



**HAL**  
open science

# Collusion and Predation Under Cournot Competition

Emilie Dargaud, Maxime Menuet, Petros G Sekeris

► **To cite this version:**

Emilie Dargaud, Maxime Menuet, Petros G Sekeris. Collusion and Predation Under Cournot Competition. 2022. hal-04079810

**HAL Id: hal-04079810**

**<https://hal.science/hal-04079810>**

Preprint submitted on 24 Apr 2023

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.



Distributed under a Creative Commons Attribution 4.0 International License

**GATE** LYON SAINT-ÉTIENNE

UMR 5824

93, chemin des Mouilles  
69130 Ecully - France

Maison de l'Université, Bâtiment B  
10, rue Tréfilerie  
42023 Saint-Etienne cedex 02 - France

<http://www.gate.cnrs.fr>  
[gate@gate.cnrs.fr](mailto:gate@gate.cnrs.fr)

WP 2213 – November 2022

# Collusion and Predation Under Cournot Competition

Emilie Dargaud, Maxime Menuet, Petros G. Sekeris

## Abstract:

This paper studies how predation strategies can affect the sustainability of collusion. We demonstrate that in the presence of few competitors collusion may be sustained at equilibrium for intermediate discount factors. In such instances predation implies that punishment strategies will yield low subgame perfect payoffs, thereby making collusion easier to sustain. For low discount factors collusion is not sustainable because of the high incentives to deviate to Cournot-Nash strategies. Moreover, for high discount factors it is always optimal to predate colluding firms, thus contrasting with much of the earlier literature showing that collusion is only achievable by sufficiently patient firms.

## Keywords:

Collusion, Predation, Cournot competition

## JEL codes:

D43, L13, L41

Working  
Paper

# Collusion and Predation Under Cournot Competition

Emilie Dargaud\*      Maxime Menuet†      Petros G. Sekeris‡

November 2022

## Abstract

This paper studies how predation strategies can affect the sustainability of collusion. We demonstrate that in the presence of few competitors collusion may be sustained at equilibrium for intermediate discount factors. In such instances predation implies that punishment strategies will yield low subgame perfect payoffs, thereby making collusion easier to sustain. For low discount factors collusion is not sustainable because of the high incentives to deviate to Cournot-Nash strategies. Moreover, for high discount factors it is always optimal to predate colluding firms, thus contrasting with much of the earlier literature showing that collusion is only achievable by sufficiently patient firms.

**Keywords:** Collusion, Predation, Cournot competition.

**JEL:** D43, L13, L41.

## 1 Introduction

Antitrust authorities closely scrutinize collusive practices since they allow firms to exert market power that would otherwise be absent, eventually resulting in reduced competition

---

\*Univ Lyon, Université Lumière Lyon 2, GATE, UMR 5824, F-69130 Ecully, France. Email: dargaud@gate.cnrs.fr. Declarations of interest: none

†LEO, University of Orleans, Orleans, France. e-mail: maxime.menuet@univ-orleans.fr. Declarations of interest: none

‡**Corresponding author:** TBS Business School, 1 Place A. Jourdain, 31000 Toulouse, France. E-mail: p.sekeristb@tbs-education.fr.

and increased prices. The literature on collusive agreements is quite extensive (see e.g. Choi and Gerlach (2015) for a recent review), and has mainly focused on industry characteristics that influence the sustainability of collusion (e.g. number of firms, entry barriers, market transparency, firms' asymmetry, product differentiation, demand fluctuation, ...). More particularly, the discount factor has been shown to play a key role: collusion arises at equilibrium only if firms are sufficiently patient not to be incentivized to reap the short run benefits of deviation.

In regards to the relationship between discount factors and cartel stability, the key connecting element is the credibility of the punishment strategies that could support collusion. Cartels are indeed unstable in principle since firms are tempted to deviate from collusive agreements (e.g. by charging a lower price than rivals) thereby provoking a cartel breakdown. For collusive agreements to be stable, colluding firms must be able to both detect the deviating firms and to punish them. Notwithstanding deviation detections that may be hampered by market opacity, the credibility question has been shown to be truly central in explaining the stability of collusive agreements. Indeed, for collusion to be an equilibrium, firms ought to be certain that any agreed punishment will be carried out if a deviation were to take place, i.e. punishments ought to be subgame perfect. Two particular types of credible punishments have been identified: grim-trigger punishment strategies, and optimal penal codes. The former type of punishment, initially proposed by Friedman (1971) consists in firms maximizing their competitive period payoff forever if a deviation from the collusive agreement were to occur. Optimal penal codes (Abreu 1988) consist in stick and carrot strategies that consider the worst possible punishments that may be history-dependent and are subgame perfect. As such, optimal penal codes allow collusion to emerge in more instances than grim-trigger strategies do.

When assessing the potential ongoing collusion in industries, competition authorities view (more) competitive settings as the benchmark against which one ought to compare ongoing practices. An implicit assumption, therefore, is that such market structure is indeed sustainable (i.e. Cournot Nash equilibria, marginal cost pricing, or any alternative competitive benchmark). Predatory practices—whereby firms predate their competitors to monopolize the market—may, however, jeopardize the very stability of the competitive benchmark,

thereby necessitating a totally different approach to the study of collusion since the sustainability of collusion and deviations from collusive agreements must be reconsidered. In instances where predation strategies destabilize the competitive equilibrium, it is important to understand whether collusion may nevertheless be sustained. As such, collusion could potentially be achieved despite the threat of monopolization.

In this paper we consider a Cournot competition setup with  $n$  credit constrained firms. Predation in our setting can emerge because firms are assumed to have (potentially mild) economies of scale, thence implying that predating firms can drive competitors to bankruptcy while still being profitable. Contrary to earlier findings, we show that with highly patient firms collusion cannot be supported at equilibrium because of the profitability of predation, i.e. because of the high discounted payoffs of monopolizing the market. Instead, collusion is shown to emerge for an intermediate range of discount factors, and provided the number of competitors is low ( $n \leq 3$ ). If firms are highly patient, predating colluding competitors will always be optimal because of the high expected flow of profits once a firm has wiped out competitors, while low discount factors imply that even when considering the harshest possible punishment strategy in case of non-compliance with the collusive agreement, the short run benefits of deviation are sufficiently high to disrupt the collusive equilibrium. For intermediate discount factors and large number of firms (i.e.  $n \geq 4$ ), the collusive profits are not sufficiently important to disincentivize predation, thus once more disrupting the collusive agreement. Last, for intermediate discount factors and few competitors ( $n \leq 3$ ) collusion can be sustained while (credibly) threatening deviating firms with the harshest available punishment which is shown to yield zero profits to the deviating firm.

The crude oil industry provides a good illustration of our theory. This industry—which constitutes the textbook example of cartel formation in Cournot competition setups—is inherently subject to increasing returns to scale, thereby opening up the space for monopolization practices. When the industry was still at its infancy in the late 1860s, Standard Oil Co. initiated aggressive predatory pricing practices, made possible through collusion with the transportation industry, i.e. the railroad industry (Granitz and Klein 1996), and thanks to the increasing returns to scale, enabling the company to eventually monopolize the vast majority of the U.S. market (McGee 1958). In 1911, the company was found to have violated

the Sherman Antitrust Act and was subsequently broken down into 34 separate companies, only later to be consolidated again in few companies some of which became the backbone of the oil cartel known as the Seven Sisters. With the oil shocks of the 1970s the OPEC cartel made its appearance, and collusive practices were observed until the late 1980s, after which the relative abundance of oil supply de facto rendering collusion more difficult to sustain. Although it is admittedly reductive to explain the entire evolution of the oil industry through the lenses of our model, the theory proposed in this paper contributes to understanding why monopolization was followed by collusion and thereafter by a situation more akin to Cournot competition. Indeed, our model predicts monopolization for high discount factors, collusion for intermediate ones, and Cournot-Nash equilibria for low discount factors. The discount factor of firms is a mapping of several firm-specific, industry-specific, and time-specific variables, but the available stock of oil constitutes a crucial determinant in the context of the oil industry as hypothesized and empirically supported by previous research establishing that the discount factor is a negative function of the available (known) stock of resources (Laibson 1997, O'Donoghue and Rabin 1999, van Veldhuizen and Sonnemans 2018). In other words, as the relative availability of oil stocks has been increasing through time with the recurrent discovery of new oil fields (Auzanneau 2015), it is fair to conjecture that the firms' discount factors have been contracting through time, thence resulting in monopolization practices, followed by collusion, and eventually Cournot-Nash competition.

This article contributes to the literature tying predation to market structure. Two important mechanisms that reduce future competition and increase profit by temporarily decreasing prices have been highlighted by scholars: the deep-pocket approach, and signalling mechanisms in the context of asymmetric information. Regarding the former mechanism, Telser (1966) shows that firms with "deep pockets" can afford the short run cost of predation in view of eventually monopolizing the market. This argument was subsequently adapted to settings where monopolists attempt retaining market power by adopting limit-pricing practices after entrants decide to challenge the incumbent (Benoit 1984). Bolton and Scharfstein (1990) show that mitigating agency problems between the firm and its financiers creates incentives for predatory behaviour. Second, predatory pricing can also be a rational signalling device in the context of incomplete information: low present prices can decrease future com-

petition by impacting competitors' beliefs about the profitability of entry (Milgrom and Roberts 1982) or by building an aggressive reputation for incumbents (Kreps and Wilson, 1982). As such predation has been demonstrated to be an effective tool to monopolize the market (Wiseman 2017), or to retain monopoly power.

Our work is closely related to the thin literature on predation and collusion. Earlier work has explored the role of discount factors on the sustainability of collusion in the presence of predation mechanisms. Kawakami and Yoshihiro (1997) show that in such setups financially constrained firms can only sustain collusive agreements for intermediate discount factors. The intuition of their result is that the firm with the deepest pockets is always able to enter in a longer war of attrition for the control of the market, thence implying that with a discount factor near one collusion will not be achievable. Kawakami (2010) extends this work by considering asymmetric information on the firms' financial constraints. We complement Kawakami and Yoshihiro (1997) and Kawakami (2010) by endogenizing the firms' payoffs, in turn demonstrating that the degree of competition—as captured by the number of firms—is a crucial determinant of the sustainability of collusion. In a setup different to ours, since their work restricts the analysis to two firms and Markov perfect equilibria, Beviá et al. (2020) derive conditions for the Cournot Nash equilibrium to be sustained when firms are financially constrained. Observe that although Beviá et al. (2020) consider the possibility of firms colluding, the sole deviations from collusion that are considered are Cournot reversion strategies, thence neglecting the possibility (that we do consider) for firms to predate each other when attempting to collude.

Wiseman (2017), in a different setup to ours, also shows that collusion is not sustainable for high discount factors because of monopolization practices. Our mechanism conducive to market monopolization rests on increasing returns to scale and the feature that firms have shallow pockets. Wiseman (2017) makes no specific assumptions in relation to returns to scale and it can be shown that his theory is robust to an explicit extension that would allow for returns to scale. A crucial difference, however, between Wiseman (2017) and our own theory is that, unlike us, Wiseman (2017) considers firms with deep pockets, i.e. firms that are able to experience a sequence of unfavourable outcomes, while we assume that firms are immediately driven to bankruptcy if they obtain negative profits in any given period.

Accordingly, Wiseman (2017) assumes some randomness in the firms’ survival probability, since a deterministic setup would always map in all firms but one immediately dropping out of the market: firms have different financial strengths (their state variable) that vary stochastically over time with Markov transitions and affect the likelihood of bankruptcy. As such, Wiseman (2017) assumes a continuum of active states describing the “strength of the firm’s financial position”, while our own setup assumes instead complete information over that attribute, i.e. there is a unique such state (beyond bankruptcy) where for all firms the strength of the financial position is nil. Thence, besides complementing the results of Wiseman (2017) by considering firms with shallow pockets, we also view our contribution as a highly tractable model highlighting in a simple setup when collusion is sustainable in spite of the availability of predation strategies. A further important distinction between these two models is to be found in our respective predictions: given the stationarity of firms’ financial constraints in our own setup, we are able to identify a non-empty parameter space—involving intermediate discount factors—where collusion is dynamically sustainable. On the contrary, in Wiseman (2017), since the firms’ financial strength is a random process and this critically defines the firms’ incentives to start a price war, it follows that when competing over an infinite time horizon, the likelihood that the market monopolizes converges asymptotically to one.

Predation has also been shown to be an effective tool to preserve market power in oligopolies. In Harrington (1989), predation may be used by colluding firms to deter potential entrants, thereby preserving market power, while also facilitating collusion. Likewise, Marshall et al. (2019) and Argenton (2020) demonstrate how firms may collusively predate competitors to preserve or increase their market power. Marshall et al. (2019) study the incentives of firms to collude and to predate potential entrants, while giving emphasis on the internal stability issue, namely, the incentives of cartel members to deviate. Last, Argenton (2020) explores the conditions under which a three-firm cartel may be predated by two of its members that subsequently collude, while demonstrating that a single firm would not predate the market because it does not have the commitment power to suffer a loss in the sacrifice phase.

In the next section we present the framework and major results. In Section 3 we discuss



some extensions and implications of our results. Last Section 4 concludes.

## 2 The Model

We consider an  $n$ -firm dynamic Cournot competition setting over an infinite time horizon with time denoted by index  $t = 1, 2, \dots$ . In any time period, the inverse market demand is given by  $p(Q_t) = a - bQ_t$ , where  $Q_t$  designates aggregate output at time  $t$ .<sup>1</sup> Denote the set of  $n$  initially active firms by  $\mathcal{I}$ . In  $t = 1$ , the  $n$  firms decide their output  $q_i \in \mathbb{R}, i = \{1, \dots, n\}$ , given the following total cost function:

$$TC(q_i) = \begin{cases} F + cq_i & \text{if } q_i > 0 \\ 0 & \text{otherwise} \end{cases},$$

where  $F > 0$  are the fixed operating costs and  $c$  is the marginal cost. We thence assume that fixed costs are bore only by active firms. At any period  $t$  a firm can go bankrupt by obtaining (weakly) negative profits, thence reflecting the fact that firms are credit constrained/have shallow pockets in our setup. If a firm is inactive or goes bankrupt at some period  $\tau$ , we assume that it leaves the market and stops being active for any  $t > \tau$ , therefore implying that our state variable is the pool of potentially active firms.<sup>2</sup> The discount factor is assumed to be symmetric and constant, and we denote it by  $\delta$ . We focus on subgame perfect Nash equilibria.

Our objective is to derive the conditions that make full collusion, i.e. collusion by all  $n$  firms in  $\mathcal{I}$  producing individually  $(\frac{1}{n})^{th}$  of the monopoly output, an equilibrium outcome.<sup>3</sup> Throughout this article we refer to collusion to designate full collusion, that is collusion by all active firms. We proceed in several steps. We begin by demonstrating that in our setup predation—defined as production that drives specific opponents to bankruptcy, and thus renders predated firms inactive because of the shallow pockets assumption—makes the

---

<sup>1</sup>For clarity reasons we omit time subscripts in what follows.

<sup>2</sup>Our focus is exclusively on market exit and we disregard potential entry, the absence of which could be explained by some non-modelled non-operating fixed costs.

<sup>3</sup>Observe that one could have equally studied the incentives of a subset of firms to collude, or the incentives of firms (possibly all) to produce higher quantities than the ones maximizing their joint profits (Haag and Lagunov 2007). We briefly discuss these possibilities in Section 3.

typical collusive agreements considered elsewhere in the literature collapse. Indeed, firms are shown to have incentives to predate competitors playing the Cournot-Nash strategy profile when collusion could have otherwise been supported as an equilibrium. We then demonstrate that even though firms may have incentives to predate colluding firms for high enough discount factors, there exists an intermediate range of discount factors for which collusion is an equilibrium.

*Cournot-Nash equilibrium*

The Cournot-Nash play is a candidate equilibrium strategy that is not conditioned on history and where all active firms maximize non-cooperatively their instantaneous profits forever. One can immediately derive the individual quantities, market prices, and individual profits compatible with the Cournot-Nash equilibrium where the firm-subscripts are omitted since quantities and profits are symmetric:

$$q^N = \frac{a - c}{b[n + 1]}; p^N = \frac{a + nc}{n + 1}; \pi^N = \frac{[a - c]^2}{b[n + 1]^2} - F, \quad (1)$$

where superscript  $N$  designates the Cournot-Nash equilibrium.

From this we deduce a necessary condition for a Cournot-Nash equilibrium to exist:

$$F < F^N = \frac{[a - c]^2}{b[n + 1]^2}. \quad (2)$$

Because of the fixed costs  $F$ , without any further parameter restriction our model could admit a number of Cournot-Nash equilibria involving a smaller number of active firms, i.e.  $l < n$ . To rule out such a possibility, and to ensure therefore that the game admits a unique equilibrium as given by (1), we impose a stricter parameter restriction on  $F$  that ensures that if only  $l < n$  firms were active, the remaining firms would have incentives to produce a strictly positive output. This condition reads as  $F < F^{max} = \frac{[a - c]^2}{4bn^2}$  and is derived in Appendix A.1.

Notice, moreover, that—in contrast to standard Cournot setups—in this model we consider an additional strategy: the predation strategy. Given the aggregative nature of Cournot competition games, we can express profits as a function of the focal firm’s quantities (first

argument) and of the competitors' aggregate output (second argument). Accordingly, the predation strategy for firm  $i$  predating firm  $j$  (for any given quantities of the remaining competitors) is defined as:

$$q_{i \rightarrow j}^{Pr} (q_1, \dots, q_j, \dots, q_n) := \arg \min_{q_i} \left\{ q_i : \pi_j(q_j, q_i + \sum_{k \neq \{i, j\}} q_k) \leq 0 \right\}. \quad (3)$$

Since (weakly) negative profits bankrupt firms, predation implies a reduction in the number of future competitors.<sup>4</sup> For the  $n$  firms to be incentivized to play non-cooperatively at equilibrium, it is thus necessary that when the competitors of firm  $i$  produce  $q_j = q^N$ ,  $\forall j \neq i$ , no firm  $i$  has incentives in predating the industry by producing quantities  $q_i = q^{PrN}$  such that  $\pi_j(q^N, q^{PrN} + [n - 2]q^N) \leq 0$ ,  $\forall j \neq i$ .<sup>5</sup> The no-predation condition for any firm  $i$  reads as:<sup>6</sup>

$$\pi^{PrN} + \frac{\delta}{1 - \delta} \pi^M < \frac{\pi^N}{1 - \delta} \Leftrightarrow \mathcal{D} = \frac{\delta}{1 - \delta} < \mathcal{D}^{PrN} = \frac{\pi^N - \pi^{PrN}}{\pi^M - \pi^N}, \quad (4)$$

where  $\pi^{PrN} = \pi^{PrN}(q^{PrN}, [n - 1]q^N)$  and  $\pi^M$  denotes the monopoly profits in the event predation successfully pushes the  $[n - 1]$  competitors to bankruptcy and is defined as:

$$\max_q \{ [a - bq]q - cq - F \}, \quad (5)$$

thus yielding:

$$q^M = \frac{a - c}{2b}; p^M = \frac{a + c}{2}; \pi^M = \frac{[a - c]^2}{4b} - F. \quad (6)$$

Next, the predation strategy for firm  $i$  is such that the following condition ought to be satisfied for any firm  $j \neq i$  producing their symmetric Cournot-Nash equilibrium output:

$$\pi_j(q^N, q^{PrN} + [n - 2]q^N) = 0 \Leftrightarrow p(q^{PrN} + [n - 1]q^N)q^N - cq^N - F = 0. \quad (7)$$

Re-arranging terms, the predation price ought to satisfy  $p^{PrN} = \frac{TC(q^N)}{q^N}$ , where  $TC(\cdot)$

---

<sup>4</sup>The assumption that firms go bankrupt with zero profit is made for expositional clarity. The results would remain qualitatively and quantitatively unaffected if bankruptcy required strictly negative profits.

<sup>5</sup>Notice that these predation quantities are the same for any predating firm given the model's symmetry, thence allowing us to drop the index  $i \rightarrow j$  to simplify notation.

<sup>6</sup>Throughout the article we use the transformation  $\mathcal{D} = \frac{\delta}{1 - \delta}$  rather than  $\delta$  to simplify the presentation of the results.

stands for total cost. We then deduce that:

$$p^{PrN} = \frac{F}{q^N} + c = \frac{Fb[n+1]}{a-c} + c.$$

Using the definition of the inverse market demand, and given the above predatory price, we then deduce:

$$q^{PrN} = \frac{2a + c[n-1]}{b[n+1]} - \frac{F[n+1]}{a-c} - \frac{c}{b}. \quad (8)$$

The associated instantaneous payoff of the predating firm is then equal to:

$$\pi(q^{PrN}, [n-1]q^N) = F - \frac{bF^2[n+1]^2}{[a-c]^2}. \quad (9)$$

Note that  $\pi(q^{PrN}, [n-1]q^N) > 0 \Leftrightarrow 0 < F < F^N = \frac{[a-c]^2}{b[n+1]^2}$ .

Replacing for the derived values of  $\pi^N$ ,  $\pi^M$ , and  $\pi^{PrN}$  in (4) allows us to re-write the no-predation condition as:

$$\mathcal{D} < \mathcal{D}^{PrN} = \frac{4}{[n+1]^2 - 4} - \frac{8b[n+1]^2F}{[a-c]^2[[n+1]^2 - 4]} + \frac{4b^2[n+1]^4F^2}{[a-c]^4[[n+1]^2 - 4]}. \quad (10)$$

Thence, if (2) and (10) are both satisfied, the game admits a non-cooperative equilibrium defined by (1), i.e. a Cournot-Nash equilibrium. In what follows we explore existence of non-cooperative equilibria for  $\mathcal{D} > \mathcal{D}^{PrN}$ .

#### *Mixed strategy equilibrium*

In the following Lemma we show that the game admits another equilibrium in the event (10) is violated:<sup>7</sup> a mixed strategy equilibrium.

**Lemma 1** *The game admits a mixed strategy subgame perfect equilibrium with zero profits for all firms if  $\mathcal{D} > \mathcal{D}^{PrN}$ .*

Proof in Appendix A.2.

---

<sup>7</sup>Although the game admits a predation equilibrium in the event  $\mathcal{D} \geq \mathcal{D}^{PrN}$ , in  $\mathcal{D} = \mathcal{D}^{PrN}$  firms are indifferent between predating the market and not, thence implying that a Cournot-Nash equilibrium exists as well. To avoid having to nuance our results for such weak inequalities because of equilibria multiplicity, we conduct the remaining analysis by inquiring whether collusion is sustainable when Cournot-Nash equilibria do not exist, i.e. when  $\mathcal{D} > \mathcal{D}^{PrN}$ .

Our proof establishes the existence of a mixed-strategy equilibrium for parameter configurations that rule out the existence of a Cournot-Nash pure strategy equilibrium. Observe, however, that this proof establishes the existence of *at least one* mixed strategy equilibrium, and as such does not preclude the existence of multiple equilibria. This lemma is of central importance, since it implies that when pure strategy Cournot-Nash equilibria fail to exist, the game will admit at least one equilibrium where players will mix over predation, Cournot and not producing.<sup>8</sup>

### *Collusion equilibrium*

We next derive the conditions for the existence of a collusive equilibrium. As previously mentioned our focus is on full collusion, thus disregarding the possibility that only a subset of firms could decide to collude. If all firms collude, their individual output, associated prices and profits, are given by:

$$q^C = \frac{a-c}{2bn}; p^C = \frac{a+c}{2}; \pi^C = \frac{[a-c]^2}{4bn} - F. \quad (11)$$

A necessary condition for collusion to be an equilibrium is that collusive profits be positive, which requires:

$$F < F^C = \frac{[a-c]^2}{4bn}, \quad (12)$$

a condition that is always satisfied since  $F^{max} < F^C$ .

Moreover, for collusion to emerge at equilibrium, it should be immune to unilateral deviations. Two types of deviations ought to be considered in our setting. First, firms may unilaterally predate the industry to push to bankruptcy their competitors. Second, we inspect whether it is profitable for firms to deviate unilaterally from the collusive agreement by maximizing their instantaneous payoffs and thereafter obtaining the mixed-strategies' continuation payoff. The following proposition establishes that collusion cannot be sustained in the presence of sufficiently patient firms.

**Proposition 1** *Collusion can never be sustained if  $\mathcal{D} > \mathcal{D}^{PrC}$ .*

---

<sup>8</sup>Observe that the logic of the argument, and the main 0-profit result are reminiscent of Tasnádi (2004) and van den Berg and Bos (2017), although our respective setups differ in fundamental ways since these authors consider firms (sequentially) deciding both quantities and prices.

**Proof.** For collusion to be sustained at equilibrium, it is necessary that no firm has incentives to unilaterally predate the industry, which reads as:

$$\pi^{PrC} + \frac{\pi^M}{1 - \delta} < \frac{\pi^C}{1 - \delta}, \quad (13)$$

where  $\pi^{PrC} = \pi(q^{PrC}, [n - 1]q^C)$ . This condition can then be expressed as:

$$\mathcal{D} < \mathcal{D}^{PrC} = \frac{\pi^C - \pi^{PrC}}{\pi^M - \pi^C}. \quad (14)$$

To derive an explicit expression for (14), we re-write the predation prices as:

$$p^{PrC} = \frac{TC(q^C)}{q^C} = \frac{F}{q^C} + c = \frac{2Fbn}{a - c} + c.$$

We then substitute this predation price in the definition of the inverse demand for the optimal collusive output  $q^C$  and set the expression equal to the above-derived term to deduce:

$$q^{PrC} = \frac{an + cn + a - c}{2bn} - \frac{2nF}{a - c} - \frac{c}{b}. \quad (15)$$

Using (15) we obtain the instantaneous profits of the predating firm:

$$\pi(q^{PrC}, [n - 1]q^C) = nF - \frac{4bF^2n^2}{[a - c]^2}. \quad (16)$$

Note that  $\pi(q^{PrC}, [n - 1]q^C) > 0 \Leftrightarrow 0 < F < \frac{[a - c]^2}{4bn}$ , and that  $\frac{[a - c]^2}{4bn} > F^{max} = \frac{[a - c]^2}{4bn^2}$ , so that profits from predating colluding firms are necessarily positive in the admissible range of  $F$  parameters.

We can next replace for the optimal values of  $\pi^C$ ,  $\pi^M$ , and  $\pi^{PrC}$  in (14), and then rewrite the condition as:

$$\mathcal{D} < \mathcal{D}^{PrC} = \frac{1}{n - 1} - \frac{4bn[n + 1]F}{[a - c]^2[n - 1]} + \frac{16b^2n^3F^2}{[a - c]^4[n - 1]} \quad (17)$$

■

Proposition 1 reverses the common finding in the literature that collusion is easier to

sustain with more patient firms. Indeed, when firms put a lot of weight on the future, their incentives to endorse the short run costs of predation—as reflected in the lower instantaneous profits obtained compared to the collusive profits—are now higher because of the high weight put on future time periods when the firm will act as a monopolist.

Since low discount factors are known to hamper collusion, and given the finding in Proposition 1, we then inspect whether collusive agreements under the threat of Cournot-Nash reversion strategies are possible. The next Corollary rules out this possibility.

**Corollary 1** *Collusion can never be sustained by Cournot-Nash reversion strategies.*

**Proof.** For collusion to be sustained by Cournot-Nash reversion strategies, it is necessary that

$$\pi^D + \frac{\delta}{1-\delta}\pi^N \leq \frac{\pi^C}{1-\delta}, \quad (18)$$

where  $\pi^D$  designates the deviation payoff that obtains by optimizing the following problem:

$$\max_{q_i} \left\{ \left[ a - bq_i - b[n-1]\frac{a-c}{2bn} \right] q_i - cq_i - F \right\}.$$

Solving yields the following quantity, price and profits to the deviating firm  $i$ :

$$q^D = \frac{[a-c][n+1]}{4bn}; p^D = \frac{an+a+3nc-c}{4n}; \pi^D = \frac{[[n+1][a-c]]^2}{16bn^2} - F. \quad (19)$$

After replacing for the values from (1), (11), and (19), inequality (18) then reads as:

$$\delta \geq \delta^C = \frac{[n+1]^2}{[n+1]^2 + 4n} \Leftrightarrow \mathcal{D} = \frac{\delta}{1-\delta} \geq \mathcal{D}^C = \frac{[n+1]^2}{4n}. \quad (20)$$

We obtain the usual result that an increase in the number of cartel firms makes collusion harder to sustain. Last, we establish that  $\mathcal{D}^{PrC} < \mathcal{D}^C$ , which completes the proof of the Corollary. Replacing for the expressions derived in (20) and (17) we aim at demonstrating that :

$$\frac{[n+1]^2}{4n} > \frac{1}{n-1} - \frac{4bn[n+1]F}{[a-c]^2[n-1]} + \frac{16b^2n^3F^2}{[a-c]^4[n-1]}.$$

The RHS of the above expressions is convex in  $F$ . Moreover when  $F = 0$  the above inequality is true for any  $n \geq 2$ . Lastly, since  $F \leq F^C$  as given by (12), if the inequality is verified

for  $F = F^C$ , it is verified for any  $F \in [0, F^C]$ , and by extension for any  $F \in [0, F^{max}]$ . It is immediate to inspect that for  $F = F^C$  the RHS of the inequality equals 0. ■

The intuition of Corollary 1 is that for any range of parameters where collusion could have been supported by Nash reversion strategies, the Cournot-Nash strategy profile is not an equilibrium since firms have incentives to predate the industry when competitors play their Cournot-Nash strategy profile. Indeed, Nash reversion strategies can only support collusion for high discount factors. Yet, high discount factors also increase the profitability of predation since the expected monopoly profits the predating firm will be able to secure will then weight more in its payoff function (Proposition 1). This in turn implies that Nash reversion strategies are not part of any subgame perfect equilibrium, and, consequently, cannot be used as a threat to deter deviations from collusion.

The interesting question then is to explore whether collusion may nevertheless be sustained using alternative punishment strategies to the Cournot-Nash equilibrium. We focus our attention on the parameter space  $\mathcal{D} > \mathcal{D}^{PrN}$  where the zero-(expected) profit mixed-strategy equilibrium has been shown to exist in Lemma 1. The next Lemma opens up the possibility for a collusive equilibrium to emerge:

**Lemma 2** *There exists a unique threshold  $\bar{F} \in ]0, F^{max}]$ , such that for  $F < \bar{F}$ ,  $\mathcal{D}^{PrN} < \mathcal{D}^{PrC}$ .*

Proof in Appendix A.3

Building on Lemma 2, we can now state the main finding of the paper that establishes the existence of collusive equilibria in predatory settings:

**Proposition 2** *For  $n \leq 3$ , there always exist a non-empty range of fixed costs and a non-empty range of discount factors in  $\mathcal{D} < \mathcal{D}^{PrC}$  such that collusion is sustained at equilibrium if the discount factor belongs to that interval.*

**Proof.**

By Lemma (2) we know that for  $F < \bar{F}$ , there always exist a non-empty range of  $\mathcal{D}$  values,  $[\mathcal{D}^{PrN}, \mathcal{D}^{PrC}]$ . To establish the proposition we thus need to show that there exists values of  $F \in ]0, \bar{F}]$  such that no deviation from full collusion exists. Bearing in mind that for



$\mathcal{D} \in [\mathcal{D}^{PrN}, \mathcal{D}^{PrC}]$  the Cournot-Nash strategy profile is not an equilibrium strategy profile, and firms have no incentives to predate the collusive agreement either. In view of Lemma 1, the unique alternative deviation from full collusion consists in a firm unilaterally deviating non-cooperatively from the full collusion strategy profile before obtaining zero discounted payoff in the ensuing subgame.

Denoting by  $\mathcal{D}^{DC}$  the  $\mathcal{D}$  value below which such deviation is preferable to collusion, we accordingly have:

$$\pi^D = \frac{\pi^C}{1 - \delta^{DC}} \Leftrightarrow \delta^{DC} = \frac{\pi^D - \pi^C}{\pi^D} \Leftrightarrow \mathcal{D}^{DC} = \frac{\pi^D - \pi^C}{\pi^C},$$

with the values of  $\pi^D$  and  $\pi^C$  being given, respectively, by expressions (19) and (11).

Consequently, a non-empty range of  $F$  parameters for which collusion is sustainable at equilibrium necessarily exists if  $F < \bar{F}$  and  $\mathcal{D}^{DC} < \mathcal{D}^{PrC}$ . Replacing by the appropriate values for  $\mathcal{D}^{DC}$  and  $\mathcal{D}^{PrC}$ , the expression  $\Xi(F) = \mathcal{D}^{PrC} - \mathcal{D}^{DC}$  is given by:

$$\Xi(F) = \frac{1}{n-1} - \frac{4bn[n+1]F}{[a-c]^2[n-1]} + \frac{16b^2n^3F^2}{[a-c]^4[n-1]} - \frac{\left[ \frac{[n+1]^2[a-c]^2}{16bn^2} - \frac{[a-c]^2}{4bn} \right]}{\frac{[a-c]^2}{4bn} - F}. \quad (21)$$

In Appendix A.4 we show that this expression is strictly convex in  $F$  for  $n \leq 4$ . Moreover, in  $F = F^{max}$ , one can easily show that  $\mathcal{D}^{PrC} = 0$ , while  $\mathcal{D}^{DC} > 0$ . Consequently  $\Xi(F^{max}) < 0$ , which implies that there exists a range of  $F$  values where collusion is an equilibrium if and only if  $\Xi(0) > 0$ . Replacing then for  $F = 0$  in the above expression, we obtain:

$$\Xi(0) = \frac{1}{n-1} - \frac{[n+1]^2}{4n} + 1,$$

which allows us to conclude that  $\Xi(0) > 0$  for any  $n \leq 3$ , while for  $n \geq 4$ ,  $\Xi(0) < 0$  ■

Proposition 2 identifies the conditions for collusion to be sustainable at equilibrium in settings where predation needs to be considered because firms have shallow pockets and can therefore be driven to bankruptcy. Corollary 1 establishes that collusion can never be sustained by Cournot-Nash reversion strategies. This results from predation ruling out Cournot-Nash equilibria for discount factors above some threshold  $\delta^{PrN}$ —since  $\mathcal{D}$  is a mono-

tonic transformation of  $\delta$ —and from the fact that this threshold is lower to the discount factor threshold above which collusion would have been an equilibrium in the absence of predation,  $\delta^C$ .

Collusion could potentially nevertheless be sustained by the harshest possible deviations the game can admit in case a firm deviates from the collusive agreement. Indeed, in the context of our study we demonstrate that for discount factors above the threshold  $\delta^{PrN}$  the game admits a mixed-strategy equilibrium yielding nil expected payoffs to all firms, and thus the lowest possible continuation payoffs in the context of our study. Observe, however, that even if this harshest punishment deters non-predatory deviations from the cooperative agreement, for collusion to be an equilibrium outcome firms must not have an incentive to unilaterally predate the market and subsequently obtain monopoly profits. We demonstrate that under the conditions stated in the Proposition 2, there exists a threshold on the discount factor,  $\delta^{DC}$ , below which firms have incentives to deviate from the collusive agreement (the harshest punishment does not work as a deterrent) and another threshold,  $\delta^{PrC}$ , above which firms have incentives to predate the collusive agreement, with the former threshold being smaller than the latter. Consequently collusion can only be sustained as an equilibrium outcome for an intermediate range of discount factors.

Two factors stand out as crucial in determining the possibility of observing collusive outcomes when firms have shallow pockets: the number of competitors and the magnitude of fixed costs. Lemma 2 establishes that for low fixed costs, it will not be profitable to predate the collusive equilibrium since predation involves reduced returns to scale and is therefore a costly operation. Moreover, for low fixed costs and few competitors ( $n \leq 3$ ), deviating non-cooperatively from the collusive strategy profile is not profitable either. Indeed, the incentives to deviate from collusion are a monotonically increasing function of the number of competitors since in setups with fewer competitors the expected collusive profits are higher. Second, deviations are less profitable for low fixed costs because, unlike the collusive outcome where the fixed costs will be supported at each time period, if firms deviate, fixed costs will only influence their (instantaneous) deviation profits since subsequent expected payoffs are zero for any fixed costs. Thence, lower fixed costs reduce the relative profitability of deviations.

It is interesting to observe that in the context of our model the range of discount parameters for which collusion is now possible does not overlap with the ones typically identified elsewhere in the literature: collusion is possible for intermediate ranges of discount factors rather than for patient firms. Indeed if firms are highly patient, predated colluding competitors will always be optimal because of the high expected flow of profits once a firm has wiped out competitors, while low discount factors reduce the importance of continuation payoffs and incentivize firms to optimize their short run payoff. For intermediate discount factors, on the other hand, the punishment payoffs are restricted to the instantaneous profits of deviating non-cooperatively from the collusive agreement, since in the ensuing subgame we establish the existence of a mixed-strategy equilibrium where all firms derive a zero expected profit. As punishments are therefore harsher, collusion is consequently easier to sustain.

### 3 Discussion

Our focus in this article is on full collusion by all firms, but milder forms of collusion could have been considered. One could have inquired the sustainability of (full) collusion by a subset of firms alone. Such equilibria are more difficult to sustain since, on the one hand, the discount factor above which predation is optimal would now be lower than  $\delta^{PrC}$  due to the non-colluding firms producing higher output than  $q^C$ , while, on the other hand, colluding firms would have higher incentives to deviate from the collusive agreement given the (now) lower instantaneous profits associated with collusion. A second approach that could have been followed would have been to consider all (or some) firms partially colluding by producing quantities lying between the fully collusive solution and the Cournot-Nash quantities. While such milder forms of cooperation are typically seen as easier to sustain (see e.g. Haag and Lagunoff 2007), this is not necessarily true in the context of our model. Let us assume a parameter configuration such that  $\mathcal{D}^{PrN} < \mathcal{D}^{PrC}$  and low enough fixed costs so that there exists a range of discount factors for which collusion is sustainable (Proposition 2), and let us then consider milder forms of collusion. Since  $\mathcal{D}^{PrN}$  is independent of the degree of collusion, we are interested in inspecting how the thresholds  $\mathcal{D}^{DC}$ —below which firms can profitably deviate from the collusive agreement—and  $\mathcal{D}^{PrC}$ —above which firms predate the

collusive agreement—are impacted. The former threshold is given by  $\mathcal{D}^{DC} = \frac{\pi^D - \pi^C}{\pi^C}$  and the effect of the degree of collusion on this expression is ambiguous: although both the numerator and denominator unambiguously decrease, the relative profitability of deviations (numerator) could shrink so much (for collusive agreements that are closer to the Cournot-Nash equilibrium quantities) that the threshold decreases. In other words, although milder collusive agreements generate a lower discounted expected payoff to firms, deviations from such agreements are also less profitable, thereby precluding some general prediction. Turning next to the other critical threshold,  $\mathcal{D}^{PrC} = \frac{\pi^C - \pi^{PrC}}{\pi^M - \pi^C}$ , here too the effect of milder forms of collusion is ambiguous. Although the denominator of this expression will be higher, the sign (and magnitude) of the numerator is unclear since both  $\pi^C$  and  $\pi^{PrC}$  will be smaller. The intuition is the following. Milder forms of collusion map into lower discounted profits for colluding firms, but predating such agreements where all firms produce higher output yields smaller instantaneous profits to the predating firm compared to the the full collusion considered elsewhere in the paper. Thence, both options (colluding or predating) become less attractive to the firm, and given the non-linearities of these expressions we cannot deduce a general rule tying milder forms of collusion to its sustainability.<sup>9</sup>

Second, we can reflect on the sustainability of collusion in the presence of predation strategies given the dynamic exit from the markets under study. Indeed, we have demonstrated that for large enough discount factors (i.e.  $\delta > \delta^{PrN}$ ) the game does not admit a pure strategy Cournot Nash equilibrium, while it admits at least one mixed strategy equilibrium where firms predate their competitors with some strictly positive probability. Under such circumstances, the number of firms will dynamically become smaller, thence implying that for low enough fixed costs and intermediate discount factors (conditions of Proposition 2), a market that initially does not admit collusive equilibria will in some future time period either admit collusive equilibria or else become a monopoly.

---

<sup>9</sup>We are able to show with numerical simulations that  $\mathcal{D}^{PrC}$  is increasing in the “mildness” of the collusive outcome, unless the agreement is very mild, i.e. firms produce quantities that are close enough to their Cournot-Nash equilibrium output.

## 4 Conclusion

In this paper we explore the possibility for firms to collude in the presence of predation. We consider a setup where firms are credit-constrained (i.e. have shallow pockets), thence implying that firms can have incentives to predate competitors if the industry admits economies of scale. In such settings, contrary to most of the literature, we demonstrate that collusion may be sustainable for intermediate discount factors alone. High discount factors increase the incentives to predate the market since collusion aims at monopolizing the market forever. Thence, the Cournot-Nash equilibrium which would have served as a punishment threat to support cooperation collapses, and collusion is no more sustainable by such Nash-reversion strategies. Moreover, firms have then strong incentives to predate colluding firms, thereby precluding collusion from being played at equilibrium. For low discount factors, on the other hand, firms have the typical incentives identified in the literature not to collude since even when considering the harshest possible punishment, if the continuation profits weight little enough in the firm's payoff, then firms will maximize their short run profits and thereby deviate from a collusive agreement. For intermediate discount factors, however, collusion will be sustained provided fixed costs are not too high and the number of competitors is not greater than 3. Indeed, for low fixed costs, it will not be profitable to predate the collusive equilibrium since predation involves reduced returns to scale and is therefore a costly operation. Moreover, for low fixed costs and few competitors ( $n \leq 3$ ), deviating non-cooperatively from the collusive strategy profile is not profitable either since few firms imply large collusive profits.

# A Appendix

## A.1 Parameter restriction on $F$

We impose a parameter restriction such that in an  $n$ -firm setup there can be no Cournot-Nash equilibrium featuring  $l < n$  firms. Consider any such candidate Cournot-Nash equilibrium. Accordingly, the optimal individual quantities, market price, and individual profits of any producing firm are given by:

$$q^N(l) = \frac{a - c}{b[l + 1]}; p^N(l) = \frac{a + lc}{l + 1}; \pi^N(l) = \frac{[a - c]^2}{b[l + 1]^2} - F,$$

where the candidate equilibrium variables are denoted as a function of  $l$  to signal that  $l$  firms are active under this strategy profile. For this to be an equilibrium, no firm out of the pool of inactive firms should have incentives to produce positive output. The best response of any such firm if it were to produce is given by:

$$q(Q^N(l)) = \frac{a - c}{2b} - \frac{Q^N(l)}{2},$$

where  $Q^N(l) = lq^N(l)$ . Replacing for the value of  $Q^N(l)$ , we deduce the maximal profits the focal inactive firm can secure by producing equals:

$$\pi(q(Q^N(l)), Q^N(l)) = \frac{[a - c]^2}{4b[l + 1]^2} - F,$$

thence implying that  $\pi(q(Q^N(l)), Q^N(l)) \leq 0$  if  $F \geq \frac{[a - c]^2}{4b[l + 1]^2}$ , in which case an  $l$ -firm Cournot-Nash equilibrium exists since no inactive firm would be incentivized to produce a positive output. Since, however,  $\pi(q(Q^N(l)), Q^N(l))$  is strictly decreasing in  $l$ , if  $F < \frac{[a - c]^2}{4b[l + 1]^2}$  for  $l = n - 1$ , i.e. if  $F < \frac{[a - c]^2}{4bn^2} = F^{max}$ , then an  $l$ -firm Cournot-Nash equilibrium cannot exist since an inactive firm has incentives to produce strictly positive output. This condition thus guarantees that the game admits a unique Cournot-Nash equilibrium with  $n$  producing firms as described in (1).

## A.2 Proof of Lemma 1

The proof of the existence of a mixed strategy subgame perfect equilibrium follows the steps of Dasgupta and Maskin (1986), and can be found in the online Appendix. In what follows we demonstrate that since a mixed strategy equilibrium exists, it must necessarily involve firms earning nil expected profits. To prove that we proceed in 4 steps where we sequentially prove that for  $\mathcal{D}^{PrN} < \mathcal{D}$ :

1. There is no pure strategy equilibrium.
2. If the equilibrium strategy involves a mix over a predation strategy and the Cournot-Nash strategy, then the predation strategy ought to predate the Cournot-Nash expected output.
3. If firms mix between the above-described two strategies, then they have unilateral incentives to assign a probability 1 to the predation strategy.
4. Consequently, the equilibrium mix must assign a strictly positive probability to not producing, thence implying nil expected payoffs.

1. By the definition of  $\mathcal{D}^{PrN}$ , the game admits no Cournot pure strategy equilibrium for  $\mathcal{D} > \mathcal{D}^{PrN}$ . Second, the game admits no pure strategy predation equilibrium. Indeed, if all  $n$  firms produce such predation quantities, then if any firm produces different quantities by deviating to its Cournot Nash strategy, its intertemporal payoff is necessarily higher since in the current time period the profits would then be higher (and thus strictly positive). For such predation strategy to be played at equilibrium then, it must be the case that it involves firms producing the exact same quantities under the predation strategy and under the Cournot Nash strategy. The predation strategy is defined as follows:

$$[a - c - bn\dot{q}^{Pr}] \dot{q}^{Pr} - F = 0,$$

which implies:

$$\dot{q}^{Pr} = \frac{[a - c] + / - \sqrt{[a - c]^2 - 4bnF}}{2bn}.$$

Considering then the  $[n-1]$  competitors of a specific (focal) firm produce these quantities, if the focal firm deviates to its Cournot-Nash strategy, its output is given by:

$$q^N([n-1]q^{Pr}) = \frac{a-c-b[n-1]q^{Pr}}{2b}.$$

Consequently,  $q^{Pr} = q^N([n-1]q^{Pr})$  if  $q^{Pr} = \frac{a-c}{b[n+1]}$ , or:

$$\begin{aligned} \frac{a-c}{b[n+1]} &= \frac{[a-c] + / - \sqrt{[a-c]^2 - 4bnF}}{2bn} \\ \Leftrightarrow \frac{[n-1][a-c]}{[n+1]} &= +/ - \sqrt{[a-c]^2 - 4bnF} \end{aligned}$$

If the right hand side term is negative, this equality cannot be satisfied, while if it is positive, it will be true if:

$$\begin{aligned} \frac{[n-1]^2[a-c]^2}{[n+1]^2} &= [a-c]^2 - 4bnF, \\ \Leftrightarrow 4bnF &= \frac{4n[a-c]^2}{[n+1]^2}, \\ \Leftrightarrow F &= \frac{[a-c]^2}{b[n+1]^2} = F^N. \end{aligned}$$

And since  $F^{max} < F^N$ , this equality can never be satisfied.

2. Having shown in the online appendix that an equilibrium exists, and since no pure-strategy equilibrium exists, there must exist a mixed-strategy equilibrium. We next show that if the equilibrium strategy involves a mix over a predation strategy and the Cournot-Nash strategy, then the predation strategy ought to predate the Cournot-Nash expected output. To show that, denote by  $q_i^e$  the expected output of any firm  $i$ , by  $q^e$  the (symmetric) expected output of any firm, and by  $Q_{-i}^e = [n-1]q^e$  the aggregate expected output of firm  $i$ 's competitors. The Cournot-Nash strategy being linear in the competitors' expected output, it can then be denoted for firm  $i$  by  $q_i^N([n-1]q^e)$ . Now, assume—contrary to what we aim to show—that the predation strategy of any firm  $i$  does not satisfy the following equality:

$$[a-c-b[q_j^N([n-1]q^e) + [n-2]q^e + q_i]] q_j^N([n-1]q^e) - F = 0.$$



Two scenarios ought to be considered.

First, if  $[a - c - b [q_j^N([n - 1]q^e) + [n - 2]q^e + q_i]] q_j^N([n - 1]q^e) - F < 0$ , then any firm implementing its Cournot-Nash strategy would obtain negative payoffs, and firm  $i$  would then have incentives to reduce its output since  $q_i$  would then neither be maximizing the period profits—an objective achieved by producing  $q_i = q^N([n - 1]q^e)$ , nor predating rivals at the minimal cost.

Second, if  $[a - c - b [q_j^N([n - 1]q^e) + [n - 2]q^e + q_i]] q_j^N([n - 1]q^e) - F > 0$ , then when firm  $i$  plays its predation strategy, either it drives the profits of rivals playing their predation strategy to zero, or not. In the former case a firm playing the predation pure strategy would either obtain negative profits when facing at least one other firm equally predating the market, or else positive profits if facing firms having played their Cournot Nash strategy, and yet lower profits than if the same firm had played its Cournot-Nash strategy. Deviating to the Cournot-Nash strategy is therefore necessarily profitable. We therefore conclude that if the equilibrium strategy involves a mix over a predation strategy and the Cournot-Nash strategy, then the predation strategy ought to predate the Cournot-Nash expected output.

3. We next characterize the mixed strategy profile described in 2. and establish that there can be no equilibrium where firms mix over these two strategies alone. The proof is conducted for  $n = 2$  firms, and can be extended to  $n > 2$  firms. Define by  $\lambda$  the probability with which firms play their Cournot-Nash strategy, and by  $[1 - \lambda]$  the probability they implement their predation strategy. Moreover, denote for brevity by  $q^N(q^e)$  the Cournot-Nash strategy when expecting the competitors to produce  $q^e$ , and by  $q^{Pr}(q^e) = q^{Pr}(q^N(q^e))$  the predation strategy that sets the profits of firms implementing their Cournot-Nash strategy to zero. Imposing symmetry and setting then  $q^e = \lambda q^N(q^e) + [1 - \lambda]q^{Pr}(q^e)$ , we must have at a mixed-strategy equilibrium:

$$\frac{\lambda}{1 - \lambda\delta} \pi(q^N(q^e), q^N(q^e)) = \lambda \pi(q^{Pr}(q^N(q^e)), q^N(q^e)) + \frac{\delta}{1 - \delta} \pi^M. \quad (22)$$

The left-hand side describes the discounted expected payoff from playing the Cournot Nash strategy in every subgame: if at any time period the firm obtains the same payoff from implementing any of the pure strategies over which it mixes at equilibrium, then the firm can

be seen as playing any pure strategy in any subgame. On the right hand side, we have the expected profit of the firm when it plays its predation pure strategy, since in such a scenario with probability  $[1 - \lambda]$  the competitor equally plays his predation strategy and the firms' expected payoff is then nil.

We first focus on the LHS of the expression. The Cournot best response function for an output  $q^e$  of the competitor is given by:

$$q^N(q^e) = \frac{a - c}{2b} - \frac{q^e}{2},$$

and replacing for the value of  $q^e$ ,

$$q^N(q^e) = \frac{a - c}{2b} - \frac{\lambda q^N(q^e) + [1 - \lambda]q^{Pr}(q^e)}{2}.$$

We thence deduce:

$$q^N(q^e) = \frac{a - c}{[2 + \lambda]b} - \frac{[1 - \lambda]q^{Pr}(q^e)}{[2 + \lambda]}.$$

Next, the predation quantities are defined as:

$$q^{Pr}(q^N(q^e)) : [a - c - b[q^{Pr}(q^N(q^e)) + q^N(q^e)]] q^N(q^e) - F = 0.$$

Or,

$$q^{Pr}(q^N(q^e)) = \frac{a - c}{b} - q^N(q^e) - \frac{F}{bq^N(q^e)}.$$

Replacing next  $q^{Pr}(q^N(q^e))$  in the above-computed expression describing  $q^N(q^e)$  and dropping  $q^e$  to simplify notations we obtain:

$$q^N[2 + \lambda - b(1 - \lambda)] - \frac{\lambda}{b}[a - c] - \frac{[1 - \lambda]F}{b^2q^N} = 0.$$

We next compute the roots of this expression to find equilibrium value of  $q^N$ , and since the lower root involves negative output,  $q^N$  is given by:

$$q^N = \frac{\frac{\lambda}{b}[a-c] + / - \sqrt{\frac{\lambda^2}{b^2}[a-c]^2 + \frac{4[2+\lambda-(1-\lambda)b](1-\lambda)F}{b^2}}}{2[2+\lambda-(1-\lambda)b]}. \quad (23)$$

Using last these values of  $q^N$  and of  $q^{Pr}(q^N)$  in (22), it can be shown using numerical simulations that for  $\mathcal{D} = \mathcal{D}^{PrN} \Leftrightarrow \delta = \frac{\mathcal{D}^{PrN}}{1+\mathcal{D}^{PrN}}$ , for any  $\lambda \in [0, 1]$ ,  $\frac{\lambda}{1-\lambda\delta}\pi(q^N(q^e), q^N(q^e)) < \lambda\pi(q^{Pr}(q^N(q^e)), q^N(q^e)) + \frac{\delta}{1-\delta}\pi^M$ . Moreover, we can equally show that increasing the value of  $\delta$  makes this inequality more likely to hold, thence ruling out the existence of a mixed strategy equilibrium where firms mix over their Cournot and predation strategies alone.

4. Last, having proved in Lemma 1 that the game always admits an equilibrium, and since the game admits no pure strategy equilibrium for the parameter configuration we are considering, it must be the case that firms play an “idle” strategy where they produce zero quantities with strictly positive probability. Consequently, since any pure strategy ought to secure same expected profits at the mixed strategy equilibrium, and given that producing zero quantities implies nil payoffs, it must be the case that the firms’ expected profits at the mixed strategy equilibrium is zero.

### A.3 Proof of Lemma 2

Using expressions (10) and (17), we can write  $\Psi(F) = \mathcal{D}^{PrC} - \mathcal{D}^{PrN}$  as:

$$\begin{aligned} \Psi(F) = \frac{1}{n-1} - \frac{4bnF[n+1]}{[a-c]^2[n-1]} + \frac{16b^2n^3F^2}{[a-c]^4[n-1]} - \frac{4}{[n+1]^2-4} \\ + \frac{8b[n+1]^2F}{[a-c]^2[[n+1]^2-4]} - \frac{4b^2[n+1]^2F^2}{[a-c]^4[[n+1]^2-4]} \end{aligned}$$

Simplifying yields:

$$\Psi(F) = \frac{n-1}{[n+1]^2-4} - \frac{4bF[n+1]}{[a-c]^2} \frac{[n^3-3n+2]}{[n-1][n+1]^2-4} + \frac{4b^2F^2}{[a-c]^4} \left[ \frac{4n^3}{n-1} - \frac{[n+1]^4}{[n+1]^2-4} \right]$$

Observe that  $\Psi(F)$  is a convex function, with  $\Psi(F)|_{F \rightarrow 0} > 0$ . Moreover, we have imposed that  $F < F^{max} = \frac{[a-c]^2}{4b[n+1]^2}$ . We substitute for  $F^{max}$  in  $\Psi$  to obtain:

$$\Psi(F^{max}) = \frac{n-1}{[n+1]^2-4} - \frac{[n^3-3n+2]}{[n+1][n-1][n+1]^2-4} + \frac{1}{4[n+1]^4} \left[ \frac{4n^3}{n-1} - \frac{[n+1]^4}{[n+1]^2-4} \right]$$

The sign of  $\Psi(F^{max})$  is then given by:

$$sign \{4[n-1]^2[n+1]^4 - 4[n+1]^3[n^3-3n+2] + 4n^3[[n+1]^2-4] - [n+1]^4[n-1]\}$$

$$\Leftrightarrow sign\{\Psi(F^{max})\} = sign\{-n^5 + n^4 - 6n^3 + 10n^2 - n - 3\}$$

And this expression is negative for any  $n \geq 0$ .

We thus conclude that there exists a unique root  $\bar{F}$  of  $\Psi(F)$  on  $]0, F^{max}]$  such that  $\Psi(F) \geq 0 \Leftrightarrow F \leq \bar{F}$ .

#### A.4 Convexity of Expression (21) for $n \leq 4$

We differentiate twice  $\Xi(F)$  as given by (21) so as to obtain:

$$\Xi(F)'' = \frac{32b^2n^3}{[a-c]^4[n-1]} - \frac{2 \left[ \frac{[n+1]^2[a-c]^2}{16bn^2} - \frac{[a-c]^2}{4bn} \right]}{\left[ \frac{[a-c]^2}{4bn} - F \right]^3}$$

Re-arranging terms and simplifying, this expression is positive if the following inequality is verified:

$$F < \frac{[a-c]^2}{4bn} \left[ 1 - \frac{n-1}{4^{1/3}n^{2/3}} \right].$$

Having assumed that  $F < F^{max} = \frac{[a-c]^2}{4bn^2}$ , the above expression is then necessarily verified in the admissible range of  $F$  parameters if:

$$\frac{1}{n} \leq 1 - \frac{n-1}{4^{1/3}n^{2/3}},$$

which reduces to  $n \leq 4$ .

## References

- Abreu, D. (1988), ‘On the theory of infinitely repeated games with discounting’, *Econometrica* **56**(2), 383–396.
- Argenton, C. (2020), ‘Colluding on excluding’, *European Economic Review* **113**, 194–206.
- Auzanneau, M. (2015), *Or Noir: La Grande Histoire du Pétrole*, La Découverte, Paris.
- Benoit, J.-P. (1984), ‘Financially constrained entry in a game with incomplete information’, *RAND Journal of Economics* **15**(4), 490–499.
- Beviá, C., Corchón, L. & Yasuda, Y. (2020), ‘Oligopolistic equilibrium and financial constraints’, *Rand Journal of Economics* **51**(1), 279–300.
- Bolton, P. & Scharfstein, D. S. (1990), ‘A theory of predation based on agency problems in financial contracting’, *American Economic Review* **80**(1), 93–106.
- Choi, J. P. & Gerlach, H. (2015), *The Oxford Handbook of International Antitrust Economics, Volume 2*, Oxford University Press, chapter Cartels and Collusion: Economic Theory and Experimental Economics, pp. 415–441.
- Dasgupta, P. & Maskin, E. (1986), ‘The existence of equilibrium in discontinuous games, i: Theory’, *Review of Economic Studies* **53**(1), 1–26.
- Friedman, J. W. (1971), ‘A non-cooperative equilibrium for supergames’, *Review of Economic Studies* **38**(1), 1–12.
- Granitz, E. & Klein, B. (1996), ‘Monopolization by “raising rivals’ costs”: The standard oil case’, *Journal of Law and Economics* **39**(1), 1–47.
- Haag, M. & Lagunoff, R. (2007), ‘On the size and structure of group cooperation’, *Journal of Economic Theory* **135**, 68–89.
- Harrington, J. (1989), ‘Collusion and predation under (almost) free-entry’, *International Journal of Industrial Organization* **7**, 389–401.

- Kawakami, T. (2010), ‘Collusion and predation under the condition of stochastic bankruptcy’, *Japanese Economic Review* **61**, 408–426.
- Kawakami, T. & Yoshihiro, Y. (1997), ‘Collusion under financial constraints: Collusion or predation when the discount factor is near one?’, *Economics Letters* **54**(2), 175–178.
- Kreps, D. M. & Wilson, R. (1982), ‘Reputation and imperfect information’, *Journal of Economic Theory* **27**(2), 253–279.
- Laibson, D. (1997), ‘Golden eggs and hyperbolic discounting’, *Quarterly Journal of Economics* **112**(2), 443–477.
- Marshall, R. C., Marx, L. M. & Samkharadze, L. (2019), Monopolization conduct by cartels. Mimeo, Duke University.
- McGee, J. S. (1958), ‘Predatory price cutting: The standard oil (n. j.) case’, *Journal of Law & Economics* **1**, 137–169.
- Milgrom, P. & Roberts, J. (1982), ‘Limit pricing and entry under incomplete information: An equilibrium analysis’, *Econometrica* **50**(2), 443–459.
- O’Donoghue, T. & Rabin, M. (1999), ‘Doing it now or later’, *American Economic Review* **89**(1), 103–124.
- Tasnádi, A. (2004), ‘Production in advance versus production to order’, *Journal of Economic Behavior & Organization* **54**, 191–204.
- Telser, L. G. (1966), ‘Cutthroat competition and the long purse’, *The Journal of Law & Economics* **9**, 259–277.
- van den Berg, A. & Bos, I. (2017), ‘Collusion in a price-quantity oligopoly’, *International Journal of Industrial Organization* **50**, 159–185.
- van Veldhuizen, R. & Sonnemans, J. (2018), ‘Renewable resources, strategic behavior and the hotelling rule: An experiment’, *Journal of Industrial Economics* **66**(2), 481–516.

Wiseman, T. (2017), 'When does predation dominate collusion?', *Econometrica* **85**(2), 555–584.

## On-line Appendix: Proof of Lemma 1

In what follows we first demonstrate the existence of a stationary equilibrium for  $n = 2$  using the theorem of Dasgupta and Maskin (1986), and we then extend the reasoning to  $n > 2$ , thereby allowing us to conclude that this game always admits an equilibrium.

Observe first that with 1 firm the game admits a unique equilibrium where the firm produces  $q^M$  and earns profits equal to  $\pi^M$ . In what follows, we therefore abstain from detailing the continuation payoffs of the monopolist, and we instead use throughout the proof  $\pi^M$  in such instances.

### B Case with $n = 2$ firms

We focus on *stationary* equilibria, namely equilibria where firms' quantities are time-invariant. Accordingly, we can write firm 1's discounted payoff, as:

$$\begin{aligned} \Pi_1(q_1, q_2) &= [a - c - b(q_1 + q_2)]q_1 - F \\ &+ \sum_{i=1}^{+\infty} \delta^i \mu_1 [a - c - b(q_1 + q_2)]q_1 - F \\ &+ \sum_{i=1}^{+\infty} \delta^i \nu_1 \pi^M, \end{aligned}$$

where,

$$\mu_1 = \mathbf{1}_{\{[a-c-b(q_1+q_2)]q_1-F>0, [a-c-b(q_1+q_2)]q_2-F>0\}},$$

and,

$$\nu_1 = \mathbf{1}_{\{[a-c-b(q_1+q_2)]q_1-F>0, [a-c-b(q_1+q_2)]q_2-F\leq 0\}},$$

with  $\mathbf{1}$  the indicator function.



The above expression can then be written as:

$$\begin{aligned}
\Pi_1(q_1, q_2) &= [a - c - b(q_1 + q_2)]q_1 - F \\
&\quad + \frac{\delta}{1 - \delta} \mu_1([a - c - b(q_1 + q_2)]q_1 - F) \\
&\quad + \frac{\delta}{1 - \delta} \nu_1 \pi^M.
\end{aligned} \tag{24}$$

Firm 2's discounted payoff can be defined likewise.

We next define the following sets:

$$\mathcal{E}_1 := \{(q_1, q_2) \in (\mathbb{R}^+)^2 \mid [a - c - b(q_1 + q_2)]q_1 - F > 0\},$$

$$\mathcal{E}_2 := \{(q_1, q_2) \in (\mathbb{R}^+)^2 \mid [a - c - b(q_1 + q_2)]q_2 - F > 0\}.$$

From (24), denoting by  $\mathcal{E}_1^c$  the complementary set to set  $\mathcal{E}_1$ , we have the following cases:

1. If  $(q_1, q_2) \in \mathcal{E}_1^c$ , then:

$$\Pi_1(q_1, q_2) = \Pi_1(q_1, q_2) = [a - c - b(q_1 + q_2)]q_1 - F.$$

2. If  $(q_1, q_2) \in \mathcal{E}_1 \cap \mathcal{E}_2$ , then:

$$\Pi_1(q_1, q_2) = \Pi_1(q_1, q_2) = \frac{1}{1 - \delta} ([a - c - b(q_1 + q_2)]q_1 - F).$$

3. If  $(q_1, q_2) \in \mathcal{E}_1 \cap \mathcal{E}_2^c$ , then:

$$\Pi_1(q_1, q_2) = [a - c - b(q_1 + q_2)]q_1 - F + \frac{\delta}{1 - \delta} \pi^M.$$

## B.1 Preliminaries: Characterizing the discontinuities

We denote the set of discontinuities of  $\Pi_1$  by  $\mathcal{D}$ . From (24), this set is just defined as:

$$\mathcal{D} = \partial\mathcal{E}_1^c \cup \{\mathcal{E}_1 \cap \partial\mathcal{E}_2^c\},$$

where  $\partial$  designates the boundary of the set, namely

$$\partial\mathcal{E}_1^c = \{(q_1, q_2) \in (\mathbb{R}^+)^2 \mid [a - c - b(q_1 + q_2)]q_1 - F = 0\},$$

and,

$$\mathcal{E}_1 \cap \partial\mathcal{E}_2^c = \{(q_1, q_2) \in (\mathbb{R}^+)^2 \mid [a - c - b(q_1 + q_2)]q_1 - F > 0, [a - c - b(q_1 + q_2)]q_2 - F = 0\}.$$

At this stage, we want to represent graphically on the  $(q_1, q_2)$ -plane the space of discontinuity.

*First*, let us focus on  $\partial\mathcal{E}_1^c$ . For any  $q_1 > 0$ , we can write:

$$[a - c - b(q_1 + q_2)]q_1 - F = 0 \Leftrightarrow q_2 = \frac{1}{b} \left[ a - c - bq_1 - \frac{F}{q_1} \right] =: f(q_1).$$

$f(q_1)$  is described by an inverted-U reaching a maximum for  $f(\hat{q}_1) = f(\sqrt{F/b}) = \frac{a-c}{b} - 2\sqrt{F/b}$ , as depicted in Figure 1. It follows that  $f(q_1)$  is a one-to-one mapping on  $(0, \hat{q}_1]$  and on  $[\hat{q}_1, +\infty)$ , respectively. For the range of admissible parameters ( $F < F^{max} = \frac{[a-c]^2}{4bn^2}$ ), we necessarily have that  $f(\hat{q}_1) \geq 0$ . Consequently,  $\partial\mathcal{E}_1^c$  can be written as the sum of two subsets where  $q_2$  is a continuous and one-ton-one function of  $q_1$ :

$$\partial\mathcal{E}_1^c = \{(q_1, q_2) \in (0, \hat{q}_1] \times \mathbb{R}^+ \mid q_2 = f(q_1)\} \cup \{(q_1, q_2) \in [\hat{q}_1, +\infty) \times \mathbb{R}^+ \mid q_2 = f(q_1)\}.$$

*Second*, let us focus on  $\partial\mathcal{E}_2^c$ . We know that the following equality,

$$[a - c - b(q_1 + q_2)]q_2 - F = 0,$$

characterizes an implicit function  $q_2 = \psi(q_1)$ . According to the above reasoning, we can

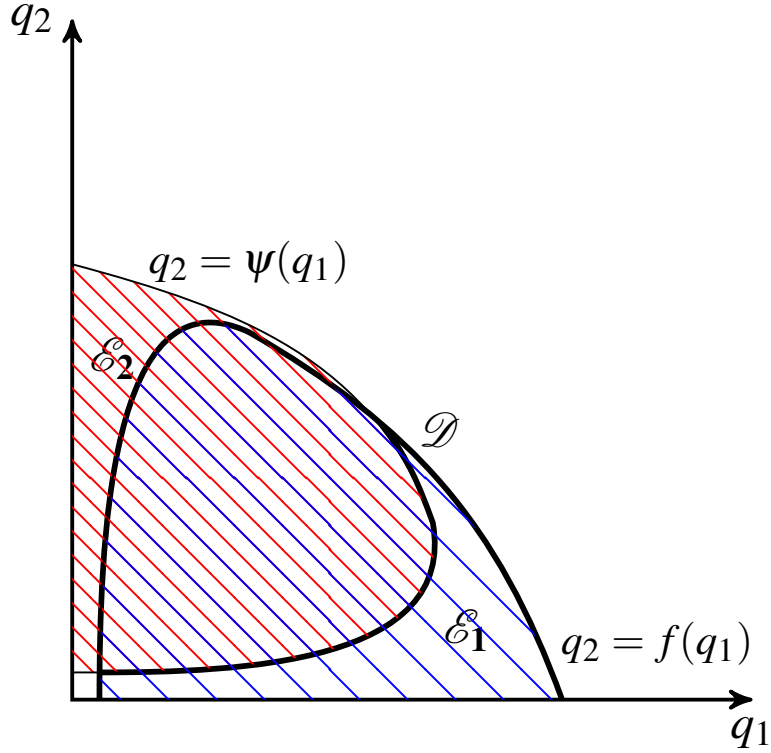


Figure 1: Graphical representation of  $\mathcal{D}$

write the set  $\mathcal{E}_1 \cap \partial \mathcal{E}_2^c$  as the sum of two subsets in which  $q_2$  is a continuous and one-to-one function of  $q_1$ . By denoting  $\tilde{q}_1$  the maximum of  $\psi$  and  $\tilde{q}_2 = \psi(\tilde{q}_1)$ , we have

$$\mathcal{E}_1 \cap \partial \mathcal{E}_2^c = \{(q_1, q_2) \in \mathcal{E}_1 \cap [0, \tilde{q}_1] \times [0, \tilde{q}_2] \mid q_2 = \psi(q_1)\} \cup \{(q_1, q_2) \in \mathcal{E}_1 \cap [0, \tilde{q}_1] \times [\tilde{q}_2, \infty) \mid q_2 = \psi(q_1)\}.$$

We graphically represent these boundaries as depicted on Figure 1.

## B.2 Dasgupta and Maskin (1986) theorem.

We now first recall the theorem of Dasgupta and Maskin (1986), before applying it to our setup.

**Theorem 5 of Dasgupta and Maskin (1986).** *Let  $A_i \subseteq \mathbb{R}$  ( $i = 1, \dots, N$ ) be a closed interval and let  $U_i : A \rightarrow \mathbb{R}^+$  ( $i = 1, \dots, N$ ) be continuous except on a subset  $A^{**}(i)$  of  $A^*(i)$ , where  $A^*(i)$  is defined as:*

$$A^*(i) = \{(a_1, \dots, a_N) \in A \mid \exists j \neq i, \exists d, 1 \leq d \leq D(i) \text{ such that } a_j = f_{ij}^d(a_i)\}.$$

Suppose  $\sum_{i=1}^N U_i(\mathbf{a})$  is upper semi-continuous and  $U_i(a_i, \mathbf{a}_{-i})$  is bounded and weakly lower semi-continuous in  $a_i$ . Then the game  $[(A_i, U_i); i = 1, \dots, N]$  possesses a mixed-strategy equilibrium.

### B.3 Applying Dasgupta and Maskin (1986)

First, let us recall how our notations relate to Dasgupta and Maskin (1986). We have  $N = 2$ ,  $A_i = \mathbb{R}^+$ ,  $a_i = q_i$ , and  $U_i = \Pi_i$ .

Second, we suppose that for each  $i$ , the discontinuities of  $\Pi_i$  are confined to a subset of a continuous manifold of dimension less than  $N$  ( $N = 2$  in this specific instance). To be precise, following Dasgupta and Maskin, for each pair of agents  $i, j \in 1, 2$ , let  $D(i)$  be a positive integer, and for each integer  $d$ , with  $1 \leq d \leq D(i)$ , let  $f_{ij}^d : \mathbb{R} \mapsto \mathbb{R}$  be a one-to-one, continuous function. Finally, for each  $i \in 1, 2$  define

$$Q^*(i) = \{(q_1, q_2) \in \mathbf{Q} \mid \exists j \neq i, \exists d, 1 \leq d \leq D(i), \text{ such that } q_j = f_{ij}^d(q_i)\}.$$

We shall suppose that the discontinuities of  $\Pi_i$  are confined to a subset  $Q^{**}(i)$  of  $Q^*(i)$ . In other words,  $Q^{**}(i)$  is the set of discontinuities of  $\Pi_i$ . As argued by Dasgupta and Maskin (p. 7), “the requirement that the function  $f_{ij}^d$  be one-to-one does not rule out a non-monotonic curve of discontinuities; since each monotonic subcurve can be made to correspond to a different  $f_{ij}^d$ .”

To apply the Theorem, we shall show that it applies to our setup for firm 1, and given the symmetry of the game, this holds true for player 2 as well. We need to ensure four conditions, which are checked in the following subsections, respectively.

#### B.3.1 Discontinuities

This point is evident, because the set of discontinuities of  $\Pi_i$  ( $\mathcal{D} = Q^{**}(1)$ ) can be written by the union of subset, in which  $q_2$  is in one-to-one continuous link with respect to  $q_1$  (see

Subsection B.1).<sup>10</sup>

### B.3.2 $\sum_{i=1}^n \Pi_i(\mathbf{q})$ is upper semi-continuous

Observe that if one firm goes bankrupt when the number of initial competitors is such that  $n = 2$ , the subgame admits an equilibrium where the surviving firm produces  $q^M$  forever after.

We can then distinguish four cases:

1. If  $(q_1, q_2) \in \mathcal{E}_1^c \cap \mathcal{E}_2^c$  then:

$$\Pi_1(q_1, q_2) + \Pi_2(q_1, q_2) = [a - c - b(q_1 + q_2)][q_1 + q_2] - 2F.$$

2. If  $(q_1, q_2) \in \mathcal{E}_1^c \cap \mathcal{E}_2$ , then:

$$\Pi_1(q_1, q_2) + \Pi_2(q_1, q_2) = [a - c - b(q_1 + q_2)][q_1 + q_2] - 2F + \frac{\delta}{1 - \delta} \pi^M.$$

3. If  $(q_1, q_2) \in \mathcal{E}_1 \cap \mathcal{E}_2$  then:

$$\Pi_1(q_1, q_2) + \Pi_2(q_1, q_2) = \frac{[a - c - b(q_1 + q_2)][q_1 + q_2] - 2F}{1 - \delta}.$$

4. If  $(q_1, q_2) \in \mathcal{E}_1 \cap \mathcal{E}_2^c$  then:

$$\Pi_1(q_1, q_2) + \Pi_2(q_1, q_2) = [a - c - b(q_1 + q_2)][q_1 + q_2] - 2F + \frac{\delta}{1 - \delta} \pi^M.$$

Recall that the definition of an upper semi-continuous mapping:  $f(x)$  is upper semi-continuous mapping in  $x_0$  iff:

$$\limsup_{x \rightarrow x_0} f(x) \leq f(x_0).$$

As usual, to ensure the upper semi-continuity we introduce a sequence  $\{q_{1m}, q_{2m}\}_{m \geq 1} \subseteq$

---

<sup>10</sup>In the words of Dasgupta and Maskin (1986), the assumption that functions  $f_{ij}^d$  need to be one-to-one aims to “exclude  $f_{ij}^d$ ’s that are vertical or horizontal on some interval” (p. 7-8). This is clearly not the case when looking at the Figure 1.

$(\mathbb{R}^+)^2$  that is convergent to  $\{q_{10}, q_{20}\}$ . First of all, it is obvious that – by construction – outside the set of discontinuities  $\mathcal{D}$ ,  $\Pi_1$  is continuous, and therefore upper semi-continuous.

We just need to focus on the set  $\mathcal{D}$ . There are two cases (see Figure B.1).

**First:**  $(q_{10}, q_{20}) \in \partial\mathcal{E}_1^c$ , i.e.,

$$[a - c - b(q_{10} + q_{20})]q_{10} - F = 0,$$

and  $\Pi_1(q_{10}, q_{20}) + \Pi_2(q_{10}, q_{20}) = \Pi_2(q_{10}, q_{20})$

$$= \begin{cases} [a - c - b(q_{10} + q_{20})]q_{20} - F & \text{if } (q_{10}, q_{20}) \in \mathcal{E}_2^c, \\ [a - c - b(q_{10} + q_{20})]q_{20} - F + \frac{\delta}{1-\delta}\pi^M & \text{if } (q_{10}, q_{20}) \in \mathcal{E}_2. \end{cases}$$

Now consider the four following subcases: (i)  $\{q_{1m}, q_{2m}\}_m \subseteq \mathcal{E}_1 \cap \mathcal{E}_2^c$ ; (ii)  $\{q_{1m}, q_{2m}\}_m \subseteq \mathcal{E}_1 \cap \mathcal{E}_2$ ; (iii)  $\{q_{1m}, q_{2m}\}_m \subseteq \mathcal{E}_1^c \cap \mathcal{E}_2$ , and (iv)  $\{q_{1m}, q_{2m}\}_m \subseteq \mathcal{E}_1^c \cap \mathcal{E}_2^c$ .

(i) We have:  $[a - c - b(q_{1m} + q_{2m})]q_{1m} - F > 0$ ,  $[a - c - b(q_{1m} + q_{2m})]q_{2m} - F \leq 0$ .

Therefore,

$$\begin{aligned} & \Pi_1(q_{1m}, q_{2m}) + \Pi_2(q_{1m}, q_{2m}) = \\ & [a - c - b(q_{1m} + q_{2m})]q_{1m} - F + \frac{\delta}{1-\delta}\pi^M + [a - c - b(q_{1m} + q_{2m})]q_{2m} - F \\ & \rightarrow [a - c - b(q_{10} + q_{20})]q_{10} - F + \frac{\delta}{1-\delta}\pi^M + [a - c - b(q_{10} + q_{20})]q_{20} - F \\ & \leq [a - c - b(q_{10} + q_{20})]q_{20} - F + \frac{\delta}{1-\delta}\pi^M \\ & \leq \Pi_1(q_{10}, q_{20}) + \Pi_2(q_{10}, q_{20}). \end{aligned}$$

(ii) We have:  $[a - c - b(q_{1m} + q_{2m})]q_{1m} - F > 0$ ,  $[a - c - b(q_{1m} + q_{2m})]q_{2m} - F > 0$ .

Therefore,

$$\begin{aligned}
& \Pi_1(q_{1m}, q_{2m}) + \Pi_2(q_{1m}, q_{2m}) \\
&= \frac{1}{1-\delta} ([a-c-b(q_{1m}+q_{2m})]q_{1m}-F) + \frac{1}{1-\delta} ([a-c-b(q_{1m}+q_{2m})]q_{2m}-F) \\
&\rightarrow \frac{1}{1-\delta} ([a-c-b(q_{10}+q_{20})]q_{20}-F) \\
&\leq \Pi_1(q_{10}, q_{20}) + \Pi_2(q_{10}, q_{20}).
\end{aligned}$$

(iii) We have:  $[a-c-b(q_{1m}+q_{2m})]q_{1m}-F \leq 0$ ,  $[a-c-b(q_{1m}+q_{2m})]q_{2m}-F > 0$ .

Therefore,

$$\begin{aligned}
& \Pi_1(q_{1m}, q_{2m}) + \Pi_2(q_{1m}, q_{2m}) \\
&= [a-c-b(q_{1m}+q_{2m})]q_{1m}-F + [a-c-b(q_{1m}+q_{2m})]q_{2m}-F + \frac{\delta}{1-\delta}\pi^M \\
&\rightarrow [a-c-b(q_{10}+q_{20})]q_{20}-F + \frac{\delta}{1-\delta}\pi^M \\
&\leq \Pi_1(q_{10}, q_{20}) + \Pi_2(q_{10}, q_{20}).
\end{aligned}$$

(iv) We have:  $[a-c-b(q_{1m}+q_{2m})]q_{1m}-F \leq 0$ ,  $[a-c-b(q_{1m}+q_{2m})]q_{2m}-F \leq 0$ .

Therefore,

$$\begin{aligned}
& \Pi_1(q_{1m}, q_{2m}) + \Pi_2(q_{1m}, q_{2m}) \\
&= [a-c-b(q_{1m}+q_{2m})]q_{1m}-F + [a-c-b(q_{1m}+q_{2m})]q_{2m}-F \\
&\rightarrow [a-c-b(q_{10}+q_{20})]q_{20}-F \\
&\leq \Pi_1(q_{10}, q_{20}) + \Pi_2(q_{10}, q_{20}).
\end{aligned}$$

Partial conclusion: the mapping  $\Pi_1(q_1, q_2) + \Pi_2(q_1, q_2)$  is upper semi-continuous on  $\partial\mathcal{E}_1$ .

**Second:**  $(q_{10}, q_{20}) \in \mathcal{E}_1 \cap \partial\mathcal{E}_2^c$ , i.e.,

$$[a-c-b(q_{10}+q_{20})]q_{10}-F > 0, [a-c-b(q_{10}+q_{20})]q_{20}-F = 0,$$

and  $\Pi_1(q_{10}, q_{20}) + \Pi_2(q_{10}, q_{20}) = \Pi_1(q_{10}, q_{20}) = [a - c - b(q_{10} + q_{20})]q_{10} - F + \frac{\delta}{1-\delta}\pi^M$ .

As before, now consider the four following subcases: (i)  $\{q_{1m}, q_{2m}\}_m \subseteq \mathcal{E}_1 \cap \mathcal{E}_2^c$ ; (ii)  $\{q_{1m}, q_{2m}\}_m \subseteq \mathcal{E}_1 \cap \mathcal{E}_2$ ; (iii)  $\{q_{1m}, q_{2m}\}_m \subseteq \mathcal{E}_1^c \cap \mathcal{E}_2$ , and (iv)  $\{q_{1m}, q_{2m}\}_m \subseteq \mathcal{E}_1^c \cap \mathcal{E}_2^c$ .

(i) We have:  $[a - c - b(q_{1m} + q_{2m})]q_{1m} - F > 0$ ,  $[a - c - b(q_{1m} + q_{2m})]q_{2m} - F \leq 0$ .

Therefore,

$$\begin{aligned} & \Pi_1(q_{1m}, q_{2m}) + \Pi_2(q_{1m}, q_{2m}) = \\ & [a - c - b(q_{1m} + q_{2m})]q_{1m} - F + \frac{\delta}{1-\delta}\pi^M + [a - c - b(q_{1m} + q_{2m})]q_{2m} - F \\ & \rightarrow [a - c - b(q_{10} + q_{20})]q_{10} - F + \frac{\delta}{1-\delta}\pi^M \\ & \leq \Pi_1(q_{10}, q_{20}) + \Pi_2(q_{10}, q_{20}). \end{aligned}$$

(ii) We have:  $[a - c - b(q_{1m} + q_{2m})]q_{1m} - F > 0$ ,  $[a - c - b(q_{1m} + q_{2m})]q_{2m} - F > 0$ .

Therefore,

$$\begin{aligned} & \Pi_1(q_{1m}, q_{2m}) + \Pi_2(q_{1m}, q_{2m}) \\ & = \frac{1}{1-\delta} ([a - c - b(q_{1m} + q_{2m})]q_{1m} - F) + \frac{1}{1-\delta} ([a - c - b(q_{1m} + q_{2m})]q_{2m} - F) \\ & \rightarrow \frac{1}{1-\delta} ([a - c - b(q_{10} + q_{20})]q_{10} - F) \\ & \leq \Pi_1(q_{10}, q_{20}) + \Pi_2(q_{10}, q_{20}). \end{aligned}$$

(iii) We have:  $[a - c - b(q_{1m} + q_{2m})]q_{1m} - F \leq 0$ ,  $[a - c - b(q_{1m} + q_{2m})]q_{2m} - F > 0$ .

Therefore,

$$\begin{aligned} & \Pi_1(q_{1m}, q_{2m}) + \Pi_2(q_{1m}, q_{2m}) \\ & = [a - c - b(q_{1m} + q_{2m})]q_{1m} - F + [a - c - b(q_{1m} + q_{2m})]q_{2m} - F + \frac{\delta}{1-\delta}\pi^M \\ & \rightarrow [a - c - b(q_{10} + q_{20})]q_{10} - F + \frac{\delta}{1-\delta}\pi^M \\ & \leq \Pi_1(q_{10}, q_{20}) + \Pi_2(q_{10}, q_{20}). \end{aligned}$$



(iv) We have:  $[a - c - b(q_{1m} + q_{2m})]q_{1m} - F \leq 0$ ,  $[a - c - b(q_{1m} + q_{2m})]q_{2m} - F \leq 0$ .

Therefore,

$$\begin{aligned}
& \Pi_1(q_{1m}, q_{2m}) + \Pi_2(q_{1m}, q_{2m}) \\
&= [a - c - b(q_{1m} + q_{2m})]q_{1m} - F + [a - c - b(q_{1m} + q_{2m})]q_{2m} - F \\
&\rightarrow [a - c - b(q_{10} + q_{20})]q_{10} - F \\
&\leq \Pi_1(q_{10}, q_{20}) + \Pi_2(q_{10}, q_{20}).
\end{aligned}$$

**Conclusion:** the mapping  $\Pi_1(q_1, q_2) + \Pi_2(q_1, q_2)$  is upper semi-continuous on  $\mathcal{E}_1 \cap \partial\mathcal{E}_2^c$ , so does on the set of the discontinuity points  $\mathcal{D}$ , and thus on  $(\mathbb{R}^+)^2$ .

### B.3.3 $\Pi_i(q_1, q_2)$ is bounded

Let  $q_2 \in \mathbb{R}^+$ . Recall that:

1. If  $(q_1, q_2) \in \mathcal{E}_1^c = \{(q_1, q_2) \in (\mathbb{R}^+)^2 \mid [a - c - b(q_1 + q_2)]q_1 - F \leq 0\}$ :

$$\Pi_1(q_1, q_2) = [a - c - b(q_1 + q_2)]q_1 - F.$$

2. If  $(q_1, q_2) \in \mathcal{E}_1 \cap \mathcal{E}_2 = \{(q_1, q_2) \in (\mathbb{R}^+)^2 \mid [a - c - b(q_1 + q_2)]q_i - F > 0, i = 1, 2\}$ :

$$\Pi_1(q_1, q_2) = \frac{1}{1 - \delta} ([a - c - b(q_1 + q_2)]q_1 - F).$$

3. If  $(q_1, q_2) \in \mathcal{E}_1 \cap \mathcal{E}_2^c$ :

$$\Pi_1(q_1, q_2) = [a - c - b(q_1 + q_2)]q_1 - F + \frac{\delta}{1 - \delta} \pi^M.$$

Note that the mapping  $\zeta : q_1 \mapsto [a - c - b(q_1 + q_2)]q_1 - F$  is continuous on  $\mathbb{R}^+$ , and described an inverted U-shaped curve, with  $\zeta(0) = 0$ , a maximum at  $\check{q}_1 = (a - c - bq_2)/2b$ , a root at  $\tilde{q}_1 > \check{q}_1$ , i.e.,  $\zeta(\tilde{q}_1) = 0$ , and  $\zeta(+\infty) = -\infty$ .

Let us introduce an upper bound on  $q_1$  denoted by  $\bar{q}_1$ , where  $\bar{q}_1 > \check{q}_1$ . We can establish the following:

1. If  $(q_1, q_2) \in \mathcal{E}_1^c$ , then  $\Pi_1(\bar{q}, q_2) \leq \Pi_1(q_1, q_2) \leq 0$ .
2. If  $(q_1, q_2) \in \mathcal{E}_1 \cap \mathcal{E}_2$ , then  $0 < \Pi_1(q_1, q_2) \leq \Pi_1(\check{q}_1, q_2)$ .
3. If  $(q_1, q_2) \in \mathcal{E}_1 \cap \mathcal{E}_2^c$ , then  $\frac{\delta}{1-\delta}\pi^M < \Pi_1(q_1, q_2) \leq \Pi_1(\check{q}_1, q_2)$ .

**Conclusion:** that the mapping  $q_1 \mapsto \Pi_1(q_1, q_2)$  is bounded on  $[0, \bar{q}_1]$ .

### B.3.4 $\Pi_i(q_i, \mathbf{q}_{-i})$ weakly lower semi-continuous in $q_i$

As Dasgupta and Maskin (1986) claim, if a function is lower semi-continuous, this function is *weakly* lower semi-continuous (footnote 14, p. 25).

By definition,  $\Pi_1(q_1, q_2)$  is lower semi-continuous in  $q_1$  if and only if, for any  $\alpha \in \mathbb{R}$ , the set  $\{q_1 | \Pi(q_1, q_2) \leq \alpha\}$  is a closed interval. This condition holds when we restrict the values of  $q_1$  to the closed interval  $[0, \bar{q}_1]$ . For example, if  $\alpha \geq \zeta(\check{q}_1) + \frac{\delta}{1-\delta}\pi^M$ , then  $\{q_1 | \Pi(q_1, q_2) \leq \alpha\} = [0, \bar{q}_1]$ , and if  $\alpha \leq 0$ , then  $\{q_1 | \Pi(q_1, q_2) \leq \alpha\} = [\check{q}_1, \bar{q}_1]$ .

### B.3.5 General conclusion

Since firms are symmetric in our setup, the exact same reasoning applies for player 2, therefore allowing us to conclude that given Dasgupta and Maskin (1986)'s Theorem 5, the game possesses a mixed-strategy stationary equilibrium on the space  $[0, \bar{q}_1] \times [0, \bar{q}_2]$ , where  $\bar{q}_2$  is defined in a similar way to  $\bar{q}_1$ .

## C Extension to $n > 2$

Having shown that an equilibrium exists for any subgame involving  $n \leq 2$  firms on restricting the space of  $q_i$  to closed interval, we will now apply a recursive reasoning to generalize the existence result to any finite  $n$ . Indeed, the problem is structurally equivalent for any finite number of firms, so that with  $n = 3$  firms the only difference for proving the upper semi-continuity of  $\sum \Pi_i(\mathbf{q})$  and the weakly lower semi-continuity of  $\Pi_i(q_i, \mathbf{q}_{-i})$  in  $q_i$  is the increased

dimensionality of the discontinuity set. The existence proof can be extended to  $n = 3$ , and recurrently to any finite  $n$ , with the same constraint on  $q_i$ .