The contribution of fuzzy sets to decision sciences

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We try to provide a tentative assessment of the role of fuzzy sets in decision analysis. We discuss membership functions, aggregation operations, linguistic variables, fuzzy intervals and valued preference relations. The importance of the notion of bipolarity and the potential of qualitative evaluation methods are also pointed out. We take a critical standpoint on the state of the art, in order to highlight the actual achievements and point out research directions for the future.

Keywords: Decision, qualitative value scales, aggregation, linguistic variables, preference relations, fuzzy intervals, ranking methods.

1. Decision in a fuzzy environment: the state of the art

The idea of using fuzzy sets in decision sciences is not surprising since decision analysis is a field where human-originated information is pervasive. The seminal paper in this area was written by Bellman and Zadeh[1] in 1970, highlighting the role of fuzzy set connectives in criteria aggregation. That pioneering paper made three main points:

- Membership functions can be viewed as a variant of utility functions or rescaled objective functions.
- Combining membership functions, especially using the minimum, can be one approach to criteria aggregation. It claimed that in some situations, criteria can be viewed as flexible constraints. It highlights a max-min paradigm for optimisation, on a par with the usual additive criteria aggregation.
- Multiple-stage decision-making problems based on the minimum aggregation connective can then be stated and solved by means of

dynamic programming.

While there is an early literature on the third item,[2] it is the second one that has been extensively developed as witnessed by the numerous papers and books on aggregation functions.[3–5] As to the first issue, not so many scholars studied membership functions in connection with measurement theory (if we except Turksen[6]), nor did they take the connection with utility functions seriously. Fuzzy set theory has been applied to decision sciences in the form of fuzzy ordering relations, linguistic variables and fuzzy intervals. We can especially point out the following:

(1) *Gradual or valued preference relations*, with two streams, one stemming from Zadeh's *fuzzy orderings* dating back to 1971,[7] and more recently studied by Fodor, Roubens, Bodenhofer and colleagues, the other being *reciprocal valued relations*, recently studied by De Baets and colleagues,[8] where the natural understanding is often probabilistic since the preference degrees between two opposite pairwise comparisons sum to 1.
(2) *Linguistic variables*[9] have been used to model preference information, so as to get decision methods hopefully closer to the user cognition.
(3) *Fuzzy interval computations and comparison techniques*[10,11] can cope with epistemic uncertainty in numerical aggregation schemes. Especially, extensions of the weighted average with ill-known weights have been proposed.[12]

What has been the contribution of fuzzy sets to decision sciences? Following the terminology of the original Bellman-Zadeh paper, fuzzy decision analysis (FDA) is supposed to take place *in a fuzzy environment*, in contrast with probabilistic decision analysis, taking place *under uncertainty.* But, what is a fuzzy environment ? It seems that many authors take it as an environment where the major source of information is linguistic, so that linguistic variables are used, which does not correspond to Bellman and Zadeh's proposal. One should nevertheless not oppose "fuzzy environment" to "uncertain environment": the former in fact often means "using fuzzy sets", while 'uncertain environment" refers to an actual decision situation: there is epistemic uncertainty due to missing information, not always related to linguistic imprecision. Actually, for many decision theory specialists, it is not clear that fuzzy sets have ever led to a new decision paradigm. Indeed, one may argue that some techniques already existed under a different terminology, and that some fuzzy decision methods are fuzzifications of standard decision techniques.

In several cases indeed, fuzzy sets have just been added to existing techniques (fuzzy AHP methods, fuzzy weighted averages, fuzzy extensions of ELECTRE-style Multiple Criteria Decision Making (MCDM) methods) with no clear benefits (especially when fuzzy information is changed into precise numbers prior to further processing, via defuzzification, which can be observed sometimes). Likewise, fuzzy preference modelling is an extension of standard preference modelling; it would benefit from a comparison to probabilistic or measurement-based preference modeling. In fact, contrary to what is often claimed in fuzzy decision analysis papers, it is not always the case that adding fuzzy sets to an existing method improves it in a significant way. Too often, these papers contain a method described step by step, with some fuzzy sets involved, but no comparison with existing methods, no formal study of the properties of the method are given. Sometimes fuzzy ratings are modelled by complex entities such as higher order fuzzy sets, which become very difficult to understand by decision-makers, as if moving to more complex rating representations could address difficulties already existing in the numerical measurement of abstract features.

2. Some prospective issues in fuzzy decision analysis

In fact, we have argued in a recent paper[14] that fuzzy decision analysis could benefit from a critical assessment of its achievements and its limitations. There are several points that would deserve specific investigations in the future.

The choice of a membership scale : do we need numerical ratings or can we live with qualitative ones ? Do we need a bipolar scale[15] where the two ends refer to the idea of *good* and *bad* and a special value inside corresponds to the idea of indifference? On the contrary one may use two opposite unipolar scales. The latter choice comes close to papers using interval-valued or so-called intuitionistic fuzzy sets. However, these approaches are often unclear because a pair of values can either represent an ill-known precise one, or can stand as evaluating pros and cons.[17] Some authors have modelled multifactorial evaluations by means of fuzzy linguistic rules and applied fuzzy logic control methods to compute global ratings. This technique is debatable because if the local rating scales are not measurable attributes, linguistic variables with triangular fuzzy intervals make no sense.[13] Perhaps in many cases a finite totally ordered scale is expressive enough. In any case, a prerequisite for a proper use of fuzzy sets in decision analysis is to have a clear intuition of what is the meaning of fuzzy membership grades in a given application,[16] and what kind of scale is appropriate. The issue of measuring membership grades[18] has recently received too sparse attention.

Fuzzy preference modelling : A number of works have been published on fuzzy preference relations. They are valued relations and there are, as recalled, above, two settings: the *reciprocal relations*, and the *valued outranking relation* (the latter often decomposed in a strict part, an equivalence part and an incomparability part[21]) which is the direct extension of reflexive Boolean relations. There is a need for a measurement approach to degrees of preference that parallels the probabilistic understanding of reciprocal relations.[19] A degree of preference in the latter case is often viewed as measuring the probability of strict preference on a bipolar scale (hence not generalizing the usual model of reflexivity), while valued outranking relations express the idea of intensity of preference on a negative scale (the top value meaning indifference[20]). Interestingly, the latter non-probabilistic view can also refer to the idea of *possibility* of preference stemming from the comparison of fuzzy intervals. Some works exist on the representation of fuzzy outranking relations in terms of fuzzy interval orderings, but the full landscape of fuzzy preference relations is far from being laid bare.

Qualitative possibility theory in decision analysis : the idea is that possibility distributions on qualitative scales can play a role similar to probability distributions, but the former explicitly accounts for incomplete information in a more faithful way than subjective probabilities. In this spirit, there exists a fuzzy counterpart of Savage theory, in a purely qualitative setting, justifying max-min and min-max aggregation schemes for decision under uncertainty and multiple-criteria decision-making.[23] More general forms of uncertainty modeling or criteria weighting schemes can be considered, using fuzzy integrals of various kinds.[24] Multiple-stage extensions, such as qualitative decision networks, have been studied,[25] and more recently partially observable possibilistic Markov decision networks.[26]

Uncertainty management in numerical decision methods : Fuzzy sets understood as possibility distributions can be useful to perform a kind of sensitivity analysis on existing decision analysis methods (for instance the AHP technique of Saaty, and all kinds of numerical aggregation schemes). However, it means that we should then not consider fuzzy-valued ratings as an alternative kind of rating, but as flexible constraints on ill-known precise values. Then extensions of compositional schemes (generalized fuzzy set operations) cannot be used without care. For instance, rather than trying to directly define fuzzy eigen-values of fuzzy set-valued preference matrices, as if they were just another kind of matrix, it makes more sense to compute the range of the eigen-values of the usual preference matrices compatible with the fuzzy specification.[27] Nevertheless, the use of fuzzy sets does not circumvent the limitations of the usual AHP method.[13] Likewise, it makes little sense to require that the sum of ill-known weights in a weighted arithmetic mean be equal to 1; what can be required is that the set of normalized weight vectors compatible with fuzzy intervals be not empty.[?] Generally speaking, before solving a decision problem with fuzzy intervals, it is better to solve it in a meaningful way using crisp intervals. If the proposed solution makes no sense with crisp intervals, it will not be better off with fuzzy intervals.

Ranking fuzzy intervals : The same considerations apply to the issue of ranking fuzzy intervals. First, defuzzification is not always the way to go, as it does away with the uncertainty and comes down to working with precise values. Then why modeling uncertainty in the first place? It is clear that if decision evaluations take the form of fuzzy intervals, the advantage of a ranking method is to lay bare the situations where an actual ranking looks difficult to reach. In fact, ranking methods for fuzzy intervals should borrow from interval ordering techniques and the comparison of random numbers (probabilistic preference, stochastic dominance) taking advantage of the fact that in the numerical setting, a fuzzy interval encodes a family of probability measures in the form of nested confidence intervals[29]

The issues raised in this short note are more fully described in the papers listed below, especially some surveys[14,30] where extensive bibliographies can be found.

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