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# Practical consensus for heterogeneous multi-agent systems with gainfluctuations via resilient sampled-data control

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This study proposes a sampled-data-based practical consensus control method for heterogeneous multi-agent systems with gain fluctuations. First, the heterogeneous multi-agent system is formulated together with randomly occurring nonlinear dynamics. Second, unlike the existing studies, a more general retarded sampled-data control approach is designed, which involves the gain fluctuations along with the actual sampling pattern and constant time delay, to achieve the practical consensus of the considered system. By utilizing the aperiodic sampling information, and looped functional approach, a Wirtinger's inequality-based discontinuous Lyapunov-Krasovskii functional is constructed to derive the delay-dependent stability condition of the closed-loop system in the form of linear matrix inequality. Finally, numerical simulation is presented to illustrate the derived theoretical results.

## 1. Introduction

A significant amount of research on a multi-agent system (MAS) can be found in the applications of formation control, an attitude of spacecraft alignment, and so on [4,10,26,28]. In the past decades, the cooperative control technique has been sufficiently developed for MAS. As in the cooperative control of MAS, one of the research topics is consensus [27], which acquires a suitable protocol for agents to reach an agreement on states. Towards this problem, the distributed controller has been designed for each agent by utilizing the agent, and its neighbor's local information only [30]. The leader-following consensus has considerable attention in recent years in which a leader is independent of the follower agents, and it has a significant impact on the information update of the follower agents [8,29]. For example, Shi and Shen [29] have inves-

tigated the leader-follower consensus problem for uncertain nonlinear MAS.

However, network connectivity also plays an essential role to achieve a consensus among the agents. In order to represent the network connectivity, the algebraic graph theory is employed in the literature under different topologies [13,21,25,32]. Further, various assumptions on system topologies have been considered by the researchers to guarantee the consensus of MAS [1,6,20,32,34]. For example, Tong et al. [32] have investigated the average consensus problem for multiple integrators over fixed and switching, undirected and connected network topologies. Cheng et al. [6] have studied the consensus problem under both fixed and switching topologies for leader-follower MAS with external disturbances. Based on the switching topology, You et al. [34] have designed a consensus protocol for leader-follower MAS with actuator saturation and time-varying delay. From the above work, the consensus analysis of MAS under the switching topology has practical significance, and it is a more general case with fixed topology as a particular case.

Nevertheless, the convergence of MAS may only attain a bounded region encompassing the equilibrium because of the ex-

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istence of the physical restrictions and communication constraints, usually named the practical consensus of MAS [3,9]. For instance, in the work of Dong et al. [9], the practical consensus problem has been established for linear time-invariant uncertain swarm systems. Recently, the practical consensus has been analyzed by Bernuau et al. [3] for homogeneous MAS with sampled-data information. In the literature above, most of the researchers have taken identical dynamics for all agents. In reality, all the agents have different dynamics due to the parameter difference or uncertainty and model mismatch; such a system is called heterogeneous MAS [7,22,38]. Compared to the study of consensus for homogeneous MAS, the heterogeneous MAS has a challenging problem since heterogeneity among the agents [7,18,23,31]. For instance, the heterogeneous MAS has proposed for linear cases in [35,36]. However, the intrinsic nonlinearities are unavoidable when analyzing the consensus problem of heterogeneous MAS, and it made much effort in the literature (see, [2,7,24] and references therein). Also, the nonlinearities may undergo randomness under a specific form in the networked environment. In this case, Wang et al. [33] introduced the Bernoulli distributed random variable to model the randomly occurring nonlinearities for the synchronization problem of complex networks. Moreover, the randomly occurring nonlinearities have been modeled for MAS in existing studies [14,17]. Therefore, we consider the issue of randomly occurring nonlinearities for heterogeneous MAS in the present study.

On the other hand, to obtain the consensus, the researchers have focused more on the sampled-data control for MAS due to the development of communication and digital technology, digital feedback and low-cost applications [3,5,11]. The results of the studies, as mentioned earlier [3,5,11], were considered MAS with identical dynamics. Further, by using the sampled-data control (SDC) technique, the consensus problem has been studied for heterogeneous MAS in the works of Zhang et al. [35]. Recently, the SDC has been designed for obtaining the practical consensus of heterogeneous MAS in the works of Ding and Zheng et al. [7]. Besides that, the fluctuation in control gain may appear for the physical systems, which leads to realizing the consensus difficulty. To mitigate the effect of fluctuation in control gain, the non-fragile(resilient) control has been designed in the works of [12,15,16]. For example, non-fragile control has been designed for the consensus of MAS with nonlinear dynamics via switching topology in the works of Kaviarasan et al. [16]. Moreover, the non-fragile control with sampled-data information has been designed for consensus of nonlinear MAS in Jiang et al. [15]. Until now, to the best of our knowledge, the practical consensus of nonlinear heterogeneous MAS has not yet been fully investigated in the literature via sampled-data-based resilient control, which is the overarching motivation of this work.

Motivated by the above literature, this paper investigates the practical consensus problem of heterogeneous MAS with nonlinear dynamics via resilient retarded sampled-data control (RRSDC). Unlike the previous studies, the randomly occurring nonlinearities, gain fluctuations, and constant signal transmission delay are taken into consideration to study the practical consensus of heterogeneous MAS. Initially, the practical consensus of heterogeneous MAS is studied via fixed topology. Then, these results are extended into the switching topology case. In order to achieve the practical consensus for heterogeneous MAS, sufficient conditions are derived in the form of linear matrix inequalities (LMIs) via Wirtinger's inequality-based discontinuous (WIBD) Lyapunov-Krasovskii functional (LKF). Finally, numerical examples are provided to demonstrate the proposed methods.

The remainder of this paper is organized as follows: Section 2 formulates the heterogeneous nonlinear MAS. Section 3 presents the practical consensus results of the proposed system. In Section 4, the validity of the presented method

is verified by the simulation results. Section 5 concludes the paper.

## 2. Preliminaries and problem formulation

### 2.1. Notations and preliminaries

Throughout this paper,  $\mathbb{R}^{n \times n}$  and  $\mathbb{R}^n$  represents the  $n \times n$  real matrix and the  $n$ -dimensional Euclidean space, respectively.  $S > 0$  ( $< 0$ ) means that  $S$  is a positive (negative) definite matrix.  $I$  denotes the identity matrix and  $\text{diag}\{\dots\}$  is a block diagonal matrix.  $Z = \text{col}\{z_1, z_2, z_3\}$  indicates the column vector of  $Z$ .  $\text{col}\{A_i\}_N$  and  $\text{diag}\{B_i\}_N$  represent the  $\text{col}\{A_1, A_2, \dots, A_N\}$  and  $\text{diag}\{B_1, B_2, \dots, B_N\}$ , respectively. The sign  $\otimes$  represents the Kronecker product of the matrix. The transpose and inverse of a matrix is represented by the superscripts " $T$ " and " $-1$ ", respectively.  $\text{Sym}\{A\} = A + A^T$  and  $\|\cdot\|$  is the Euclidean norm for given vector. The notation  $*$  is used to denote the symmetric term in a symmetric matrix.

$G = \{\mathcal{V}, E, W\}$  represents a directed weighted graph of order  $N$ , with the set of nodes  $\mathcal{V} = \{v_1, v_2, \dots, v_N\}$  and edges  $E \subseteq \mathcal{V} \times \mathcal{V}$ . An edge defined as  $(v_i, v_j)$  implies that node  $v_i$  can receive information from node  $v_j$ . The neighbors of node  $v_i$  represented as  $N_i = \{v_j \in \mathcal{V} | (v_i, v_j) \in E, i \neq j\}$ . Here,  $W \in \mathbb{R}^{N \times N}$  denotes the adjacency matrix with the entries  $w_{ii} = 0$  if any  $i$  and  $w_{ij} \neq 0$  if  $i$  receives information from  $j$ .  $\Lambda = \text{diag}\{\bar{w}_1, \bar{w}_2, \dots, \bar{w}_N\}$  is a degree matrix of  $G$  with  $\bar{w}_i = \sum w_{ij}$ . The Laplacian matrix of  $G$  is defined as  $L = \Lambda - W$ . A directed graph  $G$  is a directed spanning tree if there exists an agent, called the root, with directed paths to all other agents.

### 2.2. Problem formulation

Consider a heterogeneous MAS with  $N$  agents, where the dynamics of  $i$ th agent is

$$\dot{x}_i(t) = A_i x_i(t) + \beta(t) B_i f(x_i(t), t) + (1 - \beta(t)) C_i g(x_i(t), t) + u_i(t), \quad (i = 1, 2, \dots, N), \quad (1)$$

where  $A_i$ ,  $B_i$  and  $C_i$  are  $n \times n$  constant matrices;  $x_i(t) = (x_{i1}(t), x_{i2}(t), \dots, x_{in}(t))^T \in \mathbb{R}^n$  is the state vector with  $x_i(0) = x_i^0$  for  $i = 1, 2, \dots, N$ ;  $u_i(t) \in \mathbb{R}^n$  is the control input for all  $t \geq 0$ . The Bernoulli stochastic variable  $\beta(t)$  represents the following random events of system (1), for all  $t \geq 0$

$$\beta(t) = \begin{cases} 1 & \text{if } f(x_i(t), t) \text{ occurs,} \\ 0 & \text{if } g(x_i(t), t) \text{ occurs,} \end{cases}$$

it follows that

$$\text{Prob}\{\beta(t) = 1\} = \mathbb{E}\{\beta(t)\} = \beta_0,$$

$$\text{Prob}\{\beta(t) = 0\} = 1 - \mathbb{E}\{\beta(t)\} = 1 - \beta_0,$$

where  $\beta_0 \in [0, 1]$  denotes the probability occurrence of nonlinear functions  $f(x_i(t), t)$  and  $g(x_i(t), t)$ .

**Assumption 1.** For any  $y_1, y_2, z_1, z_2 \in \mathbb{R}^n$ , the nonlinear functions  $f, g: \mathbb{R}^n \times [0, \infty) \rightarrow \mathbb{R}^n$  satisfies the conditions as follows:

$$\begin{aligned} \|f(y_1, t) - f(y_2, t)\| &\leq \|\Gamma(y_1 - y_2)\| \\ \text{and } \|g(z_1, t) - g(z_2, t)\| &\leq \|\Upsilon(z_1 - z_2)\|, \quad \forall t \geq 0 \end{aligned} \quad (2)$$

where  $\Gamma$  and  $\Upsilon$  are known constant matrices.

The leader agent dynamics labeled as  $i = 0$  is given by

$$\dot{x}_0(t) = A x_0(t) + \beta(t) B f(x_0(t), t) + (1 - \beta(t)) C g(x_0(t), t), \quad (3)$$

where  $A, B, C$  are  $n \times n$  constant matrices and  $x_0(t) \in \mathbb{R}^n$  is the state vector with  $x_0(0) = x_0^0$ . Let the graph  $\mathcal{G}$  includes the graph  $G$  and a leader which has no neighbors. The adjacency matrix of the leader is  $M = \text{diag}\{m_1, m_2, \dots, m_N\}$  with  $m_i \geq 0$  for any  $i$ . If the leader is

a neighbor of node  $v_i$ , then  $m_i > 0$ ; otherwise,  $m_i = 0$ . Thus the Laplacian matrix of the graph  $\mathcal{G}$  represented as  $\mathcal{L} = \begin{bmatrix} 0 & \mathbf{0}_{1 \times N} \\ L_1 & H \end{bmatrix}$ , where  $L_1 = [m_1, m_2, \dots, m_N]^T \in \mathbb{R}^{N \times 1}$  and  $H = L + M \in \mathbb{R}^{N \times N}$ .

**Assumption 2.** The leader is a root of a directed spanning tree in  $\mathcal{G}$ .

**Assumption 3.** The state  $x_0(t)$  in (3) is bounded, i.e.,  $\|x_0(t)\| \leq \gamma$  for any initial condition with the constant  $\gamma > 0$ .

As in the SDC input, the sequence of holding times are  $0 = t_0 < t_1 < \dots < t_k < \dots$ ,  $\lim_{k \rightarrow \infty} t_k = +\infty$ . Then, the following non-fragile retarded SDC input is proposed via zero-order hold function

$$\begin{aligned} u_i(t) = & (K_{1i} + \Delta K_{1i}(t)) \left[ \sum_{j=1}^N w_{ij} [x_i(t - q(t)) - x_j(t - q(t))] \right. \\ & \left. + m_i [x_i(t - q(t)) - x_0(t - q(t))] \right] \\ & + (K_{2i} + \Delta K_{2i}(t)) \left[ \sum_{j=1}^N w_{ij} [x_i(t - q(t) - \rho) - x_j(t - q(t) - \rho)] \right. \\ & \left. + m_i [x_i(t - q(t) - \rho) - x_0(t - q(t) - \rho)] \right], \end{aligned} \quad (4)$$

where  $i = 1, 2, \dots, N$ ,  $q(t) = t - t_k$  for  $t \in [t_k, t_{k+1})$  and the sampling interval can be defined as  $t_{k+1} - t_k = q_k \leq \hat{q}$ . Here  $\hat{q}$  is the largest sampling interval and  $\rho$  is a constant signal transmission delay.  $K_{1i}$ ,  $K_{2i}$  are the control gain matrices. The controller gain fluctuations  $\Delta K_{1i}(t)$ ,  $\Delta K_{2i}(t)$  are assumed to satisfy the following form:

$$[\Delta K_{1i}(t), \Delta K_{2i}(t)] = \mathcal{N}_i \mathcal{F}_i(t) [\mathcal{M}_i^{(1)}, \mathcal{M}_i^{(2)}],$$

where  $\mathcal{N}_i$ ,  $\mathcal{M}_i^{(1)}$  and  $\mathcal{M}_i^{(2)}$  are known constant matrices and  $\mathcal{F}_i(t)$  denotes the unknown matrix, which satisfies  $\mathcal{F}_i^T(t) \mathcal{F}_i(t) \leq I$ . Substituting (4) into (1) yields

$$\begin{aligned} \dot{x}_i(t) = & A_i x_i(t) + \beta(t) B_i f(x_i(t), t) + (1 - \beta(t)) C_i g(x_i(t), t) \\ & + (K_{1i} + \Delta K_{1i}(t)) \left[ \sum_{j=1}^N w_{ij} [x_i(t - q(t)) \right. \\ & \left. - x_j(t - q(t))] + m_i [x_i(t - q(t)) - x_0(t - q(t))] \right] \\ & + (K_{2i} + \Delta K_{2i}(t)) \left[ \sum_{j=1}^N w_{ij} [x_i(t - q(t) - \rho) - x_j(t - q(t) - \rho)] \right. \\ & \left. + m_i [x_i(t - q(t) - \rho) - x_0(t - q(t) - \rho)] \right], \quad t_k \leq t < t_{k+1}. \end{aligned} \quad (5)$$

**Remark 1.** The SDC techniques for consensus analysis of MAS are widely established in the literature [5,11]. In the real world, it is challenging to accurately acquire the control gain parameter for MAS because of the equipment limitations and the influence of uncertain factors. In the works of [15], a non-fragile sampled-data control scheme is used to track the consensus for nonlinear MAS. Further, the RRSDC scheme has been utilized only for homogeneous MAS in [12,16]. It is worth mentioning that the RRSDC in (4) has been considered for heterogeneous MAS for the first time. When  $K_{2i} = 0$  and  $\Delta K_{2i}(t) = 0$  or  $\rho = 0$  in equation (4), the proposed resilient retarded sampled-data control scheme can be modified to resilient SDC as given by  $u_i(t) = (K_i + \Delta K_i(t)) [\sum_{j=1}^N w_{ij} [x_i(t - q(t)) - x_j(t - q(t))] + m_i [x_i(t - q(t)) - x_0(t - q(t))]]$ . Therefore, the traditional, resilient and memory SDC are the particular cases of the proposed RRSDC.

Defining the error variable as  $\tilde{x}_i(t) = x_i(t) - x_0(t)$ , for each agent  $i = 1, 2, \dots, N$ , we obtain the following from (5):

$$\begin{aligned} \dot{\tilde{x}}_i(t) = & A_i \tilde{x}_i(t) + \beta(t) B_i \tilde{f}(\tilde{x}_i(t), t) + (1 - \beta(t)) C_i \tilde{g}(\tilde{x}_i(t), t) + (K_{1i} \\ & + \Delta K_{1i}(t)) \left[ \sum_{j=1}^N w_{ij} [\tilde{x}_i(t - q(t)) \right. \\ & \left. - \tilde{x}_j(t - q(t))] + m_i \tilde{x}_i(t - q(t)) \right] \\ & + (K_{2i} + \Delta K_{2i}(t)) \left[ \sum_{j=1}^N w_{ij} [\tilde{x}_i(t - q(t) - \rho) - \tilde{x}_j(t - q(t) - \rho)] \right. \\ & \left. + m_i \tilde{x}_i(t - q(t) - \rho) \right] + h_i(x_0(t)) \end{aligned}$$

with  $\tilde{f}(\tilde{x}_i(t), t) = f(x_i(t), t) - f(x_0(t), t)$ ,  $\tilde{g}(\tilde{x}_i(t), t) = g(x_i(t), t) - g(x_0(t), t)$ , and

$$\begin{aligned} h_i(x_0(t)) = & (A_i - A) x_0(t) + \beta(t) (B_i - B) f(x_0(t), t) \\ & + (1 - \beta(t)) (C_i - C) g(x_0(t), t). \end{aligned}$$

From the above  $h_i(x_0(t_k))$  denotes the unmatched terms between follower and leader agents.

Now, let us define the vectors,

$$\tilde{\mathcal{X}}(t) \triangleq \text{col}\{\tilde{x}_1(t), \tilde{x}_2(t), \dots, \tilde{x}_N(t)\},$$

$$\tilde{f}(\tilde{\mathcal{X}}(t), t) \triangleq \text{col}\{\tilde{f}(\tilde{x}_1(t), t), \tilde{f}(\tilde{x}_2(t), t), \dots, \tilde{f}(\tilde{x}_N(t), t)\},$$

$$\tilde{g}(\tilde{\mathcal{X}}(t), t) \triangleq \text{col}\{\tilde{g}(\tilde{x}_1(t), t), \tilde{g}(\tilde{x}_2(t), t), \dots, \tilde{g}(\tilde{x}_N(t), t)\},$$

$$h(x_0(t)) \triangleq \text{col}\{h_1(x_0(t)), h_2(x_0(t)), \dots, h_N(x_0(t))\},$$

then we get

$$\begin{aligned} \dot{\tilde{\mathcal{X}}}(t) = & \bar{A} \tilde{\mathcal{X}}(t) + \beta(t) \bar{B} \tilde{f}(\tilde{\mathcal{X}}(t), t) + (1 - \beta(t)) \bar{C} \tilde{g}(\tilde{\mathcal{X}}(t), t) \\ & + K_1 \bar{H} \tilde{\mathcal{X}}(t - q(t)) + K_2 \bar{H} \tilde{\mathcal{X}}(t - q(t) - \rho) \\ & + \mathcal{N} \mathcal{F}(t) [\mathcal{M}^{(1)} \bar{H} \tilde{\mathcal{X}}(t - q(t)) + \mathcal{M}^{(2)} \bar{H} \tilde{\mathcal{X}}(t - q(t) - \rho)] \\ & + h(x_0(t)), \end{aligned} \quad (6)$$

where  $\bar{A} = \text{diag}\{A_i\}_N$ ,  $\bar{B} = \text{diag}\{B_i\}_N$ ,  $\bar{C} = \text{diag}\{C_i\}_N$ ,  $K_1 = \text{diag}\{K_{1i}\}_N$ ,  $K_2 = \text{diag}\{K_{2i}\}_N$ ,  $\mathcal{N} = \text{diag}\{\mathcal{N}_i\}_N$ ,  $\mathcal{F}(t) = \text{diag}\{\mathcal{F}_i(t)\}_N$ ,  $\mathcal{M}^{(1)} = \text{diag}\{\mathcal{M}_i^{(1)}\}_N$ ,  $\mathcal{M}^{(2)} = \text{diag}\{\mathcal{M}_i^{(2)}\}_N$ , and  $\bar{H} = H \otimes I_n$ .

From (6) the error system with the initial condition can be written as

$$\begin{aligned} \tilde{\mathcal{X}}(t) = & \bar{A} \tilde{\mathcal{X}}(t) + \beta_0 \bar{B} \tilde{f}(\tilde{\mathcal{X}}(t), t) + (1 - \beta_0) \bar{C} \tilde{g}(\tilde{\mathcal{X}}(t), t) \\ & + (\beta(t) - \beta_0) [\bar{B} \tilde{f}(\tilde{\mathcal{X}}(t), t) - \bar{C} \tilde{g}(\tilde{\mathcal{X}}(t), t)] + h(x_0(t)) \\ & + K_1 \bar{H} \tilde{\mathcal{X}}(t - q(t)) + K_2 \bar{H} \tilde{\mathcal{X}}(t - q(t) - \rho) \\ & + \mathcal{N} \mathcal{F}(t) [\mathcal{M}^{(1)} \bar{H} \tilde{\mathcal{X}}(t - q(t)) + \mathcal{M}^{(2)} \bar{H} \tilde{\mathcal{X}}(t - q(t) - \rho)] \\ \tilde{\mathcal{X}}(\theta) = & \phi(\theta), \quad \theta \in [-q^*, 0], \quad t_k \leq t < t_{k+1}, \end{aligned} \quad (7)$$

where  $q^* = \max\{\hat{q}, \rho\}$  and  $\phi(\theta)$  be the difference between the initial conditions in (1) and (3) in the interval  $[-q^*, 0]$ . Based on  $h_i(x_0(t))$ , we have

$$h(x_0(t)) = \tilde{A} x_0(t) + \beta(t) \tilde{B} f(x_0(t), t) + (1 - \beta(t)) \tilde{C} g(x_0(t), t),$$

where  $\tilde{A} = \text{col}\{\tilde{A}_i\}_N$ ,  $\tilde{B} = \text{col}\{\tilde{B}_i\}_N$ , and  $\tilde{C} = \text{col}\{\tilde{C}_i\}_N$  with  $\tilde{A}_i = A_i - A$ ,  $\tilde{B}_i = B_i - B$ , and  $\tilde{C}_i = C_i - C$ . The following can be obtained from Assumptions 1 and 3,

$$\begin{aligned} \|h(x_0(t))\| \leq & \|\tilde{A} x_0(t)\| + \|\beta(t) \tilde{B} f(x_0(t), t)\| \\ & + \|(1 - \beta(t)) \tilde{C} g(x_0(t), t)\| \\ \leq & \left( \|\tilde{A}\| + \|\beta(t)\| \|\tilde{B}\| \|\Gamma\| + \|(1 - \beta(t))\| \|\tilde{C}\| \|\Upsilon\| \right) \gamma \\ & + \|\beta(t)\| \|\tilde{B}\| \|f(x_0(0), 0)\| \\ & + \|(1 - \beta(t))\| \|\tilde{C}\| \|g(x_0(0), 0)\| \triangleq \delta. \end{aligned}$$

**Definition 1.** [7] For a given scalar  $\tau > 0$ , the heterogeneous MAS (1) and (3) are said to achieve the practical consensus in the mean square sense via the RRSDC (4) if the following condition is satisfied:

$$\lim_{t \rightarrow \infty} d(\tilde{x}(t, \tilde{x}_0), \mathcal{E}(P, \tau)) = 0,$$

where  $\tilde{x}(t, \tilde{x}_0)$  represents the error trajectory (6) with an initial condition  $\tilde{x}_0 \in \mathbb{R}^{Nn}$ ; and  $d(\tilde{x}(t, \tilde{x}_0), \mathcal{E}(P, \tau)) = \inf_{z \in \mathcal{E}(P, \tau)} \|\tilde{x}(t, \tilde{x}_0) - z\|$  represents the distance from  $\tilde{x}(t, \tilde{x}_0)$  to  $\mathcal{E}(P, \tau)$ . Here,  $\mathcal{E}(P, \tau)$  denotes an ellipsoid which defined as

$$\mathcal{E}(P, \tau) \triangleq \left\{ \tilde{x}(t) \in \mathbb{R}^{Nn} : \tilde{x}^T(t) P \tilde{x}(t) \leq \tau \right\}$$

with the matrix  $P > 0$ .

**Lemma 1.** [19] For a given matrix  $R \in \mathbb{R}^{n \times n}$ ,  $R = R^T > 0$ , and a continuously differentiable function  $\omega : [a, b] \rightarrow \mathbb{R}^n$ , the following inequality holds:

$$\int_a^b \dot{\omega}^T(s) R \dot{\omega}(s) ds \geq \frac{1}{b-a} (\Omega_1^T R \Omega_1 + 3 \Omega_2^T R \Omega_2),$$

where  $\Omega_1 = \omega(b) - \omega(a)$ , and  $\Omega_2 = \omega(b) + \omega(a) - \frac{2}{b-a} \int_a^b \omega(s) ds$ .

**Lemma 2.** [37] For any vectors  $x, y \in \mathbb{R}^n$ , matrices  $D, E$ , and  $F$  that are real appropriate dimensional matrices with  $F^T F \leq I$ , and a scalar  $\mu > 0$ , the following inequality holds:

$$2x^T D F E y \leq \mu^{-1} x^T D D^T x + \mu y^T E^T E y.$$

### 3. Main results

In this part, we will investigate the practical consensus of heterogeneous MAS with randomly occurring nonlinear dynamics (1) and (3) under a fixed and switched topology through the designed RRSDC.

For our convenience, the following notations are defined  $\mathcal{I}_l = [0_{n, (l-1)n} \quad I_n \quad 0_{n, (12-l)n}]$ , ( $l = 1, 2, \dots, 12$ ) as a block entry matrices and the other notations are defined as follows:

$$\eta_1(t) = \left[ \tilde{x}^T(t), \quad \tilde{x}^T(t_k), \quad \int_{t_k}^t \tilde{x}^T(s) ds \right]^T,$$

$$\eta_2(t) = \left[ \tilde{x}^T(t_k), \quad \tilde{x}^T(t), \quad \tilde{x}^T(t_k - \rho) \right]^T,$$

$$\eta_3(t) = \tilde{x}^T(t) - \tilde{x}^T(t_k), \quad \eta_4(t) = \left[ \tilde{x}^T(t), \quad 0, \quad \tilde{x}^T(t) \right]^T,$$

$$\eta_5(t) = \left[ \tilde{x}^T(t_k), \quad 0, \quad \tilde{x}^T(t_k - \rho) \right]^T,$$

$$\eta_6(t) = \tilde{x}^T(t) + \tilde{x}^T(t_k) - 2z(t), \quad \eta_7(t) = \tilde{x}^T(t - \rho) - \tilde{x}^T(t_k - \rho),$$

$$\eta_8(t) = \tilde{x}^T(t - \rho) + \tilde{x}^T(t_k - \rho) - 2\bar{z}(t),$$

$$z(t) = \begin{cases} \frac{1}{t-t_k} \int_{t_k}^t \tilde{x}(s) ds, & t \neq t_k \\ \tilde{x}(t_k), & t = t_k, \end{cases}$$

$$\bar{z}(t) = \begin{cases} \frac{1}{t-t_k-\rho} \int_{t_k-\rho}^{t-\rho} \tilde{x}(s) ds, & t \neq t_k \\ \tilde{x}(t_k - \rho), & t = t_k, \end{cases}$$

$$\Xi = \begin{bmatrix} \frac{X_1^T + X_1 - X_2^T - X_2}{2} & X_2 & X_3 \\ * & \frac{-X_1^T - X_1 - X_2^T - X_2}{2} & X_4 \\ * & * & X_5^T + X_5 \end{bmatrix}.$$

#### 3.1. Fixed topology

In order to derive the practical consensus of the system (7), we consider the following LKF including WIBD term  $V_w(t)$

$$V(t) = V_c(t) + V_w(t), \quad t_k \leq t < t_{k+1}, \quad (8)$$

where

$$V_c(t) = \sum_{l=1}^4 V_l(t) \text{ and } V_w(t) = V_5(t) + V_6(t)$$

with

$$V_1(t) = \tilde{x}^T(t) P \tilde{x}(t),$$

$$V_2(t) = (t_{k+1} - t) \eta_1^T(t) \Xi \eta_1(t) + (q_k - q(t)) q(t) \begin{bmatrix} z(t) \\ \bar{z}(t) \end{bmatrix}^T U \begin{bmatrix} z(t) \\ \bar{z}(t) \end{bmatrix},$$

$$V_3(t) = \int_{t_k}^t e^{a(s-t)} \eta_2^T(s) Q \eta_2(s) ds + \int_{t-\rho}^t e^{a(s-t)} \tilde{x}^T(s) Y \tilde{x}(s) ds,$$

$$V_4(t) = \int_{t_k}^t e^{a(s-t)} \tilde{x}^T(s) R_1 \tilde{x}(s) ds + \int_{t-\rho}^t e^{a(s-t)} \tilde{x}^T(s) R_2 \tilde{x}(s) ds \\ + \rho \int_{-\rho}^0 \int_{t+\theta}^t e^{a(s-t)} \tilde{x}^T(s) R_3 \tilde{x}(s) ds d\theta,$$

$$V_5(t) = \tilde{q}^2 \int_{t_k}^t e^{a(s-t)} \tilde{x}^T(s) R_4 \tilde{x}(s) ds - q(t) \eta_3^T(t) R_4 \eta_3(t) \\ - 3q(t) \eta_6^T(t) R_4 \eta_6(t),$$

$$V_6(t) = \tilde{q}^2 \int_{t_k-\rho}^{t-\rho} e^{a(s-t)} \tilde{x}^T(s) R_5 \tilde{x}(s) ds - q(t) \eta_7^T(t) R_5 \eta_7(t) \\ - 3q(t) \eta_8^T(t) R_5 \eta_8(t).$$

The following lemma is very useful for finding the practical consensus for the proposed system.

**Lemma 3.** If there exist scalars  $a > 0$ ,  $b > 0$ , and symmetric matrices  $P > 0$ ,  $Y > 0$ ,  $R_1 > 0$ ,  $R_2 > 0$ ,  $R_3 > 0$ ,  $R_4 > 0$ ,  $R_5 > 0$ ,  $Q_{22} > 0$ ,

$Q = \begin{bmatrix} Q_{11} & Q_{12} & Q_{13} \\ * & Q_{22} & Q_{23} \\ * & * & Q_{33} \end{bmatrix}$ ,  $U = \begin{bmatrix} U_{11} & U_{12} \\ * & U_{22} \end{bmatrix}$ , and any matrices  $X_l$  ( $l = 1, 2, \dots, 5$ ) such that the LKF (8) along the trajectories of (7) satisfies

$$\mathcal{W}(t) \triangleq \mathcal{L}V(t) + aV(t) - bh^T(x_0(t))h(x_0(t)) \leq 0, \quad (9)$$

then the system (7) will exponentially converge to the following ellipsoid

$$\mathcal{E}_\infty \triangleq \left\{ \tilde{x}(t) \in \mathbb{R}^{Nn} : \tilde{x}^T(t) P \tilde{x}(t) \leq \frac{b}{a} \delta^2 \right\}$$

with a decay rate  $a/2$ .

**Proof.** Multiply  $e^{at}$  on both sides of (9), we get

$$\mathcal{L}(e^{at}V(t)) = e^{at}\mathcal{L}V(t) + ae^{at}V(t) \leq be^{at}h^T(x_0(t))h(x_0(t)). \quad (10)$$

Integrating (10) from 0 to  $t > 0$ , we have

$$V(t) \leq e^{-at} \left( V(0) + \int_0^t be^{as} h^T(x_0(s))h(x_0(s)) ds \right) \\ \leq e^{-at} \left( V(0) + b\delta^2 \int_0^t e^{as} ds \right) \\ = \frac{b\delta^2}{a} (1 - e^{-at}) + e^{-at}V(0). \quad (11)$$

Since  $\tilde{x}^T(t) P \tilde{x}(t) \leq V(t)$  and (11), we have  $\lim_{t \rightarrow \infty} \tilde{x}^T(t) P \tilde{x}(t) \leq \frac{b}{a} \delta^2$ . It ensures that the system (7) exponentially converges to the ellipsoid  $\mathcal{E}_\infty$  with the decay of  $a/2$ .  $\square$

**Theorem 1.** Given positive scalars  $a, \beta_0, \hat{q}, \rho$  and under Assumptions 1–3, the MASs (1) and (3) together with RRSDC (4) is exponentially achieved the practical leader-following consensus in the converging area  $\mathcal{E}_\infty$  at a decay rate  $a/2$ , if there exist positive scalars  $b, \mu$ , symmetric matrices  $P > 0$ ,  $Y > 0$ ,  $R_l > 0$  ( $l = 1, 2, \dots, 5$ ),  $Q_{22} > 0$ ,

$Q = \begin{bmatrix} Q_{11} & Q_{12} & Q_{13} \\ * & Q_{22} & Q_{23} \\ * & * & Q_{33} \end{bmatrix}$ ,  $U = \begin{bmatrix} U_{11} & U_{12} \\ * & U_{22} \end{bmatrix}$  and any matrices  $X_l$  ( $l =$

1, 2, \dots, 5),  $S$ , diagonal real matrices  $\Theta, \bar{\Theta} > 0$ ,  $\check{K}_1 = \text{diag}\{\check{K}_{1i}\}_N$ ,  $\check{K}_2 = \text{diag}\{\check{K}_{2i}\}_N$  such that the following inequalities hold:

$$\begin{bmatrix} \bar{\Sigma}_1 + \hat{q} \Sigma_2 & \Pi \mathcal{N} \\ * & -\mu I \end{bmatrix} < 0, \quad (12)$$

$$\begin{bmatrix} \bar{\Sigma}_1 + \hat{q} \Sigma_3 & \Pi \mathcal{N} \\ * & -\mu I \end{bmatrix} < 0, \quad (13)$$

where  $\Pi = \mathcal{I}_1^T S^T + \mathcal{I}_8^T S^T$ ,

$$\begin{aligned} \bar{\Sigma}_1 = & \text{Sym} \left\{ \mathcal{I}_1^T P \mathcal{I}_8 \right\} + \mathcal{I}_1^T (aP + R_1 + R_2 + I_N \otimes \Gamma \Theta \Gamma + I_N \otimes \Upsilon \bar{\Theta} \Upsilon) \mathcal{I}_1 \\ & - [\mathcal{I}_1, \mathcal{I}_2, \mathcal{I}_3]^T \Xi [\mathcal{I}_1, \mathcal{I}_2, \mathcal{I}_3] + [\mathcal{I}_2, \mathcal{I}_8, \mathcal{I}_6]^T Q [\mathcal{I}_2, \mathcal{I}_8, \mathcal{I}_6] \\ & - e^{-a\rho} \mathcal{I}_5^T R_2 \mathcal{I}_5 + \mathcal{I}_8^T (\rho^2 R_3 + \hat{q}^2 R_4 + Y) \mathcal{I}_8 - b \mathcal{I}_{12}^T \mathcal{I}_{12} \\ & - e^{-a\rho} (\mathcal{I}_3^T Y \mathcal{I}_9 - \hat{q}^2 \mathcal{I}_3^T R_5 \mathcal{I}_9) - \mathcal{I}_{10}^T (I_N \otimes \Theta) \mathcal{I}_{10} - \mathcal{I}_{11}^T (I_N \otimes \bar{\Theta}) \mathcal{I}_{11} \\ & - e^{-a\rho} [\mathcal{I}_1 - \mathcal{I}_5]^T R_3 [\mathcal{I}_1 - \mathcal{I}_5] \\ & - 3[\mathcal{I}_1 + \mathcal{I}_2 - 2\mathcal{I}_4]^T R_4 [\mathcal{I}_1 + \mathcal{I}_2 - 2\mathcal{I}_4] \\ & - 6\text{Sym} \left\{ [\mathcal{I}_1 + \mathcal{I}_2 - 2\mathcal{I}_4]^T R_4 [\mathcal{I}_4 - \mathcal{I}_1] \right\} \\ & - [\mathcal{I}_5 - \mathcal{I}_6]^T R_5 [\mathcal{I}_5 - \mathcal{I}_6] - [\mathcal{I}_1 - \mathcal{I}_2]^T R_4 [\mathcal{I}_1 - \mathcal{I}_2] \\ & - 3[\mathcal{I}_5 + \mathcal{I}_6 - 2\mathcal{I}_7]^T R_5 [\mathcal{I}_5 + \mathcal{I}_6 - 2\mathcal{I}_7] \\ & - 6\text{Sym} \left\{ [\mathcal{I}_5 + \mathcal{I}_6 - 2\mathcal{I}_7]^T R_5 [\mathcal{I}_7 - \mathcal{I}_5] \right\} \\ & + \mu [\mathcal{M}^{(1)} \bar{H} \mathcal{I}_2 + \mathcal{M}^{(2)} \bar{H} \mathcal{I}_6]^T [\mathcal{M}^{(1)} \bar{H} \mathcal{I}_2 + \mathcal{M}^{(2)} \bar{H} \mathcal{I}_6] \\ & + \text{Sym} \left\{ -\Pi \mathcal{I}_8 + \Pi \bar{A} \mathcal{I}_1 + \beta_0 \Pi \bar{B} \mathcal{I}_{10} + (1 - \beta_0) \Pi \bar{C} \mathcal{I}_{11} \right. \\ & \left. + \Pi \mathcal{I}_{12} + (\mathcal{I}_1^T + \mathcal{I}_8^T) (\check{K}_1 \bar{H} \mathcal{I}_2 + \check{K}_2 \bar{H} \mathcal{I}_6) \right\}, \end{aligned}$$

$$\begin{aligned} \Sigma_2 = & 2 \begin{bmatrix} \mathcal{I}_1 \\ \mathcal{I}_2 \\ \mathcal{I}_3 \end{bmatrix}^T \Xi \begin{bmatrix} \mathcal{I}_8 \\ 0 \\ \mathcal{I}_1 \end{bmatrix} + a \begin{bmatrix} \mathcal{I}_1 \\ \mathcal{I}_2 \\ \mathcal{I}_3 \end{bmatrix}^T \Xi \begin{bmatrix} \mathcal{I}_1 \\ \mathcal{I}_2 \\ \mathcal{I}_3 \end{bmatrix} + \begin{bmatrix} \mathcal{I}_4 \\ \mathcal{I}_7 \end{bmatrix}^T U \begin{bmatrix} \mathcal{I}_4 \\ \mathcal{I}_7 \end{bmatrix} \\ & + 2 \begin{bmatrix} \mathcal{I}_4 \\ \mathcal{I}_7 \end{bmatrix}^T U \begin{bmatrix} -\mathcal{I}_4 + \mathcal{I}_1 \\ -\mathcal{I}_7 + \mathcal{I}_5 \end{bmatrix} + a \hat{q} \begin{bmatrix} \mathcal{I}_4 \\ \mathcal{I}_7 \end{bmatrix}^T U \begin{bmatrix} \mathcal{I}_4 \\ \mathcal{I}_7 \end{bmatrix}, \end{aligned}$$

$$\begin{aligned} \Sigma_3 = & -\text{Sym} \left\{ [\mathcal{I}_1 - \mathcal{I}_2]^T R_4 \mathcal{I}_8 \right\} - \text{Sym} \left\{ [\mathcal{I}_5 - \mathcal{I}_7]^T R_5 \mathcal{I}_9 \right\} \\ & - 3\text{Sym} \left\{ [\mathcal{I}_1 + \mathcal{I}_2 - 2\mathcal{I}_4]^T R_4 \mathcal{I}_8 \right\} \\ & - [\mathcal{I}_4, \mathcal{I}_7]^T U [\mathcal{I}_4, \mathcal{I}_7] - 3\text{Sym} \left\{ [\mathcal{I}_5 + \mathcal{I}_6 - 2\mathcal{I}_7]^T R_5 \mathcal{I}_9 \right\} \\ & - a \left\{ 3[\mathcal{I}_5 + \mathcal{I}_6 - 2\mathcal{I}_7]^T R_5 [\mathcal{I}_5 + \mathcal{I}_6 - 2\mathcal{I}_7] \right. \\ & \left. + [\mathcal{I}_1 - \mathcal{I}_2]^T R_4 [\mathcal{I}_1 - \mathcal{I}_2] + [\mathcal{I}_5 - \mathcal{I}_6]^T R_5 [\mathcal{I}_5 - \mathcal{I}_6] \right. \\ & \left. + 3[\mathcal{I}_1 + \mathcal{I}_2 - 2\mathcal{I}_4]^T R_4 [\mathcal{I}_1 + \mathcal{I}_2 - 2\mathcal{I}_4] \right\}. \end{aligned}$$

Moreover, the RRSDC gain matrices are given by  $K_1 = S^{-T} \check{K}_1$ ,  $K_2 = S^{-T} \check{K}_2$ .

**Proof.** For  $t_k \leq t < t_{k+1}$ , taking the infinitesimal operator  $\mathcal{L}$  [17] of  $V(t)$  in (8) along the state trajectories of the system (7), we get

$$\mathbb{E}\{\mathcal{L}V_1(t)\} = 2\check{\mathcal{X}}^T(t)P\check{\mathcal{X}}(t), \quad (14)$$

$$\begin{aligned} \mathbb{E}\{\mathcal{L}V_2(t)\} = & (t_{k+1} - t)\eta_1^T(t)(2\Xi\eta_4(t) + \Xi\eta_1(t)) \\ & - \eta_1^T(t)\Xi\eta_1(t) + (q_k - 2q(t)) \begin{bmatrix} z(t) \\ \bar{z}(t) \end{bmatrix}^T U \begin{bmatrix} z(t) \\ \bar{z}(t) \end{bmatrix} \\ & + 2(q_k - q(t)) \begin{bmatrix} z(t) \\ \bar{z}(t) \end{bmatrix}^T U \begin{bmatrix} -z(t) + \check{x}(t) \\ -\bar{z}(t) + \check{x}(t - \rho) \end{bmatrix}, \quad (15) \end{aligned}$$

$$\mathbb{E}\{\mathcal{L}V_3(t)\} = -a \int_{t_k}^t e^{a(s-t)} \eta_2^T(s) Q \eta_2(s) ds + \eta_2^T(t) Q \eta_2(t)$$

$$\begin{aligned} & - a \int_{t-\rho}^t e^{a(s-t)} \check{\mathcal{X}}^T(s) Y \check{\mathcal{X}}(s) ds + \check{\mathcal{X}}^T(t) Y \check{\mathcal{X}}(t) \\ & - e^{-a\rho} \check{\mathcal{X}}^T(t - \rho) Y \check{\mathcal{X}}(t - \rho), \quad (16) \end{aligned}$$

$$\begin{aligned} \mathbb{E}\{\mathcal{L}V_4(t)\} = & -a \int_{t_k}^t e^{a(s-t)} \check{\mathcal{X}}^T(s) R_1 \check{\mathcal{X}}(s) ds + \check{\mathcal{X}}^T(t) (R_1 + R_2) \check{\mathcal{X}}(t) \\ & - a\rho \int_{-\rho}^0 \int_{t+\theta}^t e^{a(s-t)} \check{\mathcal{X}}^T(s) R_3 \check{\mathcal{X}}(s) ds d\theta \\ & - \int_{t-\rho}^t e^{a(s-t)} \left( a\check{\mathcal{X}}^T(s) R_2 \check{\mathcal{X}}(s) + \rho \check{\mathcal{X}}^T(s) R_3 \check{\mathcal{X}}(s) \right) ds \\ & - e^{-a\rho} \check{\mathcal{X}}^T(t - \rho) R_2 \check{\mathcal{X}}(t - \rho) \\ & + \rho^2 \check{\mathcal{X}}^T(t) R_3 \check{\mathcal{X}}(t), \quad (17) \end{aligned}$$

$$\begin{aligned} \mathbb{E}\{\mathcal{L}V_5(t)\} = & -\hat{q}^2 a \int_{t_k}^t e^{a(s-t)} \check{\mathcal{X}}^T(s) R_4 \check{\mathcal{X}}(s) ds + \hat{q}^2 \check{\mathcal{X}}^T(t) R_4 \check{\mathcal{X}}(t) \\ & - \eta_3^T(t) R_4 \eta_3(t) - 2q(t) \eta_3^T(t) R_4 \check{\mathcal{X}}(t) \\ & - 3\eta_6^T(t) R_4 \eta_6(t) - 6\eta_6^T(t) R_4 [q(t) \check{\mathcal{X}}(t) \\ & + 2z(t) - 2\check{x}(t)], \quad (18) \end{aligned}$$

$$\begin{aligned} \mathbb{E}\{\mathcal{L}V_6(t)\} = & -\hat{q}^2 a \int_{t_k-\rho}^{t-\rho} e^{a(s-t)} \check{\mathcal{X}}^T(s) R_5 \check{\mathcal{X}}(s) ds - \eta_7^T(t) R_5 \eta_7(t) \\ & + \hat{q}^2 e^{-a\rho} \check{\mathcal{X}}^T(t - \rho) R_5 \check{\mathcal{X}}(t - \rho) \\ & - 2q(t) \eta_7^T(t) R_5 \check{\mathcal{X}}(t - \rho) - 3\eta_8^T(t) R_5 \eta_8(t) \\ & - 6\eta_8^T(t) R_5 \left[ q(t) \check{\mathcal{X}}(t - \rho) + 2\bar{z}(t) - 2\check{x}(t - \rho) \right]. \quad (19) \end{aligned}$$

Substituting the above-obtained derivatives of  $V(t)$  to inequality (9), we get

$$\begin{aligned} \mathbb{E}\{\mathcal{W}(t)\} \leq & 2\check{\mathcal{X}}^T(t)P\check{\mathcal{X}}(t) + \check{\mathcal{X}}^T(t)(aP + R_1 + R_2)\check{\mathcal{X}}(t) \\ & + 2(t_{k+1} - t)\eta_1^T(t)\Xi\eta_4(t) - \eta_1^T(t)\Xi\eta_1(t) \\ & + (q_k - 2q(t)) \begin{bmatrix} z(t) \\ \bar{z}(t) \end{bmatrix}^T U \begin{bmatrix} z(t) \\ \bar{z}(t) \end{bmatrix} \\ & + 2(q_k - q(t)) \begin{bmatrix} z(t) \\ \bar{z}(t) \end{bmatrix}^T U \begin{bmatrix} -z(t) + \check{x}(t) \\ -\bar{z}(t) + \check{x}(t - \rho) \end{bmatrix} \\ & + \eta_2^T(t)Q\eta_2(t) + a(t_{k+1} - t)\eta_1^T(t)\Xi\eta_1(t) \\ & - e^{-a\rho} \check{\mathcal{X}}^T(t - \rho) R_2 \check{\mathcal{X}}(t - \rho) - \eta_3^T(t) R_4 \eta_3(t) \\ & + \check{\mathcal{X}}^T(t) (\rho^2 R_3 + \hat{q}^2 R_4 + Y) \check{\mathcal{X}}(t) - 2q(t) \eta_3^T(t) R_4 \check{\mathcal{X}}(t) \\ & - 3\eta_6^T(t) R_4 \eta_6(t) - 3\eta_6^T(t) R_5 \eta_8(t) \\ & - 6\eta_6^T(t) R_4 [q(t) \check{\mathcal{X}}(t) + 2z(t) - 2\check{x}(t)] \\ & + \hat{q}^2 e^{-a\rho} \check{\mathcal{X}}^T(t - \rho) R_5 \check{\mathcal{X}}(t - \rho) - \eta_7^T(t) R_5 \eta_7(t) \\ & - 2q(t) \eta_7^T(t) R_5 \check{\mathcal{X}}(t - \rho) - aq(t) \eta_3^T(t) R_4 \eta_3(t) \\ & - 6\eta_8^T(t) R_5 [q(t) \check{\mathcal{X}}(t - \rho) + 2\bar{z}(t) - 2\check{x}(t - \rho)] \\ & - e^{-a\rho} \check{\mathcal{X}}^T(t - \rho) Y \check{\mathcal{X}}(t - \rho) - 3aq(t) \eta_6^T(t) R_4 \eta_6(t) \\ & - aq(t) \eta_7^T(t) R_5 \eta_7(t) - 3aq(t) \eta_8^T(t) R_5 \eta_8(t) \\ & + a(q_k - q(t))q(t) \begin{bmatrix} z(t) \\ \bar{z}(t) \end{bmatrix}^T U \begin{bmatrix} z(t) \\ \bar{z}(t) \end{bmatrix} \\ & + e^{-a\rho} \xi - bh^T(x_0(t))h(x_0(t)), \quad (20) \end{aligned}$$

where  $\xi = -\rho \int_{t-\rho}^t \tilde{x}^T(s) R_3 \tilde{x}(s) ds$ . Now, we define

$$\zeta(t) = \begin{bmatrix} \tilde{x}^T(t), \tilde{x}^T(t-q(t)), \int_{t_k}^t \tilde{x}^T(s) ds, z^T(t), \tilde{x}^T(t-\rho), \\ \tilde{x}^T(t-q(t)-\rho), \tilde{z}^T(t), \tilde{x}^T(t), \\ \tilde{x}^T(t-\rho), \tilde{f}^T(\tilde{x}(t), t), \tilde{g}^T(\tilde{x}(t), t), h^T(x_0(t)) \end{bmatrix}^T.$$

Based on the Jensen's inequality, the  $\xi$  in (20) can be written as

$$-\rho \int_{t-\rho}^t \tilde{x}^T(s) R_3 \tilde{x}(s) ds \leq -[\tilde{x}(t) - \tilde{x}(t-\rho)]^T R_3 [\tilde{x}(t) - \tilde{x}(t-\rho)]. \quad (21)$$

For a system (7), the following equation holds for any appropriate dimensional matrix  $S$ :

$$\begin{aligned} 0 = & 2\Omega(t) \left\{ -\tilde{x}(t) + \tilde{A}\tilde{x}(t) + \beta_0 \tilde{B}\tilde{f}(\tilde{x}(t), t) \right. \\ & + (1 - \beta_0) \tilde{C}\tilde{g}(\tilde{x}(t), t) + (\beta(t) - \beta_0) \\ & \times [\tilde{B}\tilde{f}(\tilde{x}(t), t) - \tilde{C}\tilde{g}(\tilde{x}(t), t)] + K_1 \tilde{H}\tilde{x}(t - q(t)) \\ & + K_2 \tilde{H}\tilde{x}(t - q(t) - \rho) + h(x_0(t)) \\ & \left. + \mathcal{N}\mathcal{F}(t) [\mathcal{M}^{(1)} \tilde{H}\tilde{x}(t - q(t)) + \mathcal{M}^{(2)} \tilde{H}\tilde{x}(t - q(t) - \rho)] \right\}, \quad (22) \end{aligned}$$

where  $\Omega(t) = \tilde{x}^T(t) S^T + \dot{\tilde{x}}^T(t) S^T$ . By utilizing Lemma 2, one can get the following inequality from (22),

$$\begin{aligned} 2\Omega(t) \mathcal{N}\mathcal{F}(t) & \left[ \mathcal{M}^{(1)} \tilde{H}\tilde{x}(t - q(t)) + \mathcal{M}^{(2)} \tilde{H}\tilde{x}(t - q(t) - \rho) \right] \\ & \leq \mu^{-1} \Omega(t) \mathcal{N}\mathcal{N}^T \Omega^T(t) + \mu \left[ \mathcal{M}^{(1)} \tilde{H}\tilde{x}(t - q(t)) \right. \\ & \quad \left. + \mathcal{M}^{(2)} \tilde{H}\tilde{x}(t - q(t) - \rho) \right]^T \\ & \quad \times \left[ \mathcal{M}^{(1)} \tilde{H}\tilde{x}(t - q(t)) + \mathcal{M}^{(2)} \tilde{H}\tilde{x}(t - q(t) - \rho) \right]. \quad (23) \end{aligned}$$

Now, the nonlinear function  $f(x_i(t), t)$  and  $g(x_i(t), t)$  satisfies the Lipschitz condition (2), which implies that for a diagonal matrix  $\Theta, \tilde{\Theta} > 0$ ,

$$\tilde{f}^T(\tilde{x}(t), t) (I_N \otimes \Theta) \tilde{f}(\tilde{x}(t), t) \leq \tilde{x}^T(t) (I_N \otimes \Gamma \Theta \Gamma) \tilde{x}(t), \quad (24)$$

$$\tilde{g}^T(\tilde{x}(t), t) (I_N \otimes \tilde{\Theta}) \tilde{g}(\tilde{x}(t), t) \leq \tilde{x}^T(t) (I_N \otimes \Upsilon \tilde{\Theta} \Upsilon) \tilde{x}(t). \quad (25)$$

Let us define  $\check{K}_1 = S^T K_1$  and  $\check{K}_2 = S^T K_2$ , then combining (20)-(25) for  $t_k \leq t < t_{k+1}$ , which gives that

$$\begin{aligned} \mathbb{E}\{\mathcal{W}(t)\} & \leq \zeta^T(t) \left\{ \Sigma_1 + (t_{k+1} - t) \Sigma_2 + (t - t_k) \Sigma_3 \right\} \zeta(t) \\ & = \zeta^T(t) \sqsupset \zeta(t), \end{aligned}$$

where  $\Sigma_1 = \tilde{\Sigma}_1 + \mu^{-1} \Pi \mathcal{N} \mathcal{N}^T \Pi^T$ . It is clear that  $\mathbb{E}\{\mathcal{W}(t)\} < 0$  if  $\sqsupset < 0$ . By the convex combination method, we obtain  $\sqsupset < 0$  for  $t_k \leq t < t_{k+1}$ . Applying Schur complement to  $\sqsupset < 0$  we get the inequalities (12) and (13). We can conclude from Lemma 3,  $\lim_{t \rightarrow \infty} d(\tilde{x}(t), \tilde{x}_0, \mathcal{E}(P, (b/a)\delta^2)) = 0$ . This completes the proof of Theorem 1.  $\square$

**Remark 2.** Note that  $\lim_{t \rightarrow t_k} V_l(t) = V_l(t_k) \geq 0$ ,  $l = 1, 3, 4$ . For the looped LKF  $V_2(t)$  satisfies  $\lim_{t \rightarrow t_k^-} V_2(t) = \lim_{t \rightarrow t_k^+} V_2(t) = V_2(t_k) = 0$ . Since  $\lim_{t \rightarrow t_k} V_c(t) = V_c(t_k) \geq 0$ , therefore  $V_c(t)$  is continuous in time. Moreover, by Lemma 1  $\lim_{t \rightarrow t_k} V_l(t) \geq V_l(t_k) = 0$ ,  $l = 5, 6$ . The positive-definiteness of  $V_l(t)$ ,  $l = 1, 3, 4$  can be easily obtained by

the positive-definite matrices  $P, Q, Y, R_1, R_2, R_3$ . When utilizing Lemma 1 and the matrix  $R_4 > 0$ , the positive-definite of  $V_5(t)$  can be obtained as follows:

$$\begin{aligned} V_5(t) & \geq q(t)^2 \int_{t_k}^t e^{a(s-t)} \tilde{x}^T(s) R_4 \tilde{x}(s) ds - q(t) \eta_3^T(t) R_4 \eta_3(t) \\ & \quad - 3q(t) \eta_6^T(t) R_4 \eta_6(t) \\ & = q(t) \left[ q(t) \int_{t_k}^t e^{a(s-t)} \tilde{x}^T(s) R_4 \tilde{x}(s) ds - \eta_3^T(t) R_4 \eta_3(t) \right. \\ & \quad \left. - 3\eta_6^T(t) R_4 \eta_6(t) \right] > 0. \end{aligned}$$

Similarly we can verified  $V_6(t)$  is positive-definite for  $(t_k, t_{k+1})$ . Also the looped functional  $V_2(t)$  satisfies  $V_2(t_k) = V_2(t_{k+1}) = 0$ , which provides that it not required to be a positive-definite, so it relaxed the stability condition.

**Remark 3.** We have discussed the practical consensus of heterogeneous MAS with randomly occurring nonlinearities under a fixed topology in the above subsection. In general, communication among the agents may be required to switch according to various practical applications. Hence, it is necessary to study the practical consensus of heterogeneous MAS with switching topology and it is summarized in the following subsection.

### 3.2. Switching topology

In this case, we consider a group of directed graph  $\mathcal{G}(\vartheta(t)) \in \{\mathcal{G}_1, \mathcal{G}_2, \dots, \mathcal{G}_l\}$ , with  $\vartheta(t)$  is the switched signal which has the values in a finite set  $T = \{1, 2, \dots, l\}$ .

**Assumption 4.** Every possible graph  $\mathcal{G}_s, s \in T$  has a directed spanning tree with a root of the leader.

The switching transmission between each agent occurs only at the sampling instant  $t_k, k \in N$ , which means that the graph  $\mathcal{G}(\vartheta(t))$  is sampled at the same sampling time  $t_k$ , i.e.,  $\mathcal{G}(\vartheta(t_k))$  and also which is constant until the next sampling. For the switching topology, RRSDC is given as follows:

$$\begin{aligned} u_i(t) & = (K_{1i}^{\vartheta(t_k)} + \Delta K_{1i}^{\vartheta(t_k)}(t)) \left[ \sum_{j=1}^N w_{ij}^{\vartheta(t_k)} [x_i(t - q(t)) - x_j(t - q(t))] \right. \\ & \quad \left. + m_i^{\vartheta(t_k)} [x_i(t - q(t)) - x_0(t - q(t))] \right] \\ & \quad + (K_{2i}^{\vartheta(t_k)} + \Delta K_{2i}^{\vartheta(t_k)}(t)) \\ & \quad \times \left[ \sum_{j=1}^N w_{ij}^{\vartheta(t_k)} [x_i(t - q(t) - \rho) - x_j(t - q(t) - \rho)] \right. \\ & \quad \left. + m_i^{\vartheta(t_k)} [x_i(t - q(t) - \rho) - x_0(t - q(t) - \rho)] \right], \quad (26) \end{aligned}$$

where  $w_{ij}^{\vartheta(t_k)}$  and  $m_i^{\vartheta(t_k)}$  are weighted coefficients of the graph  $\mathcal{G}(\vartheta(t_k))$  and  $K_{1i}^{\vartheta(t_k)}, K_{2i}^{\vartheta(t_k)}$  are the controller gain matrix depending on switching signal.  $\Delta K_{1i}^{\vartheta(t_k)}, \Delta K_{2i}^{\vartheta(t_k)}$  are the controller gain fluctuations.

From (26), the system (7) can be modified as follows

$$\begin{aligned} \dot{\tilde{x}}(t) & = \tilde{A}\tilde{x}(t) + \beta_0 \tilde{B}\tilde{f}(\tilde{x}(t), t) + (1 - \beta_0) \tilde{C}\tilde{g}(\tilde{x}(t), t) \\ & \quad + (\beta(t) - \beta_0) [\tilde{B}\tilde{f}(\tilde{x}(t), t) - \tilde{C}\tilde{g}(\tilde{x}(t), t)] + h(x_0(t)) \\ & \quad + K_{1s} \tilde{H}_s \tilde{x}(t - q(t)) + K_{2s} \tilde{H}_s \tilde{x}(t - q(t) - \rho) + \mathcal{N}_s \mathcal{F}_s(t) \\ & \quad \times \left[ \mathcal{M}_s^{(1)} \tilde{H}_s \tilde{x}(t - q(t)) + \mathcal{M}_s^{(2)} \tilde{H}_s \tilde{x}(t - q(t) - \rho) \right] \\ \tilde{x}(\theta) & = \phi(\theta), \quad \theta \in [-q^*, 0], \quad t_k \leq t < t_{k+1}, \quad (27) \end{aligned}$$

where  $\mathcal{M}_s^{(1)}$ ,  $\mathcal{M}_s^{(2)}$ ,  $\mathcal{N}_s$  are known matrices and  $\mathcal{F}_s(t)$  satisfies  $\mathcal{F}_s^T(t)\mathcal{F}_s(t) \leq I$ .  $K_{1s}$ ,  $K_{2s}$  and  $\bar{H}_s$  are the control gain matrices of  $K_1(\vartheta(t_k))$ ,  $K_2(\vartheta(t_k))$  and the weighted matrix  $\bar{H}(\vartheta(t_k))$  corresponding to  $\vartheta(t_k) = s \in T$  respectively.

**Theorem 2.** Suppose that under Assumptions 1, 3 and 4, given positive scalars  $a$ ,  $\beta_0$ ,  $\hat{q}$ ,  $\rho$  there exist scalars  $b > 0$ ,  $\mu > 0$ , symmetric matrices  $P > 0$ ,  $Y > 0$ ,  $R_l > 0$  ( $l = 1, 2, \dots, 5$ ),  $Q_{22} > 0$ ,

$$Q = \begin{bmatrix} Q_{11} & Q_{12} & Q_{13} \\ * & Q_{22} & Q_{23} \\ * & * & Q_{33} \end{bmatrix}, U = \begin{bmatrix} U_{11} & U_{12} \\ * & U_{22} \end{bmatrix} \text{ and any matrices } X_l (l =$$

1, 2, ..., 5),  $S$ , diagonal matrices  $\Theta, \bar{\Theta} > 0$ ,  $\check{K}_{1s} = \text{diag}\{\check{K}_{1s}\}_N$ ,  $\check{K}_{2s} = \text{diag}\{\check{K}_{2s}\}_N$  such that the following LMIs hold:

$$\begin{bmatrix} \bar{\Sigma}_{1s} + \hat{q}\Sigma_2 & \Pi\mathcal{N}_s \\ * & -\mu I \end{bmatrix} < 0, \quad (28)$$

$$\begin{bmatrix} \bar{\Sigma}_{1s} + \hat{q}\Sigma_3 & \Pi\mathcal{N}_s \\ * & -\mu I \end{bmatrix} < 0, \quad (29)$$

where  $\Pi = \mathcal{I}_1^T S^T + \mathcal{I}_8^T S^T$ ,

$$\begin{aligned} \bar{\Sigma}_{1s} = & \text{Sym} \left\{ \mathcal{I}_1^T P_s \mathcal{I}_8 \right\} + \mathcal{I}_1^T (aP_s + R_1 + R_2 + I_N \otimes \Gamma \Theta \Gamma \\ & + I_N \otimes \Upsilon \bar{\Theta} \Upsilon) \mathcal{I}_1 - [\mathcal{I}_1, \mathcal{I}_2, \mathcal{I}_3]^T \Xi [\mathcal{I}_1, \mathcal{I}_2, \mathcal{I}_3] \\ & + [\mathcal{I}_2, \mathcal{I}_8, \mathcal{I}_6]^T Q [\mathcal{I}_2, \mathcal{I}_8, \mathcal{I}_6] - e^{-a\rho} \mathcal{I}_5^T R_2 \mathcal{I}_5 \\ & + \mathcal{I}_8^T (\rho^2 R_3 + \hat{q}^2 R_4 + Y) \mathcal{I}_8 - \mathcal{I}_{10}^T (I_N \otimes \Theta) \mathcal{I}_{10} \\ & - 3[\mathcal{I}_1 + \mathcal{I}_2 - 2\mathcal{I}_4]^T R_4 [\mathcal{I}_1 + \mathcal{I}_2 - 2\mathcal{I}_4] \\ & - 6\text{Sym} \left\{ [\mathcal{I}_1 + \mathcal{I}_2 - 2\mathcal{I}_4]^T R_4 [\mathcal{I}_4 - \mathcal{I}_1] \right\} - b\mathcal{I}_{12}^T \mathcal{I}_{12} \\ & - 3[\mathcal{I}_5 + \mathcal{I}_6 - 2\mathcal{I}_7]^T R_5 [\mathcal{I}_5 + \mathcal{I}_6 - 2\mathcal{I}_7] \\ & - e^{-a\rho} [\mathcal{I}_1 - \mathcal{I}_5]^T R_3 [\mathcal{I}_1 - \mathcal{I}_5] - e^{-a\rho} (\mathcal{I}_9^T Y \mathcal{I}_9 - \hat{q}^2 \mathcal{I}_9^T R_5 \mathcal{I}_9) \\ & - 6\text{Sym} \left\{ [\mathcal{I}_5 + \mathcal{I}_6 - 2\mathcal{I}_7]^T R_5 [\mathcal{I}_7 - \mathcal{I}_5] \right\} \\ & + \mu [\mathcal{M}_s^{(1)} \bar{H}_s \mathcal{I}_2 + \mathcal{M}_s^{(2)} \bar{H}_s \mathcal{I}_6]^T [\mathcal{M}_s^{(1)} \bar{H}_s \mathcal{I}_2 + \mathcal{M}_s^{(2)} \bar{H}_s \mathcal{I}_6] \\ & - [\mathcal{I}_1 - \mathcal{I}_2]^T R_4 [\mathcal{I}_1 - \mathcal{I}_2] - \mathcal{I}_{11}^T (I_N \otimes \bar{\Theta}) \mathcal{I}_{11} \\ & - [\mathcal{I}_5 - \mathcal{I}_6]^T R_5 [\mathcal{I}_5 - \mathcal{I}_6] + \text{Sym} \left\{ -\Pi \mathcal{I}_8 \right. \\ & \left. + \Pi \bar{A} \mathcal{I}_1 + \beta_0 \Pi \bar{B} \mathcal{I}_{10} + (1 - \beta_0) \Pi \bar{C} \mathcal{I}_{11} + \Pi \mathcal{I}_{12} \right. \\ & \left. + (\mathcal{I}_1^T + \mathcal{I}_8^T) \check{K}_{1s} \bar{H}_s \mathcal{I}_2 + (\mathcal{I}_1^T + \mathcal{I}_8^T) \check{K}_{2s} \bar{H}_s \mathcal{I}_6 \right\}, \end{aligned}$$

$\Sigma_2$  and  $\Sigma_3$  are given in Theorem 1. Then exponentially achieved the practical leader-following consensus for the MASs (1) and (3) under the controller (26) with the convergence region  $\mathcal{E}_\infty$  is given by

$$\mathcal{E}_\infty^s \triangleq \left\{ \tilde{x}(t) \in \mathbb{R}^{Nn} : \tilde{x}^T(t) P_s \tilde{x}(t) \leq \frac{b}{a} \delta^2 \right\}, \quad (30)$$

where  $P_s \triangleq \min\{P_s : s \in T\}$  at a decay rate  $a/2$ . Moreover, the sampled-data controller gain matrices are given by  $K_{1s} = S^{-T} \check{K}_{1s}$ ,  $K_{2s} = S^{-T} \check{K}_{2s}$ .

**Proof.** Choose the same LKF in Theorem 1 by replacing  $V_1(t)$  into  $\tilde{x}^T(t) P_s \tilde{x}(t)$ . Then, define  $\check{K}_{1s} = S^T K_{1s}$  and  $\check{K}_{2s} = S^T K_{2s}$  and all other notations and proof are similar to the Theorem 1. Then, we achieve the practical leader-following consensus for the system (27) in the switching case. This proof is completed.  $\square$

**Remark 4.** The novel features and contributions of this paper lie in the following:

- The random variables are introduced to represent the stochastic nonlinearities in the heterogeneous MAS at any instant of time, which obeys the Bernoulli distribution.

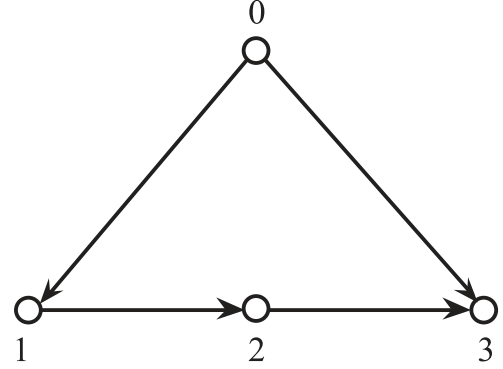


Fig. 1. Directed graph  $\mathcal{G}$ .

- Developing a control protocol for the heterogeneous MAS, the gain fluctuation and constant signal transmission delay are simultaneously considered so that the designed RRSDC scheme is insensitive in these factors.
- For the proposed scheme, the sampling intervals are time-varying and aperiodic, which is more general than the periodic sampling to analyze and achieve the mean square practical consensus for the heterogeneous MAS.
- A novel LKF, including the looped functional and WIBD term is proposed to deploy the available information on the sawtooth structure characteristic of the actual sampling pattern. In the proposed LKF, some matrices are no need to satisfy the positive-definite condition.
- The practical consensus problem for a heterogeneous MAS is investigated under fixed topology. Then, it has been extended for the switched case also.

#### 4. numerical examples

**Example 1.** Consider the generalized Chua's circuit described as in the works of Ding and Zheng [7]

$$\begin{cases} \dot{x}_1 = \kappa_1(x_2 - \alpha_0 x_1 - \nu(x_1)) \\ \dot{x}_2 = x_1 - x_2 + x_3 \\ \dot{x}_3 = -\kappa_2 x_2 - \kappa_3 x_3 \end{cases} \quad (31)$$

with nonlinear function  $\nu(x_1)$  is given by

$$\nu(x_1) = \alpha_2 x_1 + \frac{1}{2}(\alpha_1 - \alpha_2)(|x_1 + 1| - |x_1 - 1|).$$

Now, we consider three follower agents and one leader agent. The directed graph  $\mathcal{G}$  in Fig. 1 represents the topology of the system, which is a directed spanning tree with leader agent node 0. For our convenience, assume that all connecting weights are to be

$$1 \text{ and corresponding } H = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & -1 & 2 \end{bmatrix}. \text{ Consider the randomly}$$

occurring nonlinear functions in (31), the dynamics of each agent  $i$  for system (31) under the control input  $u_i$  with  $i = 1, 2, 3$  is given by

$$\begin{aligned} \dot{x}_i(t) = & A_i x_i(t) + \beta(t) B_i f(x_i(t), t) \\ & + (1 - \beta(t)) C_i g(x_i(t), t) + u_i(t), \end{aligned}$$

$$x_i(t) = [x_{i1}(t), x_{i2}(t), x_{i3}(t)]^T.$$

The resultant parameter values for the dynamics of the follower agents are given by

$$A_i = \begin{bmatrix} a_i & b_i & 0 \\ 1 & -1 & 1 \\ 0 & c_i & 0 \end{bmatrix}, B_i = \begin{bmatrix} d_i & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, C_i = \begin{bmatrix} e_i & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

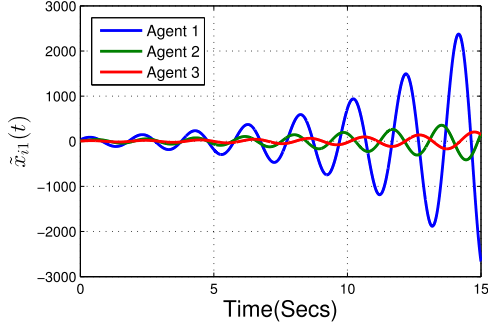


Fig. 2. State trajectories of  $\tilde{x}_{i1}(t)$  without control input.

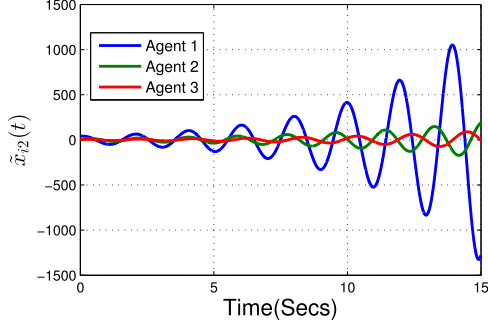


Fig. 3. State trajectories of  $\tilde{x}_{i2}(t)$  without control input.

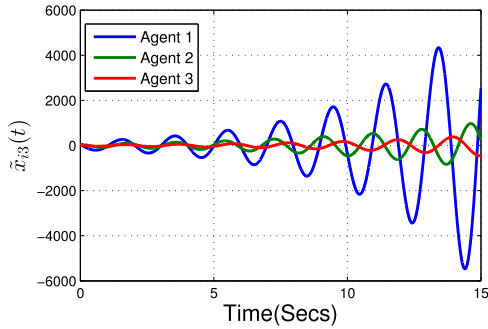


Fig. 4. State trajectories of  $\tilde{x}_{i3}(t)$  without control input.

with  $a_1 = -3.2$ ,  $a_2 = -2.5$ ,  $a_3 = -2.5714$ ,  $b_1 = 10$ ,  $b_2 = 10$ ,  $b_3 = 9$ ,  $c_1 = -14.87$ ,  $c_2 = -18$ ,  $c_3 = -14.286$ ,  $d_1 = 5.9$ ,  $d_2 = 5.833$ ,  $d_3 = 3.8571$ ,  $e_1 = 5.9$ ,  $e_2 = 5.833$ ,  $e_3 = 3.8571$ . The randomly occurring nonlinear functions are given by  $f(x_i) = g(x_i) = [0.5(|x_{i1} + 1| - |x_{i1} - 1|), 0, 0]^T$ . The leader agent parameter values are given by

$$A = \begin{bmatrix} -2.3714 & 8.9000 & 0 \\ 1 & -1 & 1 \\ 0 & -14.4860 & 0 \end{bmatrix}, B = \begin{bmatrix} 4.8571 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

$$C = \begin{bmatrix} 4.8571 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

The state trajectories for the error system (7) without control input (i.e.,  $u_i(t) = 0$ ) is plotted in Figs. 2–4. It is clearly observed that the controller must need for the agents to achieve the consensus with the leader. Now we assume  $a = 0.6$ ,  $\beta_0 = 0.8$ ,  $\hat{q} = 0.06$  and  $\rho = 0.1$ . The uncertain parameter matrices are chosen to be  $\mathcal{M}^{(1)} = \mathcal{M}^{(2)} = \mathcal{N} = 0.09I_9$  and  $\mathcal{F}(t) = I_9 \times 0.09 \sin(t)$ . According to the proposed control technique with the value of the above parameter, the following control gains are obtained by solving the LMIs (12) and

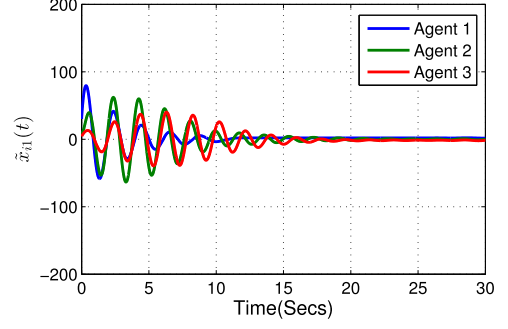


Fig. 5. State trajectories of  $\tilde{x}_{i1}(t)$  with control input (4) for each agent  $i = 1, 2, 3$ .

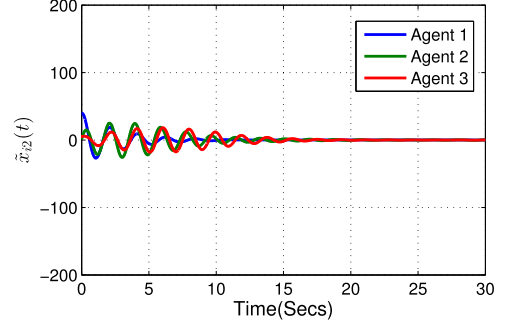


Fig. 6. State trajectories of  $\tilde{x}_{i2}(t)$  with control input (4) for each agent  $i = 1, 2, 3$ .

Table 1

Maximum sampling period  $\hat{q}$  for different values of  $\rho$  in Example 1.

$\rho$	0.10	0.15	0.20	0.30
$\hat{q}$	0.060	0.0585	0.040	0.0345

(13) in Theorem 1,

$$K_{11} = \begin{bmatrix} -0.1153 & -0.0311 & -0.0671 \\ -0.1134 & -0.4922 & -0.0720 \\ -0.0816 & -0.0176 & -0.5323 \end{bmatrix},$$

$$K_{12} = \begin{bmatrix} -0.0573 & -0.0373 & -0.0584 \\ -0.0616 & -0.3058 & -0.0132 \\ -0.0331 & -0.0376 & -0.3227 \end{bmatrix},$$

$$K_{13} = \begin{bmatrix} -0.0379 & -0.0281 & -0.0336 \\ -0.0492 & -0.1953 & 0.0157 \\ -0.0119 & -0.0072 & -0.2086 \end{bmatrix},$$

$$K_{21} = \begin{bmatrix} 0.0009 & -0.0002 & 0.0003 \\ -0.0001 & 0.0002 & 0.0011 \\ -0.0002 & -0.0002 & 0.0007 \end{bmatrix},$$

$$K_{22} = \begin{bmatrix} 0.0003 & -0.0001 & 0.0001 \\ -0.0001 & 0.0001 & 0.0003 \\ -0.0001 & -0.0001 & 0.0004 \end{bmatrix},$$

$$K_{23} = \begin{bmatrix} 0.0003 & -0.0001 & 0.0001 \\ -0.0002 & 0.0000 & 0.0004 \\ -0.0001 & -0.0001 & 0.0003 \end{bmatrix}.$$

Based on the above control gain matrices, the simulation results on the state trajectories for the error system (7) are plotted in Figs. 5–7. Evolution of random nonlinearities and control input trajectories are given in Figs. 8 and 9, respectively. Also, the maximum sampling period is calculated for different values of  $\rho$  with the above same parameter values based on Theorem 1 and it is tabulated in Table 1. We can observe from Table 1, the maximum sampling period is decreased gradually when increase the memory parameter value. From the simulation results, we conclude that the

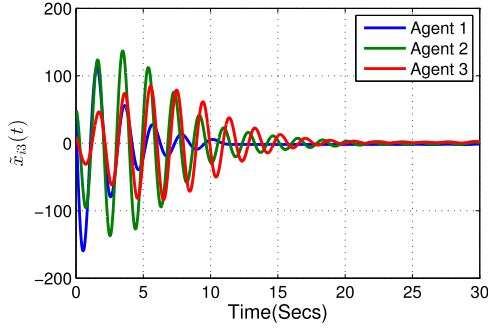


Fig. 7. State trajectories of  $\tilde{x}_{i3}(t)$  with control input (4) for each agent  $i = 1, 2, 3$ .

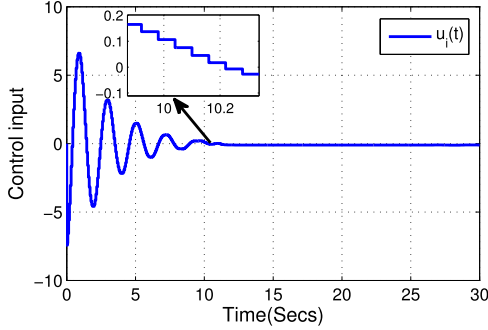


Fig. 8. Control responses of the system in Example 1.

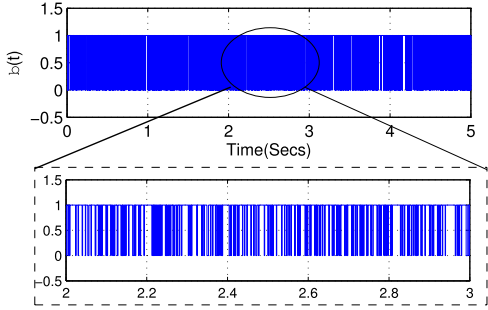


Fig. 9. Evolution of  $\beta(t)$  in Example 1.

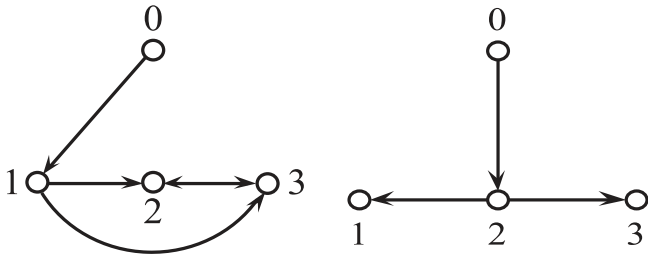


Fig. 10. Two possible switching topologies: (a) Graph  $\mathcal{G}_1$  (b) Graph  $\mathcal{G}_2$ .

practical consensus achieved by all agents together with a leader in a bounded domain.

**Example 2.** In this example, we have to demonstrate that the effectiveness of Theorem 2 for heterogeneous MAS (1) and (3). Consider the system (1) with three follower agents, and their communication topology is assumed to be two different directed graphs which are given in Fig. 10. The dynamics of each follower agents are taken by

$$A_1 = \begin{bmatrix} -10.5 & -0.3 & 1 \\ 0 & -8.3 & 0.7 \\ 0 & 0.6 & -12.5 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} -10.5 & 1 & 0 \\ 1 & -9.8 & 0.9 \\ 0 & -1 & -11.8 \end{bmatrix}, A_3 = \begin{bmatrix} -12.7 & 0 & 1.1 \\ 0.4 & -10.8 & 0 \\ 1.0 & 0 & -12.4 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 3.9 & 0 & 0.4 \\ 0 & 0.3 & 0 \\ 0.2 & 0 & 0.1 \end{bmatrix},$$

$$B_2 = \begin{bmatrix} 1.8 & 0 & 0 \\ 0 & 1.2 & 0 \\ -0.5 & 0 & 1.0 \end{bmatrix}, B_3 = \begin{bmatrix} 3.8 & 0 & 0.7 \\ 0 & -0.6 & 0 \\ 0 & 0.2 & 0 \end{bmatrix},$$

$$C_1 = \begin{bmatrix} 1.9 & 0 & 0.7 \\ 0 & 0 & 0.3 \\ 0.3 & 0 & -0.6 \end{bmatrix}, C_2 = \begin{bmatrix} 2.9 & 0 & 0.2 \\ 0 & 0.2 & 0 \\ -0.6 & 0 & 1.2 \end{bmatrix},$$

$$C_3 = \begin{bmatrix} 3.5 & 0 & 0.4 \\ 0 & 0 & -0.6 \\ 0.2 & 0 & 0.9 \end{bmatrix}.$$

The nonlinear functions are chosen to be  $f(x_i) = g(x_i) = [0.8(|x_{i1} + 1| - |x_{i1} - 1|), 0, 0]^T$ . The parameter values of the leader agent given by

$$A = \begin{bmatrix} -2.9 & 0.4 & -0.1 \\ 1 & -1.5 & 0.8 \\ -0.6 & 0.7 & -3.6 \end{bmatrix}, B = \begin{bmatrix} 7.8 & 0 & 0 \\ 0 & 3.5 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

$$C = \begin{bmatrix} 8.2 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

For two different topology, the corresponding uncertain parameter values are taken by  $\mathcal{M}_1^{(1)} = \mathcal{M}_1^{(2)} = \mathcal{M}_2^{(1)} = \mathcal{M}_2^{(2)} = 0.05I_9$ ,  $\mathcal{N}_1 = \mathcal{N}_2 = 0.01I_9$  and  $\mathcal{F}_1(t) = \mathcal{F}_2(t) = I_9 \times 0.5 \sin(t)$ . Taking  $a = 0.2$ ,  $\beta_0 = 0.8$ ,  $\rho = 0.1$  and solving the LMIs (28)-(29) with the maximum sampling interval  $\hat{q} = 0.01$ , and the corresponding RRSDC gains are

$$K_{11}^{\mathcal{G}_1} = \begin{bmatrix} -0.0055 & -0.0012 & -0.0004 \\ -0.0040 & -0.0040 & -0.0061 \\ -0.0014 & -0.0057 & -0.0051 \end{bmatrix},$$

$$K_{12}^{\mathcal{G}_1} = \begin{bmatrix} -0.0025 & -0.0011 & -0.0002 \\ -0.0020 & -0.0003 & -0.0024 \\ -0.0003 & -0.0015 & -0.0102 \end{bmatrix},$$

$$K_{13}^{\mathcal{G}_1} = \begin{bmatrix} -0.0031 & -0.0006 & -0.0015 \\ -0.0024 & 0.0015 & -0.0091 \\ -0.0010 & -0.0012 & -0.0256 \end{bmatrix},$$

$$K_{21}^{\mathcal{G}_1} = 10^{-4} \times \begin{bmatrix} -0.1982 & -0.0177 & -0.0059 \\ -0.0302 & -0.1951 & -0.0361 \\ -0.0104 & -0.0460 & -0.1935 \end{bmatrix},$$

$$K_{22}^{\mathcal{G}_1} = 10^{-4} \times \begin{bmatrix} -0.0750 & -0.0061 & 0.0008 \\ -0.0103 & -0.0817 & -0.0037 \\ -0.0003 & -0.0052 & -0.0869 \end{bmatrix},$$

$$K_{23}^{\mathcal{G}_1} = 10^{-4} \times \begin{bmatrix} -0.0594 & -0.0035 & -0.0045 \\ -0.0097 & -0.0748 & -0.0173 \\ -0.0049 & -0.0151 & -0.1644 \end{bmatrix},$$

$$K_{11}^{\mathcal{G}_2} = \begin{bmatrix} -0.0034 & -0.0007 & -0.0003 \\ -0.0026 & -0.0026 & -0.0043 \\ -0.0010 & -0.0039 & -0.0035 \end{bmatrix},$$

$$K_{12}^{\mathcal{G}_2} = \begin{bmatrix} -0.0058 & -0.0025 & -0.0007 \\ -0.0051 & -0.0006 & -0.0064 \\ -0.0009 & -0.0039 & -0.0286 \end{bmatrix},$$

$$K_{13}^{\mathcal{G}_2} = \begin{bmatrix} -0.0051 & -0.0010 & -0.0028 \\ -0.0041 & 0.0026 & -0.0164 \\ -0.0019 & -0.0024 & -0.0489 \end{bmatrix},$$

$$K_{21}^{\mathcal{G}_2} = 10^{-4} \times \begin{bmatrix} -0.1044 & -0.0082 & -0.0026 \\ -0.0116 & -0.1080 & -0.0116 \\ -0.0036 & -0.0179 & -0.1037 \end{bmatrix},$$

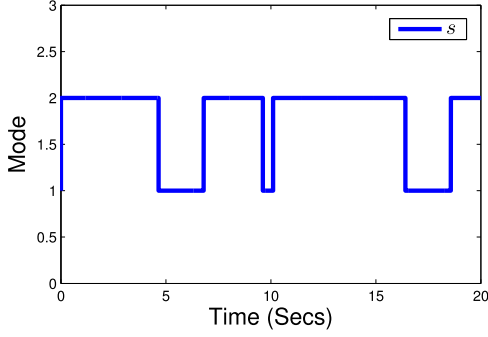


Fig. 11. Switching mode.

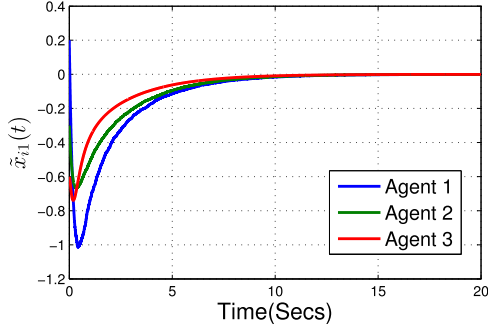


Fig. 12. State trajectories of  $\tilde{x}_{i1}(t)$  with control input (26).

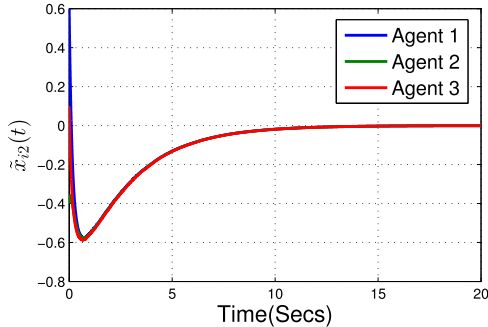


Fig. 13. State trajectories of  $\tilde{x}_{i2}(t)$  with control input (26).

$$K_{22}^{\mathcal{G}_2} = 10^{-4} \times \begin{bmatrix} -0.1398 & -0.0033 & 0.0033 \\ -0.0021 & -0.1988 & 0.0105 \\ 0.0005 & 0.0000 & -0.1027 \end{bmatrix},$$

$$K_{23}^{\mathcal{G}_2} = 10^{-4} \times \begin{bmatrix} -0.0581 & 0.0013 & 0.0034 \\ 0.0080 & -0.1279 & 0.0353 \\ 0.0016 & -0.0009 & -0.0651 \end{bmatrix}.$$

Figure 11 illustrates the switching topology for the agents, where modes 1 and 2 represent the graphs  $\mathcal{G}_1$  and  $\mathcal{G}_2$ , respectively. The state trajectories of the error system are given in Figs. 12–14, which shows that the proposed resilient retarded sampled-data controller stabilizes the considered heterogeneous MAS, and their corresponding control response for switched topology is displayed in Fig. 15. From the above discussions, one can conclude that, for heterogeneous MAS, all the follower agents achieved practical consensus together with the leader under the control input (26).

## 5. Conclusion

In this paper, the practical consensus analysis for heterogeneous MAS with randomly occurring nonlinearities under fixed and switching topologies has been investigated in a mean-square

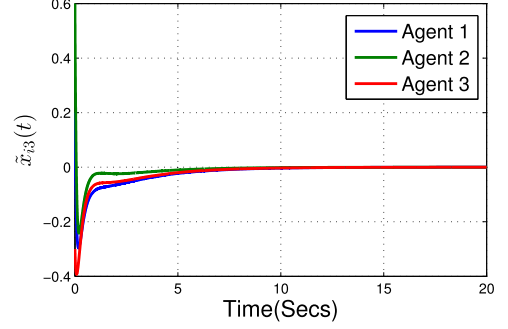


Fig. 14. State trajectories of  $\tilde{x}_{i3}(t)$  with control input (26).

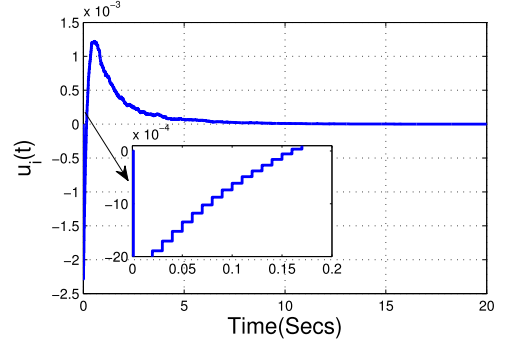


Fig. 15. Control responses of the system in Example 2.

sense. Besides, a stochastic variable has been introduced to depict the random behavior of the nonlinearities. The main advantage of the method is to design the RRSDC scheme for heterogeneous multi-agent systems, which considers the control gain variation and signal transmission delay. Based on the WIBD term and looped functional information, a novel LKF has been constructed to derive sufficient conditions for the leader-follower practical consensus of heterogeneous MAS under a fixed topology. Then, the derived conditions have been extended for heterogeneous MAS under the switching topology. At last, the effectiveness of the RRSDC design method was verified via a numerical example with its simulations. An increasing number of agents will burden the computational resource, and thus in the future, we will introduce some lemmas based on the free matrix-based inequalities to alleviate the computational burden. It would also be interesting to further consider the sampled-data control-based practical consensus problem for singular MASs and stochastic MASs with communication delays and packet dropouts.

## Declaration of Competing Interest

The authors of this manuscript declare that there is no declaration of interest on this manuscript.

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