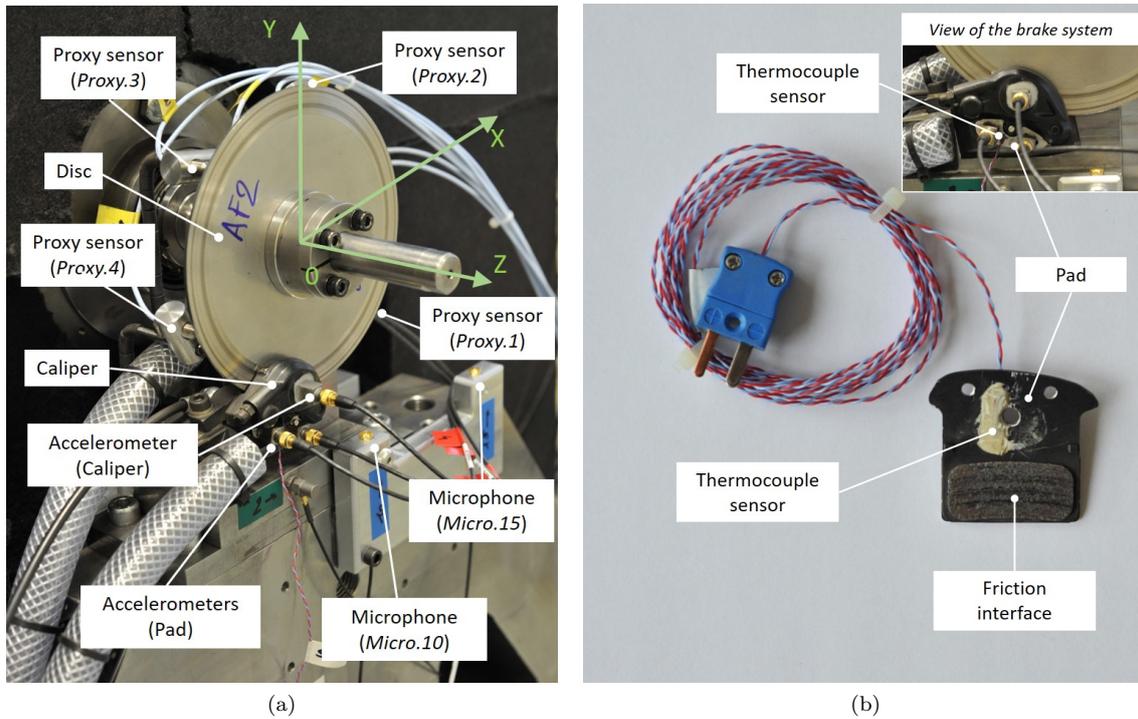


Figure 1: Experimental bench FIVE@ECL (a) brake system under study (b) visualization of the thermocouple sensor for one Shimano F03C brake pad

It should be noted that usually far-field acoustic measurements in the direction normal to the brake system are also performed by using one array composed of thirteen microphones. In the present study, squeal noise in far-field will not be analyzed due to the fact that a FLIR T640 thermal camera has been installed in place of the far-field acoustic device.

## 2.2. Previous results on the squeal phenomena main characteristics and positioning of the proposed study

The main objective of this section is to briefly describe the squeal phenomena characteristics previously investigated in [41, 42] with the aim of introducing the original scientific subject of study proposed in this paper.

As a reminder, a braking test consists on a single braking that is composed of three successive phases. It is carried out according to the following protocol:

- Phase 1: the brake system runs at a constant rotational speed without braking. This initial phase makes it possible to identify any vibrations or parasitic noises so as not to confuse them with the squeal phenomena.
- Phase 2: a rise in brake pressure is achieved up to a nominal value. During this second phase, the disc and the two pads come into contact. This leads to the appearance of friction-induced vibrations and squeal noise.
- Phase 3: the braking test is operational and the squeal characteristics based on vibrational and acoustics measurements are undertaken.

A braking test is characterized by a constant rotational speed for all the duration of the test, an initial temperature at the beginning of the first phase, a specific duration for the three phases previously described, a linear evolution of pressure values during ramp-up pressure phase 2 and a constant pressure during the last phase.

Figure 2 illustrates one typical evolution of squeal noise in far-field, previously published in [41]. Table 1 summarizes the operating conditions associated with this braking test. As previously explained in [41], the nonlinear vibratory signature of brake squeal is mainly composed of three fundamental frequencies (i.e.  $f_1=[579;584]$  Hz,  $f_2=[754;769]$  Hz and  $f_3=[2019;2044]$  Hz) and the harmonic components of the first fundamental frequency (i.e.  $nf_1$ ). The most important contributions are provided by  $f_1$ ,  $3f_1$  and  $4f_1$ . Small frequency variations are also observed during the braking test (for the interested reader the value of each squeal frequency during a braking test are given on the bottom of each subfigure of Figure 2). Based on previously conducted investigations [41, 42], it can be noted that each frequency contribution is present at different levels of contributions for the pads, the caliper and the disc. For more details on the complete and detailed analysis of squeal phenomena from friction-induced vibration and squeal noise in far-field and near-field, the reader may refer to [41]. From a temporal point of view, the repeatability of the squeal and the links between acoustic and vibration squeal events have also been discussed in [42].

Moreover, it has been observed that the squeal event (for both frequency content and the associated amplitudes) is globally stable during one braking test even if some small potential variations of the squeal characteristics in time can be detected. The original contribution of the present paper will be to analyze in detail these evolutions of the squeal signature during a test. More specifically the following general observations have been previously brought [42]:

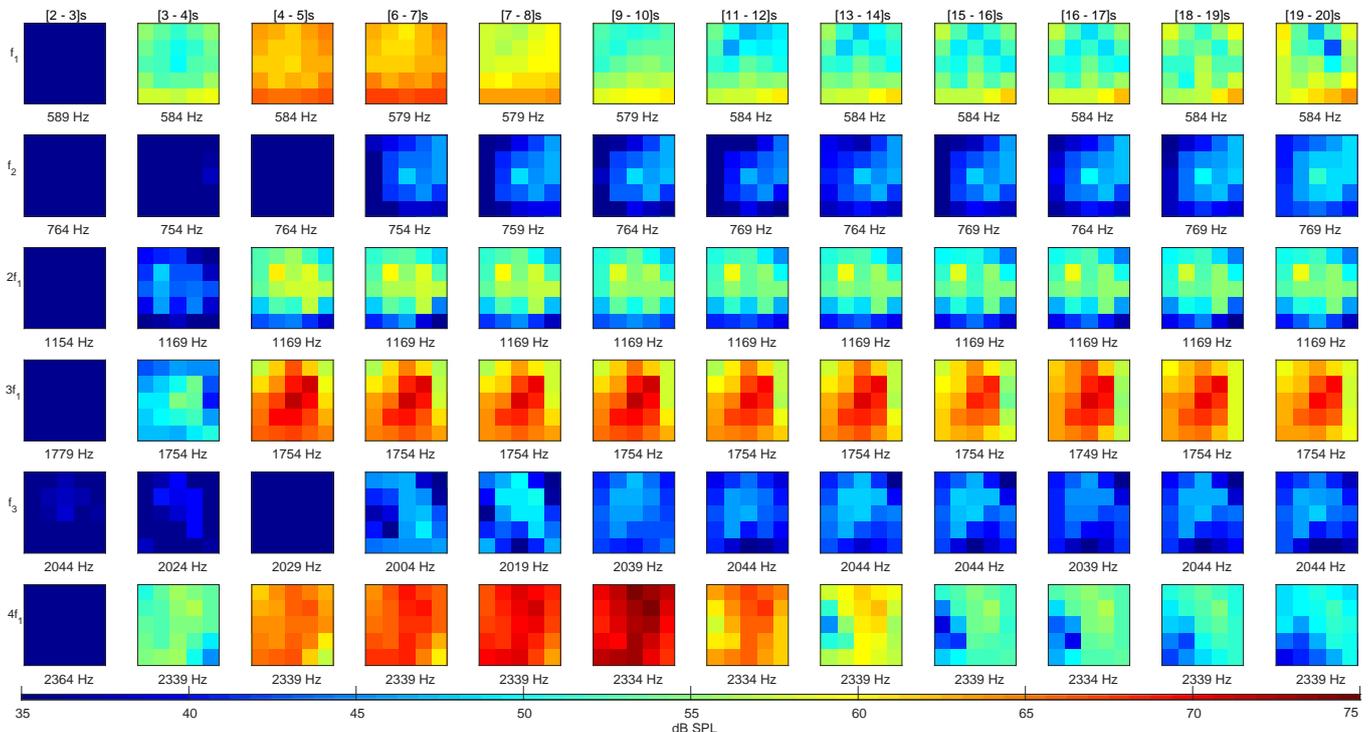


Figure 2: Evolution of the far-field noise emissions during the braking test on the frequency range [500;2500] Hz from the reference case provided in [41]

- A significant evolution of the acoustic noise in far-field during a braking test can be observed for specific frequency contributions while keeping a constant frequency signature. As illustrated in Figure 2, the acoustic levels at the first fundamental frequency  $f_1$  are high at the beginning of the braking tests and then decrease to stay at low levels until the end of the braking test. In the same way, the acoustic levels at the fourth harmonic  $4f_1$  increase initially with the highest level of intensity reached between  $t = [4; 12]$  s and then decrease slowly. We can also note that the increase and decrease levels for  $f_1$  and  $4f_1$  are not similar and identical in time. Moreover a slow increase of other frequency contributions are also visible (see for example  $f_2$  in Figure 2). This phenomenon is also noticeable on vibration measurements. For the sake of brevity all these previous results are not presented in this section, see [42] for more details or Section 3.1 where a detailed analysis will be conducted for the experimental tests performed specifically for the study proposed in this paper. It has been previously verified that these transient squeal phenomenon are repeatable on several braking tests [41, 42].
- Some frequency contributions of low level are characterized by a pronounced decrease in frequency during the braking test (see [41, 42] or Section 3.1 for more explanations). This phenomenon will be illustrated in Section 3.1.

A basic analysis has been previously put forward by the authors [42] to explain this complex and repeatable transient phenomenon appearing during squeal events. This phenomenon could be indirectly linked to a rise in temperature during a squeal test for which the pads temperature reaches 70 °C at the end of the test. The temperature increase may induces some structural modification of the brake system leading to assume that some characteristics of the squeal signature may be linked to the frequency shift of some structural modes during a braking test. This hypothesis was initially formulated without extensive study on the subject. So the major objective of the following study will be to perform further investigations on the potential links between the temperature and structural modifications of the brake system as well as the resulting effects on the transient behavior of the squeal signature.

| Case             | Pressure (bar) | Rotational speed (rpm) | Phase 1 (s) | Phase 2 (s) | Phase 3 (s) |
|------------------|----------------|------------------------|-------------|-------------|-------------|
| Reference [41]   | 9.2            | 203                    | [0;3.1]     | [3.1;6.7]   | [6.7;20]    |
| Case under study | 8.4            | 202                    | [0;2.7]     | [2.7;4.7]   | [4.7;30]    |

Table 1: Operating conditions for braking tests

### 3. Experiments and discussion

#### 3.1. Thermal characteristics during squeal events

As previously mentioned, some specific braking tests were performed using an infrared camera. This non-contact temperature measurement device detect radiation in the form of the infrared energy transmitted, emitted and reflected

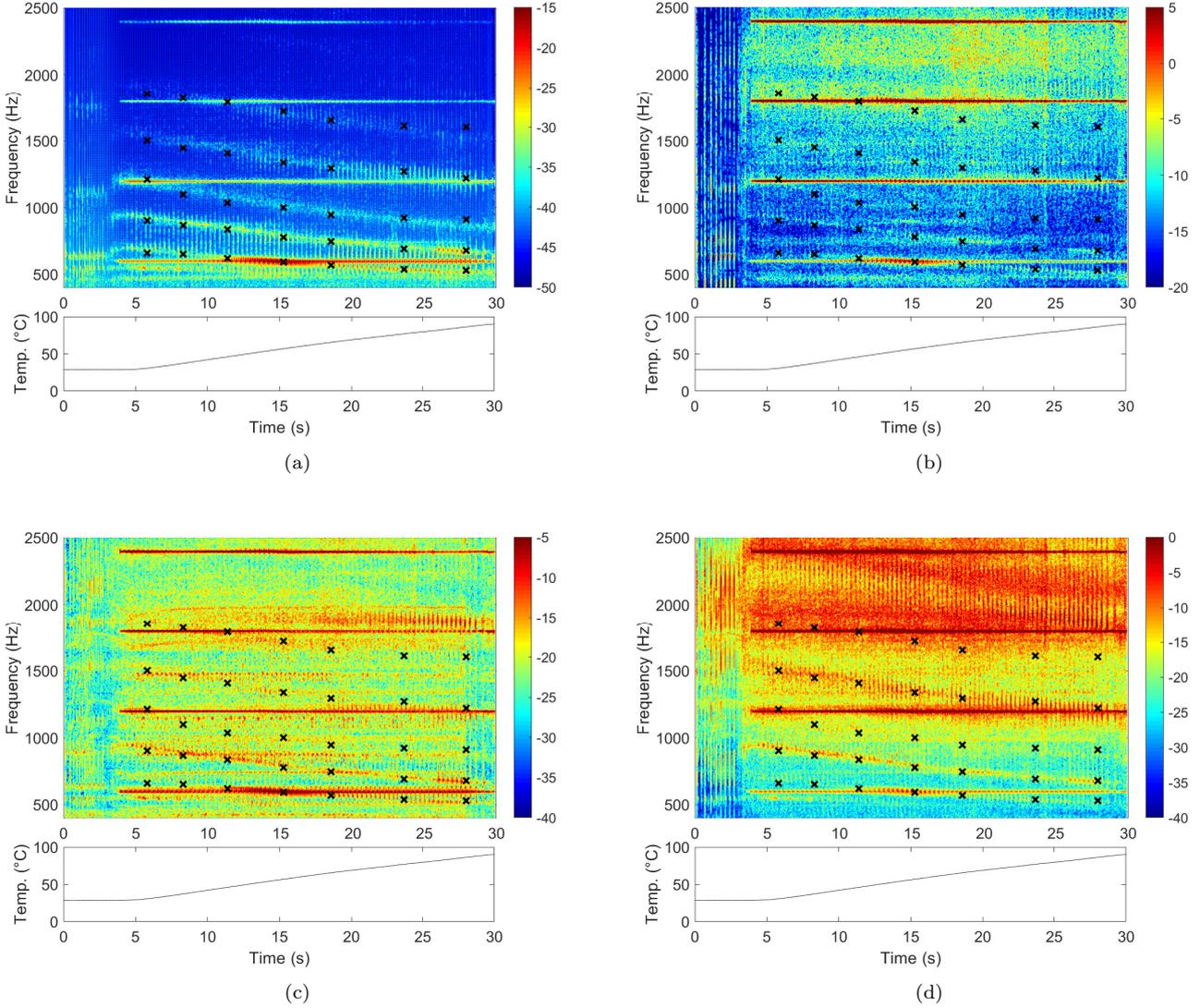


Figure 3: Spectrograms for (a) one normal displacement of the disc *Prox4*, (b) one normal acceleration of one pad *Triaxe.3.Z*, (c) squeal noise in near-field *Micro.10* and (d) one tangential acceleration of the caliper *Triaxe.5.X* (×: frequency plots for the modal analysis of the no-rotational coupled system performed in Section 3.2)

by all materials of the brake system during squeal events, the measured temperature being the sum of these three energy components. Generally, when measuring the temperature of an object, the emitted energy (i.e. the energy generated by the target) is of primary concern in comparison to the transmitted energy (i.e. the energy passing through the object) and the energy reflected from the target to the IR camera. Then, the amount of infrared energy is converted into visual displays, called thermograms, that reproduce images of that radiation. Due to the fact that the amount of radiation emitted by an object increases with temperature, thermography allows one to see variations in temperature of the target. Thanks to this IR camera, completed by the thermocouple sensor at one brake pad, a very accurate temperature reading and its evolution during braking tests can be performed on the rotating and non-rotating parts of the test bench FIVE@ECL.

The experimental protocol is globally similar to what was described in the previous Section 2.2 and in the previous works [41, 42]. All braking tests are performed at a start temperature between 25 °C and 29 °C with nominal values of 200 rpm for the rotation speed and 8.4 bars for the brake pressure. The braking test was repeated at least three times to ensure the repeatability of squeal events and to verify that the thermogram is the same for each test. The test duration is 30 s with the initial Phase 1 of 2.7 s and a rise in brake pressure of 2 s (Phase 2) to ensure steady-state behaviour of vibration and squeal noise over approximately 25 s (Phase 3), with a continuous rise in temperature up to 100 °C in the vicinity of the frictional interface. Table 1 summarizes the operating conditions associated with the braking test examined in this study.

The braking tests are not carried out for temperatures above 100 °C in order to avoid damage of IEPPE accelerometers located near the frictional interfaces of disc and pads, and possibly proximity sensors close to the disc surface. The experimental results for temperatures ranging from 25 °C to 100 °C near the frictional interface (measurement based on the thermocouple sensor) and from 25 °C to 60 °C for the disc (measurement based on the IR camera) will be sufficient for the stated objective in this study. This braking test will serve as the reference test for the rest of the

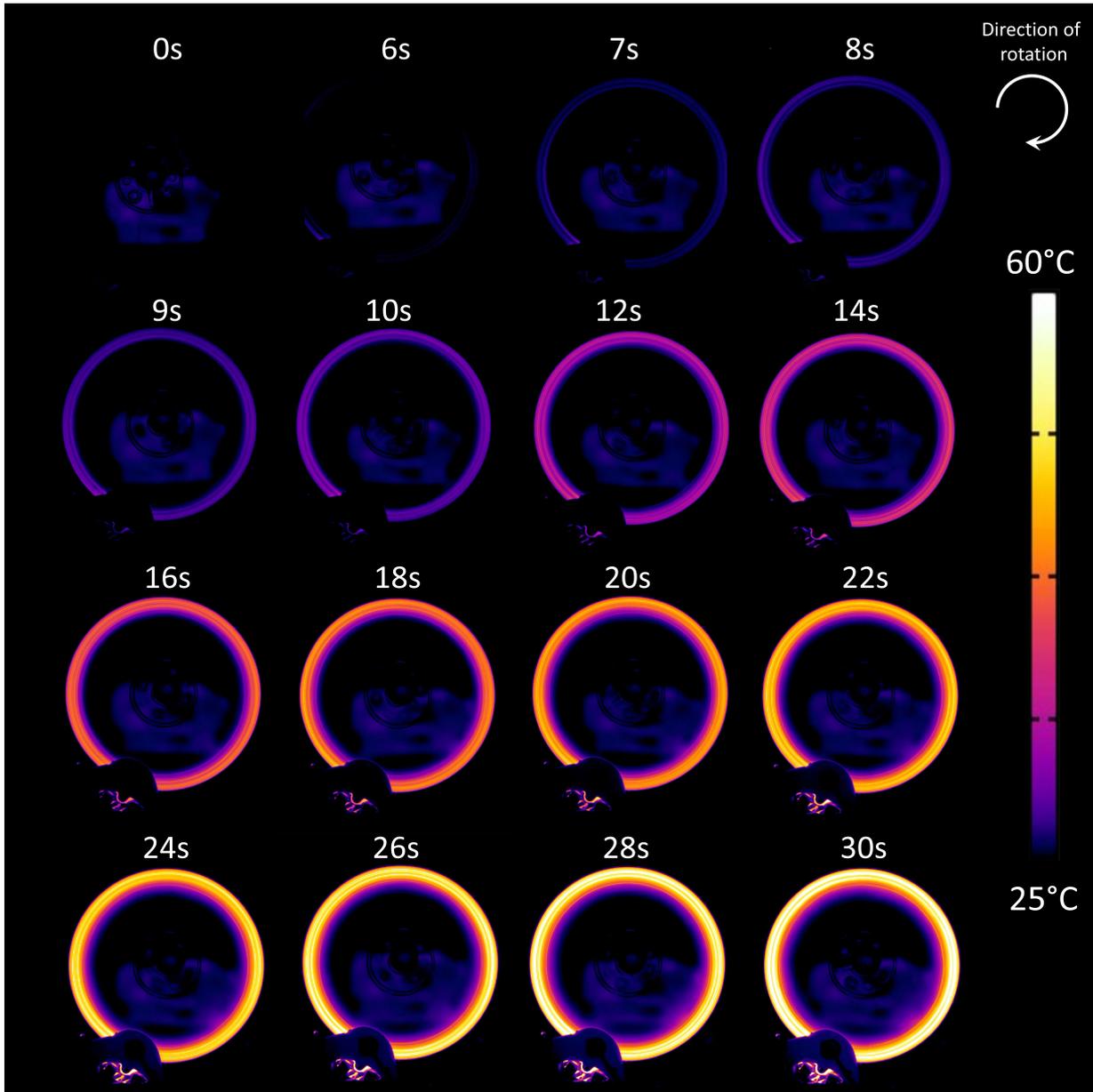


Figure 4: Thermal images during the braking test

study and to conduct comparison with the modal analysis performed in the next section.

170 Figure 3 gives the spectrograms of the transient nonlinear responses of one normal displacement of the disc (*Proxy.4*), one normal acceleration of one pad (*Triaxe.3.Z*), the pressures measurement in near-field (*Micro.10*), and one tangential acceleration of the caliper (*Triaxe.5.x*). The temperature evolution based on the thermocouple measurement is also indicated below each spectrogram. These results are of course in agreement with previous studies [41, 42]. At the beginning of braking, the squeal phenomena appears and then a global stabilization of the signal is observed. Additionally the intensity fluctuation at the first fundamental frequency  $f_1$  is observed for  $t = [10; 20]$  s (take a more precise look at the Figures 3(a) and 3(b)). The pronounced decreases in frequency over time for the two ranges [500; 700] Hz and [600; 1000] Hz that have been previously mentioned in [41, 42] are also reproduced. Moreover Figure 3(a) points out that other frequency decrease phenomena occur in the frequency ranges [900; 1200] Hz and [1200; 1600] Hz. Even if these phenomena of intensity fluctuation for  $f_1$  and decrease of some frequencies may appear to be secondary phenomena in regard to the global stable squeal signature, the objective of the following study is to provide elements of explanation on the physical phenomena involved in relation to these observed secondary phenomena and the squeal events observed on the experimental bench FIVE@ECL. It is noteworthy that the results presented in Figure 2 and those in Figures 3 correspond to two distinct tests, as indicated in Table 1, for which the timescales of the three phases are not identical, explaining the time shifting on the observed results between the previous contribution [41] and the presented study. Moreover the instantaneous friction coefficient has been calculated over the braking time when the pressure is constant (i.e. Phase 3 for  $t=[10;30]$  s). It remains constant during the brake test with  $\mu=[0.55;0.62]$ . This result is similar to the previous results proposed in [41].

185 Figure 4 illustrates some thermal images displayed by the IR camera of the brake system which is emitting or reflecting the infrared energy during these squeal events. The direction of rotation of the brake disc during the brake

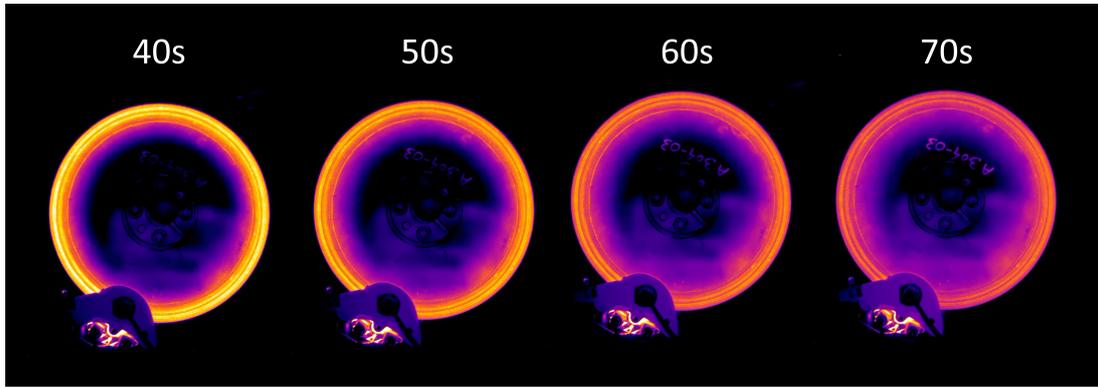


Figure 5: Thermal images after the braking test - the temperature scale is identical to those of Figure 4

| Modal test number | Color in Figure 6 | Pad ( $^{\circ}\text{C}$ ) | Disc ( $^{\circ}\text{C}$ ) |
|-------------------|-------------------|----------------------------|-----------------------------|
| 1                 | magenta           | 40                         | $\approx 25$                |
| 2                 | cyan              | 47                         | $\approx 30$                |
| 3                 | blue              | 55                         | $\approx 35$                |
| 4                 | green             | 65                         | $\approx 40$                |
| 5                 | red               | 73                         | $\approx 50$                |
| 6                 | brown             | 84                         | $\approx 55$                |
| 7                 | black             | 92                         | $\approx 60$                |

Table 2: Pad and disc temperatures for the first hammer shock (symbol  $\times$  in Figure 6) of the six modal analysis tests (no-rotational coupled system)

190 test is indicated at the top right of the figure. One classical result is that the high-temperature area is located at the friction pads. It is also clearly shown that the temperature field of the brake disc during a braking test increase continuously from  $25^{\circ}\text{C}$  to  $60^{\circ}\text{C}$  with a homogeneous temperature on one annular region. This annular region corresponds to the area that was undergoing greater friction due to the repeated moving motion of the disc over the contact area with the pad. This annular region widens during the braking test but without reaching the center of the disc. It is observed also that a higher temperature area of the disc is located at the trailing edge than at the leading edge of the pad. This is especially true for the beginning of the tests (see for example Figure 4 for  $t=[7;12]$  s).  
 195 This observation is possible thanks to a high frame rate of image acquisition (30 images per second) compared to the relatively low rotation speed of the disc (about 3.3 revolutions per second). Moreover it should be noted that the braking test was performed for only one specific rotational speed. Xiang et al. [37] demonstrated that the rotational speed may have a significant impact: for the brake dynamometer tested by Xiang et al., the higher the brake disc rotation speed, the larger the area of high temperature distribution was, and the annular region that was undergoing greater friction widened with the increase of brake disc rotation speed.  
 200

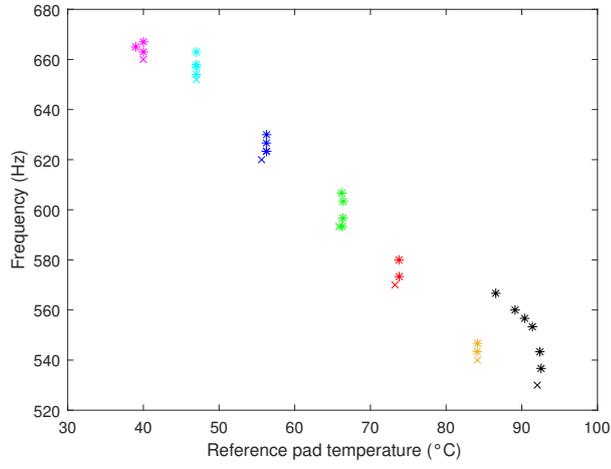
Finally it may be concluded that the thermal effects and the rise in temperature of the disc probably have no impact on the connections between the disc and the shaft. Consequently this excludes that the continuous frequency decreases observed during a braking test are due to structural modifications induced by a softening effect at the junctions between the disc and the shaft (i.e. a modification of the boundary conditions between the two subsystems). On the other hand the temperature increase of the disc is significant during a braking test. In fact the average temperature on the annular region of higher-temperature is approximately  $60^{\circ}\text{C}$ . This significant increase in temperature over the entire circular edge of the disc and over a large width (about a quarter of the disc radius) may be the cause of structural changes and therefore modification of the natural frequencies of the disc. This could be an explanation for the observed decrease for some specific frequencies during a braking test. The rest of this study will therefore attempt to answer this question.  
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### 3.2. Discussion on the potential links between observed thermal effects and modal analysis of the brake system

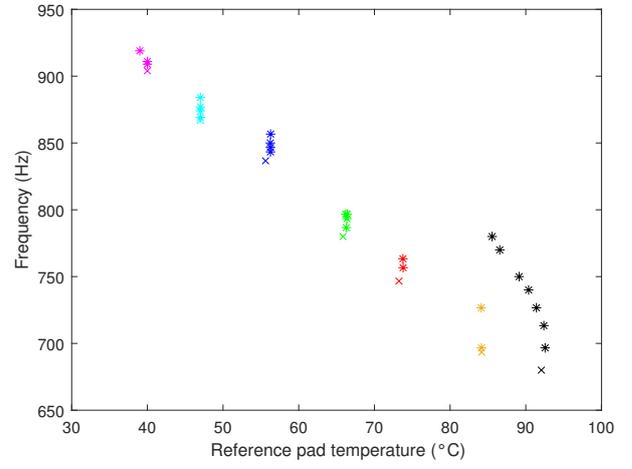
215 First of all, it is important to remember that the presence of specific squeal frequencies during a braking test cannot be directly related to the modal signature of the no-rotational brake system. Contributions such as gyroscopic and frictional effects are present during a braking test which can lead to the appearance of instabilities. So the aim

| Modal test number | Color in Figure 7 | Pad ( $^{\circ}\text{C}$ ) | Disc ( $^{\circ}\text{C}$ ) |
|-------------------|-------------------|----------------------------|-----------------------------|
| 1                 | blue              | 73                         | $\approx 45$                |
| 2                 | green             | 76                         | $\approx 50$                |
| 3                 | red               | 90                         | $\approx 60$                |
| 4                 | black             | 107                        | $\approx 70$                |

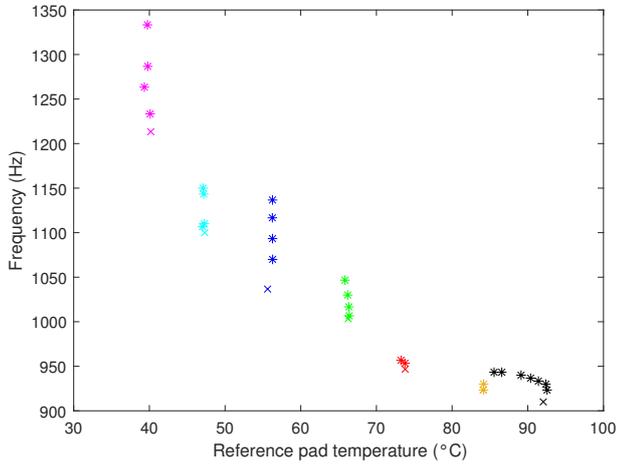
Table 3: Pad and disc temperatures for the first hammer shock (symbol  $\times$  in Figure 7) of the four modal analysis tests (uncoupled system)



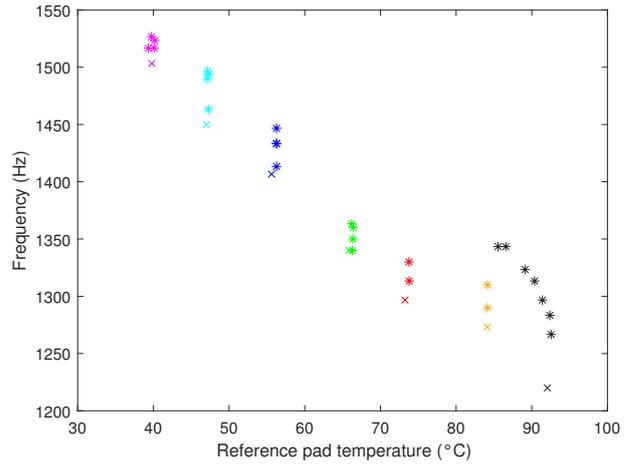
(a)



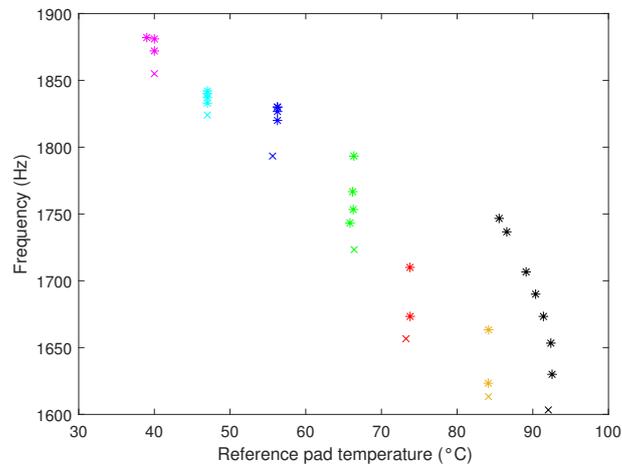
(b)



(c)

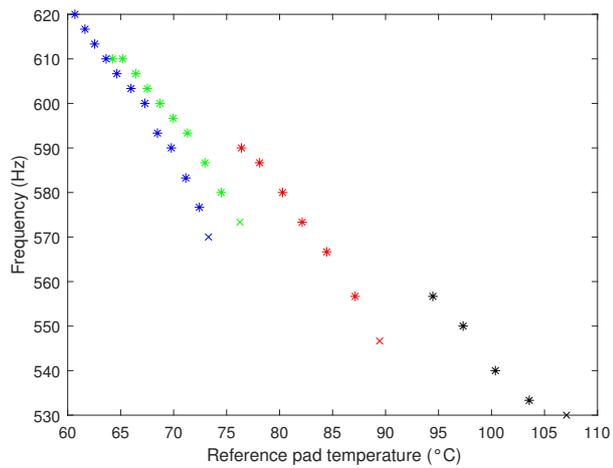


(d)

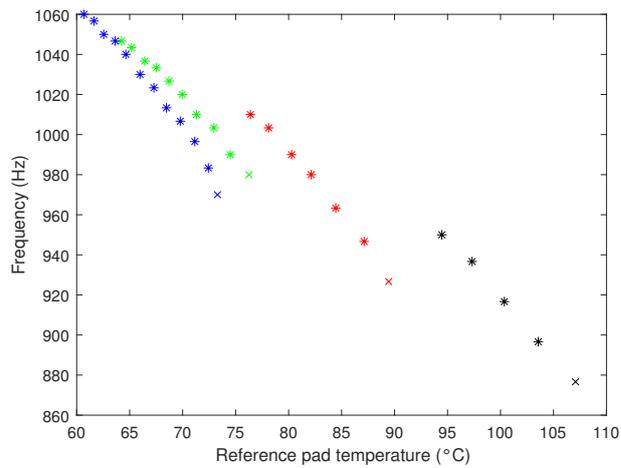


(e)

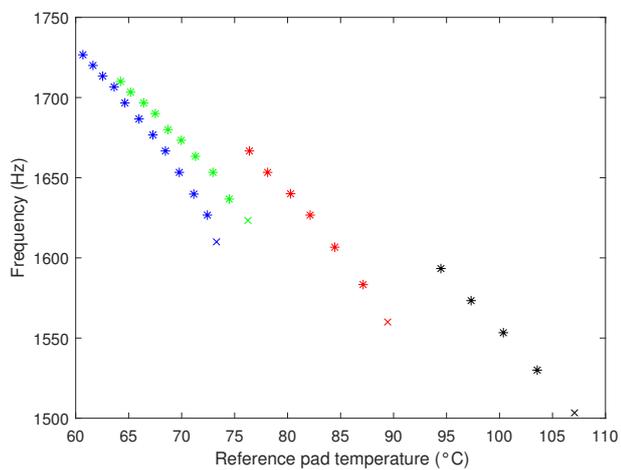
Figure 6: Evolution of the natural frequencies  $f_i^c$  for the no-rotational coupled system (a)  $f_1^c$  (b)  $f_2^c$  (c)  $f_3^c$  (d)  $f_4^c$  (e)  $f_5^c$



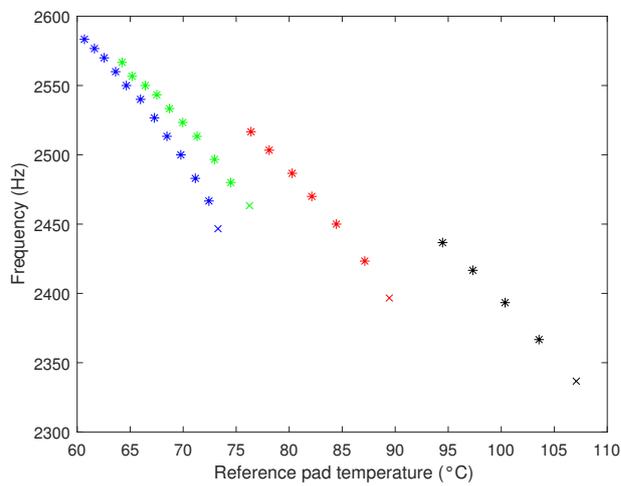
(a)



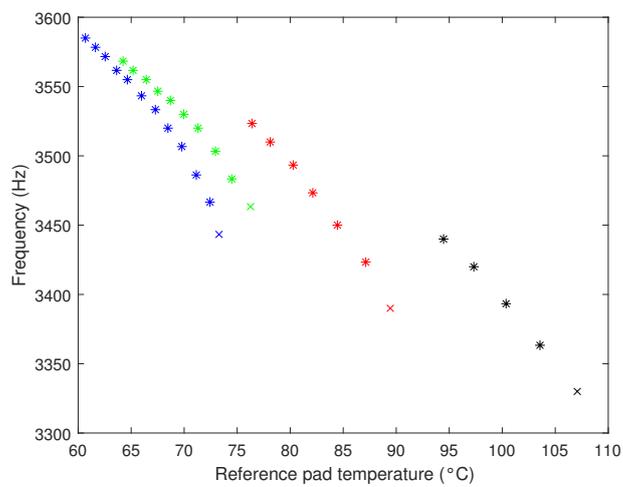
(b)



(c)



(d)



(e)

Figure 7: Evolution of the natural frequencies  $f_i^u$  for the no-rotational uncoupled system composed by the disc and the shaft (a)  $f_1^u$  (b)  $f_2^u$  (c)  $f_3^u$  (d)  $f_4^u$  (e)  $f_5^c$

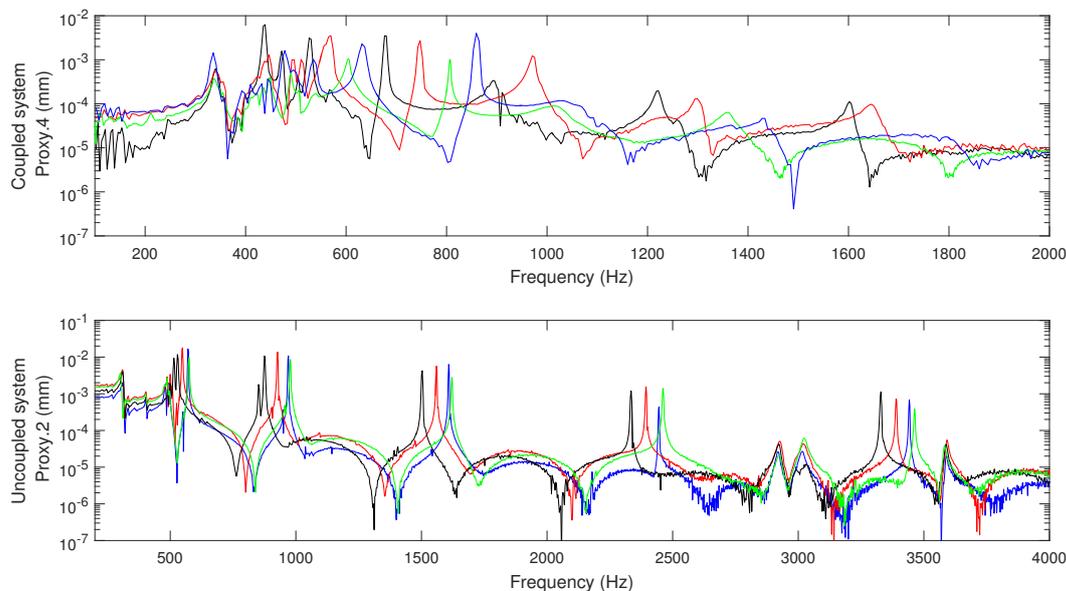


Figure 8: Response and frequency peaks of the no-rotational coupled and uncoupled systems for various temperatures; coupled system: black=92 °C, red=73 °C, green=65 °C, blue=55 °C - uncoupled system: black=107 °C, red=90 °C, green=76 °C, blue=73 °C

of this section is to better understand the effect of the increase of disc temperature on the natural frequencies of the brake system and consequently to investigate the potential links with the decrease of some squeal frequencies during a braking test. To reach such an objective, modal analysis will be conducted for two configurations at several disc temperature:

- The first configuration is the no-rotational coupled system configuration. It corresponds to the brake system when the disc does not rotate. It considers that the two pads and the disc are in contact applying a pressure force without any friction due to the no-rotation of the system. The main objective is to investigate the potential links between the evolution of the natural frequencies of the no-rotational coupled system due to thermal effects and the decreasing frequencies appearing during the squeal events. It can however be noted that, in the case of the modal analysis on the no-rotational coupled system, the rotational contribution such as the gyroscopic effects and the role of friction are not present.
- The second one corresponds to the no-rotational uncoupled system configuration: the rotating and stationary components of the brake system are not in contact (no pressure force applied). It corresponds to the separate subsystems (disc, pads and backplate) when the components are not in contact. This second configuration makes it possible more particularly to bring out the natural frequencies of the disc connected to the shaft. This will allow us to better investigate the thermal effects on the vibrational modes of the disc-shaft subsystem.

The test protocol is defined as follows:

- Excitation of the brake system is performed for a specific temperature by impacting the normal surface of the disc at one specific point (on the outer radius of the disc opposite to the friction area) with a shock-hammer.
- For a given temperature and configuration, each modal test is reproduced several times to ensure the repeatability of the results.
- For the no-rotational coupled system configuration, a classical braking test is performed until the temperature measured by the thermocouple sensor reaches a given value. The rotation of the braking system is then stopped, the pressure is maintained and the modal analysis of the no-rotational coupled system is carried out. To be noted that the temperature at the vicinity of the pad is monitored and so always available during a modal analysis. Even if this temperature does not exactly match with the disc temperature, it is assumed that monitoring this value and its evolution is efficient in the present case.
- For the no-rotational uncoupled system configuration, the braking test is carried out until the specified temperature. The rotation of the braking system is stopped and the pressure is removed so that the disc and the two pads are no longer in contact. The disc is rotated by hand so that it is in the same spatial configuration for each modal test. Then the modal analysis of the no-rotational uncoupled system is carried out.

This protocol is chosen to ensure that each modal analysis is performed on the system which has undergone the thermal effects and potential structural modifications until a chosen temperature during a braking test. Figure 5 shows thermal images after the braking test for the no-rotational brake system and by keeping the pressure force. It

is first noticed that the temperature on the hot annular region of the disc decrease slowly whereas the temperature on the center of the disc increase lightly which corresponds to a classical thermal diffusion. What should be particularly noted is that the thermal images by considering a temperature increase during the braking test are not similar to the thermal images obtained when the temperature decrease after stopping the braking test (see and compare Figures 4 and 5 for both the pad and the disc). This means that changes in temperature in the braking system and thermal exchanges are different. So a modal analysis only carried out on the brake system that cools down after a braking test from the highest temperature to the ambient temperature will not be representative of a modal analysis made according to the protocol described above. This explains the specific and mandatory protocol proposed below.

For the no-rotational coupled system configuration (no-rotational uncoupled system, respectively) seven (four, respectively) braking tests have been performed until a chosen pad temperature. Tables 2 and 3 summarize the temperature reached at the end of the braking tests at one pad via the thermocouple sensor and at the annular ring of the disc via the IR camera. In the following, we will focus only on the first five frequencies that vary with temperature. For the sake of clarity, these frequencies will be referred to  $f_i^c$  and  $f_i^u$  (with  $i = 1, \dots, 5$ ) for the coupled and uncoupled systems, respectively. This choice is obviously guided by the fact that we are interested in finding elements of explanation and relationships between the frequency evolutions of some vibration modes involving primarily the disc and the decreases frequencies observed during squeal events.

Figures 6 and 7 show the evolution of the natural frequencies  $f_i^c$  and  $f_i^u$  for both the no-rotational coupled system and the no-rotational disc connected to the shaft. The cross symbol  $\times$  indicates the first modal analysis carried out after each specific braking test while the star symbol  $*$  corresponds to the modal tests which were carried out successively with a delay of [2;5] s between each test (i.e. modal tests performed just after the modal test denoted by  $\times$ ). By analyzing all these results, some observations may be brought:

- The first five natural frequencies  $f_i^c$  of the no-rotational coupled system are drastically impacted by the pad temperature reached at the end of a braking test (see Figures 6(a), (b), (c), (d) and (e) respectively). By reporting the crosses of Figures 6 for the first five natural frequencies  $f_i^c$  of the coupled system on the four spectrograms of the braking test (see the black crosses in Figures 3), it is observed that the crosses globally follow the evolution of the decreasing frequencies previously detected during the braking test. This is more particularly visible on the spectrogram of the normal displacement of the disc (look at the concordance between the crosses and the decreasing frequency lines for the four frequency range [500; 700] Hz, [600; 1000] Hz, [900; 1200] Hz and [1200; 1600] Hz). The matching is also visible in the frequency range [500; 700] Hz and [600; 1000] Hz for the other three spectrograms. At this stage of the study it is important to recall an essential point on the limitations of this comparison and the analysis proposed: it should be mentioned that the modal analysis is performed on the no-rotational coupled system so that the rotational contribution such as the gyroscopic effects and the friction contribution are not present. As previously explained, squeal frequencies cannot be directly compared to the natural frequencies of the brake system. Therefore, one might expect a slight difference on the results of the natural frequencies of the no-rotational coupled system with the reference braking test. This result only highlights the potential links between the decreasing frequencies during braking tests and the evolutions of natural frequencies of the no-rotational coupled system (more specifically related with the vibration modes of the disc brake system).
- The first five natural frequencies  $f_i^u$  of the no-rotational uncoupled system are also drastically impacted by the pad temperature reached at the end of a braking test (see Figures 7(a), (b), (c), (d) and (e) respectively). This result indicates that the evolution of disc temperature induces a significant modification of vibratory behavior. Even if thermal effects do not drastically change the global nonlinear signature of brake squeal, the temperature increase can be a contributing factor for the small fluctuation of the squeal event whose origin corresponds to structural modification of the brake disc. So phenomena such as the amplitude fluctuation of  $f_1$  or the identified decrease frequencies during a braking test correspond to some unexpected thermal effects on squeal events.
- Showing more specifically the evolution of natural frequencies  $f_i^c$  and  $f_i^u$  for successive modal tests performed on different specific braking tests (see the symbol star  $*$  for one chosen color), we have a natural decrease of the disc and pad temperature so inducing an increase of the four natural frequencies  $f_i^c$  and  $f_i^u$ . It is observed that the evolution of the natural frequencies  $f_i^u$  of the disc are practically identical for the four braking tests (see Figures 7 and compare the magenta, green, black and red colors). Furthermore the frequencies evolution is approximately linear versus the temperature. On the contrary, in the case of the no-rotational coupled system, the frequencies evolution versus the temperature is more complex. First of all the evolution of the five natural frequencies  $f_i^c$  for a chosen test are not identical (see Figures 6 and compare curves for one color): for example, the second and five frequencies ( $f_2^c$  and  $f_5^c$ ) of the last braking test (i.e. for the highest temperature) evolve significantly with a frequency variation of 14% and 9% respectively (see Figures 6(b) and (e) respectively, black color) whereas the third frequency  $f_3^c$  hardly varies with less than 4% change (see Figure 6(c), black color). It is also observed that the temperature rises slightly after stopping the braking test.

Additionally Figure 8 shows the global amplitudes responses of the uncoupled and coupled systems in the frequency range [100;2000] Hz and [100;4000] Hz respectively, for the first modal analysis carried out after one braking test (i.e. previously denoted by the cross symbol  $\times$  in Figures 6 and 7). Of course the values of the frequency peaks are in agreement with the evolution of the natural frequencies indicated in Figures 6 and 7. First it is interesting to note that

some frequency peaks are not temperature dependent: see for example peaks at 340 Hz for the coupled system and 310 Hz, 2930 Hz, 3025 Hz and 3590 Hz for the uncoupled system. Additional studies of modal characteristics of the complete braking system have led to the conclusion that these modes correspond to vibration modes of others parts of the system such as the disc shaft. Also one can note for the coupled system a frequency interval around [400;600] Hz on which several resonance peaks appear close to each other. This phenomenon is not found for the uncoupled system for which the resonance peaks are clearly separated from each other. Due to the fact that only the modal contributions of the disc and the shaft are detected for the modal analysis of the uncoupled system, this analysis highlights more particularly the modes of coupled system which involve the potential vibratory contributions of the pads and caliper.

Thereby these detailed modal analyzes highlight very clearly the fact that the phenomena of decrease in frequencies detected on the frequency band [500;2500] Hz during squeal events are linked in part to thermal effects on the structural behavior of the brake disc. It is worth remembering at this level of the study that the main signature of squeal is invariant over the duration of a braking test, as previously discussed in [41, 42] and that we analyze here only a non-predominant evolutionary phenomenon appearing during brake squeal. Likewise, the scientific problem linked to the thermal effect at the pad-disc interface is not addressed in the context of this study. Indeed, the decrease in frequencies for the uncoupled system allows to conclude without ambiguity that the primordial phenomenon involved is linked to a structural modification of the disc under thermal effects and not to a major physical phenomenon at the frictional interface or an evolution of the pad material due to thermal effects. These results can be referred as unexpected thermal effects.

It is then also possible to explain in a more in-depth way the phenomena of fluctuation in the intensity of the amplitudes of each signature such as  $f_1$  or  $3f_1$  during the braking test. Due to the frequency decrease in time, the amplitude fluctuation of a constant signature corresponds to a combination of the stable energy contribution at a specific frequency and the complementary energy contribution coming from the decrease of the frequency. For example, the crossing between a constant frequency contribution  $f_1=[579;584]$  Hz, and the decreasing frequency from 700 Hz to 500 Hz induces an increase in amplitude of  $f_1$  for  $t=[10;20]$  s. The same phenomenon of additional energy contribution which leads to a temporary amplitude increase of frequency contribution is observable at  $3f_1$  for  $t=[8;14]$  s as illustrated in Figure 3(a). Moreover the evolution of amplitude for  $2f_1$  with a maximum for  $t=[4;8]$  s followed by a decrease of this contribution can also be related to the combination of this harmonic component  $2f_1$  with the decreasing frequency from 1200 Hz to 900 Hz (see Figure 3(a)).

Finally, the slow increase of the frequency contribution  $f_2=[754;769]$  Hz at the end of the braking test in Figure 2 can be explained by the additional participation of the second decreasing frequency from 1000 Hz to 600 Hz (see Figure 3(a)). In the same way, the fluctuation of the squeal noise in far-field at the beginning of the braking test for frequency contribution  $f_3=[2019;2044]$  Hz (see Figure 2 for  $t=[4;10]$  s) can be due to the additional contribution of the fifth decreasing frequency from 2100 Hz to 1600 Hz.

These two last result allow us to affirm that the two fundamental frequencies  $f_2=[754;769]$  Hz and  $f_3=[2019;2044]$  Hz previously identified in [41] correspond in fact to the combined contribution of descending frequencies with the fundamental constant frequency  $f_1=[579;584]$  Hz during squeal events. This corresponds to an unexpected effect of temperature evolution on squeal phenomena.

In conclusion, all these results highlight the links between the temperature and structural modifications of both the disc and brake systems as well as the resulting effects on the transient behavior of the squeal signature. In this section, no discussion is provided on the description of the disc mode shapes from modal analysis. This is due to the fact that the number of proximitor sensors used was not sufficient to rigorously reconstruct the different disc mode shapes for both the coupled and uncoupled brake systems. Therefore no study was undertaken to perform mode pairing from modal analysis.

#### 4. Conclusion, discussion and perspectives

The present work studies the origin of complex and repeatable transient phenomena appearing during squeal events using experimental data from braking tests carried out on the Friction-Induced Vibration and noise benchmark at Ecole Centrale de Lyon (FIVE@ECL). These results demonstrate that although thermal phenomena do not necessarily have a predominant impact on the overall brake squeal signature for low temperature braking tests not exceeding 100 °C, they are nevertheless visible and the cause of complex secondary transient phenomena during squeal events.

We clearly demonstrated the link between thermal effects based on temperature rise during braking tests with some frequencies decrease during squeal events and fluctuation of specific nonlinear contributions of the squeal vibrational behavior. More precisely, it has been shown that the evolution of the squeal signature can be related to structural modifications of the disc due to thermal effects and temperature rise. This result corresponds to an unexpected thermal effect due to the fact that, classically, the influence of temperature leading to an evolution of the brake squeal focuses more particularly on the material properties of the pad or the thermo-physical properties on the interfacial characteristics.

Even if the proposed study give a better understanding of the thermal effects on both squeal events and structural vibration behavior of the brake system, there are still many questions and future research must be attempted to increase the understanding of the physical mechanisms involved in the observed phenomena (i.e. decrease of some frequencies and fluctuation of some frequency contributions). Indeed, the proposed paper clearly puts forward some strong conclusions: the decrease in frequencies is not due to a softening effect at the connections between the disc

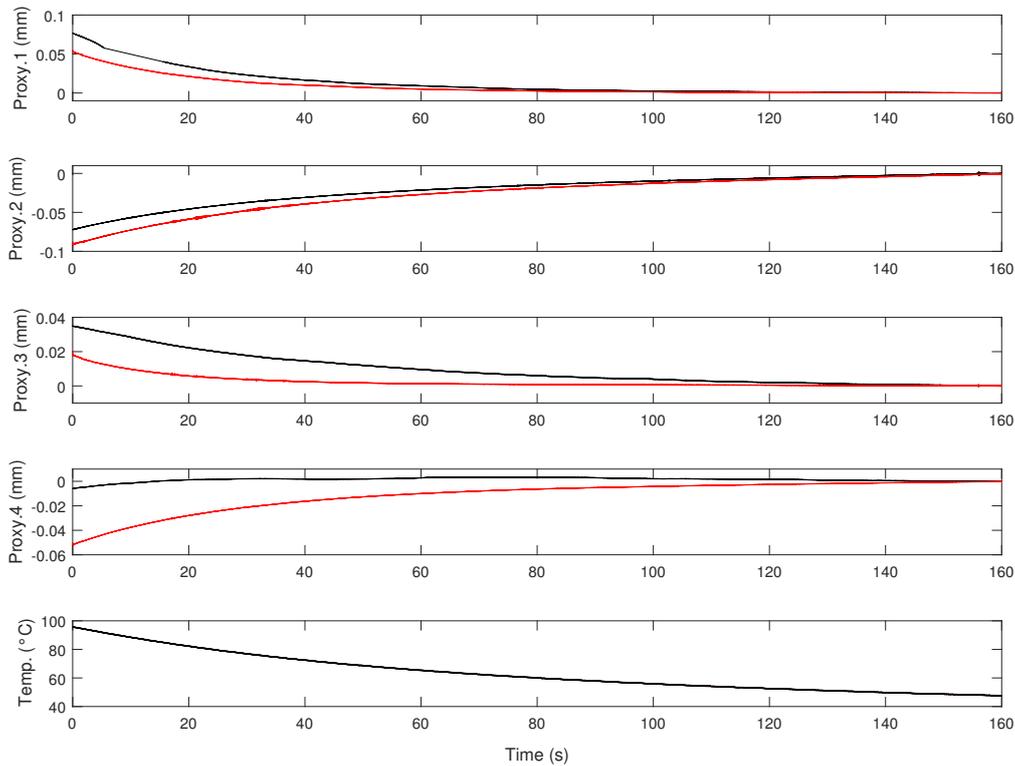


Figure 9: Evolution of the disc deflection for the coupled (black color) and no-coupled (red color) systems after braking tests

and the shaft (see Section 3.1) but it is more specifically related to changes in the structural behavior of the disk (see Section 3.2). Moreover by performing a classical theoretical and numerical analysis to calculate the eigenfrequencies [44] of the brake disc under study, it immediately appears that a variation in the Young's modulus of the disc is not sufficient to explain the significant evolution of the frequencies observed in Section 3.2. Thus the hypothesis that the thermal effects generate modifications of the Young's modulus allowing consequently the observed decrease of specific frequencies during a braking test cannot be retained.

In addition exploratory studies not presented in the study for the sake of brevity have already been put in place to make progress on this subject:

- Additional experimental tests have also been performed to undertake the evolution of the thickness of the disc during a braking test by placing two proximitor sensors face to face on each side of the disc at the location of *Proxy.2* (see Figure 1(a)). Results indicate a small decrease of 0.025 mm of the disc thickness. This allows to conclude on the fact that evolution of frequencies are not the result of a disc thickness variation of the disc at the annular circle due to thermal effects.
- Exploratory static tests indicate that the evolution of the disc deflection for the coupled and no-coupled system after braking tests (i.e. during the disk cooling phase until return to ambient temperature as previously) is not negligible with quantities of the order of 0.1 mm at most at the location of the four proximitor sensors, as illustrated in Figure 9. After a period of relaxation of approximately 150 s, the disc naturally returns to its initial form. These results show that thermal deformations generated during a braking test can be one of the possible physical phenomena related to the observed decreases in frequency during braking tests and thus the consequences of the unexpected thermal effects on squeal events observed on the experimental bench FIVE@ECL.

Even if giving an exhaustive list of topics of interest for future developments is not possible, further studies related to a better understanding of thermo-mechanical and tribological phenomena at the interfaces would undoubtedly lead to a better understanding of the existing links between vibrational squeal phenomena observed in the present study and thermal and tribological impacts.

Similarly, it could be interesting to visualize the squeal modes in order to better understand the squeal phenomena. However it is worth mentioning that the squeal modes may potentially rotate (forward or backward motions) with the appearance of traveling waves. It requires the implementation of specific devices such as rotating laser tracker measurements or acoustic holography. Such experimental devices should also be useful to perform mode pairing in time during a braking test.

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