A Welfare Analysis of Genetic Testing in Health Insurance Markets with Adverse Selection and Prevention

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“Personalized Medicine and Prevention: Can Cross-Subsidies Survive in the Health Insurance Markets?”

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Personalized Medicine and Prevention : Can Cross-Subsidies Survive in the Health Insurance Markets?[1]

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Abstract

Personalized medicine is still in its infancy, with costly genetic tests providing little actionable information in terms of efficient prevention decisions. As a consequence, few people undertake these tests currently, and health insurance contracts pool all agents irrespective of their genetic background. Cheaper and especially more informative tests will induce more people to undertake these tests and will impact not only the pricing but also the type of health insurance contracts. We develop a setting with endogenous prevention decisions and we study which contract type (pooling or separating) emerges at equilibrium as a function of the proportion of agents undertaking the genetic test as well as of the informativeness of this test.

Starting from the current low take-up rate generating at equilibrium a pooling contract with no prevention effort, we obtain that an increase in the take-up rate has first an ambiguous impact on welfare, and then unambiguously decreases welfare as one moves from a pooling to a separating equilibrium. It is only once the take-up rate is large enough that the equilibrium is separating that any further increase in take-up rate increases aggregate welfare, by a composition effect. However, a better pooling contract in which policyholders undertake preventive actions (and lower their health risk) can also be attained if the informativeness of the genetic tests increases sufficiently.

JEL Codes: D82, I18.

Keywords: discrimination risk, informational value of test, personalized medecine, pooling and separating equilibria.

1 Introduction

Personalized medicine is defined as the development of ever more accurate diagnoses, prevention actions and therapies, based on the individual characteristics of the agents. This type of medicine is made possible by the development of cheaper and more informative genetic tests. Genetic tests are still currently costly and do not provide much useful guidance for prevention.[1] Few individuals undertake a genetic test to learn about their future probability of developing a disease, except in very specific instances. As a consequence, health insurance markets offer pooling contracts where most policyholders are uninformed about their genetic propensity to develop specific diseases. In other words, there is little adverse selection based on genetic information, and also no discrimination risk caused by genetic testing.

This may change in the near future, as genetic tests become cheaper and especially as they provide more actionable information about prevention strategies to decrease the likelihood of developing certain diseases (see the many examples in Snyder, 2016). This should induce more individuals to undertake genetic tests, and will then generate more adverse selection, increasing the cross subsidies imbedded in the pooling insurance contracts. The type of contracts offered by insurers may then change, from pooling to separating contracts, where insurers try and induce informed individuals to reveal the information they have obtained. The objective of this article is precisely to understand the impact of both a higher test take-up rate, and of more informative tests, on the type of equilibrium health insurance contracts, and to assess the welfare consequences of these changes.

The existing literature dealing with genetic testing is silent on this evolution, for two reasons. First, to the best of our knowledge, most articles dealing with genetic testing assume that individuals are homogenous *ex ante* in terms of costs and benefits of testing, so that they all adopt the same decision regarding genetic tests and prevention behaviors. It is then impossible to understand the impact of continuous increases in take-up rates of genetic tests when the equilibrium take-up rate is either zero or one. Second, the literature tends to focus on separating equilibrium contracts à la Rothschild and Stiglitz (1976), although the currently observed contracts are most often pooling (see Hoy, 2006). Understanding the impact of cheaper and more informative genetic tests on insurance contracts requires that the type of contract, separating or pooling, be endogenously determined.

In this article, we consider a setting where an exogenous fraction of the population has done a genetic test.[2] The test is costless and reveals with certainty whether agents

[1] See for instance the recent statement of the American College of Medical Genetics and Genomics (ACMG): "The use of DNA-based health screening to guide preventive care in the screened individual has long been discussed, but until recently has had limited applications." (Murray et al., 2021, p. 989).

[2] We do not endogenize the decision to test or not. We come back to this point at the end of this

have a low or high probability of developing a disease. The proportion of high probability agents is common knowledge, and agents who have not done the test only know their expected probability of developing the disease. The genetic information allows to tailor a costly prevention effort, which decreases the probability of developing the disease if the individual has a deleterious genetic background. This prevention effort is observable and contractible by the competitive fringe of insurers.[3] The best known example of this setting is the recommendation for women testing positive to the BRCA1 or BRCA2 alleles, which increase the probability of developing breast cancer, to undertake a mastectomy. Snyder (2016) contains several other examples where the prevention effort consists in taking drugs or in modifying one's behavior.

We study a context of adverse selection, where policyholders are not forced by law to disclose whether they have undertaken a genetic test, and its results. This setting corresponds to the *Consent Law* regulation (used in the Netherlands and in Switzerland, for instance) where individuals are allowed but not required to divulge this information to private health insurers. We follow Wilson (1977)'s approach, where the type of contract (pooling or separating) is endogenous and depends on the proportion of high risks. More precisely, we study how the contract type is affected by two characteristics of personalized medicine: the share of agents who do the test, and the informativeness of the tests as proxied by the cost of the prevention effort which alleviates the consequences of having a deleterious genetic background. For instance, going back to the BRCA1/2 gene and the recommendation to perform surgery when tested positive, one can only hope for the development of a less psychologically costly prevention technology in the future.

We call informed (resp., uninformed) agents those who have done (resp., have not done) the test. Informed agents with low probabilities to develop the disease (denoted type L) always have an incentive to reveal this information, and obtain a cheap contract with full coverage. Informed agents with high probabilities (type H) have an incentive to pretend that they did not do the test to benefit from a cheaper contract. The type of equilibrium then refers to how informed high type agents and uninformed agents (type U) are treated: they can either be pooled (and offered the same contract), or be separated by insurers offering them a menu of contracts, with a self-selection constraint.

We first characterize sequentially separating and pooling contracts. Separating contracts offer actuarially fair prices to both types U and H and full coverage to type H. The contracts offered have to give an incentive to type H not to pretend being uninformed. This is done by tweaking the contracts on two dimensions: the classical dimension of coverage rate (offering less-than-full insurance to type U) and/or by re-

section.

[3] Most countries have a mixed health insurance market, where private insurers complement the coverage offered by public insurance. Our model focuses on the private insurance part.

quiring different prevention effort levels for types H and U. With a binary prevention effort, we then obtain three varieties of separating equilibrium, depending on the cost of the prevention effort. When this cost is low, it is unappealing to induce type H to reveal their type by preventing them from making the prevention effort, and the separating contracts require the same effort from both types of agents (together with under-coverage for type U). A similar reasoning applies when the effort cost is large, in which case none of the two contracts requires any prevention effort. When the effort cost is intermediate, preventing type U (but not type H) from doing the prevention effort, together with some under-coverage for type U, is the least costly way to separate the two types.

By contrast, a pooling equilibrium offers the same contract to both types U and H, and thus the same level of prevention effort. Following Wilson, the amount of coverage is determined by the lowest risk in the pool, here type U. When the prevention effort cost is low enough, both types are required to perform the prevention effort. When it is high enough, none is required.

We then show that, for any value of the prevention effort cost, there is a unique threshold value of the test take-up rate below (resp., above) which the equilibrium is pooling (resp., separating). This threshold varies in a complex way with the value of the prevention effort cost. The welfare impact of the genetic test take-up rate then runs as follows. As long as this rate is low enough, the equilibrium is pooling. Welfare is then affected in two opposite directions by a larger take-up rate. First, the pooling contract becomes more expensive (with a higher proportion of type H in the pool) which decreases the utility of both types U and H. Second, there is a composition effect on aggregate welfare, with fewer types U and more types L and H. We establish that this composition effect is positive, resulting in an ambiguous overall impact of a larger take-up rate on welfare. When the take-up rate is large enough that the equilibrium is separating, the only impact of a higher take-up rate is the composition effect, resulting in a higher aggregate welfare. Finally, we show that the utility of type H (and aggregate welfare) decreases discontinuously when the equilibrium changes from pooling to separating. We then obtain that a larger fraction of agents taking a genetic test has an unambiguous positive impact on welfare only if it is large enough to generate a separating equilibrium, and has an unambiguously negative impact on welfare when moving from a pooling to a separating equilibrium.

The impact of a lower prevention effort cost (used as a proxy for the efficiency of the genetic test in terms of actionable health information) depends on the value of the test take-up rate.[4] The most interesting case happens when the take-up rate

[4] Alternatively, we could have studied the impact of a larger decrease in the probability of developing the disease when the prevention effort is done, as in Bardey and De Donder (2013) for instance. This would have lead to (even) more complex formulations, without a commensurate gain in intuition.

is intermediate. In that case, decreasing the effort cost from its currently large level moves us from the current pooling equilibrium with no prevention effort to a separating equilibrium where only type H makes the effort, before switching back to a pooling equilibrium, but with effort for both U and H. The welfare impact of a lower effort cost then runs as follows: (i) it does not impact welfare when the pooling contract does not prescribe effort, and increases welfare when it does, (ii) it increases the utility of both types U and H in the separating contract, and (iii) type H's utility (and aggregate welfare) decrease discontinuously when moving from a pooling to a separating equilibrium (and *vice versa*). In a nutshell, decreasing the prevention effort cost is not always welfare improving, especially for type H when moving from the pooling to the separating equilibrium.

Finally, we study numerically how welfare is impacted when simultaneously increasing the test take-up rate and decreasing the prevention effort cost, which seems to be the most plausible scenario in practice. We obtain that welfare first decreases when the equilibrium is pooling without prevention effort, and then increases when the equilibrium is either separating with only type H making the effort, or pooling with effort.

The message delivered by this welfare analysis is then twofold. First, the move from pooling to separating equilibrium (as the test take-up rate increases, the prevention effort cost decreases, or both) is especially detrimental to the utility of the agents who are unlucky enough to discover that they have a deleterious genetic background. Increasing the test take-up rate may also decrease welfare in a pooling equilibrium, if the price effect trumps the composition effect, while it always increases welfare in a separating equilibrium (if it is large enough that the equilibrium is separating). Encouraging individuals to undertake a genetic test then may result in short run welfare losses (as long as the equilibrium is not separating), especially for type H agents. Second, even though the short run impact of cheaper and more informative tests may be to destroy cross-subsidies (by moving us from a pooling to a separating equilibrium), the long run impact may be to conserve these cross-subsidies (and moreover to induce all concerned agents to exert the prevention effort, thereby realizing the full prospects of personalized medicine), provided that the prevention effort cost decreases enough (*i.e.*, that the informativeness of the genetic tests improves enough, at the same time as these tests become cheaper).

We now turn to the related literature. Doherty and Thistle (1996) is the seminal article studying the incentive to gather information in insurance markets in the context of adverse selection. They first show that individuals do not have an incentive to acquire information (as in taking a costless genetic test for instance) when the informational status of the agent (*i.e.*, whether he has done a test or not, irrespective of its result) is observable by the insurer. They then obtain that individuals acquire information at equilibrium only if either insurers cannot observe consumers' informational status (as in the *Strict Prohibition* regulation mentioned in footnote 8 below) or if consumers are allowed to conceal or not their informational status (as in the *Consent Law* regulation studied here). They characterize the separating contracts offered to agents under different configurations of information costs and benefits.

The subsequent literature has built on Doherty and Thistle (1996) mainly by adding a prevention effort which either decreases the probability that the damage occurs (primary prevention, as in Hoel and Iversen [2002], Peter *et al.* [2017] and Bardey and De Donder [2013]) or decreases the extent of the damage when it occurs (secondary prevention, as in Crainich [2017] and Barigozzi and Henriet [2011]), and by studying different regulatory settings. Note that the existence of a prevention effort, which can be tailored to the test results, tends to increase the value of the information generated by these tests.

To the best of our knowledge, most of this literature has kept two assumptions made in Doherty and Thistle (1996). First, all individuals are *ex ante* identical (in both the individual costs and benefits of the test) and thus, at equilibrium, they either all choose to test, or no one tests. Second, they focus on separating equilibria à la Rothschild-Stiglitz. One exception to the first point is Hoel et *al.* (2006) who study the consequences for the testing decisions of introducing heterogeneity in psychological preferences (repulsion from chance). They provide an equilibrium analysis in a setting with separating equilibria. The exceptions to the second point are Hoy (2006), Hoy *et al.* (2003) and Crainich (2017), which consider more realistic settings that include a pooling equilibrium.[5] However, they do not tackle the transition from pooling to separating equilibrium that may arise endogenously as, for instance, the test take-up rate increases. Finally, Bardey *et al.* (2019) run an experiment based on a theoretical set-up where individuals are heterogenous and do not take the same decision with respect to genetic testing. They assume that agents who claim to be uninformed about their type are offered a pooling contract under Consent Law.

As explained above, we use here a more reduced form by assuming that an exogenous fraction of individuals have been tested while the rest of the population have not. Thus, we have individuals with different informational statuses at equilibrium and we provide comparative static analysis results with respect to the fraction of informed individuals and the prevention cost. Our analysis encompasses pooling and separating equilibria in a set-up à la Wilson.

Endogenizing the fraction of tested individuals would require adding another dimension of heterogeneity that would affect their choice of whether to test or not. We do that in a companion paper (Bardey and De Donder, in progress) where we assume

[5] Strohmenger and Wambach (2000) also study the impact of genetic tests in a large set of equilibrium contracts. In particular, these authors compare the "laissez-faire" regulation (allowing insurers to request genetic tests and to use their results) to strict prohibition.

that agents differ in their psychological attitude towards testing (modelled as different degrees of attraction to/repulsion from chance). We also study the type of equilibrium that may emerge in health insurance markets, *i.e.* pooling *versus* separating. However, the main complexity created by this approach is a fixed-point problem in the computation of the pooling contract, since the contract's fair premium depends on the fraction of agents tested, while the testing decision depends on the contract's premium. To solve this problem, we have to simplify our model by assuming away the prevention effort. We then obtain that the fraction of agents who choose to be tested at the equilibrium decreases monotonically with the test cost, and that the equilibrium contract is pooling above a threshold test cost and separating below. A decrease in the testing cost is Pareto improving when the contract remains separating (*i.e.*, when the test cost remains smaller than the threshold) but is welfare damaging for most agents both when the contract is pooling and when the threshold test cost is crossed so that the contract type moves from pooling to separating.

These welfare effects then confirm those obtained here when one directly varies the fraction of agents tested, even though the current paper has a more optimistic view regarding the information conveyed by genetic tests, which allows policyholders to reduce their health risk. We consider the approach adopted in the companion paper to be complementary to the one studied here, as opposed to a straight improvement, because the progress made in endogenizing the testing decision is obtained at the cost of assuming away the prevention effort. Indeed, technical improvements in what to do with the genetic information (proxied by the effectiveness of the prevention effort) seem to us at least, if not more, important to study than reductions in the cost of acquiring this information. Moreover, the impact of a lower testing cost is unsurprisingly to increase the fraction of agents tested, so that the intuition gained by endogenizing the testing decision seems rather limited.

The structure of the paper runs as follows. Section 2 presents the model while section 3 defines a generic insurance contract in this setting. Section 4 studies the separating equilibrium contracts, while section 5 analyzes the pooling equilibrium contracts. Section 6 studies which kind of contract, separating or pooling, emerges at equilibrium as a function of the proportion of informed agents and of the level of the cost of the prevention effort. Section 7 performs a comparative static welfare analysis with endogenous contract type. Section 8 concludes. Note that the (many) intermediate analytical results are denoted as Results, while the main ones are presented as Propositions. Most formal proofs are relegated to Appendices.

2 The model

The economy is composed of a unitary mass of individuals. Each individual develops a disease with some probability, with sickness modeled as the occurrence of a monetary damage of amount m. A fraction $\lambda \in]0,1[$ of individuals are of type H and have a high probability of incurring the damage, while the remaining fraction $1-\lambda$ is of type L and has a lower probability.

Individuals choose to exert or not some primary prevention effort. We assume that the prevention decision is binary and that prevention has no effect for a low probability individual, while it decreases the disease probability of type H individuals.[6] We then denote by p_L the probability of developing the disease for a type L individual (whether he exerts the effort or not), and by p_H^0 (respectively, p_H^1) the probability of developing the disease of a type H agent who does not (resp., does) exert a prevention effort. We assume that $p_H^0 > p_H^1 > p_L$. We capture the prevention efficiency through $\Delta = p_H^0 - p_H^1$, with $0 < \Delta < \bar{\Delta}$, where $\bar{\Delta} = p_H^0 - p_L$.

An exogenous fraction k of individuals has done a genetic test and is thus informed about its type (L or H), while the remaining fraction $1-k$ is not informed. We denote an uninformed agent as having type U, with a probability of developing the disease equal to

$$p_U^i = \lambda p_H^i + (1-\lambda)p_L,$$

with $i \in \{0,1\}$ denoting whether the agent exerts ($i=1$) or not ($i=0$) the prevention effort. Note that $p_U^i < p_H^i$ since $\Delta < \bar{\Delta}$ and $\lambda < 1$.

We consider a setting where individuals, whether informed or not about their type, buy health insurance from a competitive fringe of insurers. An insurance contract is composed of a premium to be paid to the insurer, and of an indemnity from the insurer to the insured in the case the disease occurs. We further assume that the prevention effort is observable, so that there is no moral hazard in our setting and insurance contracts state whether this effort is required or not.[7]

[6] It is now well established in the medical literature that "it is a combination of the genes that you have inherited and the environment that you live in that determines the outcome" (Collins 2010), so that prevention is more efficient with type H agents. For instance, for macular degeneration, "it became clear that almost 80 percent of the risk could be inferred from a combination of (...) two genetic risk factors, combined with just two environmental risk factors (smoking and obesity)" (Collins, 2010). This normalization to zero effort for type L is done for simplicity and without loss of generality, as the model could allow for a positive effort for type L, and concentrate on the additional effort provided if type H, without affecting the results.

[7] This assumption seems reasonable, since there is little empirical evidence of *ex ante* moral hazard in health insurance contracts (see Einav and Finkelstein, 2018). Moreover, one can surmise that the generalization of wearable technologies such as connected watches will make it even easier for insurers to monitor our prevention efforts in the near future.

We assume that agents are not required by law to reveal their type, but may do so if they wish to, in which case insurers may use this information. This regulation is called *Consent Law* in the context of genetic testing and corresponds to the legal environment in the Netherlands and in Switzerland. It is straightforward to see that this environment creates adverse selection: while agents of type L always show their test results in order to secure a low premium, agents who have been revealed to be of type H have an incentive to pretend that they are untested/uninformed about their risk.[8]

The timing of the model is then as follows. A fringe of profit-maximizing health insurers offer a set of insurance contracts to agents who are exogenously informed (or not) about their individual probability of incurring the health damage. Agents then buy one insurance contract, and exert the prevention effort if the latter is required by the contract. Finally, the disease occurs or not, and the payoffs are realized.

We now describe the contracts offered by the insurers.

3 Generic insurance contract

A generic insurance contract is denoted by (π, I, i) where π denotes the premium in case of health, I the indemnity (net of the premium) in case of sickness, and where $i \in \{0, 1\}$ denotes whether the contract prescribes the prevention effort or not. The premium is computed as $\pi = \alpha p m$, where α denotes the fraction of the damage m reimbursed in case it occurs, and where p is the probability that the agents buying this contract incur the damage, given whether the prevention effort is required or not. Competition forces insurers to offer actuarially fair contracts, so that the indemnity is given by $I = \alpha(1 - p)m$. The expected utility of the agent buying this contract (π, I, i) is

$$pv(d) + (1 - p)v(b) - \phi^i,$$

where $v(.)$ is a classical Bernouilli utility function ($v'(.) > 0$, $v''(.) < 0$), common to all agents, and where

$$d = y - m + I$$

[8] Observe that some countries (such as Austria, Belgium, Denmark, France, Israel, Italy and Norway) apply a stricter version of this regulation, called *Strict Prohibition*, where insurers cannot request genetic tests, cannot require applicant to provide existing tests results, and cannot use any genetic information in underwriting and rating. Such an environment also generates adverse selection, since nothing prevents the insurance companies from offering a menu of contracts in order for the different types to self select. The difference with the Consent Law legislation studied here is that type L are forbidden from showing their test results, so that they (like type H) must be separated from type U by the provision of adequate contracts. The mechanisms at play here then also apply to the Strict Prohibition legislation. Moreover, Strict Prohibition is not collusion proof, since low risk agents would like to reveal their type, and insurers may try and use proprietary artificial intelligence to screen those low risk types without regulators' knowledge (see "A.I. is changing Insurance" by S. Jeong, New York Times, April 10, 2019), resulting in the same equilibrium contracts as under Consent Law.

is the consumption level if the damage occurs while

$$b = y - \pi$$

is the consumption level when the damage does not occur, with y the individual's exogenous income. We assume that $\phi^1 = \phi$ while $\phi^0 = 0$, where the effort cost (normalized to zero if no effort is undertaken) ϕ is measured in utility terms. The assumption of a utility (rather than monetary) cost is innocuous in our binary setting. All agents have the same utility function $v(.)$, income y and potential damage m, and differ only in their probability of damage p.

In the case of complete coverage ($\alpha = 1$), we have

$$c \equiv d = b = y - pm.$$

It is straightforward that contracts offered to type L agents are not affected by adverse selection, since they are allowed to prove their type, and have an incentive to do so in order to benefit from the low premium reflecting their low disease probability p_L. By assumption, prevention has costs but no benefit when the individual is of a low type, so that the only contracts offered to type L agents entail no prevention effort, with the consumption level, denoted by c_L, given by

$$c_L = y - p_L m,$$

with the corresponding utility level

$$V_L = v(c_L). \tag{1}$$

We follow Wilson (1977)'s approach that encompasses pooling and separating equilibrium according to the adverse selection intensity at play.[9] In our context, the proportion of high risk is then given by $k\lambda$.

We assume throughout the paper that the following assumption is satisfied.

Assumption 1 *The utility function $v(.)$ exhibits constant absolute risk aversion (CARA).*

[9] See Hoy (2006) and Seog (2010, section 7.3). Under Rothschild and Stiglitz (1976) (RS hereafter), insurers conjecture that the other insurers will not react to the introduction of a new contract (*i.e.*, they have a traditional *à la Nash* reasoning). Wilson (1977) uses a more sophisticated conjecture, which is that other insurers will withdraw their contracts that become unprofitable as a result of the introduction a new contract. This conjecture provides an insurer with lower incentives to offer new contracts than under the RS conjecture. As a consequence, a RS equilibrium is also a Wilson equilibrium. But unlike in RS, a pooling equilibrium can emerge since a contract attracting only low risks is not profitable under the Wilson conjecture.

This assumption is made for simplicity as it allows to simplify some (already long and convoluted) proofs.

We now study the separating insurance contracts, before moving to the pooling contracts, and then analyzing which of the two types of contracts emerges at equilibrium.

4 Separating insurance contracts

In a separating equilibrium, the competitive fringe of insurers offer a menu of two contracts (in addition to the contract offered to type L and described above). The first contract is intended for type H agents and offers full coverage at an actuarially fair price. The second contract is intended for type U agents and is also actuarially fair. In order to prevent type H agents from buying this second contract, insurers can play on two dimensions, the coverage rate and the prevention effort, as we will see shortly.

Competition results in the (full insurance, actuarially fair) contract offered to type H to include prevention effort if and only if this contract results in a higher utility for type H than the (full insurance, actuarially fair) contract without prevention effort. We study in section 4.1 the case where the effort cost is low enough that type H agents are offered contracts with effort, and in section 4.2 the case where they do not exert effort at equilibrium.

4.1 Type H makes an effort

We first treat the case where the utility level of type H with the non-distorted (actuarially fair with full coverage) contract is higher with effort than without. This is the case if the effort cost is low enough–*i.e.*, if

$$\phi \leq \phi^S_{\max} \equiv v\left(y - p^1_H m\right) - v\left(y - p^0_H m\right). \tag{2}$$

We first characterize the separating equilibrium, and then we perform a comparative static analysis in terms of welfare.

4.1.1 Characterization of separating equilibrium

In this case, insurers offer the contract with effort to type H. Type H agents need to be separated from type U agents, since they have an incentive to pretend that they are untested in order to benefit from the lower (actuarially fair) premium offered to type U agents. This means that the contract offered to type U agents has to be distorted, which can happen along two dimensions: the continuous dimension of the coverage rate, and/or the binary dimension of the prevention effort requirement. Perfect competition among insurers ensures that the contract offered to type U agents is the one offering the highest utility level to them, conditional on not being mimicked by type H agents.

We denote by S^{11} the case where the separating contracts offered to types U and H both prescribe a prevention effort, with the first superscript indicating that U makes the effort, and the second one that H also does the effort. In that case, the incentive compatibility constraint is denoted by IC^{11} and is given by

$$V_{HS}^{11} \geq p_H^1 v\left(d_{US}^{11}\right) + \left(1 - p_H^1\right) v\left(b_{US}^{11}\right) - \phi, \tag{3}$$

where

$$\begin{aligned} V_{HS}^{11} &= v\left(y - p_H^1 m\right) - \phi, \\ d_{US}^{11} &= y + \alpha_S^{11}\left(1 - p_U^1\right) m - m, \\ b_{US}^{11} &= y - \alpha_S^{11} p_U^1 m. \end{aligned} \tag{4}$$

The LHS of (3) is the utility reaped by type H when he chooses the contract designed for him (denoted by V_{HS}^{11}), while the RHS measures type H's utility when he buys the contract with effort targeted to type U, which entails a coverage rate of α_S^{11}. Note that d_{US}^{11} (resp., b_{US}^{11}) is the consumption level in case of damage (resp., in case the damage does not occur) for the agent buying the contract designed for type U in a separating equilibrium where both types U and H exert the prevention effort.

In the case (labelled S^{01}) where the separating contract offered to type U does not prescribe a prevention effort, the incentive compatibility constraint (denoted by IC^{01}) becomes

$$V_{HS}^{01} \geq p_H^0 v\left(d_{US}^{01}\right) + \left(1 - p_H^0\right) v\left(b_{US}^{01}\right), \tag{5}$$

where

$$\begin{aligned} V_{HS}^{01} &= v\left(y - p_H^1 m\right) - \phi = V_{HS}^{11}, \\ d_{US}^{01} &= y + \alpha_S^{01}\left(1 - p_U^0\right) m - m, \\ b_{US}^{01} &= y - \alpha_S^{01} p_U^0 m, \end{aligned} \tag{6}$$

so that d_{US}^{01} (resp., b_{US}^{01}) is the consumption level in case of damage (resp., in case the damage does not occur), and α_S^{01} is the fraction of the damage reimbursed, for the agent buying the contract designed for type U in a separating equilibrium where only H exerts the prevention effort.

Lemma 1 *(a) $\alpha_S^{11} < 1$, (b) α_S^{11} is constant with ϕ, (c) $\alpha_S^{01} < 1$ unless Δ is large enough that $p_H^1 < p_U^0$ and ϕ is small enough, and (d) α_S^{01} decreases with ϕ when $\alpha_S^{01} < 1$.*

Proof. *See Appendix A.1.* ∎

It is straightforward that α_S^{11} is not affected by ϕ (as long as $\phi \leq \phi_{\max}^S$ of course): since effort is required in both type U and type H contracts, the cost of the prevention effort is borne by type H whether he separates or cheats and does not affect the underprovision of insurance to type U. Also, it is necessary in that case to underprovide insurance (*i.e.*, $\alpha_S^{11} < 1$) to separate the two types, since they are both required to undertake the prevention effort. This need not be the case when type U is not required to do the effort, provided that the effort is so efficient (Δ large enough) that $p_H^1 < p_U^0$, and not too costly (ϕ low): in that case we have $\alpha_S^{01} = 1$. If Δ is low enough and/or ϕ large enough, then the coverage rate $\alpha_S^{01} < 1$ decreases with the cost of effort ϕ, since the latter decreases the utility type H obtains when he buys the contract designed for him, which makes it more difficult to separate him from type U (and thus requires to provide less insurance to the latter).

We now compare the utility obtained by type U agents when they buy the contracts satisfying IC^{11} (equation (3) holding with equality)

$$V_{US}^{11} = p_U^1 v\left(d_{US}^{11}\right) + \left(1 - p_U^1\right) v\left(b_{US}^{11}\right) - \phi, \tag{7}$$

and the utility they obtain under IC^{01} (equation (5) holding with equality)

$$V_{US}^{01} = p_U^0 v\left(d_{US}^{01}\right) + \left(1 - p_U^0\right) v\left(b_{US}^{01}\right). \tag{8}$$

We denote by $\phi_{\min}^S$ the value of ϕ such that type U agents are indifferent between buying the separating contract with and without effort,

$$\phi_{\min}^S \equiv p_U^1 v\left(d_{US}^{11}\right) + \left(1 - p_U^1\right) v\left(b_{US}^{11}\right) - V_{US}^{01},$$

and we obtain the following result, which characterizes the equilibrium separating contracts when $\phi \leq \phi_{\max}^S$.[10]

Result 1 *(i) When $\phi < \phi_{\min}^S$, the competitive fringe offers to type U (a) a S^{11} contract requiring effort with (b) a partial coverage that is constant with the cost of effort.*
(ii) When $\phi_{\min}^S < \phi \leq \phi_{\max}^S$, the competitive fringe offers to type U (a) a S^{01} contract requiring no effort with (b) a partial coverage (c) that decreases with the effort cost.
(iii) At $\phi = \phi_{\min}^S$, the coverage rate offered with the contract requiring prevention is strictly lower than the one forgoing prevention (i.e., $\alpha_S^{11} < \alpha_S^{01}$).

Proof. See Appendix A.2. ■

[10] We slightly abuse terminology by calling a "S^{ij} contract" a contract offered to either U or H in Case S^{ij}.

The contract offered to type U prescribes the same prevention effort as for type H when the prevention cost is low enough, but prescribes no effort –unlike for H– when the prevention cost is large. If the effort cost is very low, it is too costly in utility terms for type U agents to induce type H agents not to mimic them by preventing type U agents from undertaking the effort. The separation between the two types then entirely depends on the underprovision of insurance to type U. If the effort cost is large enough, then preventing type U individuals from undertaking the effort is used, together with restrictions on coverage, to separate both types. Preventing effort in the contract designed for type U makes this contract less attractive to type H, enabling insurers to increase the coverage offered to type U, explaining the discontinuity in coverage rates when moving from contract S^{11} to S^{01}.

We now move to the welfare analysis.

4.1.2 Comparative static welfare analysis

We take as welfare function the utilitarian one where we use as weight for each type its proportion in the population of agents.[11] Namely,

$$\begin{aligned} W_S^{i1} &= (1-k)V_{US}^{i1} + k\lambda V_{HS}^{i1} + k(1-\lambda)V_L \\ &= k\left[(1-\lambda)V_L + \lambda V_{HS}^{i1} - V_{US}^{i1}\right] + V_{US}^{i1}, \end{aligned} \tag{9}$$

$i = \{0,1\}$, where utility levels for types L, U and H are given, respectively, by V_L (see equation (1)), V_{US}^{11} (see equation (7)), V_{US}^{01} (see equation (8)), V_{HS}^{11} (see equation (4)) and V_{HS}^{01} (see equation (6)).

Result 2 *In both cases S^{01} and S^{11}, welfare decreases with ϕ and increases with k.*

Proof. See Appendix A.3. ■

In both S^{11} and S^{01}, type H exerts a prevention effort, so that its utility decreases linearly with ϕ. A larger prevention cost ϕ does not affect the coverage rate offered to U in S^{11}, but decreases directly U's utility as it undertakes the effort. In case S^{01}, type U makes no effort, but receives a lower coverage rate as ϕ increases, resulting in a lower utility. In both cases, aggregate welfare then decreases with ϕ.

In a separating equilibrium, policyholders' utilities do not depend on k. The variation of k affects the welfare function only through a composition effect (increasing the

[11] See Hoy (2006) for a discussion of the equivalence between this welfare fonction and the expected utility of an individual behind the veil of ignorance, and for a welfare analysis that relies on the construction of Lorenz curves of the income distributions generated by the insurance contracts. The addition of a prevention cost in utility terms prevents us from using this approach relying on income distributions.

proportion of types L and H at the expense of type U). As shown in (9), the derivative of welfare with respect to k compares the expected payoff of knowing one's type with the payoff of remaining uninformed. The seminal paper by Hirshleifer (1971) has established that this composition effect (defined there as the "value of the information brought by a test") is negative when damage probabilities are exogenous and when individuals face a discrimination risk. We obtain here a positive composition effect, as we depart from Hirshleifer (1971)'s setting in two directions. First, remaining uninformed entails buying a contract with partial coverage, while being informed of one's type allows to buy full coverage, generating a positive impact on the composition effect. Second, the prevention decision may change with the informational status. In Case S^{11}, knowing one's type allows to save on the prevention cost of effort in case one is revealed to be of type L, generating another positive impact on the composition effect. In Case S^{01}, uninformed agents do not exert the prevention effort, so that knowing one's type now means incurring an effort cost ϕ if revealed to be of type H, with a negative impact on the composition effect. In both cases, the composition effect is positive.

4.2 Type H does not make an effort

We now move to the case S^{00} where $\phi > \phi^S_{\max}$, when type H prefers to exert no effort when offered an actuarially fair, full insurance contract, and obtains utility

$$V^{00}_{HS} = v\left(y - p^0_H m\right). \tag{10}$$

We first characterize this separating equilibrium, before studying its comparative statics properties.

4.2.1 Characterization of separating contract

In Case S^{00}, no one makes the prevention effort, and the contract offered to type U specifies a coverage rate $\alpha^{00}_S < 1$ in order to discourage type H from buying this contract. The utility attained by type U is given by

$$V^{00}_{US} = p^0_U v\left(d^{00}_{US}\right) + \left(1 - p^0_U\right) v\left(b^{00}_{US}\right), \tag{11}$$

with

$$\begin{aligned} d^{00}_{US} &= y + \alpha^{00}_S\left(1 - p^0_U\right) m - m, \\ b^{00}_{US} &= y - \alpha^{00}_S p^0_U m. \end{aligned}$$

We then obtain:

Result 3 *When $\phi > \phi^S_{\max}$, the competitive fringe offers to type U a S^{00} contract requiring no effort, where the coverage rate $\alpha^{00}_S < 1$ is independent of ϕ and corresponds to α^{01}_S when $\phi = \phi^S_{\max}$.*

Proof. See Appendix A.4. ■

When the effort cost is high enough that even type H does not exert effort, then the separation between types U and H is obtained only by providing partial coverage in a type U contract that does not require effort. This contract is then unaffected by any further increase in ϕ.

4.2.2 Comparative statics welfare analysis

Utility levels for types U and H are given, respectively, by V^{00}_{US} (see equation (11)) and V^{00}_{HS} (see equation (10)). Utilities as well as the coverage level α^{00}_S do not depend on ϕ nor on k. The welfare function becomes:

$$W^{00}_S = (1-k)V^{00}_{US} + k\lambda V^{00}_{HS} + k(1-\lambda)V_L,$$

and we obtain:

Result 4 *In Case S^{00}, welfare is not affected by ϕ, and increases with k.*

Proof. See Appendix A.5. ■

So, a larger value of k increases welfare thanks to the composition effect. The sign of this composition effect remains positive as in S^{01} and S^{11}, even though the effort cost plays no role here since no one undertakes the prevention effort. In other words, the composition effect is driven entirely by the fact that type U agents do not buy full coverage insurance, unlike types L and H.

4.3 Summary for separating contract

The following proposition summarizes all the results obtained regarding the characterization of the separating contracts.

Proposition 1 *(i) In all separating equilibria, types L and H receive an actuarially fair contract with full coverage, and type L never exerts the prevention effort.*
(ii) The type of separating contract does not depend on k, but depends on ϕ.
(iii) If $\phi < \phi^S_{\min}$, we have a S^{11} equilibrium contract, where both types U and H make the prevention effort, and where the coverage rate of type U is given by α^{11}_S, which does not depend on ϕ.

(iv) If $\phi_{\min}^{S} \leq \phi \leq \phi_{\max}^{S}$, we have a S^{01} equilibrium contract, where type H makes the prevention effort while U does not, and where the coverage rate of type U is given by α_{S}^{01}, which decreases with ϕ.
(v) If $\phi > \phi_{\max}^{S}$, we have a S^{00} equilibrium contract, where neither type U nor H makes the prevention effort, and where the coverage rate of type U is given by α_{S}^{00}, which does not depend on ϕ.

We now summarize the comparative statics results. We focus on the impact of decreasing ϕ (resp., increasing k), since this corresponds to the empirically relevant case.

Proposition 2 *(i) Decreasing ϕ (a) has no impact on utilities nor on aggregate welfare in S^{00}, (b) increases types U's and H's utilities, as well as aggregate welfare, in S^{11} and in S^{01}.*
(ii) Increasing k (a) has no impact on utilities in any separating equilibrium, but (b) increases aggregate welfare thanks to a composition effect.

The prevention effort has no impact on utilities and welfare when no one does the effort (S^{00}) since neither the coverage nor the composition are affected by ϕ. A lower value of ϕ increases the utility of both U and H in all other cases–*i.e.*, when they both make the effort (S^{11}) but also in S^{01} because U's coverage rate increases as ϕ decreases.

We now move to the pooling contracts.

5 Pooling insurance contracts

5.1 Characterization

Recall first that the pooling contract does not concern type L, who has both the legal right and the incentive to reveal his type to the insurer in order to obtain an actuarially fair contract (with the low price of p_L) with full coverage.

A unique contract is offered to the pool of agents who claim to be uninformed about their type. By definition, in a pooling contract all agents must look alike, so that they all either undertake the (observable) prevention effort ($i = 1$, which we call the P^1 case or contract) or do not make this effort ($i = 0$, corresponding to the P^0 case/contract). This pool is composed of a mass of $1 - k$ agents who are truly uninformed (since they have not taken the test), and a mass $k\lambda$ of agents whose test has revealed them as type H. The competition among insurers results in a unit price of insurance p_P^i reflecting the average risk among this pool:

$$p_P^i = \frac{1-k}{1-k(1-\lambda)} p_U^i + \frac{k\lambda}{1-k(1-\lambda)} p_H^i,$$

$i \in \{0,1\}$.

The pooling price p_P^i is lower than the actuarially fair price p_H^i for type H agents, who would then wish to buy full insurance. The coverage rate offered at equilibrium is then the one most-preferred by type U agents, and is lower than one since the pooling price is larger than p_U^i. We denote by α_P^i the equilibrium coverage rate of the pooling contract, which is obtained as

$$\alpha_P^i = \arg\max_{\alpha} p_U^i v(y + \alpha(1-p_P^i)m - m) + (1-p_U^i)v(y - \alpha p_P^i m) - \phi^i.$$

The first-order condition for the equilibrium pooling coverage rate is given by:

$$p_U^i(1-p_P^i)v'(d_P^i) - (1-p_U^i)p_P^i v'(b_P^i) = 0, \tag{12}$$

with

$$\begin{aligned} d_P^i &= y + \alpha_P^i(1-p_P^i)m - m, \\ b_P^i &= y - \alpha_P^i p_P^i m, \end{aligned}$$

respectively, the consumption levels of (type U and H) agents who buy the pooling contract when the damage does (resp., does not) occur. It is obvious from (12) that α_P^i does not depend on the value of ϕ.

We denote the utility level attained by type U in the pooling contract as

$$V_{UP}^i = p_U^i v(d_P^i) + (1-p_U^i)v(b_P^i) - \phi^i, i \in \{0,1\}$$

and the one attained by type H as

$$V_{HP}^i = p_H^i v(d_P^i) + (1-p_H^i)v(b_P^i) - \phi^i, i \in \{0,1\}.$$

What determines whether effort is prescribed or not at equilibrium for the pooling contract is the comparison of V_{UP}^0 and V_{UP}^1. Insurers want to attract the least risky type (*i.e.*, U and not H) and competition among insurers ensures that the contract offering the highest utility to type U is offered at equilibrium. We then obtain the following proposition, which summarizes all the results obtained regarding the characterization of the pooling contracts.

Proposition 3 *(i) In all pooling equilibria, type L receives an actuarially fair contract with full coverage and never exerts the prevention effort.*
(ii) The type of pooling contract depends on both k and ϕ. There exists a unique value of ϕ, denoted by $\tilde{\phi}_P(k)$, so that:
(iii) If $\phi < \tilde{\phi}_P(k)$, we have a P^1 equilibrium contract, where both types U and H make

the prevention effort, and where the coverage rate of the pooling contract bought by both types is given by α_P^1.
(iv) If $\phi \geq \tilde{\phi}_P(k)$, we have a P^0 equilibrium contract, where neither type U nor H makes the prevention effort, and where the coverage rate of the pooling contract bought by both types is given by α_P^0.
(v) Insurance coverage rate α_P^i decreases with k, with $\alpha_P^1 \geq \alpha_P^0$.

Proof. See Appendix A.6. ■

It is intuitive that the prevention effort is made only if its cost is sufficiently low. Assumption 1 implies that the demand for insurance decreases with its unit price. Prevention, by decreasing the health risk, decreases the actuarially fair insurance premium, so that agents buy more insurance. Note for future reference that the threshold $\tilde{\phi}_P(k)$ depends on k in a non trivial way.

5.2 Comparative statics welfare analysis

The welfare function is given by:

$$W_P^i = (1-k)V_{UP}^i + k\lambda V_{HP}^i + k(1-\lambda)V_L.$$

We obtain the following proposition.

Proposition 4 *(i) Decreasing ϕ (a) has no impact on utilities nor on aggregate welfare in P^0, (b) increases types U's and H's utilities, as well as aggregate welfare, in P^1.*
(ii) Increasing k (a) decreases the utilities of both types U and H, but (b) has an ambiguous impact on aggregate welfare due to the composition effect.

Proof. See Appendix A.7. ■

The effort cost does not affect any contract in P^0, is paid by no one, and has no impact on the composition of the universe of insured agents, so that aggregate welfare is not affected by ϕ. Aggregate welfare decreases with ϕ in P^1, as a more costly effort does not affect the coverage rate but decreases the utility of types U and H who both pay this cost.

Increasing k has two countervailing effects on aggregate welfare in both P^0 and P^1. First, a larger k increases the price of the pooling contract and thus reduces the utilities of types U and especially of types H.[12] Second, increasing k creates a composition impact, which is positive as in the case of separating contracts. Observe that, unlike in the separating contracts, type H suffers from underprovision of insurance in the pooling contract, but receives a better-than-fair unit price. The overall sign of the impact of k on aggregate welfare is thus analytically ambiguous.

Using a numerical example (see Appendix B), we obtain that $\partial W_P^0 / \partial k$ is negative, so that the direct negative impact of a higher k on utilities is larger than the positive composition impact. Note that testing allows to save on the prevention effort cost (if revealed to be type L), as in S^{11}, so that a larger value of ϕ increases the composition effect. Using the numerical example, we obtain that $\partial W_P^1 / \partial k$ is negative for small values of ϕ, and positive for larger values. We come back to these impacts (including by showing figures illustrating them) in section 7.2.

6 Equilibrium contracts: separating or pooling?

The objective of this section is to understand what type of Wilson equilibrium (separating or pooling) emerges as a function of k and of ϕ. What determines whether the equilibrium is pooling or separating is the utility attained by U under both contracts. Start with the separating contract. It can only be defeated if insurers can attract the lower risk (type U) while offering them a pooling contract. Hence, we have a pooling equilibrium if type U has a larger utility level with the pooling contract than with the separating contract, and *vice-versa*.

We introduce the following assumption.

Assumption 2 Δ *is not too large:*

$$\frac{p_U^1 \left(1 - p_H^1\right)}{\left(1 - p_U^1\right) p_H^1} < \frac{v'(y)}{v'(y-m)}. \tag{13}$$

Note that the LHS of (13) is increasing in Δ (so that this inequality indeed implies that Δ is not too large) and that this expression does not involve any endogenous variables, but only exogenous ones.

This assumption allows us to prove the following lemma, which shows that, when the price of insurance is large enough (close to p_H^1, with Δ not too large), type U agents prefer not to buy insurance.

Lemma 2 *Under Assumption 2,* $\alpha_P^1 = 0$ *for k close enough to one.*

[12] Type U's utility is only affected by the direct impact of a more expensive pooling contract (we apply the envelope theorem since type U chooses α_P^i). Type H agents suffer more from the higher premium (given the coverage rate), because they have a larger probability of incurring the damage than type U. They also suffer more because the already too low coverage rate is further decreased by U as k increases.

Proof. The coverage rate of the pooling contract P^1 is determined by

$$p_U^1\left(1-p_P^1\right)v'(d_P^1)-\left(1-p_U^1\right)p_P^1 v'(b_P^1)\leq 0.$$

When $k=1$, this condition becomes:

$$p_U^1\left(1-p_H^1\right)v'(d_P^1)-\left(1-p_U^1\right)p_H^1 v'(b_P^1)\leq 0,$$

which is satisfied with a strict inequality for $\alpha_P^1=0$ under Assumption 2, so that, given Proposition 3 (v), $\alpha_P^1\to 0$ for $k\to 1$. ■

We assume that Assumption 2 holds throughout the remainder of the analysis.[13]

Proposition 5 *For any given value of ϕ, there is a unique (strictly positive) threshold value of k, denoted by $\tilde{k}(\phi)$, such that a pooling equilibrium emerges if $k<\tilde{k}(\phi)$, and a separating equilibrium emerges if $k>\tilde{k}(\phi)$.*

Proof. We prove this proposition in 5 steps.
(1) Whatever the value of ϕ, the equilibrium contract is either the pooling one preferred by U (either P^0 or P^1) or the separating one preferred by U (S^{00}, S^{01} or S^{11}).

(2) The utility of U with a pooling equilibrium is strictly decreasing in k, while its utility with any separating equilibrium is independent of k.

(3) When k tends towards 0, the pooling equilibrium tends to full coverage with actuarially fair price. This is the highest utility type U can get. At the same time, the separating contract proposes either partial coverage and/or a distortion of the prevention decision. Hence, the utility of U is higher with pooling.

(4) As k tends towards 1, the price of the pooling contract increases and reflects type H's risk, which is higher than the price of the separating contract. Lemma 2 and Proposition 3 (v) establish that, when k is large enough, we have that $\alpha_P^1=\alpha_P^0=0$. We then have that U prefers the separating contract with some coverage to (the pooling contract with) no insurance.

(5) Given (2), we have the unique, strictly positive, threshold $\tilde{k}(\phi)$. ■

Observe that there are at most 6 possible comparisons of utility levels for type U, for any value of (k, ϕ), since there are 2 types of pooling (P^0 and P^1) and 3 types of separating (S^{00}, S^{01} and S^{11}) contracts. The following proposition assesses how $\tilde{k}(\phi)$ is affected by ϕ for the 6 possible comparisons.

[13] Assumption 2 is a sufficient (but not necessary) condition to prove that U prefers the separating to the pooling contract when k tends toward 1.

Proposition 6 *(a) For values of* (k, ϕ) *such that the potential equilibria are either* P^0 *and* S^{00}*, or* P^1 *and* S^{11}*, the threshold* $\tilde{k}(\phi)$ *is not affected by the value of* ϕ*.*
(b) For values of (k, ϕ) *such that the potential equilibria are either* P^0 *and* S^{01}*, or* P^0 *and* S^{11}*, the threshold* $\tilde{k}(\phi)$ *is increasing in the value of* ϕ*.*
(c) For values of (k, ϕ) *such that the potential equilibria are* P^1 *and* S^{00}*, or* P^1 *and* S^{01}*, the threshold* $\tilde{k}(\phi)$ *is decreasing in the value of* ϕ*.*

Proof. (a) The utility of U under P^0 and under S^{00} is not affected by ϕ, which then plays no role in the comparison of utilities. The utility of U under P^1 and under S^{11} is affected linearly by the value of ϕ (because the coverage rate is not affected by ϕ in P^1 nor in S^{11}), so that the ϕ term cancels out when comparing the two utilities, and ϕ plays no role in determining the value of $\tilde{k}(\phi)$.
(b) The utility of U under P^0 is not affected by ϕ, while its utility under S^{01} decreases with ϕ (because the coverage rate decreases with ϕ). Similarly, the utility of U under S^{11} also decreases in ϕ. Using the implicit function theorem, we obtain that $\tilde{k}(\phi)$ increases with ϕ.
(c) The utility of U under S^{00} is not affected by ϕ, while its utility under P^1 decreases with ϕ. Using the implicit function theorem, we obtain that $\tilde{k}(\phi)$ decreases with ϕ. The utility of U under both P^1 and S^{01} is decreasing in ϕ, so that the proof, using the implicit function theorem, is less straightforward and relegated to Appendix A.8. ■

It would be too cumbersome to try and solve all possible configurations of cases. We then restrict ourself to the configuration we obtain in the numerical example described in Appendix B. Figure 1 depicts the function $\tilde{\phi}_P(k)$ we obtain, as well as the threshold $\tilde{k}(\phi)$, in the $(k,\ \phi)$ space. We obtain that $\phi^S_{\min} < \tilde{\phi}_P(k) < \phi^S_{\max}$ for all values of k. Note that this configuration only excludes the two extreme comparisons of contracts among the six mentioned in Proposition 6: there is no value of $(k,\ \phi)$ where the pooling contract is P^1 and the separating contract S^{00} (because this would require to consider $\phi^S_{\max} < k < \tilde{\phi}_P(k)$) and where the pooling contract is P^0 and the separating contract S^{11} (because this would require to consider $\tilde{\phi}_P(k) < k < \phi^S_{\min}$). These two excluded configurations anyway seem to be very unlikely to emerge in an equilibrium. Take for instance the comparison of P^1 and S^{00}. This supposes that, for the same parameters $(k,\ \phi)$, insurers who offer separating contracts would require of both types U and H not to do the prevention effort, while if insurers were to offer a pooling contract, they would ask both types to do this same prevention effort. The other four comparisons of contracts (P^0 and S^{00}, P^0 and S^{01}, P^1 and S^{01}, and P^1 and S^{11}) all exist for some parameter values $(k,\ \phi)$ in our example.

Insert Figure 1 here

Figure 1: In (k, ϕ) space, separation between P^1, P^0, S^{11}, S^{01} and S^{00} contracts.

Observe that there is a single intersection between the thresholds $\tilde{\phi}_P(k)$ and $\tilde{k}(\phi)$. We denote by (k^*, ϕ^*) this intersection. We denote by $\tilde{k}_{00}$ (resp., $\tilde{k}_{11}$) the value of $\tilde{k}(\phi)$ when $\phi > \phi^S_{\max}$, (resp., when $\phi < \phi^S_{\min}$). Both $\tilde{k}_{00}$ and $\tilde{k}_{11}$ are independent of ϕ (provided of course that ϕ is in the relevant range) as shown in Proposition 6(a). From Figure 1, we obtain that $k^* < \tilde{k}_{00} < \tilde{k}_{11}$.

7 Comparative static welfare analysis with endogenous contract type

7.1 With respect to prevention effort cost ϕ

Compared to sections 4.1.2, 4.2.2 and 5.2, we incorporate here how the equilibrium type (S^{11}, S^{01}, S^{00}, P^0 or P^1) changes endogenously as ϕ decreases. Focusing on the typology of cases that emerges from the numerical example (see Figure 1), we obtain four different situations, depending on the value of k. The transitions we observe as we decrease ϕ are: from P^0 to P^1 (when $k < k^*$), from P^0 to S^{01} to P^1 (when $k^* < k < \tilde{k}_{00}$), from S^{00} to S^{01} to P^1 (when $\tilde{k}_{00} < k < \tilde{k}_{11}$) and from S^{00} to S^{01} to S^{11} (when $k > \tilde{k}_{11}$).

We introduce the following assumption, whose role we discuss after Proposition 7.

Assumption 3 Δ *is low enough that* $p^1_P > p^0_U$ *for the value of* k *such that* $V^1_{UP} = V^{01}_{US}$.

This assumption ensures that type U faces a lower unit price with the separating contract without effort than with the pooling contract with effort. Note that it implies that $p^1_H > p^0_U$. We then obtain the following proposition.

Proposition 7 *With the typology of cases obtained in Figure 1, and under Assumption 3, a decrease in* ϕ *weakly increases both types* U*'s and* H*'s utility, and aggregate welfare, except when* $k^* < k < \tilde{k}_{00}$ *and* ϕ *is such that the equilibrium changes from* P^0 *to* S^{01}*, in which case we have a downward discontinuity in* H*'s utility and in aggregate welfare.*

Proof. Note first that type L's utility is a constant, as it always receives full insurance at an actuarially fair price, without exerting a prevention effort, whether contracts for U and H are pooling or separating. We know from Propositions 2 and 4 that a lower value of ϕ either has no impact on types U's and H's utilities, and thus on aggregate welfare, (in cases P^0 and S^{00}) or that it increases types U's and H's utilities, and thus aggregate welfare (in cases P^1, S^{01} and S^{11}).
Note that, by definition of an equilibrium (which maximizes the utility of type U, given

the relevant constraints), the utility of type U is continuous with ϕ as one moves from one equilibrium type to another. We then concentrate on how type H's utility (and aggregate welfare) is impacted by ϕ as one moves from one equilibrium contract type to another.
We first establish that H's utility remains continuous as one moves from one separating equilibrium to another: there is no discontinuity in type H's utility as (a) one moves from a S^{01} to a S^{00} equilibrium contract, by definition of $\phi^S_{\max}$ (see equation (2)), and (b) as moves from S^{11} to S^{01}, since $V^{11}_{HS} = V^{01}_{HS}$ (see equations (4) and (6)). Hence, aggregate welfare is also continuous for these moves.
We have discontinuous increases in H's utility as one moves from P^0 to P^1 (see Result 5 in Appendix A.9.), from S^{01} to P^0 (Result 6 in Appendix A.10.) and from S^{01} to P^1 (Result 7 in Appendix A.11.). In these cases, we also have a discontinuous increase in aggregate welfare.
■

The intuition for Proposition 7 is straightforward, except for the discontinuities in H's utility as one moves from one type of equilibrium to another. Take the move from P^0 to S^{01} for instance. When U is indifferent between P^0 and S^{01}, it trades off the higher price in P^0 (since $p^0_P > p^0_U$) with the larger coverage ($\alpha^0_P > \alpha^{01}_S$ when U is indifferent between these two contracts). Note that, by the incentive constraint IC^{01}, the utility that H obtains in S^{01} is equal to the utility it would obtain if it were to buy the contract designed for U in S^{01}. Type H agents value more the higher insurance coverage in P^0 because they have a larger damage probability ($p^0_H > p^0_U$). Hence, when U is indifferent between P^0 and S^{01}, type H strictly prefers P^0 to S^{01}.

A similar intuition applies to the other two discontinuities we observe (from P^0 to P^1, and from P^1 to S^{01}), with the caveat that, we have to assume that contract P^1 is costlier than contract S^{01} when U is indifferent between the two (see Assumption 3). Note that that this assumption is a sufficient, but not a necessary, condition to have an upward discontinuity in H's utility (and aggregate welfare) as one moves from S^{01} to P^1.[14]

Figures 2 to 4 exemplify what happens, respectively, to the utility of types U and H, and to aggregate welfare, when we vary ϕ while $k^* < k < \tilde{k}_{00}$. Starting from large values of ϕ, we obtain a P^0 equilibrium, where utilities are not affected by ϕ. We then switch to the S^{01} case, with a downward discontinuity in both H's utility and aggregate welfare. As long as we remain in S^{01}, utilities and welfare increase as ϕ decreases. We

[14] There is generically a discontinuity in H's utility (and aggregate welfare) when moving from S^{01} to P^1, whether Assumption 3 holds or not, but we are unable to sign it if Assumption 3 does no hold. Note that, if the discontinuity were negative rather than positive at that point, this would further reinforce our result that a lower value of ϕ does not always increase aggregate welfare.

then switch to the P^1 contract, with an upward discontinuity in both H's utility and aggregate welfare. Utilities and aggregate welfare further increase as we decrease ϕ while remaining in the P^1 case.

Figure 2: Type U's utility as a function of ϕ, for $k^* < k < \tilde{k}_{00}$

Figure 3: Type H's utility as a function of ϕ, for $k^* < k < \tilde{k}_{00}$

Figure 4: Welfare as a function of ϕ, for $k^* < k < \tilde{k}_{00}$

7.2 With respect to test prevalence k

Focusing on the typology of cases that emerges from the numerical example (see Figure 1), we obtain five different situations, depending on the value of ϕ, with $\phi^S_{\min} < \tilde{\phi}_P(0) < \phi^* < \phi^S_{\max}$. The transitions we observe as we increase k are: from P^0 to S^{00} (when $\phi > \phi^S_{\max}$), from P^0 to S^{01} (when $\phi^* < \phi < \phi^S_{\max}$), from P^0 to P^1 to S^{01} (when $\tilde{\phi}_P(0) < \phi < \phi^*$), from P^1 to S^{01} (when $\phi^S_{\min} < \phi < \tilde{\phi}_P(0)$), and from P^1 to S^{11} (when $\phi < \phi^S_{\min}$).

We then obtain the following proposition.

Proposition 8 *With the typology of cases obtained in Figure 1, and under Assumption 3, an increase in k has the following impact on utilities and welfare:*
(a) when k is low, we are in a pooling equilibrium where a higher k decreases both types U's and H's utilities, but has an ambiguous impact on aggregate welfare.
(b) increasing k then changes the equilibrium from pooling to separating, except when $\tilde{\phi}_P(0) < \phi < \phi^*$, resulting in a downward discontinuity in both H's utility and aggregate welfare. When $\tilde{\phi}_P(0) < \phi < \phi^*$, increasing k changes the equilibrium first from P^0 to P^1, which is associated with an upward discontinuity in type H's utility and in aggregate welfare, and then from P^1 to S^{01}.
(c) Further increases in k do not change the equilibrium type (separating) and do not affect agent's utilities, but increase aggregate welfare.

Proof. We know from Proposition 5 that a low value of k is associated with a pooling equilibrium, and from Proposition 4 that types U's and H's utilities decrease with k, while the impact on aggregate welfare is ambiguous. As k reaches $\tilde{k}(\phi)$, we move from pooling to separating equilibrium, each time resulting in a downward discontinuity in H's utility (and thus in aggregate welfare): (i) from P^0 to S^{00} when $\phi > \phi^S_{\max}$ (see Result 8 in the Appendix A.12.), from P^0 to S^{01} (when $\phi^* < \phi < \phi^S_{\max}$) (see Result 6), from P^1 to S^{01} (when $\phi^S_{\min} < \phi < \tilde{\phi}_P(0)$ and when Assumption 3 holds) (see Result 7) and from P^1 to S^{11} (when $\phi < \phi^S_{\min}$) (see Result 9 in Appendix A.13.).[15] In the case where $\tilde{\phi}_P(0) < \phi < \phi^*$, the equilibrium first changes from P^0 to P^1 (with an upward jump in H's utility and in aggregate welfare, see Result 5) and then from P^1 to S^{01}. Finally, as k further increases, we remain in a separating equilibrium (see Proposition 5), where utilities are not affected by k, but where a larger value of k increases aggregate welfare thanks to a composition effect (see Proposition 2). ■

We then obtain that aggregate welfare is always non monotone in k, because of the downward discontinuity in H's utility as one moves from separating to pooling equilibrium (note that the intuition for this discontinuity is the same as the one explained after Proposition 7, namely that type H values more the larger coverage associated with the pooling equilibrium than type U). We obtain with our numerical example that welfare is decreasing with k in the P^0 equilibrium. As already discussed after Proposition 4, welfare is increasing in k in the P^1 equilibrium when ϕ is large enough (in the case where $\phi^S_{\min} < \phi < \phi^*$) but is increasing in k when ϕ is low enough ($\phi < \phi^S_{\min}$).

As an illustration, we show in Figures 5 to 7, respectively, the utility of types U and H, and aggregate welfare, as a function of k when $\tilde{\phi}_P(0) < \phi < \phi^*$.

Figure 5: Type U's utility as a function of k, for $\tilde{\phi}_P(0) < \phi < \phi^*$

Figure 6: Type H's utility as a function of k, for $\tilde{\phi}_P(0) < \phi < \phi^*$

Figure 7: Aggregate welfare as a function of k, for $\tilde{\phi}_P(0) < \phi < \phi^*$

Welfare in that case is highly non monotone in k: it first decreases with k in the P^0 contract (because of the price effect), then increases discontinuously with k when one moves from P^0 to P^1 (thanks to the increase in H's utility), then increases with k in the P^1 contract (thanks to the composition effect), decreases discontinuously with k when one moves from P^1 to S^{01} (because of the decrease in H's utility), and finally increases with k in the S^{01} contract (thanks to the composition effect).

7.3 When both ϕ and k vary simultaneously

In this section, we show numerically what could happen if k were to increase at the same time as ϕ decreases. This movement is an illustration of what could happen in the near

[15] As for Proposition 7, Assumption 3 is a sufficient condition to sign the discontinuity in H's utility (and thus in aggregate welfare) when moving from P^1 to S^{01}.

future, as more people choose to do the genetic test (higher k) and as the informational content of those tests increases (which we proxy by a decrease in the prevention effort cost ϕ).We build on the numerical example presented in Appendix B, and assume a linear relationship between ϕ and k ($\phi(k) = 0.29 - 0.8k$). We then start in the upper left corner of Figure 1, in the (k, ϕ) space, with a high value of ϕ and a low value of k, corresponding to the situation currently observed in reality, and to a P^0 contract. As k increases and ϕ decreases, we move in the south-east direction on a straight line whose slope is such that we first cross to the S^{01} equilibrium contract, and then to the P^1 one.

Figures 8 and 9 depict respectively types U's and H's utility, and aggregate welfare, as we increase k/decrease ϕ simultaneously.

Figure 8: Types U's and H's utility as a function of k when $\phi(k) = 0.29 - 0.8k$

Figure 9: Aggregate welfare as a function of k when $\phi(k) = 0.29 - 0.8k$

As long as we remain in the P^0 equilibrium, types U's and H's utilities are not affected by ϕ but decrease with k, which makes the pooling contract more expensive (and further reduces the coverage enjoyed by type H). Proposition 4 has shown that the impact of k on aggregate welfare is ambiguous (because of a positive composition effect), but we obtain on Figure 9 that welfare decreases with k (so that the negative price effect is larger than the positive composition effect). Result 6 has shown a downward jump in type H's utility (and thus in aggregate welfare) when we move from the P^0 to the S^{01} equilibrium.

As long as we remain in the S^{01} equilibrium, types U's and H's utilities are not affected by k, but decrease with ϕ. We thus obtain an unambiguous increase in both types' utilities as we increase k and decrease ϕ simultaneously, as exemplified in Figure 8. Result 2 shows that aggregate welfare decreases with ϕ and increases with k (thanks to the composition effect) in S^{01}. We then obtain that aggregate welfare unambiguously increases as we increase k and decrease ϕ simultaneously, as exemplified in Figure 9. Result 7 has shown an upward jump in type H's utility (and thus in aggregate welfare) when we move from the P^1 to the S^{01} equilibrium.

As long as we remain in the P^1 equilibrium, Proposition 4 shows that types U's and H's utilities decrease with both k and ϕ. We obtain in our numerical example that the impact of a smaller ϕ supersedes the impact of a larger k, since both V^1_{UP} and V^1_{HP} (Figure 8) increase as we increase k and decrease ϕ simultaneously. Aggregate welfare decreases with ϕ, but the impact of a larger value of k is ambiguous (see Proposition 4). We obtain on Figure 9 that aggregate welfare increases when we increase k and decrease ϕ simultaneously.

To summarize, we obtain that a larger value of k combined with a smaller value of ϕ is detrimental for types U and H and for society as long as we remain in a P^0 equilibrium, and when we move from the P^0 to the S^{01} equilibrium. The impact then becomes positive (both for U, for H and for aggregate welfare) in both the S^{01} and P^1 equilibria, and also when one moves from the former to the latter. The lowest level of utility (for H and for U) and of aggregate welfare corresponds to the combination of values of k and of ϕ that generates a change from the P^0 to the S^{01} equilibrium.

8 Conclusion

This article has studied the welfare implications on the health insurance market of the development of personalized medicine, as measured by the increase in the take-up rate of genetic tests providing more efficient and actionable prevention actions. Starting from the current low take-up rate generating at equilibrium a pooling contract with no prevention effort, we obtain that an increase in the take-up rate has first an ambiguous impact on welfare, and then unambiguously decreases welfare as one moves from a pooling to a separating equilibrium. It is only once the take-up rate is large enough that the equilibrium is separating that any further increase in take-up rate increases aggregate welfare, by a composition effect.

We also study the impact of a decrease in the prevention effort cost, taken as a proxy for the effectiveness of the genetic tests in terms of actionable health information. We obtain that decreasing this cost, starting from its current high level, moves us from the current pooling equilibrium without prevention to another pooling equilibrium with effort, with the possibility of having a separating equilibrium for intermediate values of the effort cost. Once more, the move from pooling to separating equilibrium is especially detrimental to those unlucky enough to get informed of their detrimental genetic background.

A main result of our analysis is then to stress that the long run impact of cheap genetic tests may not be to destroy cross-subsidies, provided that the increase in their efficiency in providing prevention actions is large enough.

Observe that we have used a simple utilitarian welfare function, weighting individual types by their share in the insured population at equilibrium. Moving to a welfare criterion that puts more weight on the least well-off (type H in our setting) would reinforce our conclusion that encouraging individuals to undertake a genetic test may result in short run welfare losses, as long as the equilibrium is not separating.

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Appendix A

Appendix A.1.: Proof of Lemma 1

(a) $\alpha_S^{11} < 1$ is a necessary condition for (3) to hold since $p_H^1 > p_U^1$. (b) ϕ cancels out in both the LHS and RHS of (3).
(c) IC^{01} (equation (5)) is satisfied when $\alpha_S^{01} = 1$ if and only if

$$\begin{aligned} v(y - p_H^1 m) - \phi &\geq v(y - p_U^0 m) \\ \Leftrightarrow \phi &\leq v(y - p_H^1 m) - v(y - p_U^0 m), \end{aligned}$$

with $v(y - p_H^1 m) > v(y - p_U^0 m)$ if and only if $p_H^1 < p_U^0$.
(d) If $\alpha_S^{01} < 1$, applying the implicit function theory to (5), we obtain that

$$\frac{\partial \alpha_S^{01}}{\partial \phi} = \frac{-1}{m\left[p_H^0\left(1-p_U^0\right)v'(d_{US}^{01}) - \left(1-p_H^0\right)p_U^0 v'(b_{US}^{01})\right]}, \tag{14}$$

whose sign is negative since $v'(d_{US}^{01}) > v'(b_{US}^{01})$ and $p_H^0\left(1-p_U^0\right) > p_U^0\left(1-p_H^0\right)$.

Appendix A.2.: Proof of Result 1

We first have to prove the following lemma.

Lemma 3 $\alpha_S^{01} < \alpha_S^{11}$ *for* $\phi = \phi_{\max}^S$.

Proof. Equation IC^{11} yields

$$v(c_H^1) = p_H^1 v(d_{US}^{11}) + (1-p_H^1)v(b_{US}^{11}), \tag{15}$$

while equation IC^{01}, when measured at $\phi = \phi_{\max}^S$, yields

$$v(c_H^0) = p_H^0 v(d_{US}^{01}) + (1-p_H^0)v(b_{US}^{01}), \tag{16}$$

where

$$c_H^i = y - p_H^i m.$$

Let us consider the following function $G(\alpha, \delta)$ such that

$$\begin{aligned} G\left(\alpha,\delta\right) &= v(y-(p_H^0-\delta)m) \\ &\quad - \left(p_H^0-\delta\right)v\left(y+\alpha\left(1-\left(\left(\lambda\left(p_H^0-\delta\right)+(1-\lambda)p_L\right)\right)m-m\right)\right. \\ &\quad - \left(1-\left(p_H^0-\delta\right)\right)v\left(y-\alpha\left(\left(\lambda\left(p_H^0-\delta\right)+(1-\lambda)p_L\right)\right)m\right). \end{aligned}$$

Observe that (15) corresponds to $G(\alpha_S^{11}, \Delta) = 0$ while (16) corresponds to $G(\alpha_S^{01}, 0) = 0$. Using the implicit function theorem, we have that

$$\frac{d\alpha}{d\delta} = -\frac{\partial G/\partial \delta}{\partial G/\partial \alpha}.$$

We obtain:

$$-\frac{dG/\partial\delta}{dG/\partial\alpha} = \frac{mv'(c_H^1) - \left[v(b_{US}^{11}) - v(d_{US}^{11}) + \lambda\alpha_S^{11} m \left[p_H^1 v'(d_{US}^{11}) + (1-p_H^1)v'(b_{US}^{11})\right]\right]}{m\left[p_H^1(1-p_U^1)v'(d_{US}^{11}) - (1-p_H^1)p_U^1 v'(b_{US}^{11})\right]}.$$

As $p_H^1 \geq p_U^1$, $d_{US}^{11} \leq b_{US}^{11}$ and $v(.)$ is an increasing and concave function, the denominator is positive. Then, we have $\alpha_S^{01} < \alpha_S^{11}$, if and only if:

$$mv'(c_H^1) \geq \left[v(b_{US}^{11}) - v(d_{US}^{11}) + \lambda\alpha_S^{11} m \left[p_H^1 v'(d_{US}^{11}) + (1-p_H^1)v'(b_{US}^{11})\right]\right].$$

Moreover, from IC^{11}, we know that:

$$v(b_{US}^{11}) - v(c_H^1) = p_H^1\left[v(b_{US}^{11}) - v(d_{US}^{11})\right].$$

Introducing this last expression in the previous inequality yields:

$$\begin{gathered}
m\left[v'(c_H^1) - \lambda\alpha_{US}^{11} v'(b_{US}^{11})\right] \geq v(b_{US}^{11}) - v(d_{US}^{11}) + \lambda\alpha_{US}^{11} d p_H^1 \left[v'(d_{US}^{11}) - v'(b_{US}^{11})\right] \\
\Leftrightarrow \\
mv'(c_H^1)\left[1 - \lambda\alpha_{US}^{11}\right] + \lambda\alpha_S^{11} m\left[v'(c_H^1) - v'(b_{US}^{11})\right] \\
\geq \left[v(b_{US}^{11}) - v(c_H^1)\right]\left[\frac{1}{p_H^1} - \lambda\alpha_{US}^{11} m\left[\frac{v'(d_{US}^{11}) - v'(b_{US}^{11})}{v(d_{US}^{11}) - v(b_{US}^{11})}\right]\right] \\
\Leftrightarrow \\
\frac{mv'(c_H^1)\left[1 - \lambda\alpha_{US}^{11}\right]}{v'(c_H^1) - v'(b_{US}^{11})} + \lambda\alpha_{US}^{11} m \\
\geq \frac{v(b_{US}^{11}) - v(c_H^1)}{v'(c_H^1) - v'(b_{US}^{11})}\left[\frac{1}{p_H^1} - \lambda\alpha_{US}^{11} m\left[\frac{v'(d_{US}^{11}) - v'(b_{US}^{11})}{v(d_{US}^{11}) - v(b_{US}^{11})}\right]\right].
\end{gathered}$$

The generalized mean value theorem implies that:

$$\frac{v(b_{US}^{11}) - v(c_H^1)}{v'(c_H^1) - v'(b_{US}^{11})} = -\frac{v'(\gamma)}{v''(\gamma)}$$

and

$$\frac{v'(d_{US}^{11}) - v'(b_{US}^{11})}{v(d_{US}^{11}) - v(b_{US}^{11})} = -\frac{v''(\beta)}{v(\beta)},$$

with $\gamma \in \left[c_H^1, b_{US}^{11}\right]$ and $\beta \in \left[d_{US}^{11}, b_{US}^{11}\right]$. Assuming that $v(.)$ is a CARA function, we obtain that

$$-\frac{v'(\gamma)}{v''(\gamma)} = -\frac{1}{\frac{v''(\beta)}{v(\beta)}} = K.$$

Then, the previous inequality can be rewritten:

$$\begin{array}{rcl} \dfrac{mv'(c_H^1)\left[1-\lambda\alpha_{US}^{11}\right]}{v'(c_H^1)-v'(b_{US}^{11})} + \lambda\alpha_{US}^{11}m & \geq & K\left[\dfrac{1}{p_H^1} + \lambda\alpha_{US}^{11}m\dfrac{1}{K}\right] \\ & \Leftrightarrow & \\ \dfrac{mv'(c_H^1)\left[1-\lambda\alpha_{US}^{11}\right]}{v'(c_H^1)-v'(b_{US}^{11})} & \geq & \dfrac{K}{p_H^1}. \end{array}$$

Again, the mean value theorem implies that

$$v'(c_H^1) - v'(b_{US}^{11}) = v''(\zeta)\left(p_H^1 - \alpha_S^{11}p_U^1\right)m,$$

with $\zeta \in \left[c_H^1, b_{US}^{11}\right]$. Due to the concavity of $v(.)$, we have $v'(c_H^1) \geq v'(\zeta)$. Then, a sufficient to ensure the previous inequality is:

$$\begin{array}{rcl} -\dfrac{mv'(\zeta)\left[1-\lambda\alpha_{US}^{11}\right]}{v''(\zeta)\left(p_H^1-\alpha_S^{11}p_U^1\right)m} & \geq & \dfrac{K}{p_H^1} \\ & \Leftrightarrow & \\ p_H^1\left[1-\lambda\alpha_S^{11}v'(b_{US}^{11})\right] & \geq & \left(p_H^1-\alpha_S^{11}p_U^1\right). \\ & \Leftrightarrow & \\ \lambda p_H^1 & \leq & p_U^1 \end{array}$$

which is always satisfied. Q.E.D. ■

We first prove that the equilibrium is S^{11} for $\phi < \phi_{\min}^S$ and S^{01} for $\phi_{\min}^S < \phi < \phi_{\max}^S$ (Parts (i)(a) and (ii)(a) of the statement of the Result). We proceed in three steps: (1) Type U has larger utility with contract with effort (as determined by IC^{11}) than without (as determined by IC^{01}) when $\phi = 0$; (2) Both utilities continuously decrease with ϕ, but the utility with effort decreases faster; (3) Type U has a larger utility without effort (IC^{01}) than with (IC^{11}) when $\phi = \phi_{\max}^S$.

1) **Type U has larger utility with contract with effort (as determined by IC^{11}) than without (as determined by IC^{01}) when $\phi = 0$.**

Taken at $\phi = 0$, IC^{11} and IC^{00} imply that

$$\begin{array}{rl} & p_H^1 v\left(y + \alpha_S^{11}\left(1-p_U^1\right)m - m\right) + \left(1-p_H^1\right)v\left(y - \alpha_S^{11}p_U^1 m\right) \\ = & p_H^0 v\left(y + \alpha_S^{01}\left(1-p_U^0\right)m - m\right) + \left(1-p_H^0\right)v\left(y - \alpha_S^{01}p_U^0 m\right), \end{array}$$

which is equivalent to

$$\begin{array}{rl} & p_H^1 \left[v(d_{US}^{11}) - v(d_{US}^{01})\right] + \left(p_H^1 - p_H^0\right) v(d_{US}^{01}) \\ = & v(b_{US}^{01}) - v(b_{US}^{11}) + p_H^1 \left[v(b_{US}^{11}) - v(b_{US}^{01})\right] + v(b_{US}^{01}) \left(p_H^1 - p_H^0\right) \\ \Leftrightarrow & \\ & p_H^1 \left[v(d_{US}^{11}) - v(d_{US}^{01})\right] + \left(1 - p_H^1\right) \left[v(b_{US}^{11}) - v(b_{US}^{01})\right] \\ = & \left(p_H^1 - p_H^0\right) \left[v(b_{US}^{01}) - v(d_{US}^{01})\right]. \end{array} \tag{17}$$

We need to show that

$$p_U^1 v(d_{US}^{11}) + (1 - p_U^1) v(b_{US}^{11}) \geq p_U^0 v(d_{US}^{01}) + (1 - p_U^0) v(b_{US}^{01}).$$

This inequality is equivalent to

$$\begin{array}{rl} & p_U^1 \left[v(d_{US}^{11}) - v(d_{US}^{01})\right] + \lambda \left(p_H^1 - p_H^0\right) v(d_{US}^{01}) \\ \geq & v(b_{US}^{01}) - v(b_{US}^{11}) + p_U^1 \left[v(b_{US}^{11}) - v(b_{US}^{01})\right] + \lambda \left(p_H^1 - p_H^0\right) v(b_{US}^{01}) \\ \Leftrightarrow & \\ & \lambda p_H^1 \left[v(d_{US}^{11}) - v(d_{US}^{01})\right] + \left(1 - p_U^1\right) \left[v(b_{US}^{11}) - v(b_{US}^{01})\right] \\ \geq & \lambda \left(p_H^1 - p_H^0\right) \left[v(b_{US}^{01}) - v(d_{US}^{01})\right]. \end{array}$$

Multiplying both sides of (17) by λ, we obtain

$$\begin{array}{rl} & \lambda p_H^1 \left[v(d_{US}^{11}) - v(d_{US}^{01})\right] - \lambda \left(p_H^1 - p_H^0\right) \left[v(b_{US}^{01}) - v(d_{US}^{01})\right] \\ = & \lambda (1 - p_H^1) \left[v(b_{US}^{11}) - v(b_{US}^{01})\right]. \end{array}$$

Then, the previous inequality can be rewritten:

$$\begin{array}{rcl} \lambda (1 - p_H^1) \left[v(b_{US}^{11}) - v(b_{US}^{01})\right] + \left(1 - p_U^1\right) \left[v(b_{US}^{11}) - v(b_{US}^{01})\right] & \geq & 0 \\ & \Leftrightarrow & \\ (1 - \lambda)\left(1 - p_L\right) \left[v(b_{US}^{11}) - v(b_{US}^{01})\right] & \geq & 0, \end{array}$$

which is true. Q.E.D.

2) **Both utilities continuously decrease with ϕ, but the utility with effort decreases faster.**

By definition, $V_{US}^{11} = V_{US}^{01}$ when $\phi = \phi_{\min}^S$. We now show that

$$\frac{\partial V_{US}^{11}}{\partial \phi} < \frac{\partial V_{US}^{01}}{\partial \phi}, \tag{18}$$

so that $V_{US}^{11} > V_{US}^{01}$ for $\phi < \phi_{\min}^S$ and $V_{US}^{11} < V_{US}^{01}$ for $\phi > \phi_{\min}^S$.

As α_S^{11} is constant with respect to ϕ, the inequality (18) is equivalent to:

$$\begin{array}{rcl} -m\dfrac{\partial \alpha_S^{01}}{\partial \phi} p_U^0 \left(1-p_U^0\right)\left[v'(d_{US}^{01})-v'(b_{US}^{01})\right] & \leq & 1 \\ & \Longleftrightarrow & \\ \dfrac{mp_U^0\left(1-p_U^0\right)\left[v'(d_{US}^{01})-v'(b_{US}^{01})\right]}{m\left[p_H^0\left(1-p_U^0\right)v'(d_{US}^{01})-\left(1-p_H^0\right)p_U^0 v'(b_{US}^{01})\right]} & \leq & 1 \\ & \Longleftrightarrow & \\ \left[p_U^0-p_H^0\right]p_U^0 v'(b_{US}^{01}) & \leq & v'(d_{US}^{01})\left(1-p_U^0\right)\left[p_H^0-p_U^0\right], \end{array}$$

which is always true.

To prove that $\phi_{\min}^S$ exists and is such that $0<\phi_{\min}^S<\phi_{\max}^S$, we must show that $V_{US}^{11}>V_{US}^{01}$ for $\phi=0$ while $V_{US}^{11}<V_{US}^{01}$ for $\phi=\phi_{\max}^S$.

3) **Type U has a larger utility without effort (IC^{01}) than with (IC^{11}) when $\phi=\phi_{\max}^S$.**

Formally, we need to show that for $\phi=\phi_{\max}^S$:

$$p_U^1 v(d_{US}^{11})+(1-p_U^1)v(b_{US}^{11})-\phi \leq p_U^0 v(d_{US}^{01})+(1-p_U^0)v(b_{US}^{01}).$$

Since $\phi=\phi_{\max}^S$, IC^{01} can be rewritten

$$v(c_H^0)=p_H^0 v(d_{US}^{01})+(1-p_H^0)v(b_{US}^{01}).$$

Then, taken at $\phi=\phi_{\max}^S$, the previous inequality yields:

$$\begin{gathered} p_U^1 v(d_{US}^{11})+(1-p_U^1)v(b_{US}^{11})-v(c_H^1) \leq p_U^0 v(d_{US}^{01})+(1-p_U^0)v(b_{US}^{01})-v(c_H^0) \\ \Longleftrightarrow \\ p_U^1 v(d_{US}^{11})+(1-p_U^1)v(b_{US}^{11})-\left[p_H^1 v(d_{US}^{11})+(1-p_H^1)v(b_{US}^{11})\right] \\ \leq p_U^0 v(d_{US}^{01})+(1-p_U^0)v(b_{US}^{01})-\left[p_H^0 v(d_{US}^{01})+(1-p_H^0)v(b_{US}^{01})\right] \\ \Longleftrightarrow \\ \left(p_H^1-p_L\right)\left[v(b_{US}^{11})-v(d_{US}^{11})\right] \leq \left(p_H^0-p_L\right)\left[v(b_{US}^{01})-v(d_{US}^{01})\right]. \end{gathered}$$

As $p_H^0>p_H^1$, $\alpha_S^{01}\leq\alpha_S^{11}$ (see Lemma 3) so that the previous inequality holds. Q.E.D.

Part (i) (b) is proved in Lemma 1.

Part (ii) (b) We need to prove that $\alpha_S^{01}<1$ for $\phi=\phi_{\min}^S$. The incentive condition that determines α_S^{01} is:

$$v(y-p_H^1 m)-\phi=p_H^0 v(d_S^{01})+(1-p_H^0)v(b_S^{01}).$$

Substituting ϕ by $\phi^S_{\min}$ yields:

$$\begin{aligned} & v(y-p_H^1 m) - \left[p_U^1 v(d_S^{11}) + (1-p_U^1)v(b_S^{11}) - \left(p_H^0 v(d_S^{01}) + (1-p_H^0)v(b_S^{01})\right)\right] \\ = \; & p_H^0 v(d_S^{01}) + (1-p_H^0)v(b_S^{01}). \end{aligned}$$

Introducing IC^{11} gives:

$$\left(p_H^1 - p_L\right)\left[v(d_S^{11}) - v(b_S^{11})\right] = \left(p_H^0 - p_L\right)\left[v(d_S^{01}) - v(b_S^{01})\right].$$

Since $\alpha_S^{11} \leq 1$, we also have that $\alpha_S^{01} \leq 1$.

Part (ii) (c) is proved in Lemma 1.

Part (iii): α_S^{11} and α_S^{01} are respectively determined by IC^{11} (3) and by IC^{01} (5). Combining these two conditions yields:

$$\begin{aligned} & p_H^0 v\left(d_{US}^{01}\right) + \left(1-p_H^0\right) v\left(b_{US}^{01}\right) \\ = \; & p_H^1 v(d_{US}^{11}) + \left(1-p_H^1\right) v\left(b_{US}^{11}\right) - \phi. \end{aligned}$$

Using $\phi = \phi^S_{\min}$, we obtain:

$$\begin{aligned} & p_H^0 v\left(d_{US}^{01}\right) + \left(1-p_H^0\right) v\left(b_{US}^{01}\right) \\ & - \left[p_U^0 v\left(d_{US}^{01}\right) + \left(1-p_U^0\right) v\left(b_{US}^{01}\right)\right] \\ = \; & p_H^1 v\left(d_{US}^{11}\right) + \left(1-p_H^1\right) v\left(b_{US}^{11}\right) \\ & - \left(p_U^1 v\left(d_{US}^{11}\right) + \left(1-p_U^1\right) v\left(b_{US}^{11}\right)\right). \end{aligned} \tag{19}$$

Let us consider the following function:

$$\begin{aligned} \Gamma(\alpha,\delta) = \; & \left(p_H^0 - \delta\right) v\left(y + \alpha\left(1 - \left(\left(\lambda\left(p_H^0-\delta\right) + (1-\lambda)p_L\right)\right)\right) d - d\right) \\ & + \left(1 - \left(p_H^0 - \delta\right)\right) v\left(y - \alpha\left(\left(\lambda\left(p_H^0-\delta\right) + (1-\lambda)p_L\right)\right) d\right) \\ & - \left(\lambda\left(p_H^0-\delta\right) + (1-\lambda)p_L\right) v\left(y + \alpha\left(1 - \left(\left(\lambda\left(p_H^0-\delta\right) + (1-\lambda)p_L\right)\right)\right) d - d\right) \\ & - \left(\left(1 - \left(\left(\lambda\left(p_H^0-\delta\right) + (1-\lambda)p_L\right)\right)\right) v\left(y - \alpha\left(\left(\lambda\left(p_H^0-\delta\right) + (1-\lambda)p_L\right)\right) d\right)\right). \end{aligned}$$

The equation (19) can then be rewritten as

$$\Gamma\left(\alpha_S^{01}, 0\right) = \Gamma\left(\alpha_S^{11}, \Delta\right).$$

Now, let us apply the implicit function theorem. We obtain:

$$
\begin{aligned}
\frac{d\alpha}{d\delta} &= -\frac{\partial\Gamma/\partial\delta}{\partial\Gamma/\partial\alpha} \\
&= -\frac{v(b_{US}^{11}) - v(d_{US}^{11}) + \alpha\lambda d\left(p_H^1 v'(d_{US}^{11}) + \left(1-p_H^1\right) v'(b_{US}^{11})\right)}{d\left[p_H^1\left(1-p_U^1\right) v'\left(d_{US}^{11}\right) - \left(1-p_H^1\right) p_U^1 v'\left(b_{US}^{11}\right) - \left[p_U^1\left(1-p_U^1\right)\left(v'\left(d_{US}^{11}\right) - v'\left(b_{US}^{11}\right)\right)\right]\right]} \\
&\quad + \frac{\left[\lambda\left(v(b_{US}^{11}) - v(d_{US}^{11})\right)\right] + \alpha\lambda d\left(p_U^1 v'(d_{US}^{11}) - \left(1-p_U^1\right) v'(b_{US}^{11})\right)}{d\left[p_H^1\left(1-p_U^1\right) v'\left(d_{US}^{11}\right) - \left(1-p_H^1\right) p_U^1 v'\left(b_{US}^{11}\right) - \left[p_U^1\left(1-p_U^1\right)\left(v'\left(d_{US}^{11}\right) - v'\left(b_{US}^{11}\right)\right)\right]\right]} \\
&= -\frac{\left[v(b_{US}^{11}) - v(d_{US}^{11})\right](1-\lambda) + \alpha\lambda d\left(p_H^1 - p_U^1\right)\left[\left(v'(d_{US}^{11}) - v'(b_{US}^{11})\right)\right]}{d\left(p_H^1 - p_U^1\right)\left[\left(1-p_U^1\right) v'\left(d_{US}^{11}\right) + p_U^1 v'\left(b_{US}^{11}\right)\right]} < 0.
\end{aligned}
$$

Consequently, we have $\alpha_S^{11} < \alpha_S^{01}$ for $\phi = \phi_{\min}^S$.

Appendix A.3.: Proof of Result 2

In Case S^{11}, the derivatives with respect to ϕ and k are respectively:

$$
\begin{aligned}
\frac{\partial W_S^{11}}{\partial\phi} &= -[1 - k(1-\lambda)] < 0, \\
\frac{\partial W_S^{11}}{\partial k} &= \lambda V_{HS}^{11} + (1-\lambda)V_L - V_{US}^{11}.
\end{aligned}
$$

Using the definition of α_S^{11}, we have:

$$
\begin{aligned}
&\frac{\partial W_S^{11}}{\partial k} \\
&= \lambda\left[p_H^1 v(d_{US}^{11}) + (1-p_H^1)v(b_{US}^{11})\right] + (1-\lambda)v(y - p_L m) - \left[p_U^1 v(d_{US}^{11}) + (1-p_U^1)v(b_{US}^{11})\right] + (1-\lambda)\phi \\
&= v(d_{US}^{11})(\lambda p_H^1 - p_U^1) + v(b_{US}^{11})(\lambda(1-p_H^1) - (1-p_U^1)) + (1-\lambda)v(y - p_L m) + (1-\lambda)\phi \\
&= (1-\lambda)\left(v(y - p_L m) + \phi\right) - (1-\lambda)p_L v(d_{US}^{11}) - (1-\lambda)(1-p_L)v(b_{US}^{11}) \\
&= (1-\lambda)\left[v(y - p_L m) + \phi - \left(p_L v(d_{US}^{11}) + (1-p_L)v(b_{US}^{11})\right)\right] > 0.
\end{aligned}
$$

In Case S^{01}, the derivatives with respect to ϕ and k are respectively:

$$
\begin{aligned}
\frac{\partial W_S^{01}}{\partial\phi} &= -k\lambda + (1-k)p_U^0(1-p_U^0)m\frac{d\alpha_S^{01}}{d\phi}\left(v'(d_{US}^{01}) - v'(b_{US}^{01})\right) < 0, \\
\frac{\partial W_S^{01}}{\partial k} &= \lambda V_{HS}^{01} + (1-\lambda)V_L - V_{US}^{01} - \lambda\phi > 0.
\end{aligned}
$$

Using the definition of α_S^{01}, we have:

$$\begin{aligned}\frac{\partial W_S^{01}}{\partial k} &= \lambda\left[p_H^0 v(d_{US}^{01}) + (1-p_H^0)v(b_{US}^{01})\right] + (1-\lambda)v(y-p_L m) - \left[p_U^0 v(d_{US}^{01}) + (1-p_U^0)v(b_{US}^{01})\right] \\ &= v(d_U^0)(\lambda p_H^0 - p_U^0) + v(b_{US}^{01})(\lambda(1-p_H^0) - (1-p_U^0)) + (1-\lambda)v(y-p_L m) \\ &= (1-\lambda)v(y-p_L m) - (1-\lambda)p_L v(d_{US}^{01}) - (1-\lambda)(1-p_L)v(b_{US}^{01}) \\ &= (1-\lambda)\left[v(y-p_L m) - \left(p_L v(d_{US}^{01}) + (1-p_L)v(b_{US}^{01})\right)\right] > 0.\end{aligned}$$

Appendix A.4.: Proof of Result 3

We have two incentive compatibility constraints that mirror (3) and (5), respectively, namely the one where the contract offered to U calls for a prevention effort (with coverage rate α_S^{10}), denoted by IC^{10},

$$V_{HS}^{00} \geq p_H^1 v\left(y + \alpha_S^{10}\left(1-p_U^1\right)m - m\right) + \left(1-p_H^1\right)v\left(y - \alpha_S^{10} p_U^1 m\right) - \phi, \tag{20}$$

and the one where the contract devised for type U does not call for effort (with coverage rate α_S^{00}), denoted by IC^{00},

$$V_{HS}^{00} \geq p_H^0 v\left(d_{US}^{00}\right) + \left(1-p_H^0\right)v\left(b_{US}^{00}\right). \tag{21}$$

Constraint (21) is identical to constraint (5) when $\phi = \phi_{\max}^S$, and from Lemma 1 constraint (21) binds with equality with $\alpha_S^{00} < 1$, where α_S^{00} takes the same value as α_S^{01} when $\phi = \phi_{\max}^S$. The coverage rate α_S^{00} is constant with $\phi \geq \phi_{\max}^S$ since ϕ does not appear in constraint (21). Hence, the utility that type U obtains with this contract, which we denote by V_{US}^{00} (see equation (11)), is constant with ϕ as long as $\phi \geq \phi_{\max}^S$.

Constraint (20) is identical to constraint (3) when $\phi = \phi_{\max}^S$, and from Lemma 1 constraint (20) binds with equality with $\alpha_S^{10} < 1$, where α_S^{10} takes the same value as α_S^{11} when $\phi = \phi_{\max}^S$. We denote by V_{US}^{10} the utility attained by type U with this contract,

$$V_{US}^{10} = p_U^1 v\left(y + \alpha_S^{10}\left(1-p_U^1\right)m - m\right) + \left(1-p_U^{10}\right)v\left(y - \alpha_S^{10} p_U^1 m\right) - \phi. \tag{22}$$

We know from Result 1 (2) that $V_{US}^{00} > V_{US}^{10}$ when $\phi = \phi_{\max}^S$. We now prove that this inequality remains true for any $\phi > \phi_{\max}^S$, because V_{US}^{10} decreases with ϕ.
Using the implicit function theorem on (20) holding with equality, we obtain that α_S^{10} increases with ϕ. As ϕ increases, the first term in the RHS of (20) then increases with ϕ, while the second term decreases. Comparing the RHS of (20) with (22), we see that they only differ in the weight put on the two first terms. As $p_U^1 < p_H^1$, it is easy to see that V_{US}^{10} decreases with ϕ.
Note that, when ϕ becomes large enough, we may obtain that constraint (20) holds with a strict inequality even with $\alpha_S^{10} = 1$. In that case, increasing further ϕ has no impact on α_S^{10} and thus also decreases V_{US}^{10}.

Appendix A.5.: Proof of Result 4

We have that

$$\frac{\partial W_S^{00}}{\partial k} = \lambda V_{HS}^{00} + (1-\lambda)V_L - V_{US}^{00}.$$

Using the definition of α_S^{00}, we have

$$\frac{\partial W_S^{00}}{\partial k} = \lambda\left[p_H^0 v(d_{US}^{00}) + (1-p_H^0)v(b_{US}^{00})\right] + (1-\lambda)v(y-p_L m) - \left[p_U^0 v(d_{US}^{00}) + (1-p_U^0)v(b_{US}^{00})\right],$$

so that the rest of the proof is identical to the proof of Result 2 for S^{01}, with an appropriate change of indices.

Appendix A.6.: Proof of Proposition 3

(ii) to (iv): Observe first from (12) that α_P^0 and α_P^1 do not depend on the effort cost ϕ. We then have that V_{UP}^0 is independent of ϕ, while V_{UP}^1 decreases linearly with ϕ. When $\phi = 0$, it is obvious that $V_{UP}^1 > V_{UP}^0$. Finally, we have

$$\lim_{\phi\to\infty} V_{UP}^1 = -\infty < \lim_{\phi\to\infty} V_{UP}^0.$$

(v) It is well known that CARA preferences generate a downward sloping insurance demand function: see for instance Schlesinger (2000, p.137). We now compare the coverage rates with and without prevention.
Let us rewrite the first order condition for an effort j. We have:

$$\Upsilon(\alpha,\Delta) \equiv p_U^j\left(1-p_P^j\right)v'(d_P^j) - \left(1-p_U^j\right)p_P^j v'(b_P^j) = 0.$$

The implicit function theorem gives:

$$\frac{d\alpha}{d\Delta} = -\frac{\partial\Upsilon(\alpha,\Delta)/\partial\Delta}{\partial\Upsilon(\alpha,\Delta)/\partial\alpha}.$$

Note that $\alpha_P^1 \geq \alpha_P^0$ if and only if $d\alpha/d\Delta \geq 0$. The sign of the denominator is negative due to the second-order condition. Thus,

$$\frac{d\alpha}{d\Delta} \geq 0 \Leftrightarrow \frac{\partial \Upsilon(\alpha, \Delta)}{\partial \Delta} \geq 0$$

$$\Leftrightarrow$$

$$\begin{aligned}
&\left(-\lambda\left(1-p_P^1\right)+\left(\frac{(1-k)\lambda+k}{1-k(1-\lambda)}\right)p_U^1\right)v'(d_P^1) \\
&-\left(p_P^1\lambda-\left(1-p_U^1\right)\left(\frac{(1-k)\lambda+k}{1-k(1-\lambda)}\right)\right)v'(b_P^1) \\
&+\left(1-p_P^1\right)p_U^1 v''(d_P^1)\alpha m\left(\frac{(1-k)\lambda+k}{1-k(1-\lambda)}\right) \\
&-\left(1-p_U^1\right)p_P^1 v''(b_P^1)\alpha m\left(\frac{(1-k)\lambda+k}{1-k(1-\lambda)}\right) \geq 0
\end{aligned}$$

$$\Leftrightarrow$$

$$\begin{aligned}
&\left(\frac{p_U^1}{1-k(1-\lambda)}-\lambda\left(1-p_P^1\right)\right)v'(d_P^1)-\left(p_P^1\lambda-\left(\frac{1-p_U^1}{1-k(1-\lambda)}\right)\right)v'(b_P^1) \\
&\geq \frac{\alpha m}{1-k(1-\lambda)}\left[p_U^1\left(1-p_P^1\right)v''(d_P^1)-\left(1-p_U^1\right)p_P^1 v''(b_P^1)\right].
\end{aligned}$$

First, let us work on the RHS. We have:

$$RHS \equiv \frac{\alpha m\left(1-p_U^1\right)p_P^1}{1-k(1-\lambda)}\left[\frac{p_U^1\left(1-p_P^1\right)}{\left(1-p_U^1\right)p_P^1}v''(d_P^1)-v''(b_P^1)\right].$$

Using the first order condition, we have:

$$RHS = \frac{\alpha m\left(1-p_U^1\right)p_P^1}{1-k(1-\lambda)}v'(d_P^1)\left[\frac{v''(d_P^1)}{v'(d_P^1)}-\frac{v''(b_P^1)}{v'(b_P^1)}\right].$$

Assumption 1 ensures that $RHS = 0$. Now, let us focus on the LHS. Since $v(.)$ is concave, we have $v'(d_P^1) \geq v'(b_P^1)$. Thus, a sufficient condition to ensure that $LHS > 0$ is:

$$\begin{aligned}
\frac{p_U^1}{1-k(1-\lambda)}-\lambda\left(1-p_P^1\right) &> p_P^1\lambda-\left(\frac{1-p_U^1}{1-k(1-\lambda)}\right) \\
&\Leftrightarrow \\
p_U^1-\lambda\left(1-p_P^1\right)(1-k(1-\lambda)) &> p_P^1\lambda\left(1-k(1-\lambda)-\left(1-p_U^1\right)\right) \\
&\Leftrightarrow \\
1 &> \lambda(1-k(1-\lambda)),
\end{aligned}$$

which is true. Q.E.D.

Appendix A.7.: Proof of Proposition 4

(a) ϕ plays no role in P^0, and affects neither individual utilities nor aggregate welfare. We have

$$\frac{\partial V_{UP}^1}{\partial \phi} = \frac{\partial V_{HP}^1}{\partial \phi} = -1$$

so that

$$\frac{\partial W_P^1}{\partial \phi} = -\left[1 - k\left(1-\lambda\right)\right] < 0.$$

(b)

$$\begin{aligned} \frac{\partial W_P^i}{\partial k} &= \lambda V_{HP}^i + (1-\lambda) U_L - V_{UP}^i \\ & \quad + (1-k)\frac{\partial V_{UP}^i}{\partial k} + k\lambda \frac{\partial V_{HP}^i}{\partial k}, \end{aligned}$$

where the first line is the "composition effect" whose sign is positive,

$$\begin{aligned} & \lambda V_{HP}^i + (1-\lambda) U_L - V_{UP}^i \\ = \;& \lambda\left[p_H^i v(d_P^i) + (1-p_H^i) v(b_P^i)\right] + (1-\lambda) v(y - p_L m) - \left[p_U^i v(d_P^i) + (1-p_U^i) v(b_P^i)\right] + (1-\lambda)\,\phi^i \\ = \;& v(d_P^i)(\lambda p_H^i - p_U^i) + v(b_P^i)(\lambda(1-p_H^i) - (1-p_U^i)) + (1-\lambda) v(y - p_L m) + (1-\lambda)\,\phi^i \\ = \;& (1-\lambda)\left[v(y - p_L m + \phi^i) - \left(p_L v(d_P^i) + (1-p_L) v(b_P^i)\right)\right] > 0, \end{aligned}$$

while the second line is negative, since

$$\begin{aligned} \frac{\partial V_{UP}^i}{\partial k} &= -\alpha_P^i \frac{\partial p_P^i}{\partial k} m \left(p_U^i v'(d_P^i) + \left(1 - p_U^i\right) v'(b_P^i)\right) \\ &= -\alpha_P^i \frac{\lambda\left(p_H^i - p_L\right)}{\left(1 - k\left(1-\lambda\right)\right)^2} m \left(p_U^i v'(d_P^i) + \left(1 - p_U^i\right) v'(b_P^i)\right) < 0, \end{aligned}$$

and

$$\begin{aligned} \frac{\partial V_{HP}^i}{\partial k} &= -\alpha_P^i \frac{\lambda\left(p_H^i - p_L\right)}{\left(1 - k\left(1-\lambda\right)\right)^2} m \left(p_H^i v'(d_P^i) + \left(1 - p_H^i\right) v'(b_P^i)\right) \\ + & \quad m \frac{\partial \alpha_P^i}{\partial k}\left[p_U^i (1 - p_P^i) v'(d_P^i) - \left(1 - p_U^i\right) p_P^i v'(b_P^i)\right] < 0, \end{aligned}$$

resulting in an ambiguity as to the overall sign of $\partial W_P^i / \partial k$.

Appendix A.8.: Proof of Proposition 6(c)

The threshold $\tilde{k}$ is determined by the equality $V^1_{UP}(\tilde{k},\phi) = V^{01}_{US}(\phi)$, which corresponds to

$$\begin{aligned} & p^1_U v(d^1_P) + (1-p^1_U)v(b^1_P) - \phi \\ = \; & p^0_U v\left(y + \alpha^{01}_S \left(1-p^0_U\right) m - m\right) + \left(1-p^0_U\right) v\left(y - \alpha^{01}_S p^0_U m\right), \end{aligned}$$

with α^1_P and α^{01}_S respectively given by:

$$p^1_U(1-p^1_P)v'(d^1_P) - (1-p^1_U)p^1_P v'(b^1_P) = 0$$

and

$$v(y - p^1_H m) - \phi = p^0_U v\left(y + \alpha^{01}_S \left(1-p^0_U\right) m - m\right) + \left(1-p^0_U\right) v\left(y - \alpha^{01}_S p^0_U m\right).$$

Static comparatives yield:

$$\begin{aligned} \frac{d\alpha^{01}_S}{d\phi} &= \frac{-1}{m\left[p^0_H\left(1-p^0_U\right)v'(d^{01}_{US}) - \left(1-p^0_H\right)p^0_U v'(b^{01}_{US})\right]} < 0, \\ \frac{d\alpha^1_P}{dk} &= \frac{(1-\lambda)\,\lambda\left(p^1_H - p_L\right)}{1-k\,(1-\lambda)} \frac{p^1_U v'(d^1_P) + (1-p^1_U)v'(b^1_P)}{m\left[p^1_U(1-p^1_P)^2 v''(d^1_P) + (1-p^1_U)\left(p^1_P\right)^2 v''(b^1_P)\right]}. \end{aligned}$$

Consider the following implicit function:

$$\begin{aligned} \Gamma\left(\tilde{k},\phi\right) &= p^1_U v(d^1_P) + (1-p^1_U)v(b^1_P) - \phi \\ &\quad - \left[p^0_U v\left(y + \alpha^{01}_S\left(1-p^0_U\right)m - m\right) + \left(1-p^0_U\right)v\left(y - \alpha^{01}_S p^0_U m\right)\right]. \end{aligned}$$

The implicit function theorem yields:

$$\frac{d\tilde{k}}{d\phi} = -\frac{\partial\Gamma/\partial\phi}{\partial\Gamma/\partial\tilde{k}}.$$

The numerator gives:

$$\frac{\partial\Gamma}{\partial\phi} = -1 + \frac{p^0_U\left(1-p^0_U\right)\left(v'(d^{01}_{US}) - v'(b^{01}_{US})\right)}{p^0_H\left(1-p^0_U\right)v'(d^{01}_{US}) - \left(1-p^0_H\right)p^0_U v'(b^{01}_{US})} < 0.$$

Regarding the denominator, we have:

$$\frac{\partial\Gamma}{\partial\tilde{k}} = \frac{-\alpha^1_P\lambda\left(p^1_H - p_L\right)}{1-k\,(1-\lambda)}\left[p^1_U v'(d^1_P) + (1-p^1_U)v'(b^1_P)\right] < 0.$$

We then obtain that $d\tilde{k}/d\phi < 0$.

Appendix A.9.: Result 5

Result 5 *There is an upward discontinuity in H's utility from P^0 to P^1 when U is indifferent between the two.*

Proof. We have to prove that

$$V_{UP}^0 = V_{UP}^1 \Rightarrow V_{HP}^0 < V_{HP}^1.$$

We have:

$$\begin{aligned} V_{UP}^0 &= V_{UP}^1 \\ &\Leftrightarrow p_U^1 v(d_P^1) + (1 - p_U^1) v(b_P^1) - \phi = p_U^0 v(d_P^0) + (1 - p_U^0) v(b_P^0) \\ &\Leftrightarrow \phi = p_U^1 v(d_P^1) - p_U^0 v(d_P^0) + (1 - p_U^1) v(b_P^1) - (1 - p_U^0) v(b_P^0). \end{aligned}$$

Proving that

$$V_{HP}^0 < V_{HP}^1$$

is then equivalent to proving that

$$\begin{aligned} p_H^0 v(d_P^0) + (1 - p_H^0) v(b_P^0) \quad < \quad & p_H^1 v(d_P^1) + (1 - p_H^1) v(b_P^1) \\ & - p_U^1 v(d_P^1) - (1 - p_U^1) v(b_P^1) \\ & + p_U^0 v(d_P^0) + (1 - p_U^0) v(b_P^0). \end{aligned}$$

Regrouping terms, this is equivalent to

$$\left(p_H^0 - p_U^0\right) \left[v(b_P^0) - v(d_P^0)\right] > \left(p_H^1 - p_U^1\right) \left[v(b_P^1) - v(d_P^1)\right],$$

which holds if

$$v(b_P^0) - v(d_P^0) > v(b_P^1) - v(d_P^1), \tag{23}$$

since

$$p_H^0 - p_U^0 = (1 - \lambda)(p_H^0 - p_L) > p_H^1 - p_U^1 = (1 - \lambda)(p_H^1 - p_L).$$

Note that, thanks to the concavity of $v(.)$, inequality (23) holds if $d_P^0 > d_P^1$ together with $b_P^0 - d_P^0 > b_P^1 - d_P^1$. We have that

$$b_P^0 - d_P^0 = d\left(1 - \alpha_P^0 m\right) > b_P^1 - d_P^1 = m\left(1 - \alpha_P^1 m\right),$$

since $\alpha_P^0 < \alpha_P^1$ (see Lemma 2 (b)), and that $d_P^0 > d_P^1$ since $p_P^1 < p_P^0$. ■

Appendix A.10.: Result 6

We first have to prove the following lemma.

Lemma 4 *We have that $\alpha_P^0 > \alpha_S^{01}$ when $V_{UP}^0 = V_{US}^{01}$ when Assumption 3 holds.*

Proof. *Note that α_S^{01} is not affected by k, while α_P^0 decreases with k, starting from $\alpha_P^0 = 1$ when $k = 0$ (since in that case $p_P^0 = p_U^0$). Assumption 3 implies that $p_H^1 > p_U^0$, which in turn implies that $\alpha_S^{01} < 1$ (since IC^{01} is violated when $\alpha_S^{01} = 1$). We know that V_{UP}^0 is decreasing in k while V_{US}^{01} is not affected, so that there is a unique value of k such that $V_{UP}^0 = V_{US}^{01}$. It is easy to see that $\alpha_P^0 = \alpha_S^{01}$ implies that $V_{UP}^0 < V_{US}^{01}$ (since $p_P^0 > p_U^0$ implies that $d_P^0 < d_S^{01}$ and that $b_P^0 < b_S^{01}$). We then have that $V_{UP}^0 = V_{US}^{01}$ implies that $\alpha_P^0 > \alpha_S^{01}$.*

■

Result 6 *Under Assumption 3, there is a downward discontinuity in H's utility from P^0 to S^{01} when U is indifferent between the two.*

Proof. To prove Result 6, we have to prove that

$$V_{UP}^0 = V_{US}^{01} \Rightarrow V_{HP}^0 > V_{HS}^{01}.$$

We have

$$\begin{aligned} V_{UP}^0 &= V_{US}^{01} \\ &\Leftrightarrow p_U^0 v(d_P^0) + (1-p_U^0)v(b_P^0) = p_U^0 v(d_{US}^{01}) + (1-p_U^0)v(b_{US}^{01}) \\ &\Leftrightarrow p_U^0 \left[v(d_P^0) - v(d_{US}^{01})\right] + (1-p_U^0)\left[v(b_P^0) - v(b_{US}^{01})\right] = 0. \end{aligned} \tag{24}$$

Proving that

$$V_{HP}^0 > V_{HS}^{01}$$

is then equivalent to proving that

$$\begin{aligned} & p_H^0 v(d_P^0) + (1-p_H^0)v(b_P^0) > p_H^0 v(d_{US}^{01}) + (1-p_H^0)v(b_{US}^{01}) \\ \Leftrightarrow\ & p_H^0 \left[v(d_P^0) - v(d_{US}^{01})\right] + (1-p_H^0)\left[v(b_P^0) - v(b_{US}^{01})\right] > 0. \end{aligned} \tag{25}$$

We know that $p_P^0 > p_U^0 > p_U^1$ and we know from Lemma 4 that $\alpha_P^0 > \alpha_S^{01}$ when $V_{UP}^0 = V_{US}^{01}$, which together imply that $b_P^0 < b_{US}^{01}$. From (24), we obtain that $d_P^0 > d_{US}^{01}$, so that (25) holds since $p_H^0 > p_U^0$. ■

Appendix A.11.: Result 7

We first prove the following lemma.

Lemma 5 *We have that $\alpha_P^1 > \alpha_S^{01}$ when $V_{UP}^1 = V_{US}^{01}$ when Assumption 3 holds.*

Proof. *By Assumption 3, the P^1 contract is more expensive than $S^{01}$for type U, and moreover entails that type U pays the effort cost in P^1 but not in S^{01}. For U to be indifferent, it must then be the case that P^1 offers more coverage than S^{01} (recall that the coverage level in P^1 is the most-preferred one of U, while U is rationed in S^{01} because of IC^{01}).* ∎

Result 7 *Under Assumption 3, there is an upward discontinuity in H's utility from S^{01} to P^1 when U is indifferent between the two.*

Proof. We now have to prove that

$$V_{UP}^1 = V_{US}^{01} \Rightarrow V_{HP}^1 > V_{HS}^{01}.$$

We have

$$\begin{aligned} V_{UP}^1 &= V_{US}^{01} \\ &\Leftrightarrow \;\; p_U^1 v(d_P^1) + (1-p_U^1)v(b_P^1) - \phi = p_U^0 v(d_{US}^{01}) + (1-p_U^0)v(b_{US}^{01}) \qquad (26) \\ &\Leftrightarrow \;\; \phi = p_U^1 v(d_P^1) - p_U^0 v(d_{US}^{01}) + (1-p_U^1)v(b_P^1) - (1-p_U^0)v(b_{US}^{01}). \end{aligned}$$

Proving that

$$V_{HP}^1 > V_{HS}^{01}$$

is then equivalent to proving that

$$\begin{aligned} p_H^0 v(d_{US}^{01}) + (1-p_H^0)v(b_{US}^{01}) \;\; < \;\; & p_H^1 v(d_P^1) + (1-p_H^1)v(b_P^1) \\ & -p_U^1 v(d_P^1) - (1-p_U^1)v(b_P^1) \\ & +p_U^0 v(d_{US}^{01}) + (1-p_U^0)v(b_{US}^{01}). \end{aligned}$$

Regrouping terms, this is equivalent to

$$\begin{aligned} & \left(p_H^0 - p_U^0\right)\left[v(b_{US}^{01}) - v(d_{US}^{01})\right] > \left(p_H^1 - p_U^1\right)\left[v(b_P^1) - v(d_P^1)\right] \\ \Leftrightarrow \;\; & \left(p_H^0 - p_L\right)\left[v(b_{US}^{01}) - v(d_{US}^{01})\right] > \left(p_H^1 - p_L\right)\left[v(b_P^1) - v(d_P^1)\right] \qquad (27) \end{aligned}$$

since

$$p_H^0 - p_U^0 = (1-\lambda)(p_H^0 - p_L) > p_H^1 - p_U^1 = (1-\lambda)(p_H^1 - p_L).$$

Note that $\alpha_P^1 > \alpha_S^{01}$ together with $p_P^1 > p_U^0$ imply that $b_P^1 < b_{US}^{01}$. If $d_P^1 > d_{US}^{01}$, then (27) is satisfied.
We now show that (27) is also satisfied in the case where $d_P^1 < d_{US}^{01}$. Note that, if (26) is satisfied for some $\phi > 0$, then we have that

$$\begin{aligned}
& p_U^1 v(d_P^1) + (1-p_U^1)v(b_P^1) > p_U^0 v(d_{US}^{01}) + (1-p_U^0)v(b_{US}^{01}) \\
\Leftrightarrow\ & \left(\lambda p_H^1 + (1-\lambda)p_L\right) v(d_P^1) + \left(1 - \left(\lambda p_H^1 + (1-\lambda)p_L\right)\right) v(b_P^1) > \\
& \left(\lambda p_H^0 + (1-\lambda)p_L\right) v(d_{US}^{01}) + \left(1 - \left(\lambda p_H^0 + (1-\lambda)p_L\right)\right) v(b_{US}^{01}) \\
\Leftrightarrow\ & p_L v(d_P^1) + \lambda\left(p_H^1 - p_L\right) v(d_P^1) + (1-p_L)v(b_P^1) - \lambda\left(p_H^1 - p_L\right) v(b_P^1) > \\
& p_L v(d_{US}^{01}) + \lambda\left(p_H^0 - p_L\right) v(d_{US}^{01}) + (1-p_L)v(b_{US}^{01}) - \lambda\left(p_H^0 - p_L\right) v(b_{US}^{01}) \\
\Leftrightarrow\ & \lambda\left[\left(p_H^1 - p_L\right)\left(v(b_P^1) - v(d_P^1)\right) - \left(p_H^0 - p_L\right)\left(v(b_{US}^{01}) - v(d_{US}^{01})\right)\right] < \qquad (28) \\
& p_L v(d_P^1) + (1-p_L)v(b_P^1) - \left[p_L v(d_{US}^{01}) + (1-p_L)v(b_{US}^{01})\right].
\end{aligned}$$

If $d_P^1 < d_{US}^{01}$, then the RHS of (28) is negative, and so is its LHS, so that (27) is also satisfied. Q.E.D. ■

Appendix A.12.: Result 8

We first prove the following lemma:

Lemma 6 *We have that $\alpha_P^0 > \alpha_S^{00}$ when $V_{UP}^0 = V_{US}^{00}$.*

Proof. *Observe first that $\alpha_S^{00} < 1$ (this is already in Result 3, and is easy to establish since the effort level is the same –nil– for H and U in that case, so that the only way to prevent H from mimicking U is by under-providing insurance to U) while $\alpha_P^0 = 1$ when $k = 0$ (since in that case $p_P^0 = p_U^0$). We know that α_P^0 is decreasing in k while α_S^{00} is not affected by k, and that V_{UP}^0 is decreasing in k while V_{US}^{00} is not affected, so that there is a unique value of k such that $V_{UP}^0 = V_{US}^{00}$. It is easy to see that $\alpha_P^0 = \alpha_S^{00}$ implies that $V_{UP}^0 < V_{US}^{00}$ (since $p_P^0 < p_U^0$ implies that $d_P^0 < d_S^{00}$ and that $b_P^0 < b_S^{00}$). We then have that $V_U^P = V_{US}^{00}$ implies that $\alpha_P^0 > \alpha_S^{00}$.* ■

We now prove the following:

Result 8 *There is a downward discontinuity in H's utility from P^0 to S^{00} when U is indifferent between the two.*

Proof. We have to prove that

$$V_{UP}^0 = V_{US}^{00} \Rightarrow V_{HP}^0 > V_{HS}^{00}.$$

We have

$$\begin{aligned} V_{UP}^0 &= V_{US}^{00} \\ &\Leftrightarrow p_U^0 v(d_P^0) + (1-p_U^0)v(b_P^0) = p_U^0 v(d_S^{00}) + (1-p_U^0)v(b_S^{00}) \\ &\Leftrightarrow p_U^0 \left[v(d_P^0) - v(d_S^{00})\right] + (1-p_U^0)\left[v(b_P^0) - v(b_S^{00})\right] = 0. \end{aligned} \tag{29}$$

We have that

$$\begin{aligned} & V_{HP}^0 > V_{HS}^{00} \\ \Leftrightarrow \quad & p_H^0 v(d_P^0) + (1-p_H^0)v(b_P^0) > v(y-p_H^0)m = p_H^0 v(d_S^{00}) + (1-p_H^0)v(b_S^{00}) \end{aligned} \tag{30}$$

where the equality comes from IC^{00} (equation 21). The inequality (30) can be reformulated as

$$p_H^0 \left[v(d_P^0) - v(d_S^{00})\right] + (1-p_H^0)\left[v(b_P^0) - v(b_S^{00})\right] > 0.$$

We know from Lemma 6 that $\alpha_P^0 > a_S^{00}$ when $V_{UP}^0 = V_{US}^{00}$ which, together with $p_P^0 > p_U^0$, implies that $b_P^0 - b_S^{00} < 0$. We then obtain from (29) that $d_P^0 - d_S^{00} > 0$, and thus the inequality (30) is satisfied since $p_H^0 > p_U^0$. ■

Appendix A.13.: Result 9

We first prove the following lemma:

Lemma 7 *We have that $\alpha_P^1 > \alpha_S^{11}$ when $V_{UP}^1 = V_{US}^{11}$.*
Proof. *Same as Proof of Lemma 6, changing all superscripts 0 by 1.* ■

We are now in a position to prove:

Result 9 *There is a downward discontinuity in H's utility from P^1 to S^{11} when U is indifferent between the two.*

Proof. We have to prove that

$$V_{UP}^1 = V_{US}^{11} \Rightarrow V_{HP}^1 > V_{HS}^{11}.$$

We have

$$\begin{aligned} & V_{UP}^1 = V_{US}^{11} \\ \Leftrightarrow \quad & p_U^1 v(d_P^1) + (1-p_U^1)v(b_P^1) - \phi = p_U^1 v(d_{US}^{11}) + (1-p_U^1)v(b_{US}^{11}) - \phi \\ \Leftrightarrow \quad & p_U^1 \left[v(d_P^1) - v(d_{US}^{11})\right] + (1-p_U^1)\left[v(b_P^1) - v(b_{US}^{11})\right] = 0. \end{aligned} \tag{31}$$

We have that

$$
\begin{aligned}
& V_{HP}^{1} > V_{HS}^{11} \\
\Leftrightarrow\ & p_H^1 v(d_P^1) + (1 - p_H^1) v(b_P^1) - \phi \\
>\ & v(y - p_H^1) m - \phi = p_H^1 v(d_{US}^{11}) + (1 - p_H^1) v(b_{US}^{11}) - \phi,
\end{aligned} \tag{32}
$$

where the equality comes from IC^{11} (equation 3). The inequality (32) can be reformulated as

$$
p_H^1 \left[v(d_P^1) - v(d_{US}^{11}) \right] + (1 - p_H^1) \left[v(b_P^1) - v(b_{US}^{11}) \right] > 0.
$$

We know from Lemma 7 that $\alpha_P^1 > a_S^{11}$ when $V_{UP}^1 = V_{US}^{11}$ which, together with $p_P^1 > p_U^1$, implies that $b_P^1 - b_{US}^{11} < 0$. We then obtain from (31) that $d_P^1 - d_{US}^{11} > 0$, and thus the inequality (32) is satisfied since $p_H^1 > p_U^1$. ■

Appendix B: Numerical example

This example is based on the following parameter values: $p_L = 0.1$, $p_H^0 = 0.6$, $\lambda = 0.3$ (so that $p_U^0 = 0.25$), $\Delta = 0.25$ (so that $p_H^1 = 0.35$ and $p_U^1 = 0.175$), $y = 5$, $m = 3$, and $v(x) = \sqrt{x}$. With these parameters, we obtain that

$$
\begin{aligned}
k^* &= 0.077 < k_{00} = 0.331 < k_{11} = 0.376, \\
\phi_{\min}^S &= 0.044 < \tilde{\phi}_P(0) = 0.054 < \phi^* = 0.057 < \phi_{\max}^S = 0.199.
\end{aligned}
$$

Figure 1 : Separation between P^0, P^1, S^{00}, S^{01} and S^{11} contracts in (k, ϕ) space

Out[ ]=

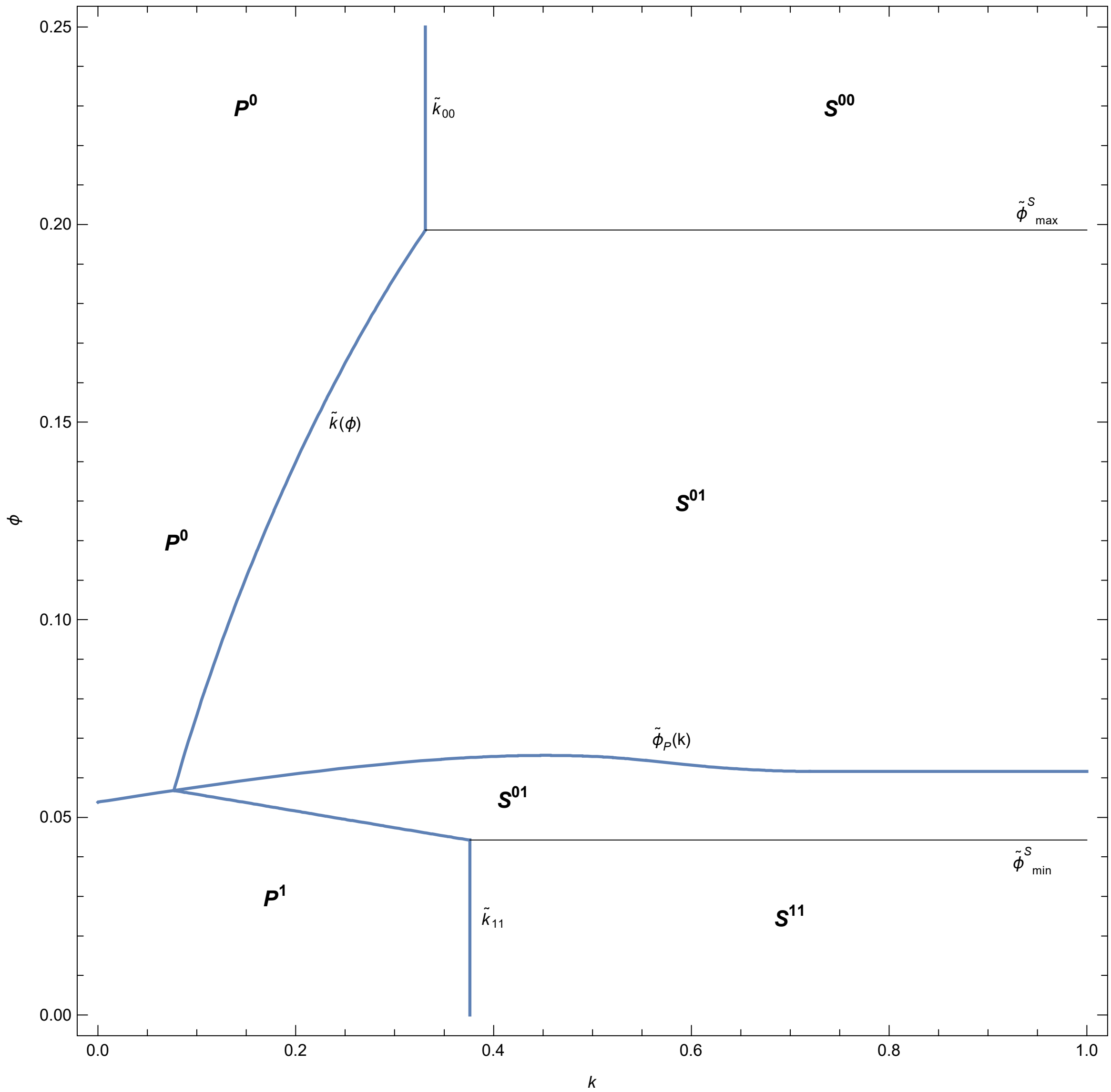

Figure 2 : Type U ' s utility as a function of φ, for $k^* < k < k_{00}$

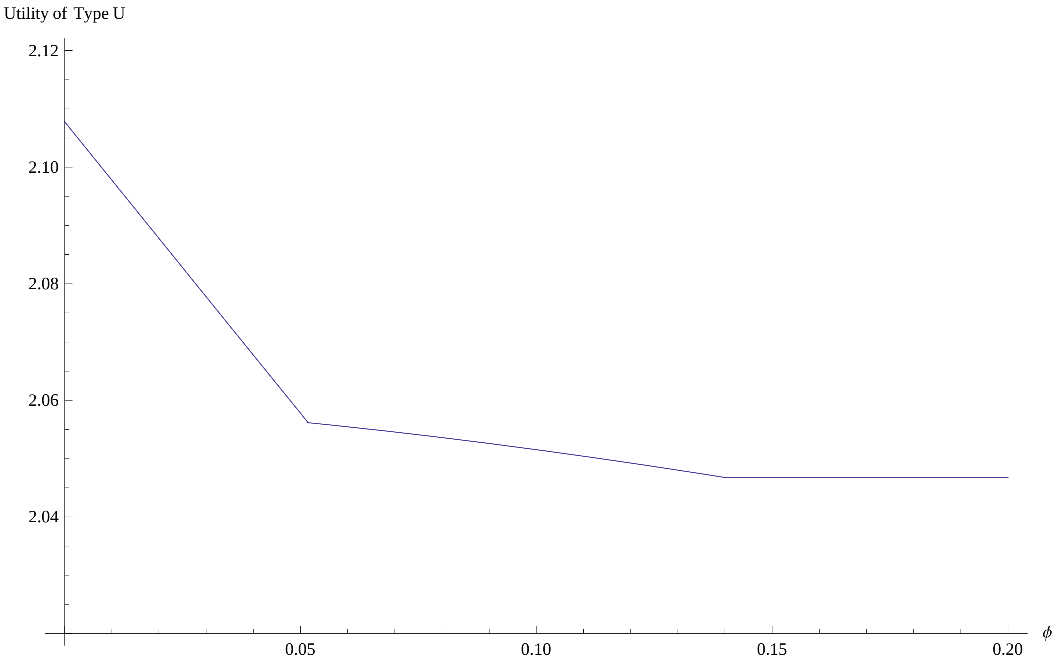

Figure 3 : Type H ' s utility as a function of φ, for $k^* < k < k_{00}$

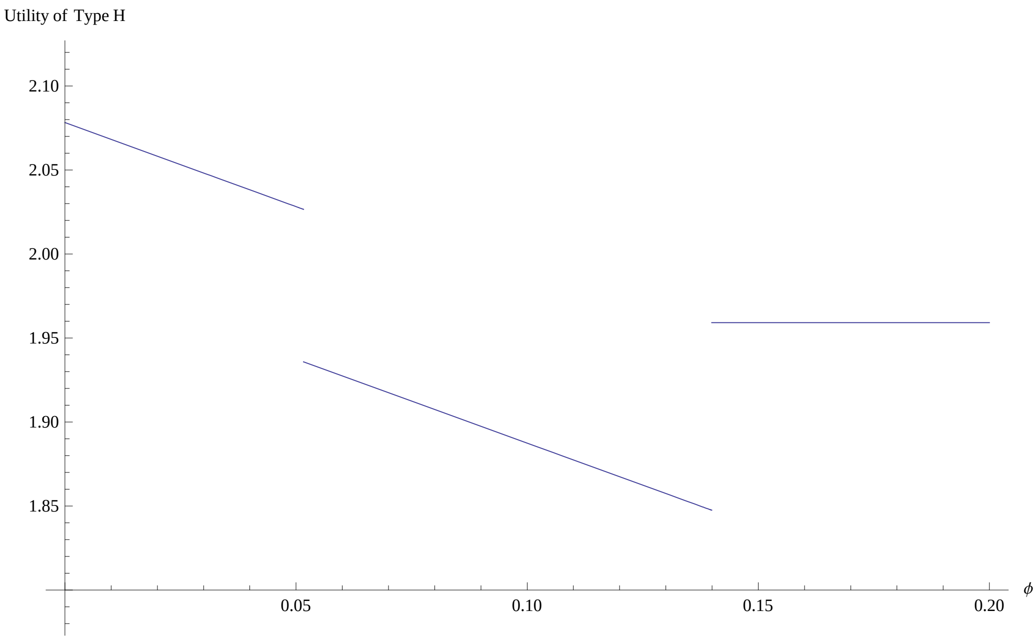

Figure 4 : Aggregate welfare as a function of φ, for $k^* < k < k_{00}$

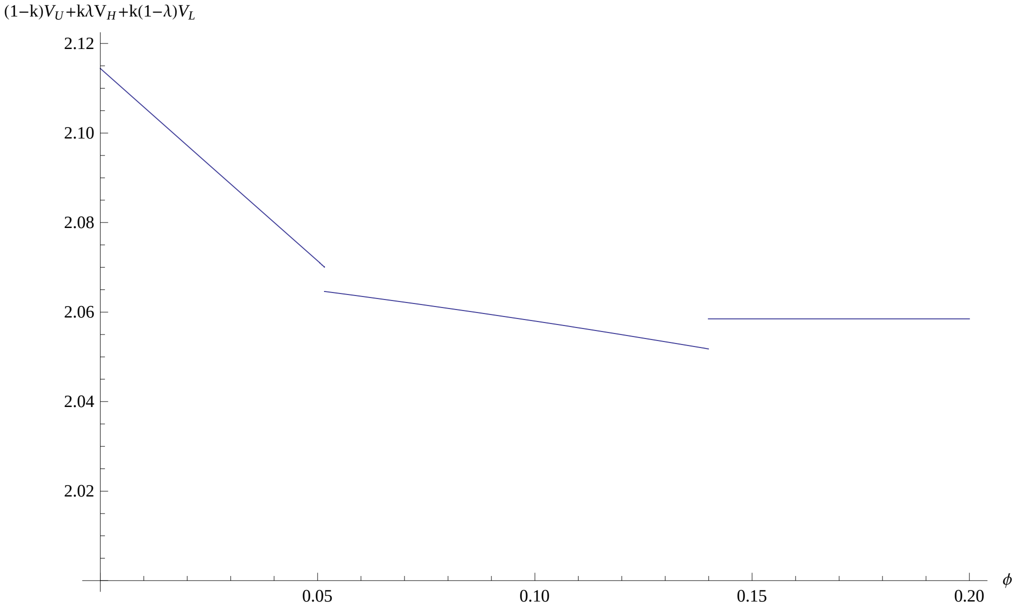

Figure 5 : Type U ' s utility as a function of k, for $\tilde{\phi}_P(0) < \phi < \phi^*$

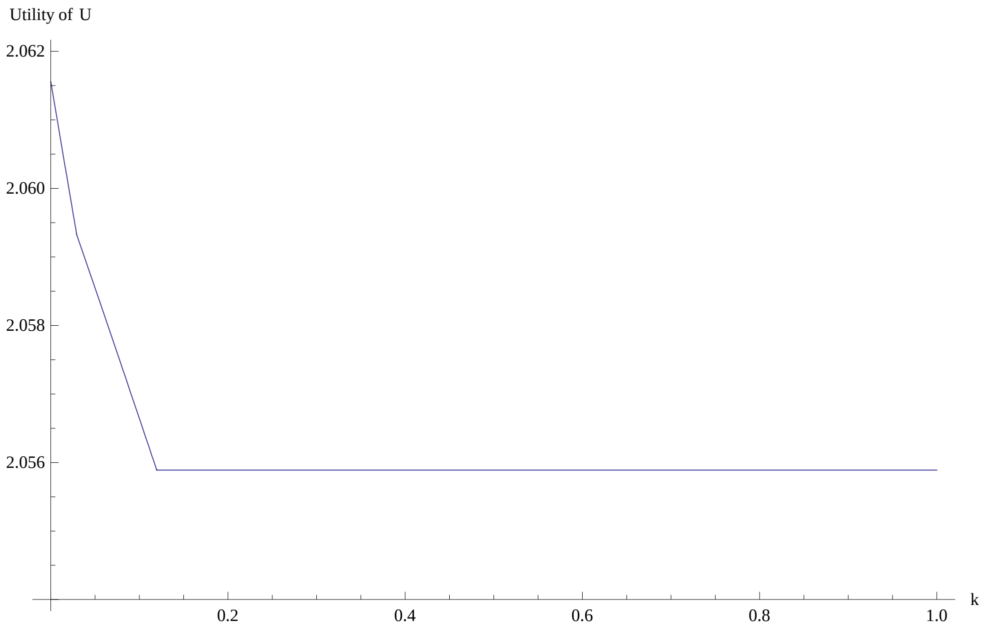

Figure 6 : Type H ' s utility as a function of k, for $\tilde{\phi}_P(0) < \phi < \phi^*$

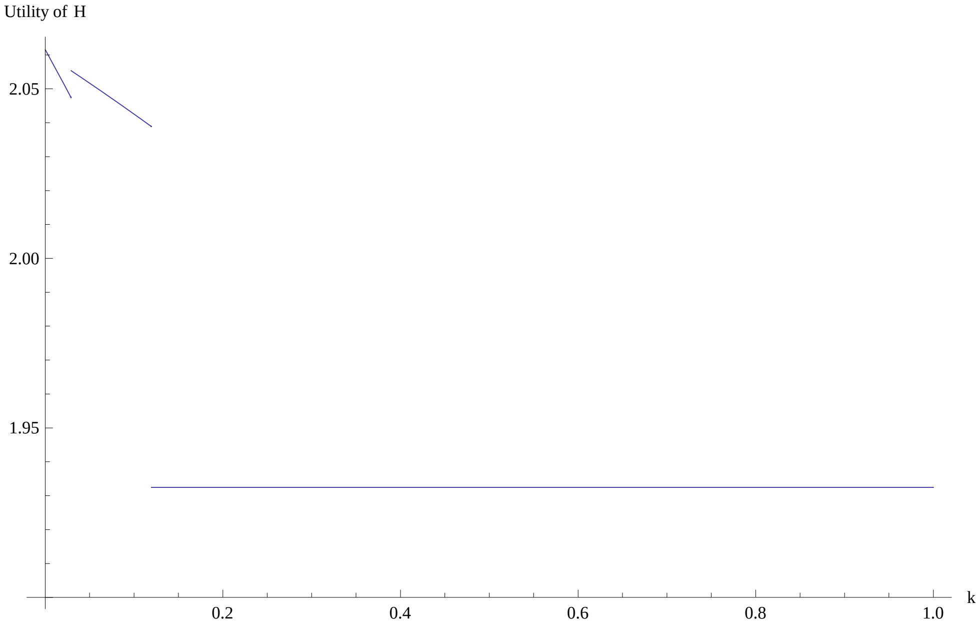

Figure 7 : Aggregate Welfare as a function of k, for $\tilde{\phi}_P(0) < \phi < \phi^*$

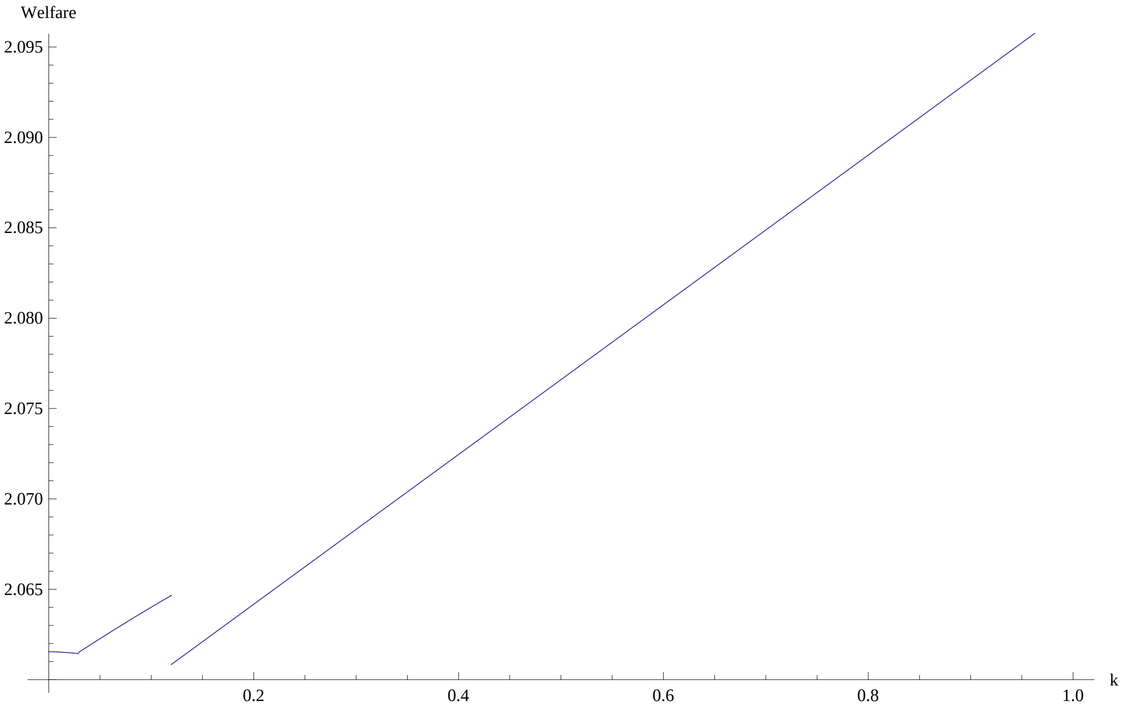

Figure 8 : Utility of U (blue) and H (red) as a function of k when ϕ[k] = 0.29 - 0.8 k

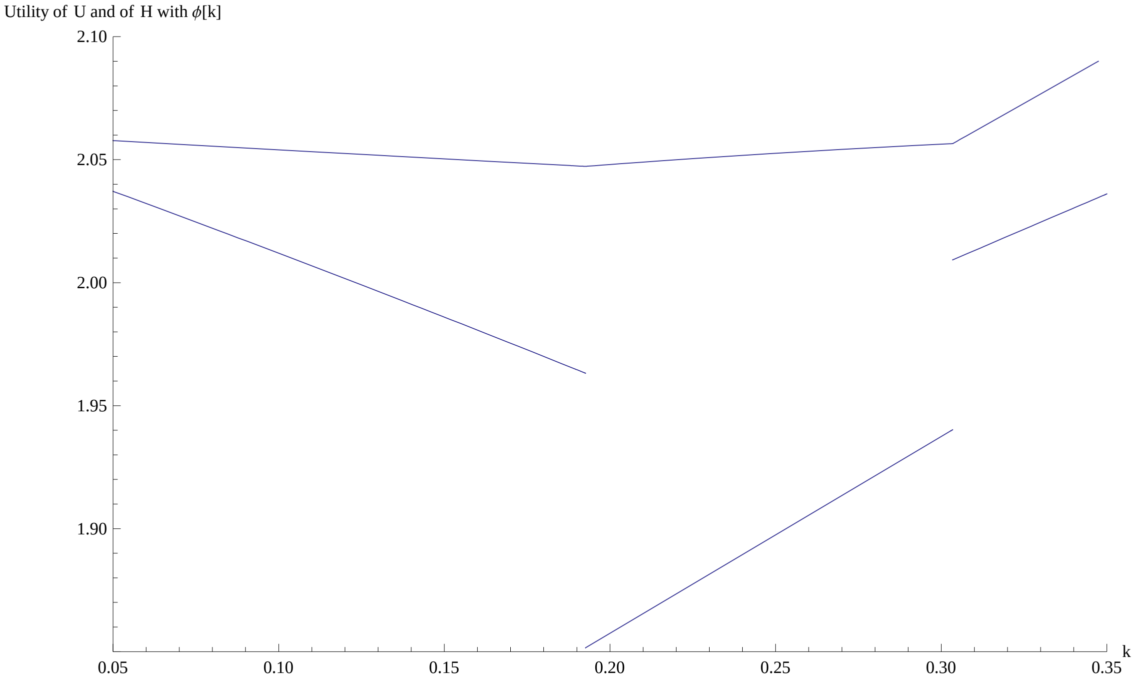

Figure 9 : Aggregate Welfare as a function of k when ϕ[k] = 0.29 - 0.8 k