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# Genetic Algorithms for Solving the Pigment Sequencing Problem

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## Abstract

Lot sizing is important in production planning. It consists of determining a production plan that meets the orders and other constraints while minimizing the production cost. Here, we consider a Discrete Lot Sizing and Scheduling Problem (DLSP), specifically the Pigment Sequencing Problem (PSP). We have developed a solution that uses Genetic Algorithms to tackle the PSP. Our approach introduces adaptive techniques for each step of the genetic algorithm, including initialization, selection, crossover, and mutation. We conducted a series of experiments to assess the performance of our approach across some multiple trials using publicly available instances of the PSP. Our experimental results demonstrate that Genetic Algorithms are practical and effective approaches for solving DLSP.

## Keywords

Genetic algorithm; production planning; pigment sequencing problem; lot sizing.

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## I INTRODUCTION

Lot sizing problems involve determining which items to produce, when to produce them, and which machine to use in order to meet customer demand while also achieving financial goals. These problems are complex, as they often require producing multiple types of items while balancing conflicting objectives, such as minimizing production and stocking costs while also meeting customer needs. Different types of lot-sizing problems have been studied in the literature. In recent years, researchers such as Houndji et al. [20] and Ceschia et al. [25] have focused on an NP-Hard variant known as the Pigment Sequencing Problem. This problem is included in the CSPLib repository [9] and involves producing multiple items on a single machine with limited capacity (one item per period). The planning horizon is discrete and finite, with predefined stocking and setup costs for each item.

Like many Discrete Lot Sizing and Scheduling Problems, the Pigment Sequencing Problem can be formalized and solved with Genetic Algorithms. Genetic Algorithms are heuristic search methods inspired by the natural evolution of living species. Based upon the concept of the survival of the fittest, genetic algorithms are able, over multiple generations, to find the best solution to a problem. Several studies [14] [29] [16] have shown how efficient they could be in solving optimization problems. In the paper, we present a search method that relies on genetic algorithms and experiment with this approach. The results indicate that Genetic Algorithm-based methods are a promising solution for addressing Discrete Lot Sizing and Scheduling Problems like the Pigment Sequencing Problem.

This paper is organized as follows: Section 2 exposes some background on the Discrete Lot Sizing and Scheduling Problems and Genetic Algorithms (GAs); Section 3 presents the problem to be solved (the Pigment Sequencing Problem) and shows an instance of the problem; Section 4 gives details on our method based on genetic algorithms; Section 5 presents some experimental results obtained from the implementation of our approach; and Section 6 concludes this paper and provides some perspectives.

## II BACKGROUND

### 2.1 Discrete Lot Sizing and Scheduling Problems (DLSP)

The PSP belongs to the category of Discrete Lot Sizing and Scheduling Problems (DLSP). The PSP is a problem in which the total capacity available for a period is used to produce one item. The origin of the multi-item DLSP traces back to Fleischmann (1990) [2]; in which a branch-and-bound procedure is presented using Lagrangean relaxation for determining lower bounds and feasible solutions. The relaxed problems are solved by dynamic programming, yielding optimal solutions or at least feasible solutions with tight lower bounds in a few minutes. Cattrysse et al [5] introduced a dual ascent and column generation heuristic to solve a DLSP with setup times formulated as a Set Partitioning Problem (SPP). Later, Van Hoesel et al. [6] formulated DLSP as an integer programming problem and presented two solution procedures: the first procedure based on a reformulation of DLSP as a linear programming assignment problem, with additional restrictions to reflect the specific (setup) cost structure; the second procedure based on dynamic programming (DP).

Besides, Jordan et al. [8] solved a Discrete Lot Sizing and Scheduling Problem for one machine with sequence-dependent setup times and setup costs as a single machine scheduling problem and which they termed the batch sequencing problem. This batch sequencing problem is solved with a branch-and-bound algorithm which is accelerated by bounding and dominance rules. Much later, Miller and Wolsey [12] formulated the DLSP with setup costs not dependent on the sequence as a network flow problem. They exposed some MIP formulations for various modifications (with backlogging, safety stock, and initial supply). Moreover, several more MIP formulations and variants have been proposed and discussed by Pochet and Wolsey [15]. Gicquel et al. [17] exposed a formulation in which they derived valid inequations for the DLSP with several items and sequential setup costs and periods. This formulation is a modification of the problem proposed by Wolsey [11]. A new approach is again proposed by Gicquel et al. [18] to the modelization of the DLSP with several items and sequential setup costs and periods that considers relevant physical attributes such as color, dimension, and the level of quality. This allowed them to effectively reduce the number of variables and constraints in the MIP models. Houndji et al. [20] introduced a new global constraint they named *stocking cost* to solve the PSP with Constraint Programming. They tested it on new instances and published it on CSPLib (Gent and Walsh [9]). The experimental results showed that *stocking cost* is effective in filtering compared to other constraints used mainly in the community of Constraint Programming. In addition, Ceschia et al. [24] used the Simulated Annealing (SA) to solve the PSP. They introduced an approach along with a statistically-principled tuning procedure that guides the local search and used it to solve new instances available in the Ophub repository. Their solver was able to find near-optimal solutions in short time for all instances, including those that are not solved by state-of-the-art methods [25]. More recently, Park et al [34] proposed a framework for solving the DLSP using reinforcement learning in which they formalized the scheduling process as a sequential decision-making problem with the Markov decision process.

## 2.2 Genetic Algorithms and Optimization problems

Genetic Algorithms are stochastic search algorithms that mimic living species' natural evolution and reproduction mechanisms. They were proposed for the first time by John Holland [4] in 1970. One of the main principles of these algorithms is the concept of the "*survival of the fittest*", which states that one individual whose features fit the best the environment is more likely to survive. Goldberg et al [1] introduced the concept of Messy Genetic Algorithms which solved problems by combining relatively short, well-tested building blocks to form longer, more complex strings that increasingly cover all features of a problem. This approach stood in contrast to the usual fixed-length, fixed-coding genetic algorithm. By emulating natural mechanisms, Genetic Algorithms assure the evolution of a population over several generations with concepts such as *Initialization* [21], *Selection* [23], *Crossover* [27], or *mutation* [33] as shown in Figure 1.

Several studies explored the application of genetic algorithms in the context of optimization problems. A. Kimms [7] introduced a mixed-integer programming formulation for the multi-level, multi-machine proportional lot sizing and scheduling problem and presented a genetic algorithm to solve that problem. Later, J. Duda [13] presented a study of genetic algorithms applied to a lot sizing problem, which has been formulated for an operational production planning in a foundry. Three variants of genetic algorithm were considered, each of them using special crossover and mutation operators as well as repair functions. Moreover, Xie et al [10] proposed heuristic genetic algorithms for lot sizing problems by designing a domain-specific encoding scheme for the lot-sizes and by providing a heuristic shifting procedure as the decoding schedule. More recently, Larroche et al [32] dealt with a complex production planning problem with lost sales, overtimes, safety stock and sequence dependent setup times on parallel and unrelated machines by developing a genetic algorithm that combines several operations already defined in the literature to solve the problem.

## III PROBLEM DEFINITION

Several studies addressed the Pigment Sequencing Problem - PSP (see, for example, [24, 31]). It can be described as a problem that requires to produce various items on one machine with predefined setup costs. Setup costs are necessary for the transition from an item  $i$  to another item  $j$  so that  $i \neq j$ . Often, the production planning needs to meet the customer orders while:

- not exceeding the production capacity of the machine;
- minimizing the setup and stocking costs.

Without loss of generality, it is assumed that only one item is produced per period and all orders are normalized i.e., the machine's production capacity is restricted to one item per period and  $d(i, t) \in \{0, 1\}$  with  $i$  the item and  $t$  the period. The PSP is a production planning problem with the following specifications: a discrete and finite planning horizon, some capacity constraints, a deterministic and static order, several items, small buckets, setup costs, only one level, and without shortage.

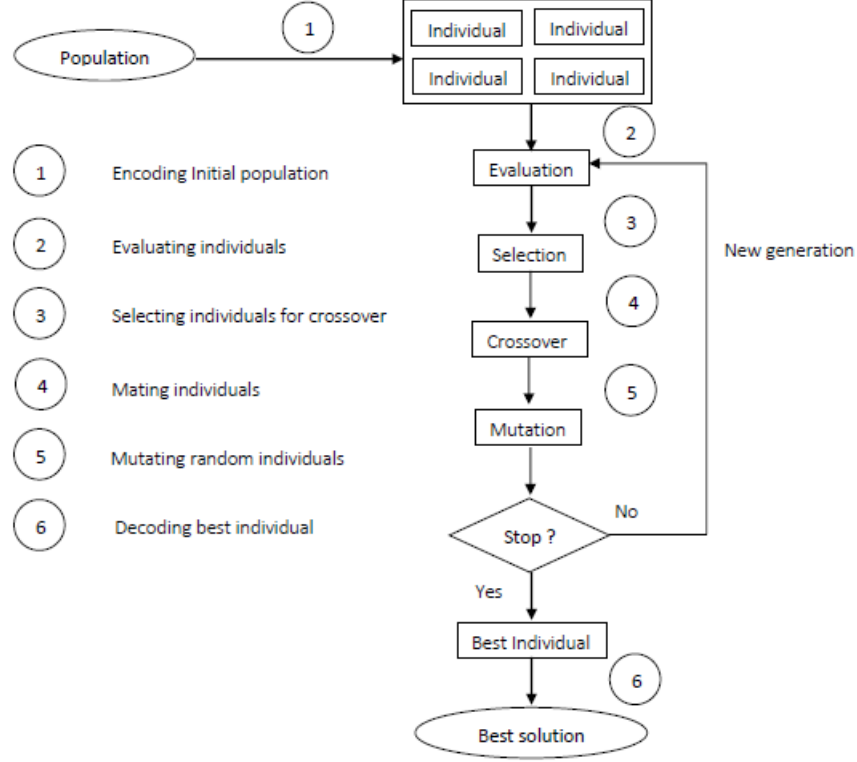


Figure 1: Flow chart of Genetic Algorithms.

Formally, the problem can be formulated as [15] :

$$\min \sum_{i,j=0}^M \sum_{t=0}^P q^{i,j} X_t^{i,j} + \sum_{i=0}^M \sum_{t=1}^P h^i s_t^i \quad (1)$$

$$s_0^i = 0, \forall i \in M \quad (2)$$

$$x_t^i + s_{t-1}^i = d_t^i + s_t^i, \forall i \in M, t \in P \quad (3)$$

$$x_t^i \leq y_t^i, \forall i \in M, t \in P \quad (4)$$

$$\sum_i y_t^i = 1, \forall i, j \in M, t \in P \quad (5)$$

$$X_t^{i,j} \geq y_{t-1}^i + y_t^j - 1, \forall i, j \in M, t \in P \quad (6)$$

with the following indices and index sets:

- $M$ : set of item indices,  $i, j \in M$  and  $M \subseteq \mathbb{N}$ ;
- $P$ : set of period indices,  $t \in P$  and  $P \subseteq \mathbb{N}$ ;

the parameters:

- $h_i$ : the holding cost of the item  $i$  with  $i \in M$ ;
- $q^{i,j}$ : the changeover cost from item  $i$  to item  $j$  with  $i, j \in M$ ;

and the following variables:

- $x_t^i$ : binary production variable that is 1 if item  $i$  is produced in period  $t$ , 0 otherwise;
- $y_t^i$ : binary setup variable that is 1 if the machine is set for the production of item  $i$  in period  $t$ , 0 otherwise;
- $d_t^i$ : binary variable that is 1 if item  $i$  is ordered in period  $t$ , 0 otherwise;

- $X_t^{i,j}$ : binary changeover variable that is 1 if in period  $t$ , we transitioned from the production of item  $i$  to the one of item  $j$ , 0 otherwise;
- $s_t^i$ : integer variable that represents the number of item  $i$  stored in the period  $t$ ,  $s_t^i \in \mathbb{R}_+$ .

The goal is to minimize the overall stocking and setup costs as expressed by (1). Constraint (2) clearly states that there is no initial stock. Constraint (3) expresses the rule of flow conservation. Constraint (4) aims to get the setup variable  $y_t^i$  to equal 1 if the item  $i$  is produced in the period  $t$ . Constraint (5) ensures the machine is always set to produce an item. Therefore,  $y_t^i$  is bound to take the value that minimizes the changeover cost. Furthermore, if there is no production in the period  $t$ ,  $y_t^i = y_{t-1}^i$  or  $y_t^i = y_{t+1}^i$ . Thus, it is interesting to set up the machine for production even if there is no item to produce. Constraint (6) sets values to changeover variables. If  $y_{t-1}^i$  and  $y_t^i$  equal 1, then  $X_t^{i,i}$  is bound to equal 1 otherwise  $X_t^{i,j}$  would equal 0 thanks to the goal function that minimizes the changeover cost.

**Example:** Consider the following relatively easy problem:

- Number of items:  $NI = 2$ ;
- Number of periods:  $NT = 5$ ;
- Order per period. Be  $d(i, t)$  the order of item  $i$  in the period  $t$ :  $d(1, t) = (0, 1, 0, 0, 1)$  and  $d(2, t) = (1, 0, 0, 0, 1)$ ;
- Stocking cost. Be  $h(i)$  the stocking cost of the item  $i$ ,  $h(1) = h(2) = 2$

Let  $x^T$  be the production planning representing a potential solution to the problem. It is an array of size  $NT$ . A possible solution to the problem is  $x^T = (2, 1, 2, 0, 1)$  with a cost of  $q(2, 1) + q(1, 2) + q(2, 1) + 2h(2) = 15$ . The optimal solution is  $x^T = (2, 1, 0, 1, 2)$  with a cost of  $q(2, 1) + q(1, 2) + h(1) = 10$ .

## IV OUR APPROACH

In this section, we present each aspect of our implementation of genetic algorithms to solve the PSP.

### 4.1 Genetic representation

When implementing genetic algorithms to solve a problem, finding the proper representation for the individual is important and influences the efficiency of the whole algorithm. One of the most straightforward representations used in genetic algorithms is the one used by John Holland [3]: the bit-array representation where a chromosome is represented by a string of bits containing 0 and 1 to express if an item  $i$  has been produced at a given period  $t$  as pictured on Figure 2.

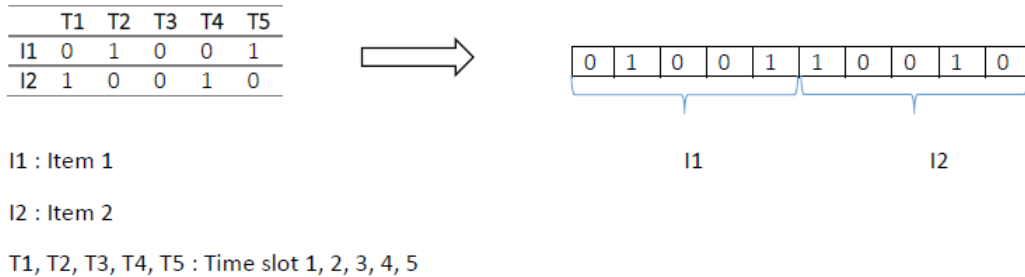


Figure 2: Chromosome bit-array representation.

Although correct, this representation significantly increases the complexity of the whole algorithm forcing us to go through a list of  $nT * nI$  items with  $nT$ : *the number of periods* and  $nI$ : *the number of items*. All of this prompted the emergence of another representation, as used by Mirshekarian et al. [30], in which the chromosome is represented by a string of integers of the length of the planning horizon ( $nT$ ). In this string, each integer corresponds to the item's index produced at the exact period and 0 otherwise as pictured in Figure 3. Thus, the complexity is considerably reduced.

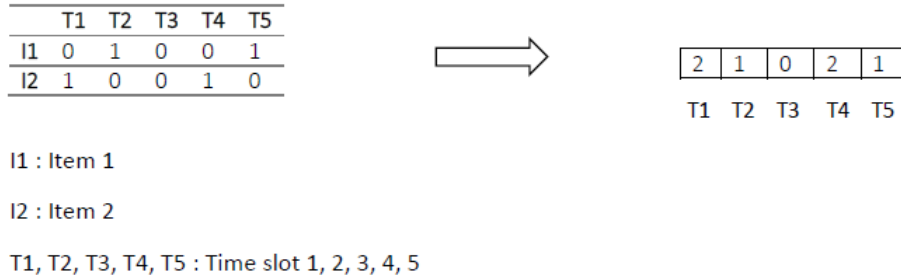


Figure 3: Chromosome final representation.

## 4.2 Initialization

As stated earlier, the initialization process consists of generating the initial population. We have opted for the heuristic algorithm based on the breadth-first search technique described in Algorithm IV.1. The process starts at the end of the planning horizon and backtracks to the first production period. The goal is to seed the best possible individuals for the initial population. At every step of the process, the algorithm determines which of the subsequent child nodes are the best to expand. This process produces better individuals than random seeding [21], helping bootstrap the overall search process.

## 4.3 Evaluation

Each chromosome is evaluated before proceeding to the selection. The cost estimation is a key input to the selection process and the element that our genetic-based algorithm seeks to minimize. In our study, the cost function is based on the aforementioned MIP formulation (1) and can be stated as follows:

$$F(x) = \sum_{i,j=0}^M \sum_{t=0}^P q^{i,j} X_t^{i,j} + \sum_{i=0}^M \sum_{t=1}^P h^i s_t^i \quad (7)$$

It consists of two costs:

- the setup cost  $\sum_{i,j=0}^M \sum_{t=0}^P q^{i,j} X_t^{i,j}$ : the sum of the setup costs for all periods;
- the stocking cost  $\sum_{i=0}^M \sum_{t=1}^P h^i s_t^i$ : the sum of the stocking time slots of all items multiplied by the stocking cost  $h^i$  of each item  $i$

---

**Algorithm IV.1** Population initialization algorithm.

---

```
1   BEGIN
2   READ Expected_Population_Size, PSP_Instance
3   SET population to []
4   SET queue to firstNode
5   SET popCounter to queue length
6   WHILE population length is less than Expected_Population_Size
7     IF queue is empty THEN
8       BREAK
9     SET node to popFirst(queue)
10    DECREMENT popCounter
11    IF node is leafNode THEN
12      ADD node chromosome to population
13      CONTINUE
14    ENDIF
15    FOR child in node children (PSP_Instance)
16      APPEND child to queue
17      INCREMENT popCounter
18      IF popCounter is greater than Expected_Population_Size THEN
19        BREAK
20    ENDFOR
21  ENDWHILE
22  END
```

---

#### 4.4 Selection

The selection operator we chose to implement is based on the process commonly known as the "Roulette wheel" [23]. Hence, each chromosome is given a probability of being selected based on its fitness. Therefore, the fittest chromosome is given the highest chance. Then, a selector is used to pick two chromosomes based on their probability. Those chromosomes will mate and produce offspring. We evaluate each chromosome based on the data provided by each instance and for each item (stocking cost and setup cost) (7). The higher the cost, the less fit the chromosome is and the lower the probability of being chosen is too. In practice, the fitness of each chromosome in a population is computed (8) relative to the cost of the fittest chromosome of this population (9)

$$M = \max(c), \forall c \in P \quad (8)$$

$$p_i = ((M + 1) - B_i) / \sum_c ((M + 1) - B_c) \quad (9)$$

along with the following variables:

- $M$ : the cost of the fittest chromosome of the population  $P$ ;
- $p_i$ : the "Roulette wheel" probability of the chromosome  $i$ ;
- $B_i$ : the production cost of the chromosome  $i$ ;



## 4.5 Crossover

In the crossover, the two chromosomes obtained from the selection process are mated only if it has been randomly decided so. A random number is drawn, and if it is below the crossover rate, the crossover occurs. In the implementation (Algorithm IV.2), we mate two chromosomes to produce one offspring, which consists in iteratively moving Chromosome 1 towards Chromosome 2 while reducing its production cost and therefore, improving its fitness. This method is inspired by the principle of the heuristic crossover as described by Umbarkar et al. [22]. We ensure the generated offspring is a new chromosome in the sense that it has never been encountered before. This crossover implementation is interesting because it improves the overall fitness score of the population over the generations. The process is best illustrated by the following example (each chromosome is represented with its cost):

**Parent 1:** (2, 2, 1, 1, 3, 0, 2, 0) : 592 -> the one chosen for yielding the offspring

**Parent 2:** (0, 2, 2, 2, 3, 1, 0, 1) : 375

**Offspring (Step 1):** (2, 2, 1, 1, 0, 3, 2, 0) : 580

**Offspring (Step 2):** (2, 2, 1, 0, 1, 3, 2, 0) : 570

**Offspring (Step 3):** (2, 2, 0, 1, 1, 3, 2, 0) : 560

**Offspring (Final Step):** (2, 0, 2, 1, 1, 3, 2, 0) : 545

---

**Algorithm IV.2** Crossover operator algorithm.

---

```
1   BEGIN
2   READ chromosome1, chromosome2, crossoverRate, PSP_instance
3   SET randomValue to random()
4   SET distanceD to distance(chromosome1, chromosome2)
5   IF randomValue is less than crossoverRate THEN
6       FOR neighborChromosome in random shuffle(chromosome.neighbors(PSP_instance))
7           IF distance(neighborChromosome, chromosome2) is less than distanceD and
8              neighborChromosome is new THEN
9               IF neighborChromosome.cost is less than chromosome.cost THEN
10                  CALL crossover with neighborChromosome, chromosome2, crossoverRate
11                     and PSP_instance
12              ENDIF
13          ENDFOR
14      ENDIF
15      CALL localSearch with chromosome1, chromosome2 and PSP_instance
16  END
```

---

## 4.6 Mutation

Once the crossover is performed, the random process of mutation takes place. For each offspring obtained from the crossover, it is randomly decided whether or not a chromosome should undergo a mutation. A mutation occurs if the randomly drawn number is below the mutation rate. The algorithm checks, for each randomly picked chromosome gene, if it is possible to switch place with another nearby gene. Plainly, it is about checking if it is possible to produce an item at another period other than the one it is currently produced without violating the constraints of the instance as described by Algorithm IV.3. Not only does it have to respect the constraints, but this process also has to ensure the generated chromosome is a new chromosome in the sense that it has never been encountered before. This condition allows for the exploration of new

areas of the search space. The process is best illustrated by the following example:

**Input Chromosome:** (2, 2, 1, 0, 1, 3, 2, 0) -> with a possible mutation (switch) of periods 2 and 3

**Result of mutation:** (2, 2, 0, 1, 1, 3, 2, 0)

---

**Algorithm IV.3** Mutation operator algorithm.

---

```
1   BEGIN
2     READ chromosome, mutationRate, PSP_instance
3     SET randomValue to random()
4     IF randomValue is less than mutationRate THEN
5       FOR neighborChromosome in random shuffle(chromosome neighbors(PSP_instance))
6         IF neighborChromosome is new THEN
7           RETURN neighborChromosome
8       ENDFOR
9     ENDIF
10    RETURN None
11   END
```

---

#### 4.7 Application of the Hybridization concept

The hybridization concept suggests combining two search methods to produce better results. As shown by Gopal et al [19], local search and Genetic Algorithms are two complement solutions. Genetic algorithms perform well on the global scale because they are capable of quickly finding promising regions, but they take a relatively long time to find the optima in those regions. Local search can find the local optima with high accuracy and fast convergence but suffers from the problem of foot hills. This justifies the implementation of a local search in our study. This local search (Algorithm IV.4) is performed every time the crossover cannot generate a new offspring. It is a variant of the *Hill climbing* method. The algorithm searches in a large neighborhood of Chromosome 1 towards Chromosome 2 to see if a better result can be found. This algorithm is also helpful as it prevents getting stuck at some local optima. The figure 4 provides some measurements backing our use of a local search algorithm to refine individuals and improve the quality of the solutions. On the axis x, are displayed the labels of the CSPlib repository's instances [5.2] used to prove this; and on the axis y, are shown the average gap between the found solutions and the optimal ones. The blue bars (AverageGap1) picture this average gap while using the local search algorithm and the red ones present this average gap (AverageGap2) without any use of a local search algorithm whatsoever. Hence, the hybridization improves the quality of the found solution across the board on average by over 81.5%.

#### 4.8 Termination

We define that the algorithm stops once it cannot improve the best solution found so far over a given number of generations. In our case, this number is 5. We call these generations idle generations.

## V EXPERIMENTAL RESULTS

In this section, we first present the tools used in the implementation and tests, then the instances on which we performed our approach of Genetic Algorithms and the parameters we defined. Finally, we expose the experimental results obtained from the tests.

---

**Algorithm IV.4** Local search algorithm.

---

```
1 BEGIN
2   READ chromosome1, chromosome2, PSP_instance
3   SET distanced to distance(chromosome1, chromosome2)
4   FOR neighborChromosome in random shuffle(chromosome neighbors(PSP_instance))
5     IF distance(neighborChromosome, chromosome2) is less than distanced
6       and neighborChromosome is new THEN
7       IF neighborChromosome cost is less than chromosome1 cost THEN
8         RETURN neighborChromosome
9       CALL localSearch with neighborChromosome, chromosome2 and PSP_instance
10    ENDIF
11  ENDFOR
12  RETURN None
13 END
```

---

## 5.1 Tools

Our approach (available at [28]) is implemented using Python, specifically version 3.6 and on a computer with the following specifications:

- Operating system: Linux Ubuntu 18.04.6 LTS ;
- Processor: Intel® Core™ i5-8250U CPU @ 1.60GHz \* 8 ;
- Memory: 11.6 GiB ;
- Type of the operating system: 64 bits ;
- Graphics: Intel® UHD Graphics 620 (KBL GT2) ;

## 5.2 Benchmarks

From our literature review and the extent of our knowledge, the Pigment Sequencing Problem has two publicly available benchmarks. Houndji et al. proposed some publicly available instances (and their corresponding best solutions) in the CSPLib. Some of these instances are characterized by a number of periods of  $NT=20$ , a number of items of  $NI=5$ , and a number of orders of  $ND=20$ . Others are characterized by a much higher number of periods (100 or 200) and a higher number of items (10 or 15): pigment100a, pigment100b, pigment200a.

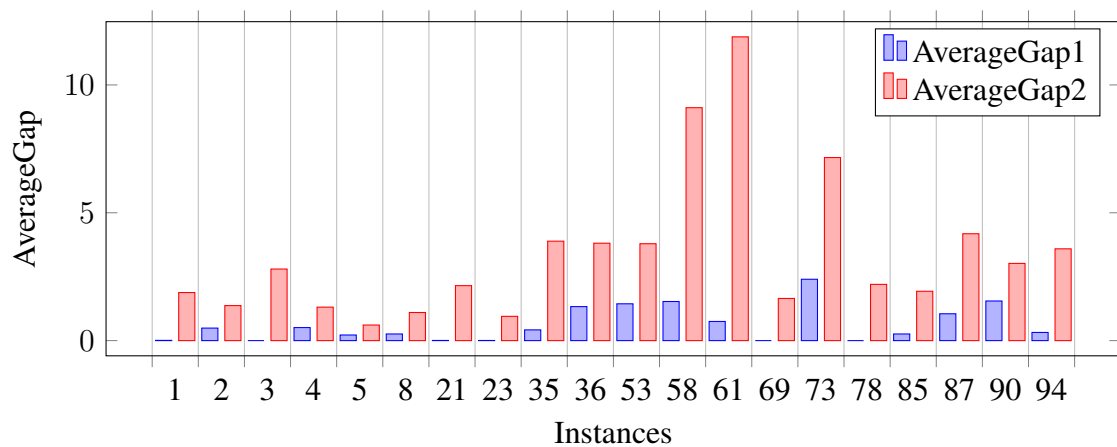


Figure 4: Chart of the average gaps with (AverageGap1) and without (AverageGap2) local search.

Later, in their study of the PSP and seeking to apply their Simulated Annealing approach on some more complex instances of the problem, Ceschia et al [24] developed a parameterized generator that receives as input the number of items  $m$ , the number of periods  $n$ , and the density of requests  $\delta$  (i.e., total request divided by  $n$ ) and produces a random instance with those features. For our tests and given that we are in an early phase of our application of Genetic Algorithms to the Pigment Sequencing Problem, we set, for the present paper, to resolve the instances available in the CSPLib repository, test and compare our approach on this benchmark with 20 instances picked for the test.

### 5.3 Parameter tuning

The performance of Genetic Algorithms is greatly affected by the settings of their parameters. These parameters, along with the population size, are specifically the probabilities of crossover and mutation. Several studies [35] have explored the impact of these different parameters on the quality of the solutions. The following notions can be derived from these studies:

- Crossover is made, in hope that new chromosomes will have good parts of old chromosomes. Hence, the crossover probability, which is the controlling parameter here, is expected to be a high value but not too high to let some part of the population survive to the next generation.
- The mutation probability, which is the parameter that determines the likelihood that an individual will undergo the mutation, is expected to be a low value. A high value of mutation probability tends to prevent the population from converging to an optimum solution.
- The population size, which is the number of individuals in the population, tends to slow the algorithm when too high and shrink the exploration space otherwise.

All these parameters are dependent on the problem being solved. However, for the sake of our study, we randomly pick some instances from the CSPLib repository and draw from the state of art to set the range of each of these parameters for our tuning exercise as follows: the mutation probability [0.05, ..., 0.15] with a step of 0.01, the crossover probability [0.75, ..., 0.9] with a step of 0.1, the population size [25, ..., 40] with a step of 1 with 10 trials over each instance.

The only performance characteristic is the accuracy of the solution. The figure 5 pictures the distribution of the error rate symbolizing the performance of the algorithm throughout the parameter tuning. This distribution is left skewed showing that, for most of the values of the population size, the mutation rate and the crossover rate; the error rate is fairly low (lower than 0.001). However, the computation of the correlation coefficient of the error rate with these same variables (the population size, the mutation rate and the crossover rate respectively 0.063, -0.052 and -0.19) exposes a weak correlation of these parameters with the error rate that we attribute to the fairly stochastic nature of the overall scheme. Nevertheless, after multiple iterations, a recurring set of values emerged and can be represented as follows:

- Size of the population  $L_p$ : 30;
- Probability of mutation  $P_m$ : 0.05;
- Probability of crossover  $P_c$ : 0.9.

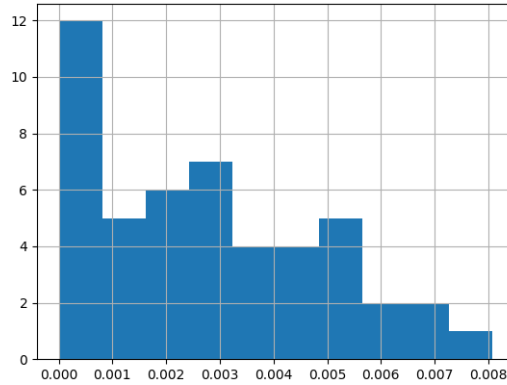


Figure 5: Histogram of the error rate representing the algorithm’s performance during the parameter tuning.

## 5.4 Results

Once our parameter values are set and to test our approach, we draw a very miscellaneous set of 20 instances from the CSPLib repository and run it ten times over each instance using the aforementioned parameter values [5.3] to configure every run.

For each instance, after ten runs, we write down the solutions found and determine the best solution among them and the time spent searching for it. The first table 1 compiles the results of the experiment on the instances from the CSPLib repository. For each instance (represented as Instance NI-NT), we note the optimal solution, the time spent by the CP algorithm to reach it, the best solution found by our approach over ten runs, and the corresponding time, along with the gap between the global optimum and the best solution, the coefficient of variation of the solutions and the mean time of the search.

When analyzing these results, it appeared essential to proceed with a statistical analysis due to the stochastic nature of Genetic Algorithms. From the results of Table 1, we notice that our approach of Genetic Algorithms has successfully spotted the global optimum for most instances (3/4 of the tested instances). For the remaining 1/4 of the instances, our approach identified a solution close to the global optimum (on average 2.008% close) . We suspect these instances to have a little bit more convoluted search space. On all the instances though, our approach finds the global optimum or a solution close to this one quite easily with a gap between the global optimum and the found solution not exceeding 7.3% and on average of 0.502%. We would expect our approach to suffer from the increase of the number of periods (100 periods and 200 periods). However, the results from the instances p100a, p100b, p100c and p200a show a trend similar to those with fewer periods (15, 20, 30 periods). On these larger instances, our approach succeeds in finding the global optimum or a solution close to it (0.611% close on average) . Given that Genetic Algorithms are stochastic methods and having tested each instance 10 times,

<sup>1</sup>the global optimum as available in Csplib repository

<sup>2</sup>the time (in seconds) spent by CP to find the global optimum [31]

<sup>3</sup>the best solution found by our approach

<sup>4</sup>the time (in seconds) spent by our approach to find its best solution

<sup>5</sup>the gap between the global optimum and the best solution found by our approach

<sup>6</sup>the coefficient of variation of all the solutions found over 10 trials

<sup>7</sup>the mean time (in seconds) of all the 10 trials

<b>Instance</b>	<b>Opt<sup>1</sup></b>	<b>CP time<sup>2</sup></b>	<b>GA Best<sup>3</sup></b>	<b>GA time<sup>4</sup></b>	<b>Gap<sup>5</sup></b>	<b>Coef var.<sup>6</sup></b>	<b>Mean time<sup>7</sup></b>
<b>1 5-20</b>	1377	9.14	1377	1.838	0%	0.031	2.305
<b>3 5-20</b>	1107	2.946	1107	1.294	0%	0	1.765
<b>5 5-20</b>	1471	0.235	1471	0.949	0%	0.294	0.858
<b>8 5-20</b>	3117	25.352	3117	2.815	0%	0.285	2.583
<b>23 5-20</b>	1473	15.039	1473	1.418	0%	0.021	1.798
<b>36 5-20</b>	1493	121.909	1502	2.98	0.6%	0.756	2.495
<b>58 5-20</b>	1384	2.347	1386	2.767	0.1%	1.508	2.462
<b>69 5-20</b>	1619	1.223	1619	1.487	0%	0	1.757
<b>78 5-20</b>	1297	16.187	1297	1.173	0%	0	1.434
<b>85 5-20</b>	2113	9.404	2113	2.766	0%	0.242	2.954
<b>90 5-20</b>	2449	23.811	2449	1.861	0%	1.36	2.288
<b>94 5-20</b>	1403	11.726	1403	1.683	0%	0.763	2.207
<b>p15b 10-15</b>	1486	12	1486	6.819	0%	0.558	3.521
<b>p15c 10-15</b>	1583	16	1583	1.675	0%	0.133	2.03
<b>p30a 5-30</b>	1119	124	1201	1.51	7.327%	0.414	1.817
<b>p30c 10-30</b>	1707	156	1731	1.741	1.405%	0	2.351
<b>p100a 10-100</b>	1323	60	1323	4.863	0%	0.239	7.853
<b>p100b 10-100</b>	1962	10	1974	8.004	0.611%	2.863	9.067
<b>p100c 15-100</b>	1982	143	1982	10.358	0%	0.182	15.749
<b>p200a 15-200</b>	2324	854	2324	28.61	0%	0	32.848
<b>Average</b>	-	80.715	-	4.33	0.502%	0.482	5.007

Table 1: Experimental results on 20 CSPLib instances.

we analyze the coefficient of variation of all the solutions found for each instance. We note that this metric, which measures the dispersion of the found solutions around a mean, doesn't exceed a maximum value of 1.508. This helps us infer that over the 10 trials for each instance, our approach has consistently found a solution quite close to the mean. These results are to be put in perspective with the ones of the CP implementation shown in Table 1. We recall again here that the CP is a paradigm for solving combinatorial problems by using constraints to reduce the set of values that each problem's variable can take. This implementation (Houndji et al [26]) of CP as shown here, has successfully found the optimum for all these instances.

Overall, these results suggest that our approach of Genetic Algorithms can easily find, over multiple trials, a solution quite close to the global optimum for this type of instances of the PSP (the ones proposed by Houndji et al. [26]).

## VI CONCLUSION AND PERSPECTIVES

In this paper, we have solved the Pigment Sequencing Problem (PSP), a Discrete Lot Sizing and Scheduling Problem (DLSP), using Genetic Algorithms. We have exposed the basic concepts supporting the implementation of Genetic Algorithms. Solving a Discrete Lot Sizing and Scheduling Problem with Genetic Algorithms is met with some exciting challenges, including the good design of the chromosome and the right choice in implementing aspects such as the selection, the initialization, the crossover, and the mutation. We have experimentally shown that using Genetic Algorithms' approaches to solving a DLSP can be a promising research area.

As further works, we would like to dive deeper into designing and experimenting with new approaches of crossover and mutation. It would also be interesting to test our approach on more complex instances or variants of DLSP.

## REFERENCES

### Publications

- [1] d. Goldberg Korb. “Messy Genetic Algorithms: Motivation, Analysis, and First Results”. In: *Complex Systems* 3 (1989), pages 493–530.
- [2] Fleischmann. “The discrete lot-sizing and scheduling problem”. In: *European Journal of Operational Research* 44 (Feb. 1990), pages 337–348.
- [3] J. Holland. “Computer programs that “evolve” in ways that resemble natural selection can solve complex problems even their creators do not fully understand”. See also URL <http://www.abc.edu+>. 1992.
- [4] J. H. Holland. “Adaptation in natural and artificial systems : an introductory analysis with applications to biology, control, and artificial intelligence”. In: *MIT press* (1992).
- [5] K. Cattrysse Salomon and V. Wassenhove. “A Dual Ascent and Column Generation Heuristic for the Discrete Lotsizing and Scheduling Problem with Setup Times”. In: *Management Science* 39 (1993), pages 477–486.
- [6] S. Van Hoesel Kuik. “The single-item discrete lot-sizing and scheduling problem: Optimization by linear and dynamic programming”. In: (1994).
- [7] A. Kimms. “A Genetic Algorithm for Multi-Level, Multi-Machine Lot Sizing and Scheduling”. In: (Oct. 1996).
- [8] D. Jordan. “Discrete Lotsizing and Scheduling by Batch Sequencing”. In: *Management Science* 44 (May 1998), pages 698–713.
- [9] I. P. Gent and T. Walsh. “Csplib : a benchmark library fr constraints”. In: *Springer* (1999), pages 480–481.
- [10] J. D. Jinxing Xie. “Heuristic genetic algorithms for general capacitated lot-sizing problems”. In: *Computers Mathematics with Applications* 44 (July 2002), pages 263–276.
- [11] L. A. Wolsey. “Solving multi-item lot-sizing problems with an mip solver using classification and reformulation”. In: *Management Science* 48(12) (2002), pages 1587–1602.
- [12] A. J. Miller and L. A. Wolsey. “Tight mip formulation for multi-item discrete lot-sizing problems”. In: *Operations Research* (2003), pages 557–565.
- [13] J. Duda. “Lot-Sizing in a Foundry Using Genetic Algorithm and Repair Functions”. In: *European Conference on Evolutionary Computation in Combinatorial Optimization* 3448 (Oct. 2005), pages 101–111.
- [14] J. J. d. M. M. J. F. Gonçalves and M. G. C. Resende. “A hybrid genetic algorithm for the job shop scheduling problem”. In: *European journal of operational research* (2005), pages 77–95.
- [15] L. A. W. Yves Pochet. “Production planning by mixed integer programming”. In: *Springer Science and Business Media* (2006).
- [16] J. Goren Tunali. “A review of applications of genetic algorithms in lot sizing”. In: *Journal of Intelligent Manufacturing* 21 (May 2008), pages 575–590.
- [17] M. M. Céline Gicquel and Y. Dallery. “On the discrete lot-sizing and scheduling problem with sequence-dependent changeover times”. In: *Operations Research Letters* (2009), pages 32–36.

- [18] M. M. Céline Gicquel Nicolas Miègeville and Y. Dallery. “Discrete lot sizing and scheduling using product decomposition into attributes”. In: *Computers and Operations Research* (2009), pages 2690–2698.
- [19] G. Gopal. “Hybridization in Genetic Algorithms”. In: *International Journal of Advanced Research in Computer Science and Software Engineering* 3 (Apr. 2013), pages 403–409.
- [20] V. R. Houndji, L. W. Pierre Schaus, and Y. Deville. “The stockingcost constraint”. In: *International conference on principles and practice of constraint programming* (2014), pages 382–397.
- [21] B. Kazimipour, X. Li, and K. Qin. “A Review of Population Initialization Techniques for Evolutionary Algorithms”. In: July 2014.
- [22] P. S. A.J Umbarkar. “Crossover operators in genetic algorithms: a review”. In: *ICTACT Journal on soft computing*, 29 (Oct. 2015).
- [23] N. Saini. “Review of Selection Methods in Genetic Algorithms”. In: *International Journal Of Engineering And Computer Science* (2015).
- [24] L. D. G. S. Ceschia and A. Schaerf. “Solving discrete lot-sizing and scheduling by simulated annealing”. In: (2016).
- [26] V. R. Houndji. “Cost-based filtering algorithms for a capacitated lot sizing problem and the constrained arborescence problem”. PhD thesis. Université Catholique de Louvain, 2017.
- [27] P. Kora and P. Yadlapalli. “Crossover Operators in Genetic Algorithms: A Review”. In: *International Journal of Computer Applications* 162 (Mar. 2017), pages 34–36.
- [29] Y. Koken Raghavan. “A genetic algorithm based heuristic for dynamic lot sizing problem with returns and hybrid products”. In: *Computers Industrial Engineering* 119 (May 2018), pages 453–464.
- [30] S. Mirshekarian and G. Süer. “Experimental study of seeding in genetic algorithms with non-binary genetic representation”. In: *Journal of Intelligent Manufacturing* 29 (Oct. 2018).
- [31] S. P. Houndji V.R. and W. L. “The item dependent stockingcost constraint”. In: *Constraints* 24 (2019), pages 183–209.
- [32] O. B. Benoît Le Badezet François Larroche and G. Massonnet1. “A Genetic Algorithm for a capacitated lot-sizing problem with lost sales, overtimes, and safety stock constraints”. In: (Oct. 2021).
- [33] Katoch, C. Sourabh, and V. Sumit Singh Kumar. “A review on genetic algorithm: past, present, and future”. In: *Multimedia Tools and Applications* 80 (Feb. 2021), pages 8091–8126.
- [34] Y. J. J. Tae Jong Park. “Discrete Lot-Sizing Problem of Single Machine based on Reinforcement Learning Approach”. In: *International Symposium on Semiconductor Manufacturing Intelligence* (2022).
- [35] F. Vlasov Khomchenko. “Parameter tuning of a genetic algorithm for finding central vertices in graphs”. In: *Journal of Physics: Conference Series* ().

## Software Project

- [25] [SOFTWARE] S. Ceschia, *Bibliothèque d’instances PSP*, 2017.
- [28] [SOFTWARE] Tafsir GNA, *PspSolver* version 0.1, Feb. 1, 2017.