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# Continuous PDE Dynamics Forecasting with Implicit Neural Representations

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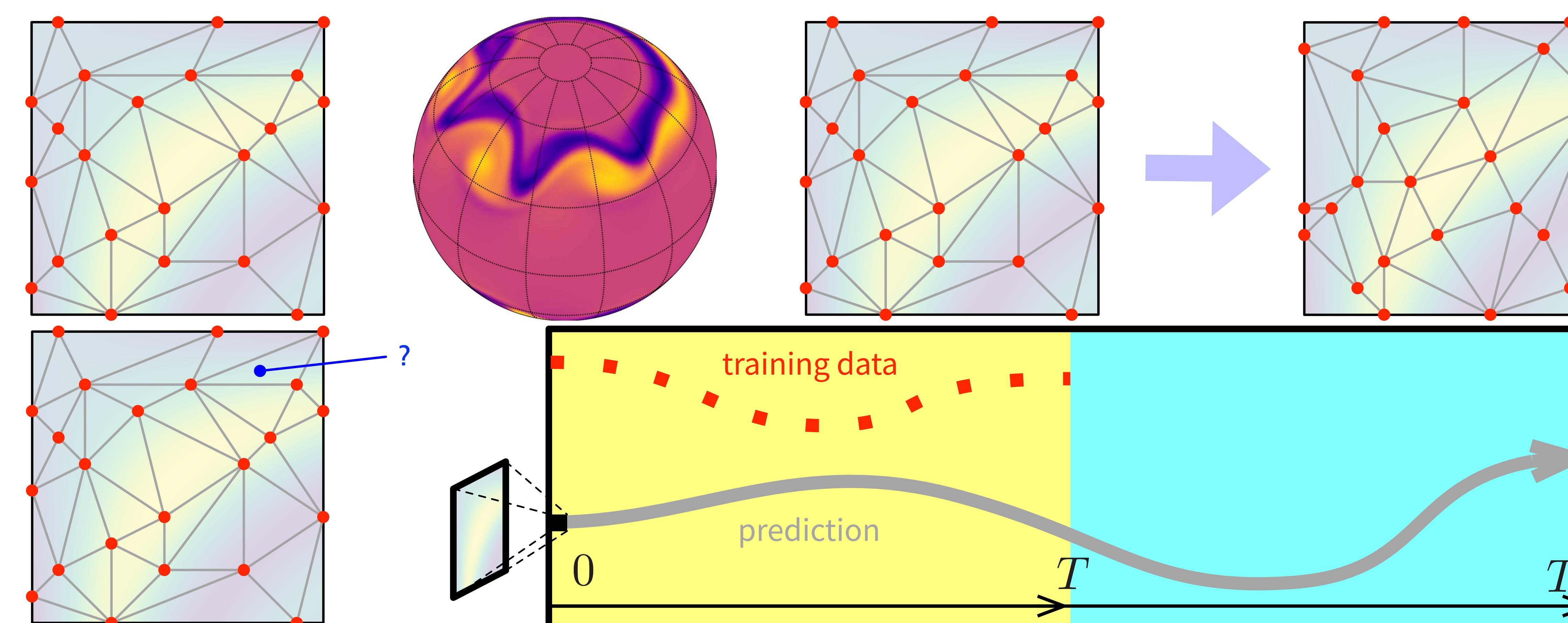
## MOTIVATION

- Data-driven modeling of Partial Differential Equations (PDEs):

$$\frac{\partial v(x, t)}{\partial t} = F\left(x, v(x, t), \frac{\partial v(x, t)}{\partial x}, \frac{\partial^2 v(x, t)}{\partial x^2}, \dots\right).$$

- Variety of neural PDE forecasters... but with limitations.
- Spatial and temporal flexibility requirements.

- [1] Chen et al. Neural ordinary differential equations. *NeurIPS* 2018.  
[2] Brandstetter et al. Message passing neural PDE solvers. *ICLR* 2022.  
[3] Li et al. Markov neural operators for learning chaotic systems. *arXiv*, 2021.  
[4] Lu et al. Learning nonlinear operators via DeepONet based on the universal approximation theorem of operators. *Nature Machine Intelligence*, 2021.  
[5] Sitzmann et al. INRs with periodic activation functions. *NeurIPS* 2020.  
[6] Fathony et al. Multiplicative filter networks. *ICLR* 2021.



Model	Ref.	Space		Time		PDE agnostic, new initial conditions
		Train / test grid independence	Free-form grid and topology	Evaluation at unobserved locations	Time continuous	
Discrete NODE	[1]	✗	✗	✗	✓	✓
MP-PDE	[2]	✗	✓	✗	✓	✓
NO	[3]	✓	✗	✗	✓	✓
DeepONet	[4]	✗	✓	✓	✗	✓
INR	[5, 6]	✓	✓	✓	✓	✗
DINO	Ours	✓	✓	✓	✓	✓

⇒ DINO solves prior limitations by combining INRs and ODEs.

## DINO: SPACE-TIME-CONTINUOUS FORECASTING

### INFERENCE

- INR-based decoder with hypernet:

$$(\text{DEC}) \forall t, \tilde{v}_t = I_{h_\phi}(\alpha_t).$$

- Encoding via auto-decoding, without explicit encoder NN:

$$(\text{ENC}) \alpha_0 = \text{auto-dec}(v_0).$$

Inverse problem solved by gradient descent:

$$\alpha_0^k = \alpha_0^{k-1} - \eta \nabla_{\alpha_t} \ell_{\text{dec}}(\phi, \alpha_t^{k-1}; v_t), \\ \alpha_0 = \alpha_0^K.$$

- Neural ODE as dynamics model:

$$(\text{DYN}) \frac{d\alpha_t}{dt} = f_\psi(\alpha_t).$$

### TWO-STAGE TRAINING PROCESS

Separate training of model components:

1. Train (ENC)-(DEC):

- hypernetwork  $h_\phi$ ;
- latent state  $\{\alpha_t\}_t$ .

2. Train (DYN) on the learned  $\{\alpha_t\}_t$ :

- dynamics  $f_\psi$ .

- Stable training of model components.

## FOURIERNET AND AMPLITUDE MODULATION

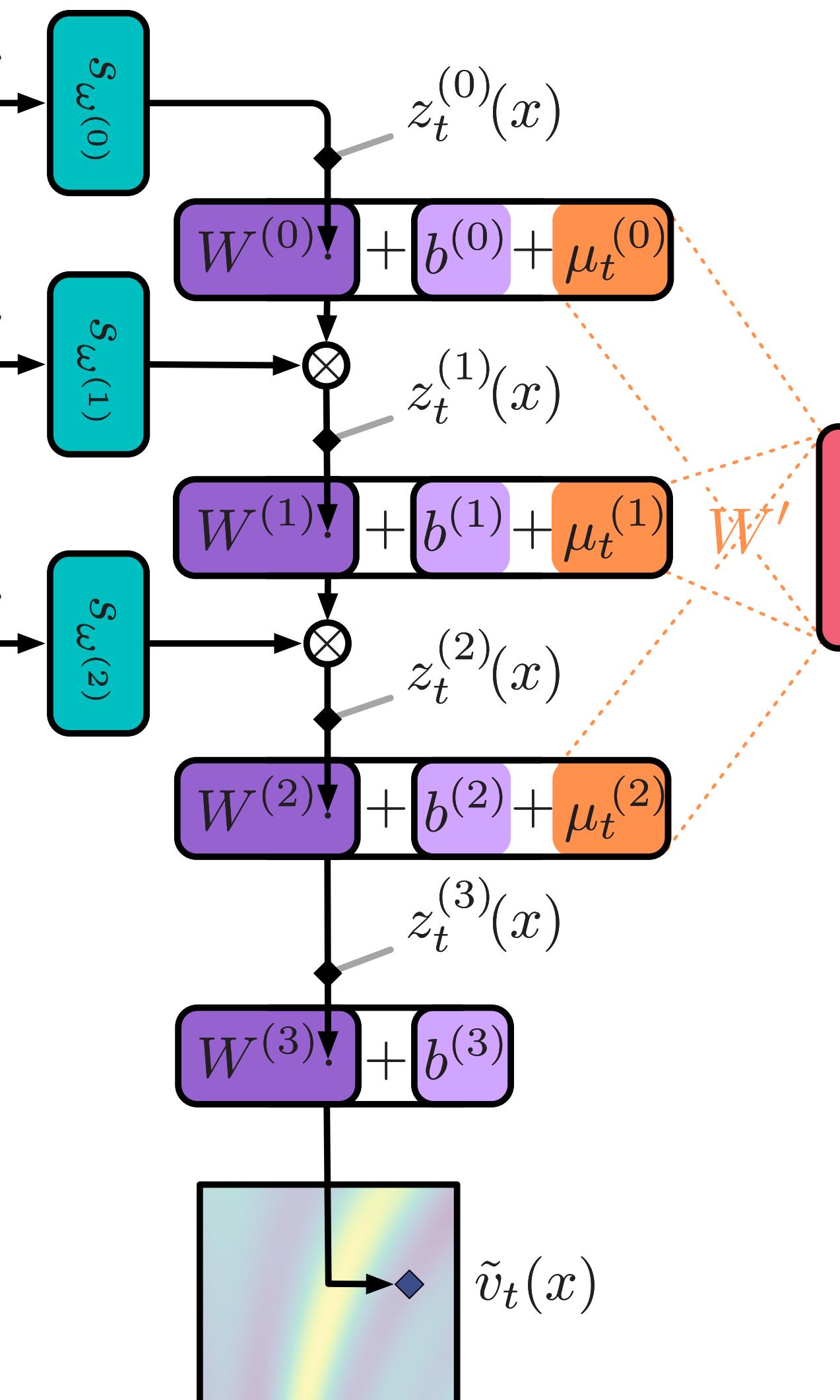
- We modulated a Multiplicative Filter Network (Fathony et al., 2021) with a linear hypernet from the latent state  $\alpha_t$ .
- We use a FourierNet:

$$s_{\omega(t)}(x) = [\cos(\omega(t)x), \sin(\omega(t)x)].$$

**Proposition 1** (Separation of time and space variables).

$$\tilde{v}_t(x) = \sum_m \underbrace{c^{(m)}(\alpha_t)}_{\text{time-dependent coefficient}} \times \underbrace{s_{\gamma(m)}(x)}_{\text{spatial basis element}}.$$

→ Inductive bias for better time extrapolation.



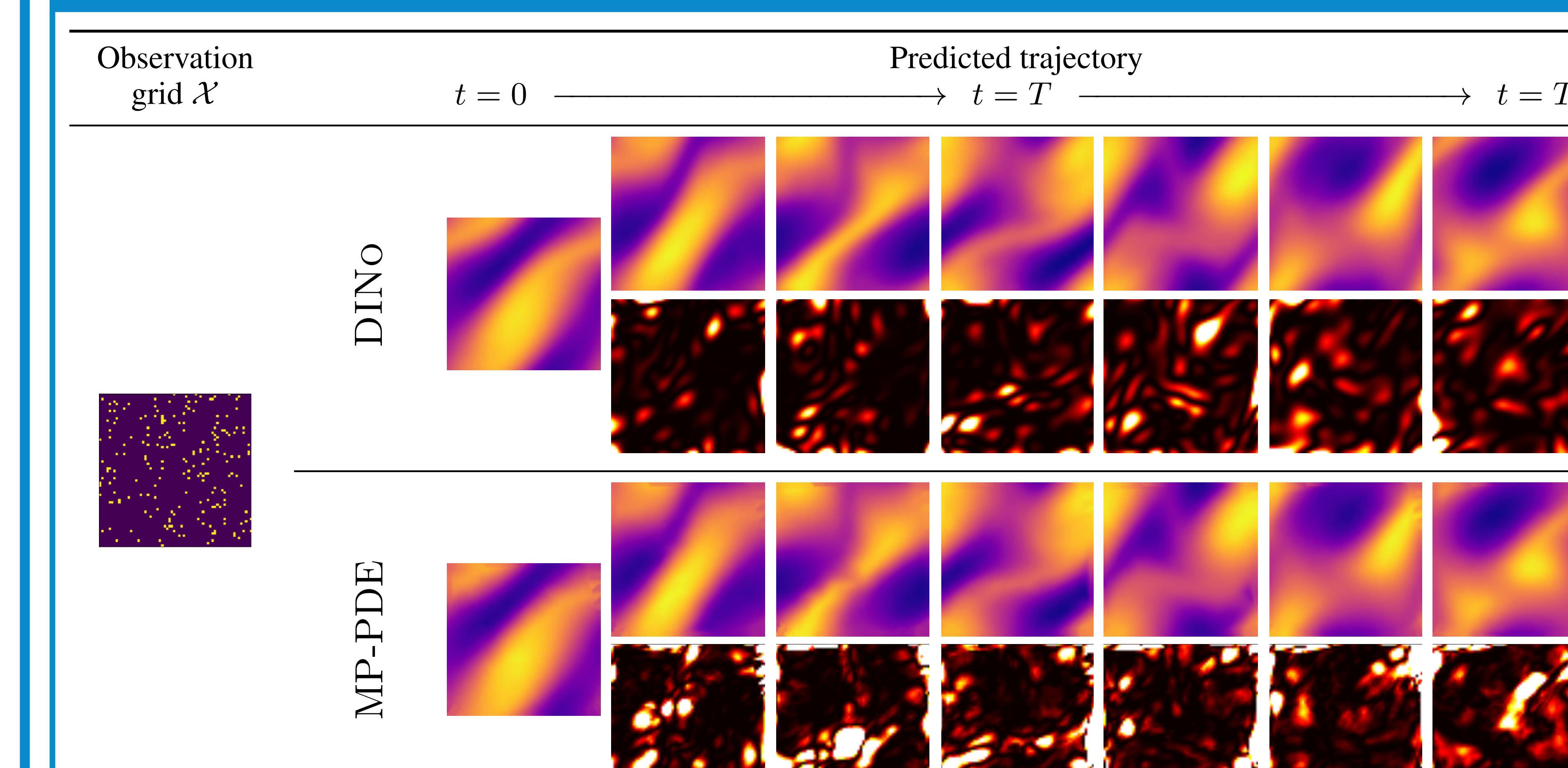
## MORE INFORMATION



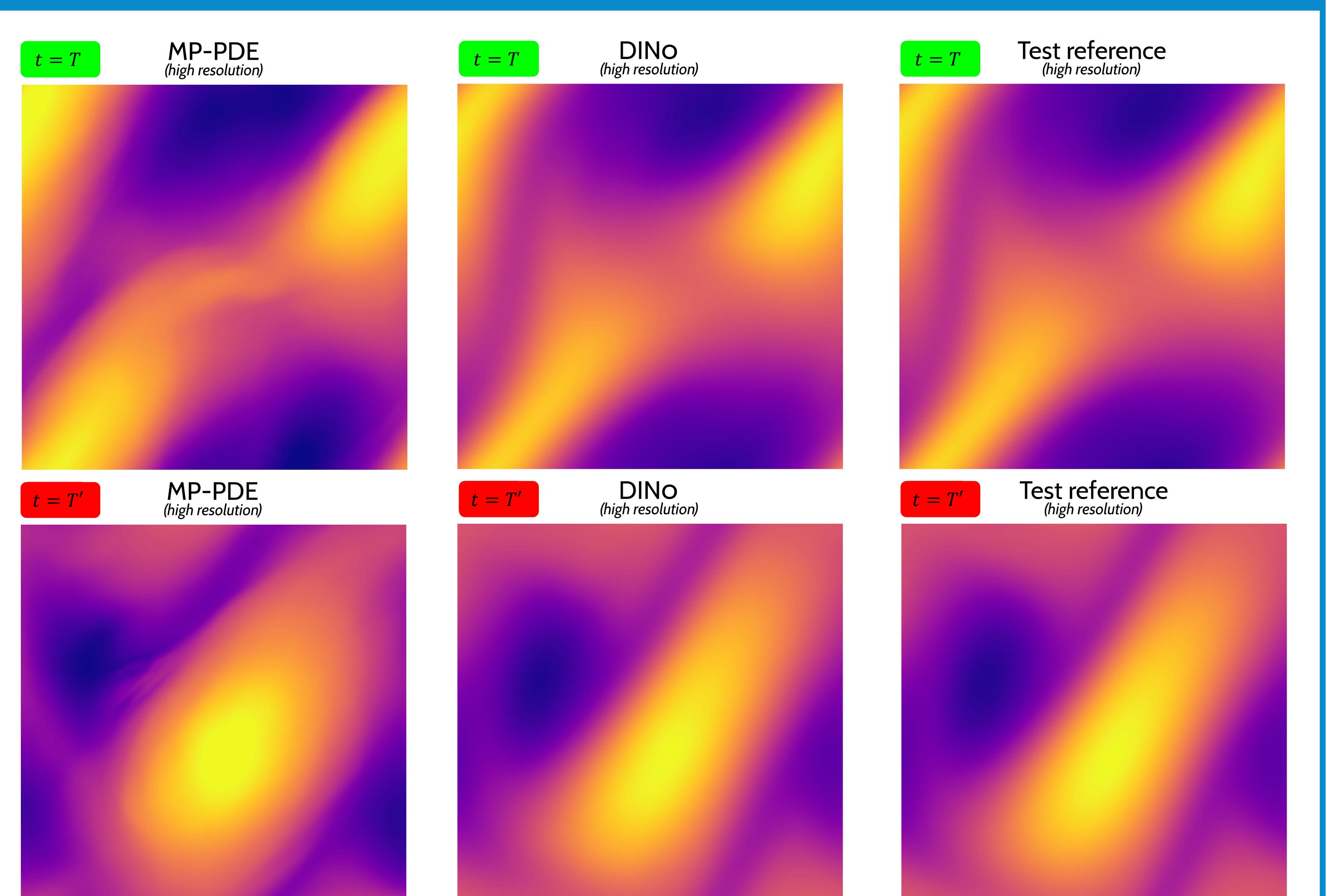
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## DIVERSE EVALUATION SCENARIOS

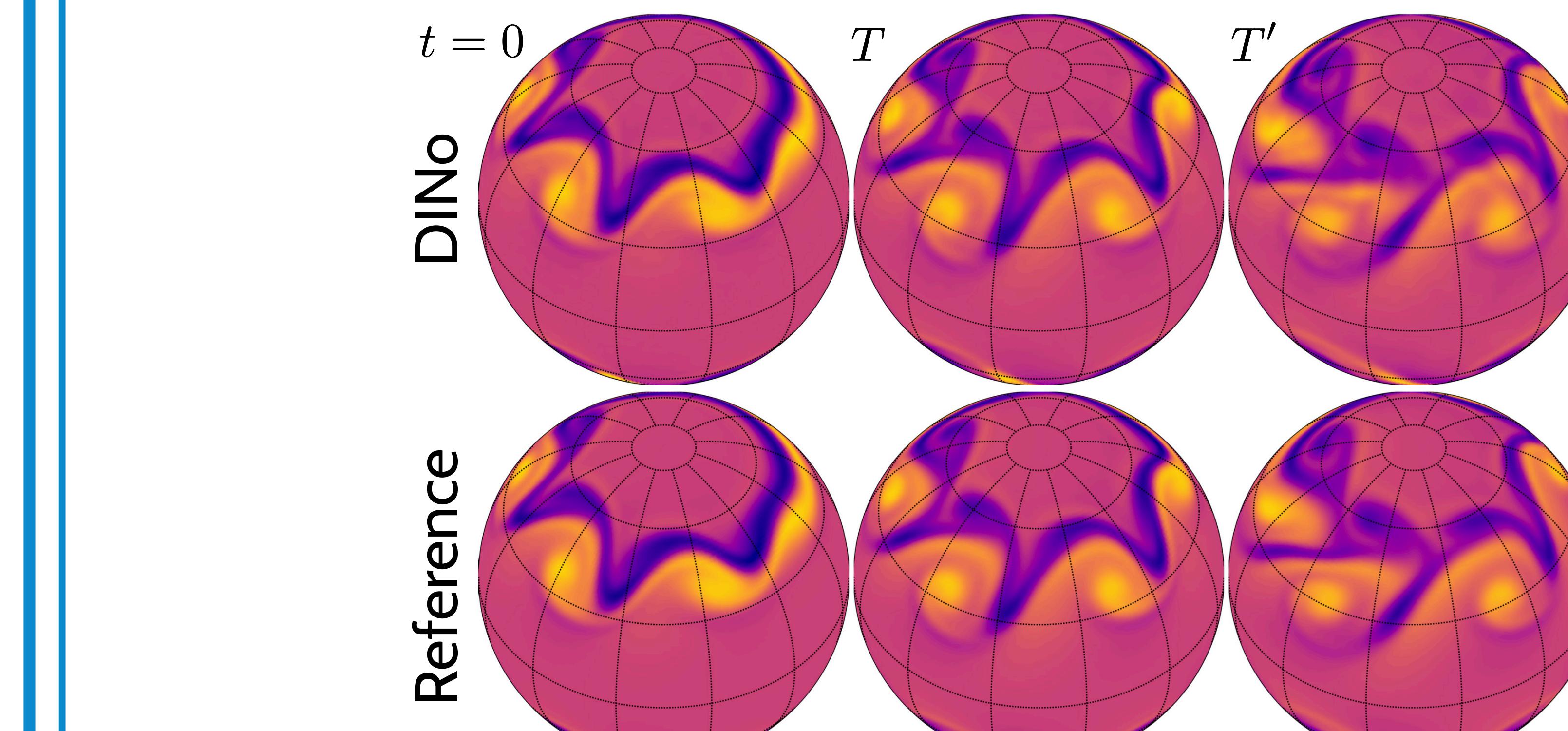
### COMPARISON TO STANDARD INTERPOLATION



### GENERALIZATION TO HIGHER RESOLUTION



### MODELING PDEs ON MANIFOLDS



### FINER TIME RESOLUTION

Model	In-t	Out-t
I-DINO (linear)	$3.459 \times 10^{-4}$	$5.598 \times 10^{-4}$
I-DINO (quadratic)	$2.165 \times 10^{-4}$	$4.473 \times 10^{-4}$
DINO (ODE solve)	<b><math>2.151 \times 10^{-4}</math></b>	<b><math>4.388 \times 10^{-4}</math></b>

