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Designs, analysis, and applications of nonlinear energy sinks

Hu Ding · Li-Qun Chen

Abstract Nonlinear energy sink (NES) is an appropriately designed nonlinear oscillator without positive linear stiffness. NES can suppress vibrations over a wide frequency range due to its targeted energy transfer characteristics. Thus, investigations on NES have attracted a lot of attention since a NES was proposed. Designs, analysis, and applications of NESs are still active since different configurations are needed in various practical circumstances. The present work provides a comprehensive review of state-of-the-art researches on NESs. The work begins with a survey of the generation of a NES and its important vibration control characteristics. The work highlights possible complex dynamics resulting in a NES coupled to a structure. The work also summarizes some significant design on the implements of optimal damping effects and the offsets of NES shortcomings. Then, the work details the applications of NESs in all engineering fields. The concluding remarks suggest further promising directions, such as NESs for multidirectional vibration reduction, NESs with nonlinearities beyond the cubic, potential deterioration caused by a NES, low-cost NESs, NESs for extremely low frequency range, and NESs integrated into active

vibration controls. There are 383 references in the review, including some publications of the authors.

Keywords Nonlinear energy sinks · Targeted energy transfer · Nonlinear vibration absorber · Passive control · Vibration suppression

1 Introduction

Vibration control in engineering has attracted the attention of a lot of scientists and engineers for many years. Passive vibration absorption and isolation are popular vibration reduction approaches in the engineering. Linear dynamic vibration absorbers have been demonstrated to be effective to mitigate unwanted vibrations at a specific frequency. A large number of research papers are dedicated to understanding, improving, and testing these relatively simple vibration suppression devices. However, most of these linear absorbers are only effective on very narrow excitation frequency bands. Thus, the linear designs need to be improved via nonlinearity. A recent example is a linear tuned mass damper enhanced by nonlinear stiffness [1]. In the past two decades, a nonlinear oscillator without linear stiffness, termed as a nonlinear energy sink, has received much attention due to its passive vibration reduction characteristics. A NES is generally regarded as a typical system with

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‘intrinsic nonlinearity,’ which is ‘nonlinearizable.’ In other words, any linear approximations of the NES system are unable to capture its essential dynamic characteristics. A fundamental characteristic is that a NES, as an essentially nonlinear system, can response over a relatively wide frequency range. The behaviors of an essentially nonlinear system strongly depend on the energy stored in the system. In the early stage, scientists are interested in studying the targeted energy transfer between oscillators [2], especially describing localized normal modes in a two-degree-of-freedom (two-DOF) system [3], a three-degree-of-freedom (three-DOF) system [4], and a chain of nonlinear coupled oscillators [5]. Later, it was discovered that the vibrational energy can be ‘pumped’ to a nonlinear oscillator that is a predetermined part in the system, and the energy is finally dissipated in the oscillator [6–8]. In fact, the nonlinear oscillator is defined as a nonlinear energy sink, serving as a passive device to reduce vibrations [9]. The irreversibility of vibration energy transfer can definitely improve vibration reductions. Afterward, investigations on nonlinear energy sinks have consistently been observed as an active research field. In particular, novel designs and various applications have encouraged the growth of the field to a high level till today.

Lee et al. [10] accomplished a comprehensive and insightful review on passive nonlinear targeted energy transfer (TET) mechanisms with the emphasis on analytical methods and practical applications to vibration reductions prior to 2008. They covered fundamental theoretical aspects of TET, including the essentially nonlinear governing dynamic mechanisms for TET, presented experimental results to validate the TET phenomenon, and discussed some engineering applications of TET. Vakakis et al. [11] treated theoretical methods, computational approaches and experimental setups of targeted nonlinear energy transfer applied to discrete and continuous nonlinear oscillators, and demonstrated the broad applicability of targeted nonlinear energy transfer to a wide range of problems. Gendelman [12] paid a special contribution to review generic local and global bifurcations of the dynamic responses in systems with NESs and revealed possibilities to predict such bifurcations on a qualitative level of analysis. The review illustrated developments and described related theoretical, numerical, and experimental findings. In addition, some excellent works introduced cross-scale energy transfer

phenomena in nonlinear dynamic systems [13, 14] and acoustic systems [15]. Lu et al. [16] reviewed three types of nonlinear dissipative devices, namely NES, nonlinear viscous damper, and particle impact damper. The present review will emphasize designs, analysis conclusion, and applications of NESs as nonlinear dissipative devices in order to enhance vibration reduction in various engineering fields from the perspective of nonlinear dynamics.

This remaining part of the review is organized as follows. Section 2 presents basic characteristics of NESs for vibration reductions. Section 3 treats complex nonlinear dynamic behavior caused by the introduction of NESs. Section 4 gives an overview of important works related to NES designs. Section 5 surveys significant applications of NESs in civil engineering, mechanical engineering, aerospace and aeronautical engineering, and other fields. Section 6 ends the review with concluding remarks on future promising topics.

2 Basic characteristics of NESs for vibration reductions

In the early days, researchers focused on the energy transfer between linear modes of weakly nonlinear systems, including multidegree-of-freedom (multi-DOF) discrete systems [17, 18] and continuum systems [19, 20]. They revealed the coupling dynamics between linear modes when the natural frequencies of the linear modes satisfy a certain relationship. Noteworthy, based on the coupling of the in-plane and the out-of-plane motions of a cantilever beam, Nayfeh and Nayfeh found interactions between high- and low-frequency modes, while natural frequencies may have not any special relationship [21]. It should be noted that the early modal coupling works generally considered the transfer of vibration energy back and forth between two or more linear modes. It was not until 2001 that the energy between the two nonlinear oscillators can be targeted to generate irreversible transfer [22]. Under a precise condition of nonlinear resonance response, a specific amount of energy can be transferred from a linear oscillator to another weakly coupled nonlinear oscillator. To be specific, targeted energy transfer occurs because the frequencies of the donor and the acceptor oscillators remain almost equal.

In the same year, Gendelman [6] presented a numerical study on the nonlinear energy pumping phenomenon in a weakly coupled and highly asymmetric system, and it is an essentially nonlinear (nonlinearizable) oscillator linearly coupled (also called as grounded configuration) with a linear oscillator. The study showed that transition energy can be ‘pumped’ to the predetermined nonlinear oscillator under certain conditions. Then, numerical evidence was presented for energy pumping from two- and three-DOF impulsively excited and weakly viscously damped vibrating systems to essentially nonlinear oscillators. Moreover, energy pumping was found as a resonance capture phenomenon on a 1–1 resonant manifold, and analytical approximations were constructed for the energy pumping [8]. Vakakis [23] developed an analytical formulation for inducing passive energy pumping in impulsively loaded linear periodic chains of oscillators with a nonlinear attachment. For the first time, the notion of ‘nonlinear energy sinks’ was proposed. The nonlinear energy pumping is enhanced by nonlinear normal modal analysis of the underlying conservative system [24]. It should be noted that both targeted energy transfer and energy pumping refer to the phenomenon that energy is irreversibly transferred. In essence, the phenomenon of irreversible energy transfer corresponds to the controlled one-way channeling of the vibrational energy to a passive nonlinear energy ‘sink.’ Then, the vibrational energy is localized and diminished in time due to damping dissipation. The above-mentioned studies are all focused on the vibrational energy transfer of impulsive excitation and transient response.

In 2003, the effects of a grounded NES on the steady-state vibration of a linear primary system were theoretically and experimentally analyzed [25]. Essentially, cubic nonlinear spring was realized experimentally by a transversely vibrating steel wire. The experimental schematic is shown in Fig. 1. The work [25] for the first time demonstrated the ability of the NES to absorb steady-state vibration energy over a relatively wide frequency range. It is an attractive advantage over classical linear vibration absorbers. The investigation [25] pointed out that a NES is most effective when it possesses essentially nonlinear stiffness, since it is capable of resonating with any natural mode of the linear system. It may be referred as a 0:m internal resonance condition. Interestingly, a light damping in the NES could enhance the NES

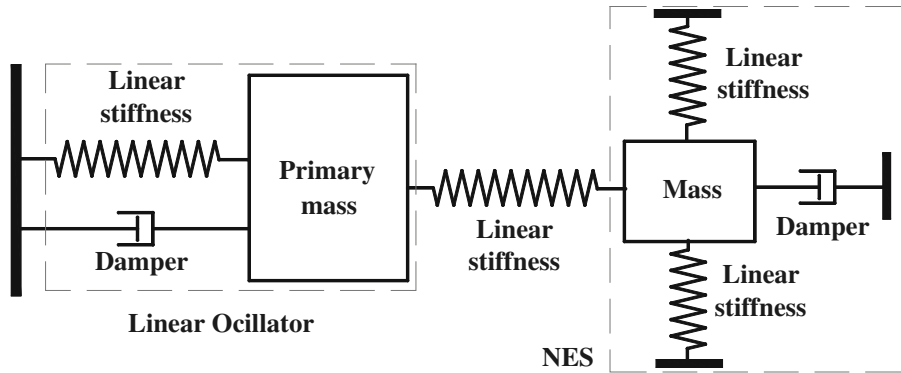
capacity of absorbing vibration energy from the linear system. In [25], the interaction force $F(t)$ between the primary oscillator and the NES is expressed as

$$F(t) = -k_L(x_P - x_N) - c_N\dot{x}_N - k_Nx_N^3 \quad (1)$$

where t is the time, k_L is the coupled linear stiffness, k_N is the cubic nonlinear stiffness coefficient provided by the vertical springs, c_N is the NES damping coefficient, and x_P and x_N are the displacements of the primary mass and the mass of the NES, respectively.

There are other significant early works on NESs. Vakakis et al.[26] investigated the dynamic interaction of a finite linear chain of coupled oscillators with the weakly coupled nonlinear attachment. In the limit case of the coupling parameter tending to zero, Gendelman et al. [27] analytically studied the degenerate bifurcation structure of the undamped system and thus yielded the necessary conditions for the occurrence of nonlinear energy pumping. In 2005, based on transient responses of a linear oscillator linearly connected to a NES, McFarland et al. confirmed experimentally two theoretical predictions; nonlinear energy pumping in coupled structures can only occur for the input energy above a certain threshold. There is an optimal energy input at which a maximum portion of the vibration energy is absorbed and dissipated in the NES [28]. Moreover, energy pumping occurs at a single ‘fast’ frequency near the eigenfrequency of the linear mode. Gendelman et al. [29] demonstrated theoretically and experimentally that the linear damped primary system exhibits a quasiperiodic response over a range of the external forcing amplitude. Then, Kerschen et al. [30] showed experimentally that during transient resonance capture that the linear subsystem and nonlinear subsystem are in a resonance state, the vibration frequency varies with time, and it leads to passively targeted energy transfer. Musienko et al. [31] proposed two mechanical devices for energy pumping, namely a type of gear box system enabling energy pumping with super-harmonic resonance and a nonlinear subsystem comprised of two linearly coupled cubic oscillators linearly attached to the main system. Malatkar and Nayfeh [32] explored rich dynamics of a structure weakly linearly coupled to a NES and revealed possibly negative effects of the NES on vibration reduction. They found that the original simple structure may exhibit complex dynamic behaviors and NES may increase the

Fig. 1 Schematic of the experimental apparatus in [25]



vibration response of the structure over specific frequency ranges [32]. In a rebuttal, Vakakis and Bergman [33] stated that optimization is needed to ensure maximum energy transmission from a linear subsystem to a NES in the frequency range of interest. Malatkar and Nayfeh [34] further showed an increase rather than a decrease in the vibration amplitude of a forced linear oscillator after adding a NES. Based on a rigorous regular perturbation theory, Dantas and Balthazar [35] observed other general dynamic phenomena of a linear oscillator coupled to a nonlinear oscillator in addition to energy pumping. They found vibration energy transfer from the linear oscillator to the nonlinear oscillator or vice versa depending on initial conditions. However, if the mass of the nonlinear oscillator is adequately small, then the linear oscillator always loses energy in the presence of resonance and the absence of external excitation [35]. Costa et al. [36] also found that the energy transfer direction between the linear and nonlinear oscillators depends on initial conditions. Therefore, it remains an open issue whether the weakly linearly coupled NES is suitable for passive vibration suppression [37].

Vakakis et al. [38] revisited resonant interactions of a NES from the view of traveling waves and showed that the absence of linear stiffness in a NES allows the NES to resonate instantaneously with the standing wave and the incident wave. Vakakis [9] proposed shock isolation designs implemented by a NES coupled with the primary system in order to achieve robust energy pumping. It is very important that the NESs possess zero linear stiffness in the proposed designs [38]. The design is also known as an ungrounded NES [39]. It is an initial step to nonlinear energy sinks without linear stiffness. The proposed NES can lead to robust energy pumping and

significant reduction in undesired vibration of the primary system. Lee et al. [40] investigated complicated dynamics of a primary system with an ungrounded NES as shown in Fig. 2 and found that periodic orbit structures of undamped oscillators influence significantly damped dynamics. Moreover, the nonlinear oscillator with a lightweight has the capacity of strong resonant interactions with the linear primary system in a wide frequency range. The interaction force between a cubic NES and the linear system is expressed as

$$F(t) = -c_N(\dot{x}_N - \dot{x}_P) - k_N(x_N - x_P)^3 \quad (2)$$

For a primary system undergoing initial impact, Manevitch et al. [41] analytically illustrated that the NES without a linear anchor spring is most efficient.

In order to reduce the required mass in a NES for passive vibration control, researchers have proposed various solutions. A lightweight ungrounded NES was proposed [40, 42, 43]. Gendelman et al. [42] concluded that for a dual oscillator system with essential mass asymmetry, energy pumping may be achieved as long as the damping and the coupling terms are properly designed. Both linear coupling and nonlinear coupling between the oscillators are acceptable. However, nonlinear coupling requires less mass of the attachment. McFarland et al. [44] experimentally

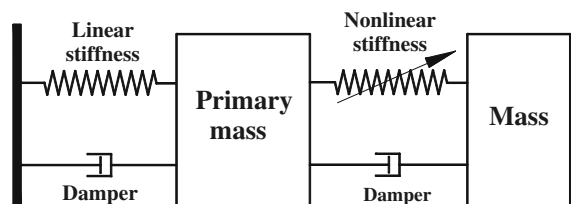


Fig. 2 Two-DOF system with essentially nonlinear stiffness

investigated targeted energy transfers in transient dynamics of a linear oscillator coupled to a lightweight NES. The experimental setup was modeled as a two-DOF system as shown in Fig. 3. Experimental measurements satisfactorily agree with numerical and analytical analysis. The simple system exhibits rather complicated nonlinear behaviors. For the setup, if the mass ratio between a NES and a primary structure is less than 2%, targeted energy transfer does not take place. Zang et al. [45] proposed a lever-type NES to reduce necessary additional masses in NESs. They demonstrated the superiority of the lever-type NES with suitable fulcrum position to conventional NESs. The global bifurcations were analyzed to explore the existence of periodic and aperiodic motions of a structure combined with a lever-type NES [46]. Since an inerter with a small physical mass results in a large dynamic inertance, the inerter can be used to improve the performance of the NES or to reduce the weight of the NES. Zhang et al. [47] introduced an inerter between a primary structure and a NES mass. The inerter provided with an effective mass effect for the proposed NES-inerter design by converting the relative translational motion to the rotational motion. Both the amplitude suppression effect and the NES vibration energy absorption demonstrate that the NES-inerter is more effective in suppressing vibration than the conventional NES. Zhang et al. [48] proposed an inertial NES by replacing the mass by an inerter and found that the inertial NES could enhance the performance of the vibration suppression and reduce the additional mass. The inertial NES design was adopted to eliminate multimodal resonances of plates, and it was proved that the inertial NES with a tiny

mass can achieve a great efficiency [49]. Javidiale-saadi and Wierschem [50] proposed a NES-inerter by equipping an inerter between the NES physical mass and a fixed point and found that the effectiveness of NES-inerter can be significantly improved compared to traditional NESs. In 2020, by superposing the quasi-zero stiffness system and the inertial NES together, Zhang et al. [51] proposed a combined vibration control technique. The combined NES was described numerically to overcome drastic low-frequency vibration of the quasi-zero stiffness system due to its small dynamic stiffness.

Various dynamic properties have been investigated. Gourdon and Lamarque [52] examined the robustness of energy pumping by treating nonlinear parameters as uncertainties. Gendelman and Starosvetsky [53] distinguished two qualitatively different types of quasiperiodic responses in a periodically forced linear oscillator with the NES attached. Panagopoulos et al. [54] explored the robustness of vibrational energy targeted transfer to change initial condition. Based on a two-DOF main system nonlinearly coupled with a NES, Kerschen et al. [55] found theoretically and experimentally that the NES can resonate with both modes and extract energy from the main system. Gendelman et al. explored the attractors of a harmonically excited linear primary system with a NES and the possible use of the NES in vibration suppression [56, 57]. They determined the parameter domains of coexistence of various attractors and then compared the efficiency of the NES with that of a linear absorber. These works demonstrated the necessity of careful tuning of the NES and the limit of the efficient range of external forcing amplitude [56, 57]. Zang and Chen

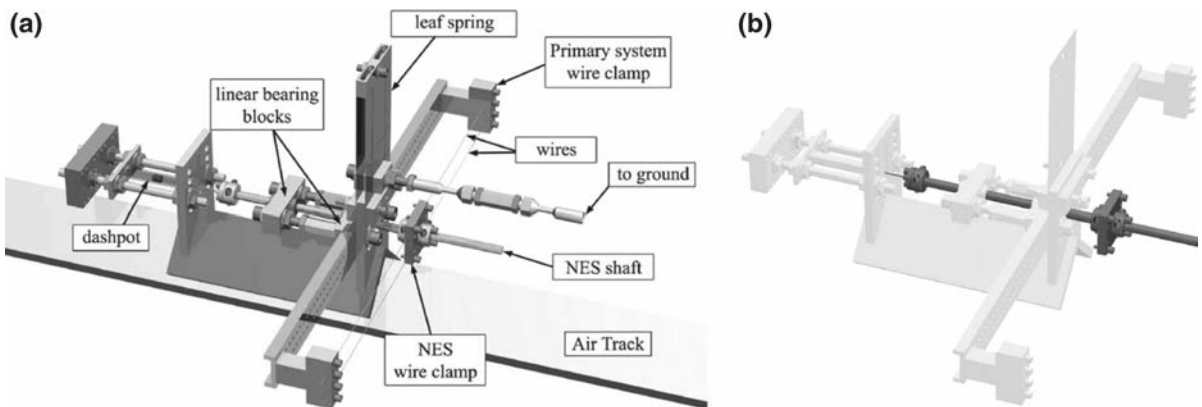


Fig. 3 **a** Schematic of the experimental fixture; **b** mass repartition gray: the primary system; black: NES [44]

[58] investigated complex dynamic behaviors of a linear oscillator subjected to a harmonic excitation and coupled with a NES. Chaos is numerically identified via phase trajectory maps, power spectra, and Poincaré maps. They demonstrated that a NES may create dynamic complexity [57]. Based on nonlinear output frequency–response functions, Yang et al. [59] proposed a vibration transmissibility to evaluate the isolation performance of a NES and found the smaller NES cubic nonlinear stiffness leads to the smaller transmissibility of the primary system and the better vibration suppression performance over all resonant frequency regions. Dekemele et al. [60] derived algebraic performance measures for the speed of both targeted energy transfer and resonance capture cascading for both single-modal and the multimodal vibrations and therefore can quickly assess the effect of linear and nonlinear terms in the NES connecting stiffness on the performance of the targeted energy transfer.

In summary, a nonlinear energy sink is an essential nonlinear oscillator that can efficiently and robustly suppress vibration. It is attached to main systems by weakly linear stiffness or only nonlinear stiffness. The essential nonlinear oscillator can generate the unidirectional and irreversible vibration energy transfer from the main system to the NES over a rather broad frequency range. Once the vibrational energy is localized to a NES, it remains passively confined there instead of ‘spreading back’ to the primary system. Therefore, it is possible to eliminate effectively the confined energy by passive or active means. In addition, another advantage of a NES is its relative simplicity and wide applicability to a range of vibration and shock suppressions required in engineering practice. Meanwhile, the addition of a nonlinear oscillator to a primary system complicates its vibration responses. Under certain unfavorable circumstances, the forced vibration of the primary system will be deteriorated. In order to utilize the advantage and to overcome the disadvantage, the research on NES is still a hot topic.

3 Complex nonlinear dynamics of the coupled system

The introduction of a nonlinear energy sink produces a new nonlinearity in the coupled system of the primary

system and the NES. It may complicate nonlinear dynamic behaviors of the coupled system. The resulting complicated behaviors are usually regarded as a negative effect for vibration reductions via NESs. The complicated dynamics includes coexistence of multiple attractors and possibility of chaotic responses in a linear harmonically forcing oscillator with a NES [56, 57]. Gendelman analyzed the degenerate bifurcations of nonlinear normal modes of the unforced system and revealed a rather rich variety of the bifurcation scenarios [61, 62]. Quinn et al. [63] reported complex nonlinear transitions associated with nonlinear mechanism of targeted energy transfer and found the central role of impulsive orbits in efficient targeted energy transfers. Starosvetsky and Gendelman [64] discovered two types of local bifurcations of periodic solutions. Kerschen et al. [65] revealed the surprising complexity of the coupled oscillators undergoing a co-dimension 3 bifurcation. Starosvetsky and Gendelman [66] investigated two-DOF weakly coupled linear oscillators with identical linear frequencies, one of the oscillators with a NES, and found a single important component of the vibration response with regard to the 1:1:1 resonance. Vakakis [67] explored the dynamic interactions between traveling waves propagating in a linear lattice and a local attached NES and found relaxation oscillations, countable infinities of subharmonic orbits, and chaotic motions. For a linear system weakly linearly coupled to a NES, Savadkoobi et al. [68] investigated transient chaos and dynamic behaviors in order to explain qualitatively targeted energy transfer and the complicated basins of attraction of the two stable fixed points. Andersen et al. [69] analyzed a countable infinity of resonance manifolds of the dynamics to understand a strongly nonlinear damped transition corresponding to sustained nonlinear resonance scattering. This transition is only realized in the neighborhood of the low-frequency branch of the impulsive orbit manifold of the corresponding Hamiltonian dynamic system. Moreover, the sustained resonance scattering can be eliminated by the strong 1:3 resonance capture and can only be implemented for sufficiently weak damping [69]. Luongo and Zulli [70] proposed a mixed multiple scale and harmonic balance method to derive the differential equations describing the slow-flow and fast-flow dynamics of a multi-DOF system equipped with a NES and excited by a harmonic external force. The results of

equilibrium points and strongly modulation responses showed a good agreement between the outcomes of the singular perturbation and direct numerical integration. Sapsis et al. [71] characterized the damping properties and effective stiffness of a linear oscillator coupled with a NES. Significant nonlinear coupling between distinct linear modes is introduced by NESs. In a conservative system, because nonlinear beat phenomena result in recurring energy exchanges, there is no irreversible energy transfer between substructures [11]. Therefore, the existence of damping is a prerequisite of targeted energy transfer. Moreover, a small linear stiffness component in a NES does not qualitatively change the vibration response, while it perturbs slightly the response.

Savadkoobi et al. [72] investigated energy pumping phenomenon of a linear sinusoidal forcing oscillator coupled with a nonsmooth NES with the account of the weight. They showed that a simple nonlinearity was not destroyed by the gravitational effect [72]. Chen et al. [73] focused theoretically and experimentally on dynamic effects of the weight on the vertical vibration of a primary oscillator coupled with a cubic NES. They found that there are qualitative and quantitative differences between cases with and without the weight. Moreover, the quantitative difference decreases as the excitation amplitude increases. The static equilibrium of the primary oscillator and NES (x_{Ps}, x_{Ns}) is defined by the following equation

$$\begin{aligned} k_P x_{Ps} + k_N (x_{Ps} - x_{Ns})^3 &= -m_P g, \\ k_N (x_{Ns} - x_{Ps})^3 &= -m_N g \end{aligned} \quad (3)$$

where k_P is the stiffness of the linear primary oscillator, m_P is the mass of the primary oscillator, m_N is the mass of the NES, and g is the gravitational acceleration. The interaction force with cubic nonlinearity between the primary oscillator and the NES is given by [73]

$$\begin{aligned} F(t) &= -c_2 (\dot{x}_N - \dot{x}_P) \\ &\quad - 3k_N \left(\frac{m_N g}{k_N} \right)^{2/3} (x_N - x_{Ns} - x_P + x_{Ps}) \\ &\quad + 3k_N \left(\frac{m_N g}{k_N} \right)^{1/3} (x_N - x_{Ns} - x_P + x_{Ps})^2 \\ &\quad - k_N (x_N - x_{Ns} - x_P + x_{Ps})^3 \end{aligned} \quad (4)$$

The gravitational effect of the mass in the NES is accounted for in the study of vibration suppression of a composite plate structure [49]. The study demonstrated that the inertial NES is a good way to overcome the gravitational effects of the NES mass.

Charlemagne et al. [74] studied an asymmetric two-DOF oscillator. A linear primary oscillator was nonlinearly connected to a grounded nonlinear oscillator, and the both oscillators were subjected to external excitations. A time multiscale method was applied around 1:1:1 resonance to obtain complex geometry of the slow invariant manifold and equilibrium and singular points of the two-DOF oscillator [74]. Li et al. [75] developed an asymptotically analytical method to explore the transient 1:1 resonance of the modulated response of a vibro-impact NES. Theoretical and numerical results reveal five different response regimes. An experiment validated the existence of the different response regimes and different routes to chaos. The study showed that targeted energy transfer depends not only on the presence or absence of a 1:1 resonance, but also on impulse intensity during transient resonance capture [75]. Chouvion [76] suppressed forced steady-state response of a nonideal boundary beam with an attached NES and found detached resonant curves under certain tuning parameters. Zang et al. [77] proposed a generalized transmissibility defined by the ratio of root-mean-squares of output and input in order to evaluate the performance of a NES on a two-DOF linear system. Phase portraits, frequency spectrum, and Poincaré map were used to reveal the occurrence of quasiperiodic motions when periodic responses predicted by the harmonic balance method are unstable [77]. Based on the invariant manifold approach, Bitar et al. [78] introduced an extended form of Manevitch's complex variables and proved the approximations improved by the account for the higher harmonic. Stochastic reduced-order models were applied to analyze stochastic stability of a two-DOF NES system subjected to random excitation [79]. Single-walled carbon nanotubes were studied from the view of the nonlinear resonance interaction between bending and circumferential flexure modes [80]. Several NESs were coupled to a multi-DOF unstable vibrating system to stabilize it with the predicted steady-state regimes [81]. The unscented Kalman filter was used to determine the model parameters of an

experimental NES device with a geometric nonlinearity in its stiffness and a friction-based nonlinearity in its damping [82]. Bitar et al. [83] demonstrated that the presence of the third harmonic improves approximations via an extended complexification method.

As more and more complex NES devices are being designed to suppress vibration of more and more complex engineering structures, investigations on nonlinear dynamics of coupled systems will continue to attract more and more attention.

4 Design schemes of NESs

4.1 Vibration control

Although some effective approaches have been proposed to produce nonlinear restoring forces in NESs, the explorations on alternative mechanisms to achieve nonlinear restoring forces and different configurations of NESs are still of great practical interests in order to enhance vibration reduction efficiency and implement adaptability to various engineering applications.

4.1.1 Configuration designs of NESs

Similar to the design configuration in [44], symmetric design has been used to achieve nonlinear stiffness in NES. Gourdon et al. [84, 85] implemented experimentally the cubic nonlinearity with two linear springs. The mechanical schematic is shown in Fig. 4.

As summarized in [28], when prestress forces in linear springs are approximately equal to zero, the linear stiffness in the direction perpendicular to the spring is enough small to be neglected. Therefore, the Taylor series expansion yields the cubic nonlinear restoring force

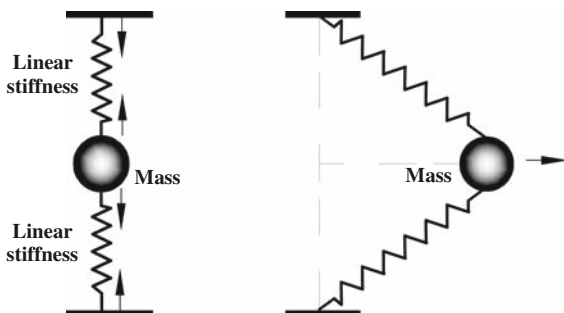


Fig. 4 Design of the cubic nonlinearity with two linear springs

$$F(t) = -\frac{k}{L^2}(x_N - x_P)^3 = -k_N(x_N - x_P)^3 \quad (5)$$

where k and L are the stiffness and the length of the linear spring, respectively. Qiu et al. [86, 87] achieved a strongly cubic nonlinear stiffness without a linear part, using two symmetric conical springs with two precompressed cylindrical springs. Qiu et al. [88] constructed theoretically and experimentally pure cubic nonlinear stiffness in a NES with variable pitch springs to reduce vibration of a linear primary oscillator. Zhang et al. [89] designed NES nonlinear stiffness with NiTiNOL-Steel wire ropes.

Luo et al. [90] developed and validated experimentally an innovative type two-stage NES device by utilizing pairs of elastomeric polyurethane foam bumpers. They combined geometric nonlinearity with material nonlinearity. Cubic nonlinear stiffness was implemented by hybrid-shaped bumpers, and biquadratic and higher power nonlinear stiffness with pyramid-shaped bumpers. The elastomeric bumpers NES was successfully applied to a model six-story building subject to an impulse-like loading and high levels of input energy were rapidly dissipated and redistributed to achieve vibration suppression [90]. Wierschem et al. [91] used shaped elastomeric polyurethane bumpers to produce nonlinear restoring forces and showed experimentally that the resulting NES is capable of reducing significantly the vibration response of the structure under shocks due to ground motions.

4.1.1.1 Vibro-impact NESs Georgiadis et al. [92] designed a shock isolation by using two NESs with nonsmooth nonlinear stiffness elements and showed the shock isolator capable of absorbing rapidly a major portion of the unwanted shock energy. Since vibro-impact nonlinearities are promising candidates for fast passive energy absorption by targeted energy transfer occurring on a fast timescale, both smooth and nonsmooth stiffness nonlinearities were used to design NESs. Nucera et al. [93] proposed an easily produced ungrounded piecewise-linear oscillator, called as nonsmooth vibro-impact NES, as shown in Fig. 5. Since the vibro-impact NES can resonantly interact with multiple modes over broad frequency bands, the local NES can affect the global dynamics of the primary system [93]. Lamarque et al. [94] used a small mass nonsmooth NES to suppress free and

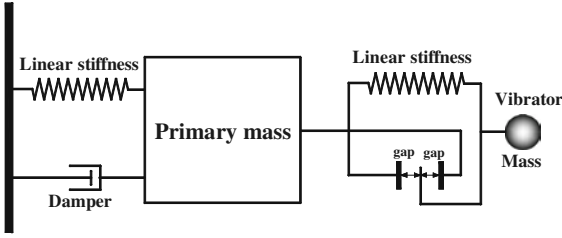


Fig. 5 Schematic of an ungrounded piecewise-linear oscillator

forced oscillations of a primary nonsmooth system by strongly modulated responses. Lee et al. [95] revealed periodic and quasiperiodic motions as well as passive targeted energy transfer and examined the effects of the clearances, the coupling stiffness, and the NES mass on the vibro-impact shock absorption. Gendelman developed an approximate analytic procedure for analysis of transient vibration response of a primary linear system with vibro-impact NESs and proposed an approach to design efficiently the NESs [96]. NES was realized as a straight cavity inside the primary system where the impacting particle can freely move, as shown in Fig. 6. The obvious advantage of this design is no extra space is required for the additional oscillator.

Later, Gendelman and Alloni [97] observed chaotic strongly modulated response regime for very small external forces, and the observation makes it possible for a vibro-impact system to harvest vibratory energy. Gourc et al. theoretically and experimentally studied dynamic response of a two-DOF oscillators comprising a linear primary oscillator subjected to a harmonic displacement and an embedded vibro-impact NES [98, 99]. The collision was modeled via a simplified shock theory. The vibro-impact NES was realized with a free moving ball in a cavity with a predesigned gap. The ball mass was less than 1% of the linear oscillator mass. The multiscale method and the invariant manifold approach were applied in the case of 1:1

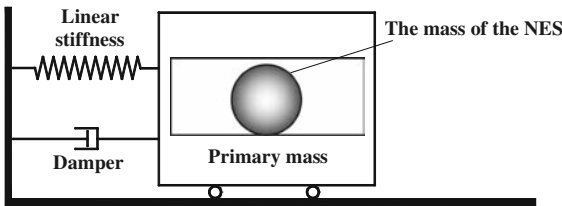


Fig. 6 Sketch of a linear primary system and a vibro-impact NES

resonance to predict theoretically the existence of the strongly modulated response that was experimentally confirmed [98, 99]. Li et al. [100] revealed chaotic characteristics of a system with a vibro-impact NES by identifying Lyapunov exponents from experimental data. Lu et al. [101] reviewed particle damping technology involving a single-particle impact damper, a multiunit impact damper, a particle damper, and a multiunit particle damper. Farid et al. [102] proposed a hybrid vibro-impact NES consisting of a traditional tuned mass damper and a vibro-impact NES that can work for excitations of a relatively large energy range.

In order to understand no 1:1 resonance regimes, Li et al. [103] compared a cubic NES with a vibro-impact NES attached to a linear primary oscillator subject to transient forcing and periodic forcing. An analytical approach combining the invariant manifold method with the multiscale method was extended to seek a general equation governing the variation in motion along the slow invariant manifold. No 1:1 resonance was numerically demonstrated in the case of the excited targeted energy transfer under a transient force and the strongly modulated response under a periodic force. The experimental results confirmed the existence of the strongly modulated response resulted from the vibro-impact NES [103]. Gendelman [104] concluded that NESs with soft or piecewise-linear stiffness may achieve better targeted energy transfer. Lamarque et al. [105] revealed that an abrupt jump from a upper stable branch to a lower stable branch accounts for majority energy transferring from a primary system to a piecewise-linear NES. Starosvet-sky and Gendelman [106] proposed a NES with piecewise-quadratic-type nonlinear damping to eliminate the existence of dangerous periodic regimes, which are regarded as a shortcoming in NES applications. In order to mitigate shock loadings, Wang et al. [107] designed a discontinuous vibro-impact vibration absorber linearly connected to a primary system, as shown in Fig. 7.

Al-Shudeifat et al. [108] proposed a single-sided highly asymmetric type of vibro-impact NES. Single-sided vibro-impacts lead to high-frequency spreading of shock energy due to significant energy dissipation during inelastic impacts. It is possible to develop a single-sided vibro-impact NES to reduce rapidly the first one or two cycles of responses of a two-DOF primary structure over a broad range of input energy.

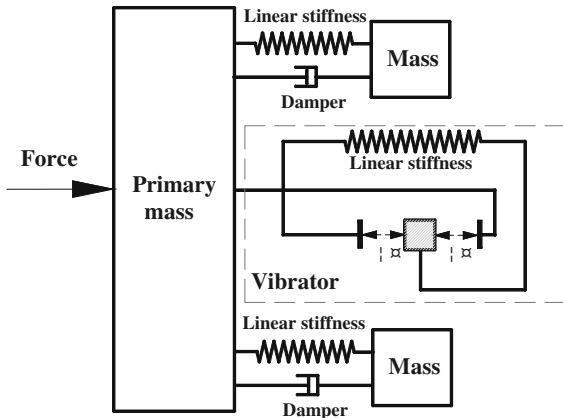


Fig. 7 Schematic of a discontinuous vibro-impact vibration absorber

Moreover, in contrast to a double-sided symmetric vibro-impact NES, an asymmetric vibro-impact NES maintains its high shock mitigation performance even for very strong shock excitations. Therefore, the capacity for effective shock mitigation of asymmetric vibro-impact NESs does not deteriorate with the increasing shock severity [108]. Li et al. [109] studied the vibration energy transfer mechanism of a single-sided vibro-impact NES. They proposed a vibro-impact vibration reduction factor to evaluate the energy dissipation performance of a single-sided vibro-impact NES.

Al-Shudeifat [110] proposed a dead-zone NES with employing symmetric piecewise nonlinearities, as shown in Fig. 8. By flexible design of the dead-zone NES, the NES efficiency was considerably enhanced for moderate and severe energy inputs. The first and second cycles of oscillation of the linear primary system were significantly affected even for high initial input energy. The dead-zone NES was found to be

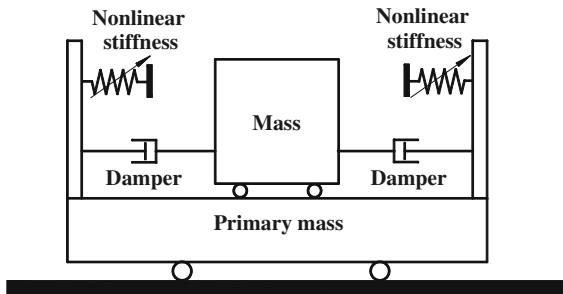


Fig. 8 Schematic of a symmetric piecewise-nonlinear NES

more robust than linear absorbers and some other types of NESs.

Moore et al. [111] applied an empirical mode decomposition technique to decompose responses of a linear oscillator and a vibro-impact NES into smooth components and nonsmooth components and found a significant effect of higher-order harmonics and higher-order transient resonance captures on the vibro-impact response.

Wei et al. [112] proposed a NES with both cubic nonlinearity and nonsmooth nonlinearity. The cubic nonlinearity was set as a spring connecting a linear primary system to the NES, and the nonsmooth nonlinearity was introduced by vibro-impacts on bilateral barriers. At the optimal targeted energy transfer state, the vibro-impact cubic NES suppress vibration more efficient than a cubic NES [112]. Yao et al. [113] proposed a NES with a cubic nonlinear stiffness spring and piecewise-linear stiffness springs, and they showed numerically and experimentally no strongly modulated response behavior under either too large or too small excitations and thus vibration suppressions only effective for moderate exciting forces. Wei et al. arranged in parallel a cubic NES and a vibro-impact NES to construct an efficient composed NES with both advantages of cubic and nonsmooth nonlinearities [114]. Chen et al. proposed a nonsmooth NES with descending stiffness for expanding effective excitation range [115]. The nonsmooth NES has a higher cubic nonlinear stiffness in the initial range, and the stiffness is reduced as its amplitude exceeds the initial range [115, 116].

4.1.1.2 Rotary NESs Hubbard et al. designed a compact NES for reducing vibration of a flexible

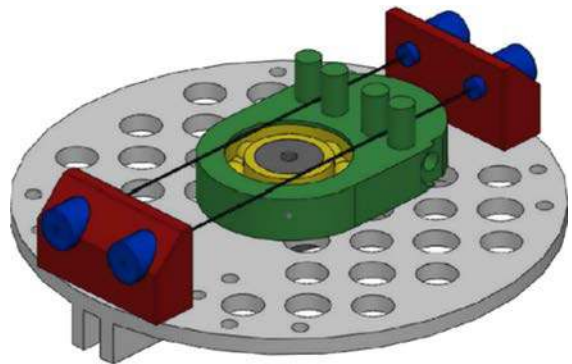


Fig. 9 Schematic of the compact NES [117]

swept wing [117]. The NES performed angular oscillations, as shown in Fig. 9. The NES mass may pivot at the shaft so that the posts transversely deflect the steel wires.

Gendelman et al. [118] theoretically and experimentally investigated a device with a rotating eccentric mass inertially coupled to a primary mass, shown in Fig. 10. The rotator can revolve with any frequency and resonate with any mode of the primary mass and thus can efficiently serve as a rotator NES causing targeted energy transfer in the system. The rotator NES is simple and compact, but its nonlinear dynamics is much more complicated than that of a nonrotating design. When the total energy of the inertially coupled system is large enough, the vibration response of the rotator is usually chaotic [119]. Sigalov et al. [120] demonstrated that the nonlinear inertial coupling could lead to interchanges between regular and chaotic dynamic behavior. Shapes of nonconventional nonlinear normal modes were determined by phase locking of oscillatory and rotatory motions of the rotator with integer ratios of characteristic frequencies. This new type of nonlinear normal modes, as stable or unstable equilibrium points, can drastically change the nonlinear behaviors of the system [120]. Farid and Gendelman [121] adopted a conventional damped pendulum as a NES to mitigate the transient response of impulsive loads, and the NES works over a broader initial energy range than regular rotational

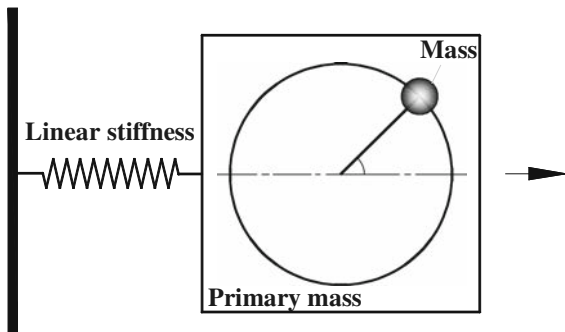
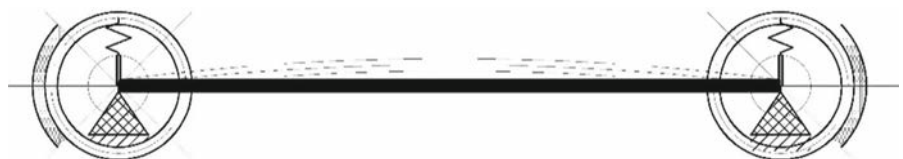


Fig. 10 Top view sketch of dynamic system comprising a primary mass and an attached eccentric rotator

Fig. 11 Mechanical model of the rotational NES attached to the end of a beam



NESs do. They examined the contribution of the gravity to the damped targeted energy transfer from the primary system to the pendulum NES. A properly designed pendulum NES acts almost as a usual tuned mass damper for small initial energy and a rotational NES for large initial energy and therefore can overcome the major shortcoming of traditional NESs [121]. Replacement of the rigid pendulum NES coupling arm by an elastic arm can achieve robust performance of passive nonlinear energy transfer and dissipation [122]. AL-Shudeifat et al. [123] utilized a lightweight rotating NES to dissipate the vibration energy of a primary two-DOF system, and they compared energy dissipation via the inertial NES with other-type NESs. Yao et al. [124] proposed a NES consisting of positive coil springs and a negative stiffness magnetic spring in order to reduce the vibration response of a rotor system. Yao et al. [125] developed experimentally a grounded NES with piecewise-linear stiffness in order to suppress lateral vibration in rotor systems. The NES is with cubic nonlinear stiffness resulting from piecewise-linear stiffness of elastic rods and achieves a wide-band vibration suppression and multiple resonance peaks suppression. The optimal vibration suppression rate is numerically 78% and experimentally 68% [125]. Saeed et al. [126] proposed a hybrid NES design based on the synergetic synthesis of the rotary and impact-based NESs in a single rotary-impact NES.

Mao et al. [127] proposed a novel torsional NES to suppress bending vibrations of an elastic beam with simply supported boundaries. The NES was assembled at the beam end. The vibration energy of the structure was absorbed through the rotation angle at the beam end, as shown in Fig. 11. Therefore, the interaction torque $J(t)$ of the rotational NES is given by

$$J(t) = -c\theta_{E,t} - \frac{k \left[\sqrt{R^2 + r^2 - 2Rr \cos(\theta_E - w_{E,x})} - L \right] Rr \sin(\theta_E - w_{E,x})}{\sqrt{R^2 + r^2 - 2Rr \cos(\theta_E - w_{E,x})}} \quad (6)$$

where r is the length of the rocker arm. R is the radius of the inner ring connected with the rocker arm by a linear spring with damping c , stiffness k , and length L , and θ_E and w_E are, respectively, the rotation angle and the transverse displacement of the beam end. Based on

$$J(t) = -c_N \dot{\theta}_{E,t} - \frac{k \left[\sqrt{(R-r)^2} - L \right] Rr (\theta_E - w_{E,x})}{\sqrt{(R-r)^2}} - k \left\{ \frac{\left[\frac{R^2 r^2}{\sqrt{(R-r)^2}} - \frac{\sqrt{(R-r)^2} - L}{6} Rr \right]}{\sqrt{(R-r)^2}} - \frac{\sqrt{(R-r)^2} - L}{2\sqrt{(R-r)^2} (R-r)^2} R^2 r^2 \right\} (\theta_E - w_{E,x})^3 + O\left((\theta_E - w_{E,x})^5\right) \quad (7)$$

the Taylor expansion, the expression of the interaction torque can be replaced by

4.1.1.3 NESs with nonlinear damping Andersen et al. [128] documented a NES with geometrically nonlinear viscous damping, as shown in Fig. 12. The NES restoring and damping force is given by

$$F(t) = -c_N (\dot{x}_N - \dot{x}_P) - 2c_{NL} (x_N - x_P)^2 (\dot{x}_N - \dot{x}_P) - k_N (x_N - x_P)^3 \quad (8)$$

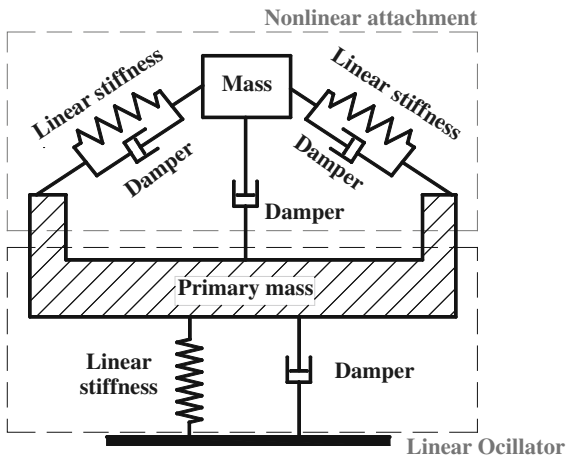


Fig. 12 Schematic of the geometrically nonlinear viscous damping

where c_N and c_{NL} are, respectively, the damping coefficients of the vertical and the horizontal springs. Due to 1:3 resonance capture, they found the geometrically nonlinear viscous damping leading to dynamic instability of a system composed of a linear primary

oscillator and a NES. Moreover, the system performs continuous resonance captures over certain energy and frequency ranges. Kong et al. [129] studied dynamics of single-DOF and two-DOF NESs with geometrically nonlinear damping. Liu et al. [130] explored the complex dynamics of a two-DOF system comprised of a primary linear oscillator and a lightweight NES with strong geometrically nonlinear stiffness and damping. Nonlinear stiffness and damping consisted of two inclined linear spring-damper elements. Unwanted large-amplitude steady-state responses of the linear oscillator coexisting with desirable small-amplitude responses can be reduced and even eliminated completely [130].

Ahmadabadi and Khadem [131] utilized a NES with nonlinear damping to annihilate large-amplitude periodic responses of a forced two-DOF vibration system. Hydraulic dampers were used to produce piecewise-quadratic nonlinear damping. This work showed the sensitive dependence of vibration responses on the NES parameters. Under appropriate parameters, the NES can excite the strongly modulated responses region and remove simultaneously all large-amplitude periodic branches in both system modes [131].

Silva et al. [132] proposed equivalent electrical circuits for electromechanically coupled systems with various types of stiffness and damping nonlinearities. They developed an experimentally validated the piezoelectric-based NES [133]. The electrical circuit

consists of a negative capacitance shunt and a cubic nonlinear capacitance realized via operational amplifiers. The linear term coefficient is less than 98% of the coefficient without the negative capacitance. They investigated the effects of the piezoelectric-based NES on the vibration of an electromechanically coupled cantilever beam and found significant vibration attenuation achieved over a wide frequency range [133].

4.1.1.4 Multi-DOF NESs Single-DOF NESs suppress vibration and mitigate shock in a wider frequency band than linear vibration absorbers. Nevertheless, a single-DOF NES may be quite sensitive to the external forcing magnitude and the overall energy level [134]. More precisely, a single-DOF NES achieves highly targeted energy transfer efficiency only over a relatively narrow range of external forcing amplitude. Passive targeted energy transfer from a linear discrete system of coupled oscillators to an attached multi-DOF NES has been considered [11]. Benarous and Gendelman [135] proposed a two-DOF NES including a purely cubic oscillator and an internal rotator, as shown in Fig. 13. The comparison showed that the two-DOF NES is superior to the single-DOF NES of the same weight for all energy ranges above the excitation threshold [135].

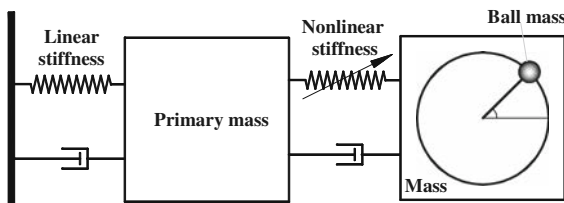
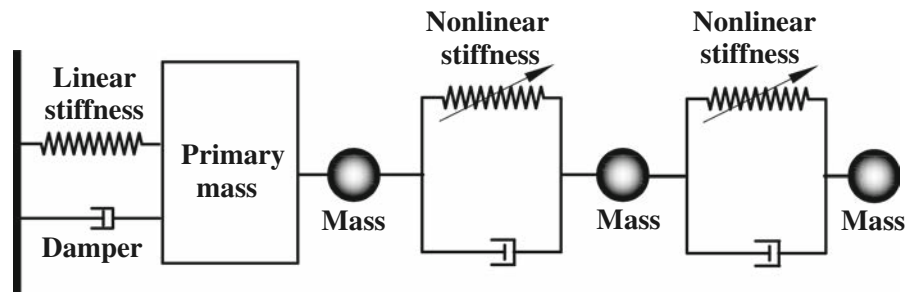


Fig. 13 Sketch of a two-DOF NES that comprises a purely cubic oscillator with an internal rotator

Fig. 14 Schematic of multi-DOF essentially nonlinear attachments linearly coupled to a linear primary system



Panagopoulos et al. analyzed transient responses of a linear system coupled with multi-DOF essentially nonlinear attachments [136, 137]. Multimodal vibrational energy of the linear system is pumped to the attachments with their multiple nonlinear modes simultaneously interacting with multiple modes of the linear primary system [136, 137]. Tsakirtzis et al. [138] demonstrated that a multi-DOF nonlinear attachment that is essentially a NES, shown in Fig. 14, can transfer irreversibly and dissipate up to 90% of input energy in a linear primary structure in some cases. Charlemagne et al. [139] investigated multiple nonlinear lightweight oscillators linearly coupled to a linear primary oscillator and a multi-DOF linear primary system, respectively. Charlemagne et al. [140] explored the dynamics of a chain of oscillators with cubic on-site restoring forces coupled to a linear oscillator forced harmonically.

In order to enhance passive targeted energy transfer performance, Gendelman et al. [141] designed a strong asymmetry two-DOF NES composed of two equal masses connected by small cubic nonlinear stiffness with a parallel linear viscous damper. Large nonlinear stiffness is used to connect the two-DOF NES to a primary linear system excited by an impulse. Grinberg et al. [142] showed that two-DOF NESs enhance targeted energy transfer for periodic excitations as well, but the coupling between a linear primary system and a NES may lead to a multitude of possible responses and even chaotic behaviors. Steady-state dynamic behaviors of a single-DOF nonlinear primary system coupled with a single-DOF NES were compared with those coupled with a two-DOF NES [143]. Wierschem et al. [144] investigated experimentally on broadband targeted vibration energy transfer between a two-DOF primary system and a two-DOF NES.

Li et al. [145] designed a NES with two parallel vibro-impact oscillators in for a linear primary oscillator excited periodically and transiently and demonstrated analytically and experimentally the effectiveness of the two vibro-impact NESs as long as the two vibro-impact oscillators were activated with two impacts.

4.1.1.5 Bi-stable NESs Al-Shudeifat [146] designed a bi-stable NES via negative stiffness implemented through a snap-through configuration with geometric nonlinearity, as shown in Fig. 15. This configuration of two linear springs (black) may produce negative linear and positive nonlinear stiffness terms during the NES mass motion. The bi-stable NES enhances considerably the performance of targeted vibration energy transfer and local fast energy dissipation. For broadband initial energy, with a proper initial position of the NES mass relative to the primary mass, the bi-stable NES was able to dissipate 97–99% of the input energy of the primary oscillator. The resultant force of damping and restoring of the bi-stable NES is given by

$$F(t) = -c_N(\dot{x}_N - \dot{x}_P) - 2k(1 - L/L_0)(x_N - x_P) - k(L/L_0^3)(x_N - x_P)^3 + O((x_N - x_P)^5) \quad (9)$$

where L_0 is the length of the spring at the unstable equilibrium position (compressed, gray). The configuration of the unloaded springs generates nonlinear stiffness $k(L/L_0^3)$ and negative linear stiffness $2k(1 - L/L_0)$, where $L > L_0$.

Manevitch [147] proposed a bi-stable NES with negative linear and positive cubic nonlinear stiffness, as shown in Fig. 16, and examined the beneficial effects of the bi-stability on nonlinear targeted energy transfer from a linear excited impulsively primary

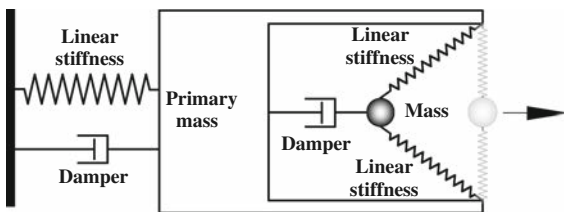


Fig. 15 Single-DOF linear structure coupled with the negative stiffness bi-stable NES

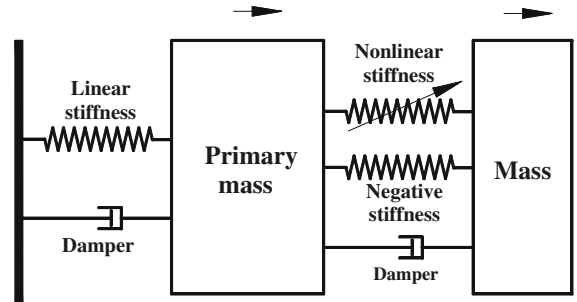


Fig. 16 NES model consisting of a lightweight attachment, a negative linear stiffness, and a cubic nonlinear stiffness

system to the appropriately designed bi-stable NES. Comparison with other types of NESs reveals significant advantages in the overall energy dissipation rate and the targeted energy transfer efficiency. The resonance capture regime of 1:1 and super-harmonic 1:3 can be realized in order to achieve strong energy exchange between the linear primary system and the bi-stable NES [147].

Romeo et al. [148] investigated numerically strongly modulated oscillations and chaotic cross-well oscillations in a linear system with a bi-stable NES. Romeo et al. [149] addressed the mechanisms leading to intense energy exchanges between a bi-stable NES and a linear primary oscillator excited by a low-energy impulse. The bi-stable NES is implemented via a buckled beam with a midpoint mass. The buckled beam pre-compression defines the relationship between negative linear stiffness and positive cubic stiffness. A pure cubic nonlinear NES is with a well-defined input energy threshold below which there is neither significant energy absorption nor dissipation. The bi-stable NES can overcome the threshold limitation of the pure cubic nonlinear NES [149].

Fang et al. [150] designed a continuous bi-stable or multistable NES of a cantilever with a free end mass controlled by a magnetostatic force and found experimentally and theoretically that the bi-stable or multistable NES can achieve high and strongly robust targeted energy transfer efficiency for broad-range impacts. In their work, the bi-stable and the multistable NESs result from modulating the nonlinear restoring force, and the NES damping is implemented by a partial constrained layer damping. Habib and Romeo [151] tuned and optimized a bi-stable NES termed as TBNES in order to mitigate vibrations of two coupled symmetric linear oscillators. They

claimed that the absorption capability of the TBNES is higher than that of a purely cubic NES.

Yang et al. [152] constructed a bi-stable NES with negative stiffness to enhance passive and adaptive vibration reduction for a pipe conveying fluid. They claimed that the bi-stable NES can absorb vibration energy in a higher decay rate, with higher robustness, with a smaller threshold, and higher energy dissipation efficiency. The boundary between chaotic and targeted energy transfer regimes was numerically determined by calculating the Lyapunov exponent [153].

Yao et al. [154] presented numerically and experimentally a tri-stable NES with negative stiffness and positive piecewise-linear stiffness. The negative stiffness was produced by permanent magnets, and the piecewise-linear stiffness was produced by a combination of elastic beams. The demonstrated the tri-stable NES able to attenuate small vibrations with the amplitude less than 1 mm. A bi-stable NES with buckled beam [155] and a multistable NES with piecewise-linear stiffness and magnet negative stiffness [156] were developed to suppress the vibration of an unbalanced rotor system. A piezoelectric NES was designed by introducing in-phase actuation to compensate for or to enhance the external vibration loads, so that the NES is triggered operating in high attenuation efficiency [157]. A magnetic bi-stable NES was proposed for structural seismic control, and the NES used appropriately configured permanent magnets to generate noncontact and smooth bi-stable restoring force [158].

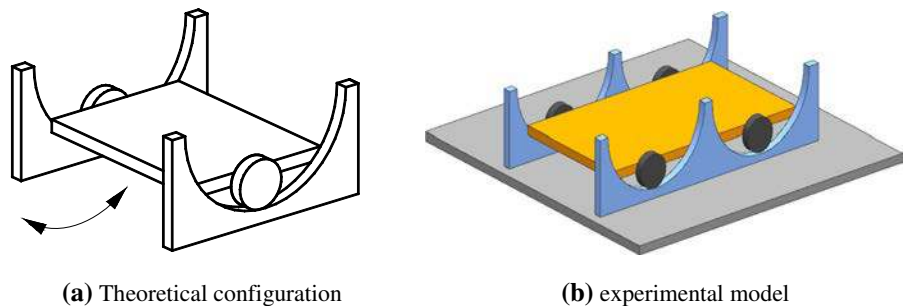
4.1.1.6 Track NESs Wang et al. [159, 160] designed a track NES, as shown in Fig. 17. The shape of the track is assumed to be smooth and symmetric. The damping and restoring force of the track NES is

$$F(t) = -c_N \dot{x}_N - m_N \left\{ \left[\frac{dh(x_N)}{dx_N} \right] \ddot{x}_N + \frac{dh(x_N)}{dx_N} \frac{d^2h(x_N)}{dx_N^2} \dot{x}_N^2 + g \frac{dh(x_N)}{dx_N} \right\} \quad (10)$$

where $h(x_N)$ is the shape of the track. They investigated analytically and optimized numerically the track NES acting on a two-DOF primary structure and examined experimentally the NES performance for the structure under impulsive and seismic excitations. The track NES facilitates energy transfer from lower modes to higher modes where the vibration can be dissipated at a faster rate and thus achieves the significant vibration attenuation of the structure response over a broad range of input frequencies [159, 160]. Lu et al. [161] proposed an optimal design process for the track NES and verified experimentally the vibration mitigation effects on a large-scale five-story steel frame structure. The track NES was capable of working in a wide frequency range with a small mass ratio to the primary structure under seismic excitations [161]. Wang et al. [162] proposed a single-sided vibro-impact track NES to mitigate vibrations of seismic and impulsive excited multistory buildings and illustrated numerically and analytically that the proposed NES can be more efficient than a smooth track NES or a tuned mass damper in reducing the vibrations over a wide range of input energy and primary system stiffness.

Wang et al. [163] demonstrated numerically that a two-phased arc NES mitigates vibrations of an eight-story base-isolated primary structure with little installation limitation and high mitigation effectiveness. The softening nonlinear restoring force of the two-phased arc NES was produced through the auxiliary mass moving on a track with two phases. The force-displacement relationship of the optimal two-phased arc NES was consistent with the hysteretic behavior of

Fig. 17 Schematic of a track NES



the primary structure. The low-phase track was tuned to accommodate the pre-yield dynamics of the primary structure, while the high-phase track was adapted for the post-yield dynamics [163]. As effective control strategies, a track NES and a single-sided vibro-impact track NES were numerically used to mitigate the seismic response of a 32-story high-rise building [164].

4.1.1.7 Parameter adjustable NESs In order to improve vibration suppression, some parameter adjustable NESs were proposed. Since the optimal targeted energy transfer for impulse excitations depends on input energy levels, it is impossible to optimize NES passive controls for any inputs. Wei et al. [165] derived the optimal equations for a NES acting on a linear primary structure based on optimal targeted energy transfer. They used the 1:1 resonance manifold to determine the optimal phase difference between the NES and the structure as $\pi/2$ for the undamped system [165]. Under periodic loadings, the suppression efficiency of an optimized NES is much higher than that of a conventional NES. In the view of active vibration controls, a NES can serve as an actuator to maintain the vibration suppression of a system at the optimal target energy transfer. Viguié and Kerschen [166] proposed a qualitative tuning methodology to impose the NES frequency–energy dependence so that of the dependence a single-DOF structure is equal to that of a multi-DOF structure. Anubi and Crane [167] designed and analyzed an active variable stiffness suspension, a vertical strut, and a horizontal strut controlled by a hydraulic actuator and found the suspension serving as an active NES reducing effectively the vibration energy transmitting from the road disturbance to the sprung mass. Bellizzi et al. [168] proposed a NES involving an active damper with a time delay. The time delay was included in the damping model to account for the time required to change the damping characteristics. They showed that the introduction of a relatively small time delay can lead to significant changes in the response regimes [168].

Anubi and Crane [169] proposed a suspension system with a semi-active variable stiffness. The suspension system consisted of horizontal and vertical struts controlled semi-actively by magnetorheological dampers. The vertical magnetorheological damper force was designed to track a skyhook damper force,

and the control algorithm of the horizontal magnetorheological damper forces was algorithmically controllable to adopt the NES concept to transfer the vibration energy to a control mass effectively. Simulation results demonstrated the significant improvement in the proposed semi-active system in ride comfort over traditional skyhook controllers.

Lamarque et al. [170] proposed a smooth cubic nonlinear NES with time-periodically varying stiffness around a constant value to improve strongly modulated responses and vibration mitigations of a primary system. Foroutan et al. [171] presented a tuned bi-stable NES connected to the ground by a spring with linear and nonlinear local stiffness.

4.1.1.8 Other types of NESs Avramov and Mikhlin [172, 173] proposed an essentially nonlinear absorber based on a snap-through truss, as shown in Fig. 18. The restoring force is given by

$$F(t) = -k \left[2 - \frac{L}{\sqrt{(L \cos \theta - x_p)^2 + x_p^2}} - \frac{L}{\sqrt{L^2 \cos^2 \theta + x_p^2}} \right] x_N \quad (11)$$

where θ is the angle of the coupling spring. The vibration absorber is attached to a linear primary system by purely quadratic nonlinear stiffness. They found that the system energy is concentrated on the nonlinear absorber if the localized nonlinear normal mode is realized. Moreover, the localized mode is stable over the wide range of the excitation frequency, while the nonlinear absorber has linear stiffness due to the asymmetry of the snap-through truss. In the cases of quasiperiodic and periodic responses, Avramov and

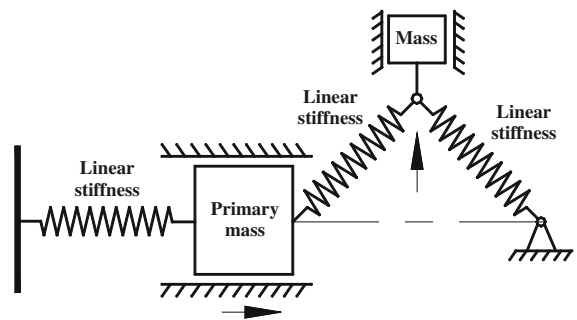


Fig. 18 Mechanical system of a linear primary system attached to a snap-through truss

Gendelman [174] demonstrated that the vibration amplitude of the primary system is moderate and less than that of the truss.

Laxalde et al. [175] proposed a nonlinear absorber with a small mass and a softening hysteretic damping modeled by a Bouc–Wen model and showed that a properly designed absorber can act as a nonlinear energy sink and nonlinearity and damping are important for the efficiency of the absorber. Tsiatas and Charalampakis [176] constructed a hysteretic NES by connecting a purely hysteretic spring and a linear spring in parallel and found that the NES performs effectively when the linear spring is with negative stiffness. Tsiatas and Karatzia [177] investigated the reliability of the hysteretic NES in shock mitigation. Savadkoohi et al. [178] built a NES with smooth cubic nonlinear stiffness and Bouc–Wen type nonsmooth hysteresis nonlinear stiffness to localize the vibration energy of a linear primary oscillator.

Febbo and Machado [179] proposed a strongly nonlinear vibration absorber with a saturable nonlinearity. The saturable nonlinearity was modeled by using a finite extensibility nonlinear elastic potential. The proposed nonlinear absorber was attached to a single-DOF linear/nonlinear primary oscillator subjected to an external harmonic excitation. The results showed that the proposed absorber is efficient for a strongly nonlinear primary system, but it is with the possible disadvantage that large-amplitude quasiperiodic vibrations coexist with unstable periodic vibrations [179].

Vakakis et al. [180] treated both symmetric and highly asymmetric configuration NESs focusing on the interaction of traveling waves in a linear periodic oscillator chain with the NESs. They found that the asymmetric NES performs well with the strong nonlinear stiffness in one direction and the weakly restored linear stiffness in the opposite direction. Al-Shudeifat [181] utilized a strongly nonlinear asymmetric magnetic force to design a NES. The asymmetric nonlinear magnetic force was generated by two pairs of permanent magnets, as shown in Fig. 19. Compared with the symmetric stiffness-based NESs, this asymmetric design can provide with a considerable enhancement in the shock attenuation for broadband energy input [181].

Charlemagne et al. [182] presented a NES with local and global nonlinear stiffness. The NES global nonlinear stiffness interacts with the primary

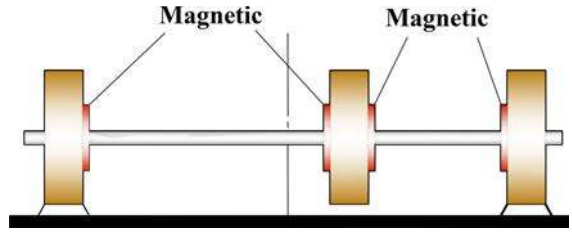


Fig. 19 Asymmetric magnetic NES configuration

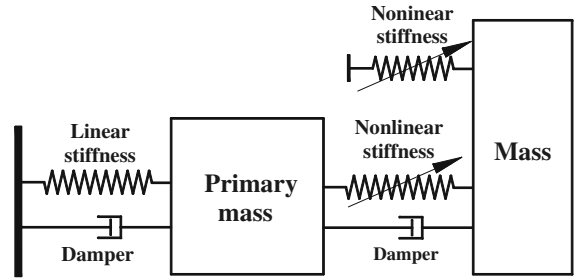


Fig. 20 Schematic of a grounded NES composed of local and global nonlinear stiffness

oscillator, while the local nonlinear stiffness depends only on the NES response, as shown in Fig. 20. A time multiscale method was utilized around 1:1:1 resonance to determine the slow invariant manifold. The existence of the NES local nonlinear stiffness changed the slow invariant manifold. The NES damping and cubic restoring force is given by

$$F(t) = -c_N(\dot{x}_N - \dot{x}_P) - k_N(x_N - x_P)^3 - k_G x_N^3 \quad (12)$$

where k_G is the local nonlinear stiffness coefficient.

Al-Shudeifat [183] introduced a set of nonlinear coupling restoring forces, with a nonnegative component and a real-power component, to enhance a NES. The proposed NES provides with hardening and softening stiffness in each oscillation cycle and reduces more vibration of the primary oscillator than a NES with purely cubic stiffness does. Lo Feudo et al. [184] proposed a magnetic absorber with adjustable linear and nonlinear stiffness coefficients. Depending on the coefficient of the linear stiffness, the absorber can be configured as three types, namely a nonlinearly tuned vibration absorber, a cubic NES, and a bi-stable NES. Their results showed no easy general guidelines to select one of the three types [184]. Luo et al. [185] realized physically essential nonlinear NES stiffness via a pyramidal shape polyurethane elastomeric spring with an increasing

cross-sectional area from top to bottom. The pyramidal elastomeric spring was numerically optimized via a component-level finite element to achieve the required essentially nonlinearity. The computational model of the polyurethane elastomeric spring was verified by experimental results [185]. Li et al. [186] used noncircular gears and normal torsion springs to design nonlinear springs with prescribed torque–angle relationships.

Mahmoudkhani [187] adopted a flexural beam as a pendulum to construct an auto-parametric system by attaching vertically it to a linear primary oscillator with a low stiffness torsional spring. The second natural frequency of the flexural beam was tuned to be close to the natural frequency of the linear primary oscillator. The quasiperiodic oscillation of the flexible beam pendulum absorber is beneficial to vibration absorption. Farid and Gendelman [188] modeled a partially liquid-filled tank as an oscillatory system with NESs. Dekemele et al. [189] designed a NES with adjustable nonlinear stiffness. Chen et al. [190] found that the increasing excitation enhances the NES efficiency with properly reducing the nonlinear stiffness.

4.1.2 Optimization of NESs

4.1.2.1 Energy transfer efficiency Wei et al. [165] found that the efficiency of an optimal NES was much larger than that of a conventional NES. In order to optimize NES designs, some analytical methods have been developed to analyze the effects of NES parameters on the NES efficiency [191]. Tripathi et al. [192] obtained near-optimal parameters for a NES acting on a class of weakly damped multi-DOF primary systems. In order to design an optimal NES with the largest efficiency, Manevitch et al. [193, 194] investigated analytically, numerically, and experimentally energy pumping in a strongly inhomogeneous two-DOF oscillator. For the purpose of determining the NES parameters to achieve the best vibration attenuation of a primary system, the efficient vibration energy pumping process was treated as a damped beating with strong vibration energy transfer, and thus a criterion for optimization was proposed. Optimal parameters of a base-excited viscoelastic isolation system and nonlinear energy sink were determined based on vibration mitigation criteria and broader stability ranges [195].

Nguyen and Pernot [196] applied the method of multiple scales and Manevitch’s complexification method to bring a clear insight in designing practically optimal nonlinear stiffness for a NES. Based on transient dynamic of a linear oscillator linked either to a NES or to a network of several NESs in parallel, the energy dissipation rate was used to assess the efficiency and the robustness of vibration reductions. Optimal criterion was obtained to design the best NES for attenuating free vibrations of a linear primary oscillator [196]. Qiu et al. [197, 198] studied the optimal design of a bi-stable NES for the vibration suppression of a linear primary oscillator excited periodically. They proposed an optimal design criterion and a corresponding parameter configuration for the bi-stable NES.

For initial energy near a homoclinic orbit of an underlying Hamiltonian system, a complexification averaging technique was applied to realize optimal targeted energy transfer [199]. Transient responses were examined to determine analytically the parameter region of the optimal targeted vibration energy transfer from a single-DOF forced primary oscillator to the NES. The appropriate parameters of the forced primary oscillator parameters lead to effective or even optimal vibration energy transfer. For sufficiently small viscous damping, damped transitions depend strongly on the corresponding topological structure of periodic and quasiperiodic orbits of the Hamiltonian system [199]. Based on the investigation on impulse orbits, system parameters can be changed to produce the best passive target energy transfer [200]. By transforming chaotic and periodic motions of the systems into stable orbits, Chavarette et al. [201] applied a linear optimal control theory to control the vibration of ideal and nonideal structures linearly coupled to an essentially nonlinear oscillator.

Pham et al. [202] demonstrated that it is possible to control simultaneously two 1:1 resonances of a three-DOF system, two coupled one-DOF linear oscillators attached to a NES. They utilized splitting harmonics and iterative techniques to explore dynamic behaviors of the three-DOF system and to examine the effects of the system parameters. For a two-DOF system of a harmonically excited linear oscillator coupled strongly with a NES, Gourc et al. [203] found theoretically and experimentally an optimal area in the forcing amplitude–nonlinear stiffness plane, where strongly modulated responses occur with a small

amplitude resonance threshold. The complexification-averaging technique was used to determine the existence zone of stable strongly modulated response. The NES was optimized to achieve as small mass ratio as 1% [203]. Lin and Oguamanam [204] investigated the targeted energy transfer efficiency or the energy transfer rate between a single-DOF primary system and a NES. A wavelet transforms modulus maxima method was applied to extract the phase information in resonance captures. The results showed a negative correlation between the targeted energy transfer efficiency and the duration of transient dynamics remaining in the 1:1 resonance capture regime (the 1:1 phase lock mode). The stronger nonlinear element leads to the larger targeted energy transfer efficiency [204]. In order to reduce vibrations of a linear Euler–Bernoulli beam, Parseh et al. [205] applied complex-averaging and arc-length continuation methods to optimize a NES with the emphasis on the robustness against changing excitation force amplitude. The optimization was performed by varying different NES parameters, and the optimal parameters were determined. Results demonstrated that the NES designed for the maximum possible forcing amplitude can reduce significantly the vibration not only for the designed force but also for forces with smaller amplitude. Therefore, the optimization for the maximum value of expected forcing amplitude is essentially robust [205].

Based on the identification of the design and aleatory space regions corresponding to significantly different NES efficiencies, Boroson and Missoum proposed an algorithm for optimizing uncertainty NESs including random design and aleatory variables via Kriging approximations (a support vector machine) and Monte Carlo simulations [206]. Boroson et al. [207] optimized multiple-uncertainty NESs configured in parallel in order to mitigate the excitation limitation and to maximize the efficiency range. The main idea is to extend the excitation range with the high NES efficiency via parallel configuration of NESs. They announced that the optimization enabled a sharp increase in the mean efficiency value [207].

By adding a relatively small excitation with a close frequency or a random narrow-band signal, Starosvet-sky and Gendelman [208] investigated analytically and numerically strongly modulated response regimes in a forced primary system coupled with a NES. With random initial conditions, Sapsis et al. [209]

considered a linear system with a NES in the presence of stochastic excitations to both the linear system and the NES. The stochastic factor not only maintains but also enhances the optimal targeted energy transfer regime predicted by the deterministic analysis. Xiong et al. [210] examined vibration absorption of a mechanical system coupled with a NES under narrow-band stochastic excitations. The quasiperiodicity response regime was determined by via complex-averaging and frequency-detuning methodology. Compared with the optimal linear absorber, the NES could work better in suppressing the vibration under the narrow-band stochastic force excitation [210]. Oliva et al. [211] employed sensitivity analyses to determine optimal parameters of an ungrounded NES configuration for reducing vibrations of a single-DOF and multi-DOF structure under random Gaussian white noise base excitations. Xue et al. [212] used a NES to reduce vibration of a linear system excited by a harmonic and random base excitation.

4.1.2.2 Vibration suppression efficiency Vibration suppression efficiency of a NES can be defined by comparing the maximum vibration amplitude of a primary system with the NES and the counterpart without the NES. Vibration suppression efficiency can serve as a goal for the optimizations of NESs. It was found optimal parameters of a NES depend on primary systems acted by the NES [49, 213].

For a harmonically forced linear oscillator coupled with a NES, Starosvet-sky and Gendelman explored bifurcations of vibration responses via the mass ratio to depict qualitative changes for the growing NES mass and to provide with a threshold estimation of the NES mass [214]. Vaurigaud et al. investigated a four-degree-of-freedom (four-DOF) primary structure with two parallel NESs at the last DOF [215–217]. Parallel NESs devices were optimized via bifurcation analysis. The investigation revealed that parallel NESs are more efficient in the senses of smaller masses, smaller nonlinear stiffness, and more activation levels. Optimal parallel NESs were utilized to control a four-story prototype structure. Under seismic excitations, the two parallel NESs suppressed effectively the vibrations in expected modes of the structure.

Nucera et al. [218] optimized a vibro-impact NES on the lowest floor and smooth cubic NES on the top floor to suppress significantly seismic responses of a three-story shear frame. They demonstrated

experimentally that the vibro-impact NES excites higher structural modes. Hence, the seismic energy is transferred from low-frequency to high-frequency ranges. Since higher-frequency modes are with smaller amplitude response, the optimized NES combination is capable of reducing the required NES mass at the top floor and the structural response amplitude. It was observed that a vibro-impact NES is more effective if its response regime has two impacts per cycle of a primary structure with the NES [219]. Ahmadi and Attari [220] used a baseline record and a harmony search strategy to optimize a vibro-impact NES with a specified mass ratio to find the best stiffness ratio and clearance. The vibro-impact NES was applied for seismic mitigation of medium-rise steel moment frames shocked by eight historic earthquakes. Li et al. [221] proposed a procedure to optimize the design of a vibro-impact NES. The slow invariant manifold was analytically obtained. Activation characteristics of the vibro-impact NES were investigated based on the analytical expression of the slow invariant manifold. The activation characteristics were numerically and experimentally validated. The activation characteristics were used to propose a design procedure of the vibro-impact NES, and the procedure was applied to control the chatter during a turning process [221]. Li et al. [222] further utilized an asymptotical method to explore the vibro-impact NES optimization of targeted energy transfer under periodic and transient excitations. Theurich et al. [223] proposed a design criterion for optimizing a tuned vibro-impact NES. Optimal clearances for transient and periodic excitations were, respectively, determined from two transition points of the slow invariant manifold, and the optimal clearances were theoretically verified and experimentally validated. They found that the design criteria can not only predict the effective target energy transfer at the resonant frequency, but also achieve the optimal performance [223].

4.2 Acoustics control

Acoustics is actually beyond direct applications of targeted energy transfer in mechanical engineering [224]. For example, passive damping is inefficient in the low frequency range. Cochelin et al. [225] and Bellet et al. [226] observed experimentally targeted energy transfer via a nonlinear energy sink (NES) in

acoustics. They attached an essentially cubic oscillator with nonlinearity produced by a thin circular viscoelastic membrane to an acoustic linear system of a tube with air inside by a box with air inside as a weak coupling between the nonlinear oscillator and the linear system. The membrane can tune itself to different resonance frequencies over a wide range. Using the membrane acoustic NES, Shao and Cochelin [227] observed targeted energy transfer phenomenon of an acoustic medium inside a parallelepiped cavity under harmonic forcing. The acoustic NES was mounted on one wall of the cavity. The stability analysis of nonlinear normal modes led to the desired working zone for the NES [227]. Bellizzi et al. [228] employed a simple thin circular viscoelastic membrane as a NES and linearly coupled it on an acoustic medium (a primary system) subjected to quasiperiodic excitations. The acoustic medium was modeled as a two-DOF linear system. The research showed that a single NES can efficiently reduce two resonance peaks, even if the two peaks are simultaneously excited, incommensurable, and well separated. In order to reduce a low frequency resonance, Wu et al. [229] sought the suitable position and membrane parameters for a desired NES working zone. Bellet et al. [230] proposed multiple nonlinear membrane absorbers placed in parallel for enhancing the robustness and the efficiency of targeted energy transfer in acoustics. An experimental setup with two membranes showed that different NESs have clearly different activation thresholds and the NESs have additional effects on the efficiency and the possibilities of targeted energy transfer. Wu et al. [231] demonstrated that the desired working zone can be enlarged by additional membranes. At low frequencies and high sound levels, Chauvin et al. [232] experimentally investigated the acoustic properties of a nonlinear vibro-acoustic absorber involving a clamped thin membrane made of Latex. It removed up to 90% of the incoming acoustic wave near a frequency that depends on the source level. Bryk et al. [233] proposed experimentally a new hybrid electro-acoustic nonlinear membrane absorber composed of a thin circular viscoelastic membrane whose linear stiffness was experimentally controlled via a feedback loop. Then, they applied the hybrid electro-acoustic NES to achieve targeted energy transfer in a concrete building for low-frequency noise reduction [234]. Shao et al. [235] controlled the broadband low frequency noise

inside a 3D acoustic cavity by a nonlinear membrane. They investigated the effects of the membrane prestress on the targeted energy transfer and the desired NES working zones.

In order to reduce sound radiation, energy pumping phenomenon was used to suppress vibrations in radiating modes. Mattei et al. [236] utilized two beams coupled by a spring to make a double-leaf panel. One of the two beams serves as a primary system, and the other is a thin buckled beam with a mass fixed at its center functioning as a bi-stable NES. When the bi-stable NES vibrates chaotically around its two equilibriums, most energy in the primary structure was reduced. Experimental and numerical results demonstrated the stronger attenuation of vibrations in the first two modes. Therefore, the bi-stable NES can reduce the vibration of the primary structure with multiple resonances.

Bitar et al. [237] converted a loudspeaker into an electroacoustic absorber by connecting a passive nonlinear shunt circuit acting as a NES at the loudspeaker transducer terminals. The results showed that the nonlinear shunt can change the acoustic characteristics of the loudspeaker in a broad acoustic absorption frequency range.

5 Applications of NESs

Investigations have highlighted the advantages of NESs for vibration reductions, revealed NES-induced complex dynamics, and implemented experimentally different design configurations. Thus, NESs are possibly and feasibly applied to reduce vibrations in a variety of engineering fields.

5.1 Vibration attenuation in civil engineering

5.1.1 Multistory structures

In order to apply it to civil engineering, Gourdon and Lamarque [238] investigated efficiencies of energy pumping of a piecewise-linear system and a chaotic system. Vibrations of a building were attenuated based on two building models weakly coupled to a nonlinear oscillator, respectively. The numerical results showed robustness of energy pumping for slightly modified parameters. Based on a multi-DOF NES coupled to a single-DOF linear oscillator, Gourdon and Lamarque

[239] demonstrated that the involvement of multiple nonlinear modes leads to the increase of the energy area with energy pumping phenomenon because the use of multiple nonlinear modes allows energy pumping to occur for lower energy. The multi-DOF NES is a nonlinearly coupled NES chain. Irreversible passive energy pumping characteristics of NESs have been theoretically and experimentally demonstrated in efficient suppression of a single-story [84] and a four-story [85] building vibrations under earthquake excitations. Based on a two-DOF system linked to a NES, Pham et al. [240] explained clearly the completion between two nonlinear modal localizations. Manevitch's complexification technique was adopted to reveal slow dynamics of the three-DOF system from fast oscillating response. The first triggered resonance was predicted in the transient regime. A two-story building model was investigated to understand multiple targeted energy transfer mechanisms in the competition [240]. Quinn et al. [241] demonstrated the effects of a NES on vibration responses of a two-story structure subjected to an impulse. They predict analytically and verify experimentally the efficacy of the NES to reduce significantly the response of the two-DOF system under a shock excitation. Al-Shudeifat et al. [123] investigated numerically and experimentally the targeted energy transfer effects of an inertially coupled rotating NES attached to a two-story model building. Chen et al. [242] integrated a NES with friction, a sliding mass, and negative stiffness produced by geometrical nonlinearities. The seismic performance of the negative stiffness NES on a one-story shear frame was compared with those of a cubic NES and a linear tuned mass damper. The comparisons showed competitive and robust attenuation of the negative stiffness NES [242]. Lu et al. [161] utilized a track NES to mitigate seismic response of a five-story steel frame structure. They observed the dependence of the energy transfer on the modal shapes of the structure and the mounting positions of the track NES. Wang et al. mitigated vibration responses of multistory buildings by designing various track NESs [159, 160, 162, 163]. Nonlinear magnetic forces were used to design a magnetic NES for reducing vibrations of a three-story structure [184]. Free vibrations of the structure were significantly reduced under an impulsive shock. Dekemele et al. [189] solidified the feasibility of a NES in mitigating single and multimodal vibrations in real-life structures.

Based on discontinuous stiffness nonlinearity, Nucera et al. [93] considered an ungrounded vibro-impact NES attached to a two-degree-of-freedom linear primary system. The work demonstrated the feasibility of targeted energy transfer by the piecewise-linear oscillator for seismic mitigation. Moreover, complicated dynamics, such as sustained nonlinear resonance scattering phenomenon, occurred in the linear primary system with the vibro-impact NES [93]. They performed a series of experimental tests with a three-story shear-frame structure under seismic excitations in the form of two different historic earthquakes [218]. It was shown that targeted energy transfers can extract rapidly a significant portion of seismic energy from the shear-frame structure through to the attached vibro-impact NES [218]. In order to improve seismic mitigation for a three-story shear frame, Nucera et al. [243] combined a vibro-impact NES acting on the first floor with a smooth cubic stiffness NES acting on the third floor. The results showed that the vibro-impact NES can reduce initial peaks of the frame response to strong near-field earthquakes and the smooth NES can reduce the frame vibrations after the initial peaks. Therefore a vibro-impact NES and a smooth NES can drastically reduce the building seismic response [243].

Based on single-sided vibro-impacts, Al-Shudeifat et al. [244] investigated passive vibro-impact NESs for shock mitigation of a nine-story large-scale structure. By controlling the inherent damping of the NESs, the optimized NESs could scatter up to 80% of the input shock energy from the fundamental mode to higher structural modes. Large-scale experimental and numerical researches on NESs for mitigating structural responses under seismic shock [245] and blast excitation [246] were performed. Four NESs employing smooth essentially nonlinearity and two single-sided vibro-impact NESs employing nonsmooth nonlinearity were installed at the top two floors of a large-scale building structure model. The study demonstrated that structural responses caused by earthquake ground motions can be mitigated by synergistic effects of a system of NES devices. Lou et al. [185] tested an elastomeric NES for a large-scale nine-story model building structure under impulse-like seismic loadings. Experimental results showed that the properly designed elastomeric NES can be effectively, rapidly, and robustly mitigate structural vibrations caused by floor accelerations, displacements, or column strains.

Wang et al. proposed an asymmetric nonlinear energy sink to mitigate the unwanted responses of building structures. The asymmetric NES, configured based on a cubic NES, consists of an auxiliary mass connected to the primary structure through a linear and nonlinear springs. These two springs are statically balanced at a deformed position, producing an asymmetric restoring force [247].

5.1.2 Structural elements

5.1.2.1 Rods Georgiades et al. [248] explored nonlinear resonant interactions between a primary structure and a NES. The primary structure is a damped and forced dispersive linear finite rod, and the NES is essentially attached to the end of the rod. The investigation revealed passive, broadband, and irreversible targeted energy transfer from the rod to the NES. Supposing the primary system to be a linear elastic rod on a continuous linear elastic foundation, Panagopoulos et al. [249] provided computational tools for multiscaled analysis of complex multifrequency transitions of the system. They identified the frequencies at which there were the dominant resonant interactions between the rod and a NES attached to the rod.

5.1.2.2 Beams In order to suppress vibration of a cantilever beam under shock excitations, Ahmadabadi and Khadem [250] investigated performances of grounded and ungrounded configurations of NESs. They explored bifurcations and topological structure of nonlinear normal modes of both systems to find conditions of targeted energy transfers in each NES configuration. It is found that the ungrounded configuration produces targeted energy transfer more effectively with about 89% energy dissipation. No essential energy transfer to the grounded NES was observed. The finding demonstrated the importance of nonlinear normal modes in the occurrence of a one-way irreversible energy transfer for continuous systems [250]. Yang and Wang [251] investigated numerically and experimentally the reduction in linear velocity responses of a damped cantilever beam with a vibro-impact NES. Unstable velocity responses of the beam were observed.

As NESs had gradually manifested their advantages in suppressing vibrations of discrete systems, Georgiades and Vakakis [252] took the lead in using NESs

to reduce vibrations of flexible structures. A NES was suspended between two boundaries of an elastic beam. The NES was numerically demonstrated to be simple, light, and efficient in vibration suppression of flexible structures. An appropriately designed NES can passively absorb and locally dissipate most shock energy of the elastic beam, up to an optimal value of 87% [252]. The interaction between a forced vibrating nonlinear beam and a NES was studied with the account for the first two natural vibration modes of the beam [253, 254]. Numerical results showed that an appropriately designed NES can passively reduce the displacement of a bridge with a train up to 43%, where the bridge was modeled as an Euler–Bernoulli beam and the train as a set of successive moving loads [255]. Samani et al. [37] assessed vibration reduction performances of several kinds of nonlinear absorbers applied to a simply supported linear Euler–Bernoulli beam excited by a moving loading or vehicle.

Parseh et al. [205] used a NES to reduce harmonically forced vibrations of linear Euler–Bernoulli beams under four kinds of boundary conditions and compared the performance of the NES with that of a linear tuned mass damper [256]. Chen et al. [257] compared the absorption performance of single and parallel NESs on the vibration of a simply supported beam. They found that the performance of the parallel NESs is superior and able to eliminate higher branch responses. Mao et al. [127] demonstrated that a novel rotational NES can efficiently suppress the transverse vibration of a flexible beam with simply supports. A NES was adopted to mitigate vibration of rotating beams with accounting for the centrifugal stiffening effect [258].

A locally attached NES can suppress transient vibrations of an elastic beam mounted on springs [259]. The efficiency of the NES was examined based on an adaptive Newmark method. Using a wave approach with the advantage of reduced computational costs, Chouvion explored the effects of both the cubic nonlinear stiffness attached to the NES mass and the boundary torsional stiffness on the NES efficiency of suppressing nonlinear vibrations of a beam [76]. It was found that an uncertainty in the boundary model may strongly impact on the maximum beam amplitude observed over a frequency range. Kani et al. [260] optimized the NES efficiency of vibrational energy dissipation for a nonlinear continuous primary system via both sensitivity analysis and particle swarm

optimization method. The work showed that there is targeted energy transfer between the nonlinear beam and the NES for sufficiently strong input energy. Parseh et al. [261] investigated steady-state responses of a nonlinear Euler–Bernoulli beam and a Timoshenko beam, respectively, attached to a NES. An optimization procedure was performed by varying different NES parameters. The NES dissipation element highly impacted the reduction in the nonlinear structural vibration. Besides, multivalued frequency response phenomenon was observed in regimes of low excitation frequency. Chen et al. [262] suppressed the first modal resonance of a truss core sandwich beam via a grounded NES placed in the interior of the beam. The grounded NES is sufficiently compact to be installed in a limited space allowed by engineering structures. The increasing NES mass enhances the resonance suppression of the beam subjected to both impulse and harmonic loads. The larger NES damping works better for the beam under harmonic loads but worse for the beam under impulse loads. A NES was used to suppress nonlinear vibration of a laminated composite beam with the account of the environment moisture and the temperature [263].

In order to mitigate vibrations of a cable-stayed bridge, Izzi et al. investigated targeted energy transfer devices grounded by a linear spring via an analytical formulation of the complex generalized eigenvalue problem [264]. Weiss et al. [265] used an absorber with a nonsmooth forcing function to control a targeted vertical vibration mode of a cable.

5.1.2.3 Plates Georgiades and Vakakis [266] first explored nonlinear modal interactions and broadband targeted energy transfer phenomena between a thin elastic plate on an elastic foundation and three types of NES attachments, namely a single ungrounded NES, a set of two single-DOF NESs attached at different points of the plate, and a single multi-DOF NES. They investigated systematically strong modal interactions between the NESs and different plate modes, the threshold of maximum energy level, and the dominant resonance captures. It is found that the NES efficiency remains robust with changing NES mass, even for extremely small mass values. Moreover, the target energy transfer efficiency is very sensitive to the NES damping coefficient [266]. Using a NES, Taleshi et al. [267] controlled nonlinear vibration of a simply supported thin plate under a harmonic excitation.

Passive targeted energy transfers were optimized by varying the NES parameters and locations. Complex-averaging and arc-length continuations techniques were applied to obtain steady-state responses of the thin plate. The NES designed for specific excitation amplitudes performs better than a TMD, but the NES robustness is lower than the TMD with the increasing excitation amplitude [267]. An inertial NES was used to eliminate multimodal resonances of a composite plate [49]. It was found that the inertial NES is efficient in vibration elimination for both low-order and high-order resonances. However, optimal parameters of the inertial NES on different resonance response of the composite plate are not the same.

Sun et al. [268] utilized a NES to reduce the vibration of a linear elastic plate under flywheel excitations. Zhang et al. [269] suppressed significantly the severe vibration of a wind excited composite laminated plate via a NES. Chen et al. [270] placed a NES inside a truss core sandwich plate to dissipate the vibration energy excited by a half-wave shock.

5.1.3 Circular cylinder

Mehmood et al. [271] utilized a NES to control the vortex-induced vibrations of a freely oscillating circular cylinder. They illustrated the existence of a critical mass ratio, below which the NES is unable to suppress the vibration of the cylinder. In addition, effective control requires minimal NES damping [271]. Tumkur et al. [272] proposed a two-DOF equivalent reduced-order model to reproduce the dynamics of a coupled fluid–cylinder–NES system. The two-DOF self-excited oscillation model was assumed for laminar vortex-induced vibration of a sprung rigid circular cylinder coupled with a NES. The two-DOF system alternates internally between regular dynamics and chaotic bursts. A 1:1:1 resonance capture into a slow invariant manifold leads to targeted energy transfer from the circular cylinder to the NES [272]. Employing complexification/averaging and the multiscale method, Blanchard et al. [273] demonstrated that a slow invariant manifold leads to targeted energy transfer from a two-dimensional circular cylinder undergoing vortex-induced vibration to a NES rotator. Capture into the slow invariant manifold corresponds to transient circular cylinder stabilization, whereas escape from the slow invariant manifold

results in chaotic bursts [273]. Blanchard et al. [274] employed proper orthogonal decomposition on a slowly decaying portion to study the energy transfer and redistribution in order to understand the effects of a rotational NES on coherent flow structures. Dai et al. [275] mitigated vortex-induced oscillations of a circular cylinder by using a passive NES. Depending on the values of the NES parameters, the coupled flow cylinder and the NES system exhibits periodical, aperiodic, and multiple stable responses. The authors found that the increasing NES mass results in a right shift of the lock-in region and the presence of multiple stable responses enhances the energy pumping effect [275]. Chen et al. [276] studied the effects of a passive NES on vortex-induced and wake-induced vibration of elastically supported single-DOF and two-DOF circular cylinders. Fluid–structure interaction simulations revealed the vibration of two-dimensional circular cylinders to assess low Reynolds number vortex-induced. The NES mass significantly affects the amplitude of the vortex-induced vibration and the lock-in areas. The NES nonlinear stiffness has virtually no effects on the shift of the lock-in areas. The increasing nonlinear stiffness leads to the significantly reducing amplitude of the vortex-induced vibration response of the cylinder for an appropriate NES mass [276]. An internal dissipative rotational NES was applied to a 3D linearly sprung cylinder undergoing vortex-induced vibration was investigated [277]. With linearly damped rotational motion, the rotational NES mass was allowed to rotate at constant radius about the cylinder axis. Blanchard and Pearlstein [278] suppressed vortex-induced vibration of a linearly sprung circular cylinder by a rotational NES, consisting of a mass rotating about the axis of the cylinder and a dissipative element damping the rotational motion of the mass.

Dai et al. [279] suppressed the galloping vibration of an elastically mounted square prism, an aeroelastic system, via a NES. The galloping force was modeled based on the quasi-steady hypothesis. Increasing the NES damping can result in a significant increase in the galloping onset speed. The normal form of the Hopf bifurcation was derived in order to identify the instability and to determine the effects of the NES stiffness. The results demonstrated that the galloping vibration of the square prism can be efficiently reduced by the NES. The larger NES stiffness, the larger NES damping, and the smaller NES mass lead

to the more vibration reduction of the square prism [279]. Guo et al. [280] used a NES to suppress an adaptive and passive galloping of a suspended linear cable. Based on a one-term Galerkin truncation, harmonic balance analysis showed efficient inhibition of the limit cycle oscillation of the cable due to nonlinear wind loading. Mamaghani et al. [281] investigated passive vibration control of a doubly fixed Euler–Bernoulli beam under a sinusoidal flow using a NES. They determined the conditions required to generate the Hopf and saddle-node bifurcations. Blanchard et al. [282] used a rotational NES to mitigate vibration of a linearly sprung circular cylinder pasted by two-dimensional flow. The rotational NES was inertially attached to the cylinder. Nankali et al. [283] introduced an ungrounded NES to suppress chatter instability caused by nonlinear regenerative cutting force in a turning process of a single-DOF machine tool. Franzini et al. [284] adopted an essential and rotative nonlinear vibration absorber to suppress parametric instability of a cylinder. The rotative absorber was assembled onto the cylinder by a linear dashpot.

5.2 Vibration suppression in mechanical engineering

5.2.1 *Self-excited systems*

Suppressing limit cycle oscillations in the van der Pol oscillator, Lee et al. [285] pioneered the application of NES to control vibration of a self-excited oscillator. They proposed both grounded and ungrounded NESs. The grounded NES was attached to the van der Pol oscillator by a weak linear spring. The transient dynamics showed that 1:1 and 1:3 transient resonance captures suppressed the limit cycle oscillations by nonlinear energy pumping. For appropriately designed system parameters, subcritical Hopf bifurcation occurs to ensure the unique existence of a global trivial attractor. The bifurcation analysis revealed that instability of the self-excited oscillator can be robustly and completely eliminated by NESs. Therefore, the NES can be applied to suppress instability in self-excited systems [285]. Gendelman and Bar found that a NES produces different types global bifurcations including canard explosion, Shil’nikov bifurcation, and heteroclinic bifurcation as well as multiple basins of attraction in self-excitation regimes [286]. Domany

and Gendelman [287] applied the averaging method and the multiscale method to explore efficiently global dynamics of Van der Pol–Duffing oscillator with a NES. The approximate analytic procedure allows predicting a multitude of response regimes in the self-excited system with essential nonlinearities. Bergeot and Bellizzi [288] used a network of several parallel NESs to mitigate dynamic instability of a Van de Pol oscillator.

Viguie et al. [289] proposed a prototypical drill string–NES to stabilize the dynamics of a drill-string system. The NES takes the form of a discrete torsional oscillator, including a disk connected to the drill string system by an essentially nonlinear spring and a viscous damper. The study showed that the NES can completely eliminate the instability in a relatively wide range of parameters and can achieve partial limit cycling suppressions [289]. Ahmadabadi and Khadem [290] applied a NES to annihilate self-excited oscillations of a drill string with real system parameter values. They presented a slightly modified experiment and a torsional vibration model. Results demonstrated that different components of the drill string coupled with the NES can help to acquire better responses. Bergeot et al. [291] adopted two ungrounded NESs to control passively friction-induced vibrations due to the mode coupling in braking systems. They predicted the critical friction coefficient values for these steady-state responses [291]. The concept of targeted energy transfer by means of NESs was adopted to mitigate the squealing noise of a friction system with uncertain parameters [292]. Gourc et al. [293] used a NES with purely cubic stiffness to reduce chatter vibration in turning process based on a nonlinear cutting law. Gourc et al. [294] theoretically and experimentally investigated the benefits of a vibro-impact NES to mitigate passively the chatter instability in a turning process. An asymptotic analysis revealed dynamic behaviors of the system coupled with the vibro-impact NES. A chatter mitigation mechanism by successive beating was discovered. The original experimental procedure was proposed to validate this beneficial dynamic behavior of the vibro-impact NES [294].

5.2.2 *Rotational structures*

The vibration energy of a rotational system can be irreversibly transferred to a NES. Bab et al. [295] demonstrated the efficiency of a NES on the vibration

mitigation of a rotating structure under a mass eccentricity force. The NES has linear and cubic nonlinear stiffness and a linear damping. The condition of occurrence of strongly modulated responses was analytically derived from the mixed multiple scales–harmonic balance method. Results showed that by the desired rotor rotational speed for the strongly modulated response can be tuned by changing the linear NES stiffness [295]. Bab et al. [296] further investigated the effects of a number of NESs on mitigation of the vibration of a rotor supported by journal bearings with nonlinear suspension subjected to a mass eccentricity force. Two smooth NESs were located on the bearing body in the perpendicular directions. Bab et al. [297] extended the investigation on the effect of a number of smooth NESs located on the disk and bearing for mitigating the oscillation of a rotor-blisk-journal bearing system under a mass eccentricity force. A genetic algorithm was adopted to optimize the NES parameters.

Guo et al. [298] compared the whirling vibration reduction by a NES with that by a TMD in rotor systems. The whirling vibration at the critical speed can be mitigated by passive targeted energy transfer of NESs. In order to achieve the best vibration reduction performance, numerical simulations were performed to optimize the NES parameters. The work demonstrated that a NES does not require knowing a prior unbalance value, and the fact is an advantage of a NES to a tuned mass damper.

Motato et al. [299] applied two parallel NESs to reduce broadband torsional oscillations generated by internal combustion engines and transmitted into lightly damped drivetrain primary structures. Haris et al. [300] numerically studied the torsional oscillation attenuation of a clutch disk in a typical drivetrain configuration by using multiple NESs coupled in parallel. The oscillations of the transmission input shaft were significantly reduced over a wide range of response frequencies with rather small NES inertia. Dolatabadi et al. analyzed a pendulum NES performance in a piston model of an internal combustion engine [301]. In a wide engine speed range, the NES with a mass ratio of 12.5% dissipates more vibration energy than conventional methods do. Tehrani and Dardel [302] utilized a tuned mass damper and a NES to prevent the contact between the disk and a stator. They stated that the tuned mass damper and the NES have exactly the same positive effects on the nonlinear

vibrations of a Jeffcott rotor system. Tehrani and Dardel [303] applied a NES to mitigate vibration of a flexible bladed rotor system. The NESs were attached to both the tips of the blades and the disk. The research showed that the performance of removing the contact was limited to small clearance between the blades and the stator. For a rotor with three, four, or five blades, two different clearances were examined [304]. Ahmadabadi [305] used a lightweight NES to mitigate undesired vibrations in engine crankshaft systems over a wide frequency range. The most effective design results in dissipating up to 38% vibratory energy. Taghipour et al. [306] compared a linear tuned mass damper, a conventional NES, and a NES including both linear and nonlinear stiffness in vibration mitigation of a Jeffcott rotor primary system under nonlinear restoring forces. They found that the NES with both linear and nonlinear stiffness is best in damping vibration and providing with a wide stability range. Based on a gas generation section of a gas turbine, Bab et al. [307] adopted a smooth NES to control the vibration of a continuously rotary system during a nonstationary rotating motion rubbing at disk positions. The results showed that the NES can reduce the vibration of the gas turbine over a wide speed range. The NES was installed on a drivetrain experimental rig driven by an electric motor through a universal joint connection placed at an angle, thus inducing the second-order torsional oscillations. The NES leads to significant vibration reduction away from resonances and near resonances over a relatively broad frequency range [308].

Bergeot et al. studied the capacity of a NES to suppress a rotor instability termed a helicopter ground resonance [309, 310]. The NES was coupled to the fuselage in an ungrounded style. A helicopter model was built by using Coleman transformation and binormal transformation successively. Combination of the complexification-averaging and the geometric singular perturbation yielded an analytical explanation of partial/complete suppression through strongly modulated responses for the helicopter ground resonance, and the analytical results were numerically validated via direct numerical integration. Bergeot et al. [311] attached each NES on a helicopter blade. Focusing on a single helicopter rotor blade in a hovering flight state, Ebrahimzade et al. [312] compared the effects of linear vibration absorbers with that of a NES on the rotor blades' aeroelastic behaviors.

Bab et al. [258] used a NES to mitigate vibrations of a rotating Euler–Bernoulli beam subjected to an external periodic excitation. They determined conditions of a saddle-node bifurcation, a Hopf bifurcation, and strongly modulated responses. When strongly and weakly modulated responses simultaneously occurred, the NES parameters were in the best range for the vibration mitigation. Moreover, the rotating beam tip was the best position for attaching the NES.

5.2.3 Axially moving structures

Axially moving structures belong to typical gyroscopic continua. NESs can be used to reduce transverse vibrations of axially moving structures. Zhang et al. [313] applied a NES to suppress the unwanted vibration of an axially moving string with transverse wind loads. A two-term Galerkin truncation was utilized. Luongo and Zulli [314] applied a NES to reduce forced vibration amplitude of a nonlinear elastic string in internal resonance conditions. The multiscale method and the harmonic balance method were directly applied to reveal the NES beneficial effect on reduction in the vibration amplitude of the string. In the case without internal resonance, an elastic string was investigated subject to both a steady wind and a 1:1 or 1:3 resonant harmonic force [315]. Zulli and Luongo [316] utilized a NES to control vibrations of a nonlinear taut elastic string without internal resonance. Combining the multiscale and the harmonic balance method, the partial differential governing equation was directly solved in order to obtain vibration response and to present a tool to optimize the NES position and parameters. Periodically, weakly and strongly modulated responses were numerically calculated in terms of frequency–response curves, invariant manifolds, and time series [316]. With varying axial speed, Zhang et al. [317] suppressed vibrations of axially moving beams using the targeted energy transfer theory. The performances of a single NES and two parallel NESs on stability were compared. The research found that the performance of the parallel NESs is good or even better with much less total mass. A single NES was proposed to suppress forced vibrations [318] or thermal shock-induced vibrations of an axially moving beam [319].

5.2.4 Fluid-conveying pipes

A fluid-conveying pipe is a gyroscopic continuum. NESs have been applied to suppress the pipe vibrations. In the following works, a pipe-conveying fluid was modeled as an Euler–Bernoulli beam. Yang et al. [320] applied the targeted energy transfer theory to suppress excessive vibrations of a pipe conveying fluid by using a NES. They showed that the NES is an efficient, lightweight, robust passive vibration absorber for eliminating vibrations of a pipe. The NES extracted vibrational energy from each vibrating mode of the pipe over a wide range of the parameters [320]. As the most efficient vibration suppression regime, Mamaghani et al. [321] determined the required conditions for the strongly modulated responses phenomenon in the first resonance of a pipe-conveying fluid subjected to an external harmonic load. Based on the one-term Galerkin truncation, the efficiency of the optimal NES in vibration suppression of the first-order natural mode resonance was compared to that of the optimal linear absorber. The results showed that the NES is more efficient in a large frequency range around the first natural frequency [321]. Zhao et al. [322] suppressed the nonlinear vibration of a pipe conveying fluid under harmonic displacement excitation by using a NES. Yang et al. [152] suppressed the vibration of a pipe with a bi-stable NES and realized a maximum energy absorption efficiency of 98.19%. Zhou et al. [323] used a NES to mitigate flutter instability of a cantilevered fluid-conveying pipe. Khazaee et al. [324] investigated stochastic optimization of multiple NESs configured in parallel and series configurations for a simply supported pipe conveying fluid.

5.3 Vibration mitigation in aeronautics and astronautics

5.3.1 Whole spacecraft

Yang et al. [325] designed conceptually and implemented experimentally a NES to suppress the vibration of whole spacecraft in launching and to keep its natural frequencies almost unchanged. Their analytical, numerical, and experimental work found that the performance of the vibration suppression is improved for the increasing NES mass, the increasing NES viscous damping, the increasing excitation amplitude,

and the decreasing nonlinear stiffness. The NES vibration reduction for the whole spacecraft was revisited from the view of the power flow analysis, and the investigation yields qualitatively same outcomes [326]. Yang et al. [327] proposed an asymmetric NES force which was generated by two pairs of aligned permanent magnets to improve the reliability of space system.

5.3.2 Wings

Lee et al. numerically and analytically [328] as well as experimentally [329] investigated the inside-installed NES suppression of aeroelastic instabilities (limit cycle oscillations) in a two-DOF rigid wing model. Three main mechanisms for suppressing aeroelastic instability were revealed as recurring burst out and suppression, intermediate suppression, and complete elimination. The underlying dynamic mechanisms governing them were demonstrated to be series of resonance captures. The investigation showed that lightweight NESs with negative offsets can form the basis of economical and robust designs for the suppression of limit cycle oscillations in the wing. The key issue is to design a NES producing 1:1 resonance captures with the aeroelastic modes. The limit cycle oscillations suppression is more robust with the smaller NES nonlinear stiffness or damping and the larger NES mass. In addition, the NES improved the stability of the wing without introducing undesirable new dynamics. Lee et al. [330] suggested that an appropriately designed multi-DOF NES can greatly improve the robustness of instability suppression. For enhancing robustness, it is crucial to control of the occurrence of the lower bifurcation point of limit cycle oscillations above a Hopf bifurcation point. The multi-DOF NES achieves the same suppression performance with the smaller total masses as a few single-DOF NESs under identical disturbances [330]. Using a time-domain nonlinear system identification based on the equivalence between empirical and analytical slow flows, Lee et al. [331] utilized targeted energy transfer mechanisms to suppress aeroelastic instability. The aeroelastic system is coupled with a single-DOF or multi-DOF NES. In order to demonstrate different targeted energy transfer mechanisms and to investigate robustness of aeroelastic instability suppression, nonlinear modal interactions were revealed by constructing frequency–energy plots.

Hubbard et al. [117] investigated computationally and experimentally a rotary NES at the tip of a flexible swept wing and found the induced strongly nonlinear effects reducing drastically the response of the selected wing modes responsible for limit cycle oscillations triggering and development. Strongly nonlinear resonance interactions between the NES and the second bending mode of the wing were observed for enough energy imparted to the system. Therefore, it is feasible to use the rotary NES to suppress or to prevent aeroelastic instability [117]. Vaurigaud et al. [332] introduced a concept of limiting phase trajectories to predict the asymptotic behavior of suspend-span bridge coupled with a NES, and the bridge was modeled as a two-DOF system with a constant wind excitation. By considering the unsteady aerodynamic effects, Guo et al. [333] suppressed the limit cycle oscillations of a two-DOF airfoil by using a NES. A harmonic balance method was applied to examine the effects of the limit cycle oscillation suppression on a supercritical or subcritical Hopf bifurcations of the aeroelastic system. Studies showed that when the NES moves from the nose to the trailing edge, it helps to suppress the limit cycle oscillations of the airfoil. With the increase in the NES nonlinear stiffness, the amplitude of limit cycle oscillations decreases in the supercritical Hopf bifurcation and increases in the subcritical Hopf bifurcation. The transition condition in complete energy exchange and localization was exactly formulated in terms of limiting phase trajectories to improve the understanding of triggering mechanisms of the aeroelastic instability suppression by the NES. Bichiou et al. [334] utilized a NES to control the nonlinear vibration response of a pitching–plunging airfoil section. The NES can reduce the plunge and pitch vibration when the primary system shows supercritical behavior. Moreover, in the transition from a subcritical bifurcation to a supercritical bifurcation, the NES may be more effective with the well-designed mass and location along the airfoil. However, the NES damping is limited to a small freestream velocity region. Setting two NESs in the wing, Zhang et al. [335] found that the NESs can control the resonances of two modes of the wing and the energy of the heave mode and the pitch mode are, respectively, transferred to each NES. Considering the freeplay and cubic nonlinearity in pitch, Guo et al. [336] utilized a NES to suppress the aeroelasticity of an airfoil with a control surface in an

incompressible flow. The limit cycle oscillations were obtained via the harmonic balance method. It was found that the NES increases both the linear flutter speed of the airfoil without freeplay and the nonlinear flutter speed of the airfoil with freeplay [336]. A NES was optimized to mitigate limit cycle oscillations of a two-DOF airfoil with nonlinear stiffness in pitch and heave [337]. A NES placed at the leading edge and another NES at the trailing edge reduced flow-induced vibrations of a two-dimensional airfoil [338]. The numerical simulations demonstrated that the NESs fail to suppress vibrations of the airfoil as the total energy exceeds the suppression range of the NESs.

In order to improve the critical flutter condition and to mitigate transonic aeroelastic instability of a wind-tunnel model wing, Hubbard et al. [339] designed a prototype of a winglet-mounted lightweight NES. The properly designed NES was coupled to the tip of the wing. The aeroelastic response modes of the wing with a NES coupled to the tip were identified and tested for achieving robust suppression of transonic aeroelastic instability. Computational and experimental results indicated that the stability was improved over broad conditions and the dynamic pressure increase was about 40% in at the critical stability condition [340]. It was experimentally demonstrated that the NES had a profound influence on the second bending mode of the wing. However, when the critical aeroelastic mode transitions from bending to torsion, the designed NES may have a destabilizing effect. For the first time, Pacheco et al. [341] used a NES to suppress panel flutter and reduce the intensity of the limit cycle oscillation. The geometric nonlinearity of the plate and the supersonic aerodynamic loads were considered. By balancing the aerodynamic work partially, the NES can either suppress flutter or mitigate limit cycle oscillations. Yan et al. [342] evaluated the usability of NESs in controlling inherent nonlinear transonic flutter. They found that it is optimal to install the NES mass toward the trailing edge of the airfoil. The NES can suppress the transonic flutter completely and partially for different NES parameters. In the parametric region of the complete suppression, the NES may cause the wing to a new equilibrium position. Moreover, NES has an optimal nonlinear stiffness coefficient and a minimum critical damping coefficient. On the other hand, in the parametric region of the partial suppression, the vibration depends on the initial conditions. A NES was used to suppress a three-

dimensional trapezoidal wing flutter and to mitigate the nonlinear aeroelastic responses [343].

5.4 Energy harvesting with NESs

In the past decade, research on vibration energy harvesting has attracted much attention. Vibration sources such as human motion and ocean waves are potentials providing with mechanical energy that can be harvested for producing electricity. A linear energy harvesting device can only collect ambient vibration energy that matches its resonance frequency. Therefore, the bandwidth of the effective operating frequency is usually limited to a specific range. Moreover, it is impossible to cover the random vibration frequency of external sources. If the ambient vibration frequency slightly deviates from the resonant frequency of the energy harvesting device, the resulting power output of harvesters would be reduced drastically. NES efficiently controls structural vibration through one-way irreversible nonlinear energy pumping. Moreover, NES has the characteristic of not being affected by the external excitation frequency [344]. These advantages should be able to be used for harvesting vibrational energy [345, 346].

5.4.1 Piezoelectric-based energy harvesters

Ahmadabadi and Khadem [347] first tried to combine vibration suppression with vibration energy harvesting via a NES. They attached a system comprising a NES and a piezoelectric-based vibration energy harvester to an elastic beam under shock excitations, considering both grounded and ungrounded configurations of the harvester. The system parameters were globally optimized both to maximize the dissipated energy by the NES and to increase the harvested energy. A satisfactory amount of the vibrational energy was harvested as the electric power [347]. In order to achieve simultaneous broadband energy harvesting and vibration reduction, Zhang et al. [348] designed a piezoelectric energy harvesting system based on a NES. Both experimental and numerical investigations demonstrated that the proposed apparatus possesses the targeted energy transfer and the initial energy dependence. Li et al. [349] proposed a NES–piezoelectric system to suppress vibration and to harvest vibrational energy. The NES–piezoelectric system was attached to a two-DOF primary oscillator under a

shock excitation. Continuous energy transition was observed when the targeted energy transfer was triggered by nonlinear beats [349]. A NES-based piezoelectric vibration energy harvester was presented to achieve broadband harvesting [350]. Ahmadabadi and Khadem [351] implemented optimal vibration suppression and energy harvesting by using collocated NESs and piezoelectric elements coupled to a beam under shock excitation. In order to maximize the dissipated energy by the NESs, the system parameters were globally optimized. Multiple NESs enabled more efficient nonlinear energy pumping than a single NES [351]. Li et al. [352] integrated vibration suppression and piezoelectric energy harvesting for a primary oscillator under a harmonic excitation. Raj and Santhosh [353] investigated an excited harmonically primary oscillator coupled with a piezoelectric energy harvester and an absorber. The stiffness of both the primary oscillator and the absorber has linear and cubic nonlinear terms. The investigation indicated the importance of including stiffness nonlinearities for performance enhancement. Moreover, for the considered parameter values, the absorber and harvester system with both linear and nonlinear components performs better than a NES [353]. Zhang et al. [354] proposed a novel multifunctional nonlinear composite structure consisting of functionally graded beams and piezoelectric sheets to achieve the ideal energy harvesting effect under the thermal environment. Geometrical and material nonlinearities were integrated to design a novel NES coupled to a piezoelectric-based vibration energy harvester subjected to a shock load [355]. The captured impact energy was dissipated by the NES damper and the hysteresis behavior of the Bouc–Wen model and was collected by the piezoelectric system. Remick et al. [356] designed an impulsive vibration energy harvesting system composed of two oscillators coupled with essentially nonlinear stiffness. A weakly damped linear primary oscillator was coupled to a weakly damped lightweight oscillator through a piezoelectric cable. The piezoelectric cable generated a cubic nonlinear stiffness. Under impulsive excitations, the system dynamics exhibited transient resonance captures. Therefore, the response of the harvesting element showed high-frequency ‘bursts’ or instabilities. Then, sustained energy transferred from the excited primary oscillator to the energy harvester. The baseline minimum performance excitation conditions

and optimal performance conditions were identified. A related experiment was reported [357].

5.4.2 *Magnetolectric energy harvesters*

Kremer and Liu [358] described a novel apparatus in detail to achieve the essential nonlinearity and energy harvesting. In order to minimize the mechanical damping, a pair of permanent magnets was mounted on an axially preloaded elastic beam fixed at both the ends. Permanent magnets and elastic beam, respectively, acted as the mass and spring of a NES. Both linear coupling and nonlinear coupling between the NES and the primary system were introduced. Several key parameters and relationships were experimentally determined. Simulations and experiments concluded that the apparatus, with the characteristics of a NES, is capable of energy localization, broadband harvesting energy, and broadband vibration absorption. The best vibration energy absorption occurred when the initial energy threshold (for the NES to be engaged) is just passed and a nonlinear beating and targeted energy transfer appear. Harmonically forced responses were studied [359]. Remick et al. [360] built an energy harvesting device with a NES. The NES was coupled to a primary oscillator via a piano wire, a neodymium magnet, and an inductance coil. The vibrational energy was harvested by the electromagnetic elements. The work obtained experimentally the high-frequency dynamic instability for a system subject to a single impulsive load caused by transient resonance captures. This instability led to a large-amplitude relative vibration between the primary oscillator and the NES. Therefore, this high-frequency instability was beneficial for vibration energy harvesting. The primary system subjected to repeated impulsive excitations was studied in [361].

In order to realize vibration suppression and energy harvesting, Fang et al. [362, 363] integrated a NES and a giant magnetostrictive material. The numerical results showed that the target energy transfer leads to a very efficient suppression for the vibration response of the linear primary system. Due to the Villari effect, the localized vibrational energy can be converted into the magnetic field energy and then converted into the electrical energy. Zhang et al. [354] proposed a lattice sandwich structure consisting of a NES and a giant magnetostrictive material for harvesting vibration energy and controlling vibration. The NES and the

energy harvester with levitation magnetolectric were integrated in the whole spacecraft system [364]. A NES enhanced by a giant magnetostrictive material energy harvester was proposed to suppress nonlinear aeroelastic responses of a cantilevered trapezoidal plate in hypersonic airflow [365].

5.4.3 Bi-stable energy harvesters

An oscillator with a bi-stable configuration has received a lot of attention because the oscillator is able to absorb high vibrational energy when it snaps through from one stable equilibrium to another stable equilibrium, providing substantial motion over a wide range of input frequencies. Romeo et al. [149] demonstrated that a negative stiffness component in a bi-stable NES gives rise to two passive energy transfer mechanisms from a linear primary oscillator to the NES at a lower energy regime, namely in-well nonlinear beats and chaotic cross-well oscillations. Chiacchiari et al. [366] designed a bi-stable electromagnetic energy harvester for harvesting small-amplitude and broadband vibration energy of an excited primary system. Having electromechanical coupling elements, the harvester is a lightweight, weakly damped oscillator with cubic nonlinear and negative linear stiffness. Energy harvesting capabilities of systems with and without negative linear coupling stiffness were compared. The comparison illustrated significant enhancements in total vibrational energy harvested and energy harvesting efficiency due to the addition of bi-stability [366]. Pennisi et al. [367] proposed a theoretical design and its experimental realization of a magnetic-strung NES attached to an energy harvester. The mass of the NES is a magnet. It was shown that the bi-stable configuration seems to be a promising way to harvest vibrational energy.

5.4.4 Vibro-impact energy harvesters

Darabi and Leamy [368] introduced a clearance-type NES in order to trap impact energy and to increase the net harvested energy. Both subdomains of the clearance-type NES contain displaceable degrees of freedom. It was analytically, computationally, and experimentally demonstrated for the optimization of the clearance-type NES to enhance energy harvesting performance. The proposed NES uses a clearance-type nonlinearity to transfer the wave energy generated by

an impact from the primary subdomain to the secondary subdomain, whereupon the energy is captured and harvested [368]. Lai et al. [369] designed an energy harvesting device with dielectric elastomers based on vibro-impact mechanical motions of a ball moving freely inside a cylinder with dielectric elastomers membranes at both sides. They found rich nonlinear dynamic behavior of the proposed system. Moreover, the energy harvesting device is suitable for low excitation frequency range.

5.5 Other applications of NESs

Starosvetsky and Gendelman [370] investigated the performance of a NES attached to a forced multi-DOF linear primary system. In the particular system, the quasiperiodic motions of the NES turn out to be favorable to vibration absorption. However, the drawbacks of the NES are noted which include its inefficiency for the low amplitude of excitation.

Felix et al. [371] focused on a nonideal structure linearly coupled with an essential nonlinear oscillator. They examined the dynamic interaction between the structure and the oscillator as well as the bifurcations of parameters from the view of vibration attenuation.

Motivated by the study of vibrating cable stayed-bridges, Thanh et al. [372] analyzed combination resonance of a primary basic quadratic nonlinear system coupled with a NES. They found the NES more efficient than a basic tuned mass damper. Lamarque and Savadkoobi [373] revealed the energy exchange between a NES and a primary oscillator with Bouc–Wen-type hysteresis behaviors. They detected the invariant manifold of the nonlinear oscillators at fast timescale and equilibrium points and fold singularities at the slow timescale. This work warned that the NES should be properly designed in order to prevent the primary structure to present uncontrolled and undesirable behavior [373]. Zulli and Luongo [374] used a NES to control the vibration of a Duffing oscillator under a dual-frequency harmonic excitation, which is used to induce 1:1 and 1:3 resonances. The general beneficial effects of the NES were found under 1:1 resonance conditions. Under the 1:3 resonance condition, some specific initial conditions may cause some dangerous oscillations [374]. Sun and Chen [375] used a NES to control the vibration of a nonlinear primary system harmonically excited. Steady-state responses were analytically derived from the complexification-

averaging method and confirmed numerically. It was found that as the excitation magnitude increases, the NES may lose its efficiency over a certain range of frequencies due to the presence of high-response branches. However, the failure can be eliminated by reducing appropriately the NES nonlinear stiffness [375]. Alisverisci et al. [376] applied a NES to absorb energy from an electromechanical device with a pendulum arm. They illustrated that the NES is capable of diminishing chaotic behaviors. Hurel et al. [377] investigated vibratory energy exchanges between a pendulum and a NES. This work explained how to control passively the oscillation of the pendulum through the NES positioned at a given direction.

Hasan et al. [378] explored analytically and numerically the passive targeted energy transfer in weakly coupled granular media with the focus on nonlinear dynamic mechanisms. They considered two weakly coupled uncompressed granular chains composed of elastic spherical beads with Hertzian interactions and supported by linear elastic foundations. Zhang et al. [379] modeled homogeneous granular chains mounted on an elastic foundations. Periodic half-sine pulses were applied to one of the chains. Passive pulse redirection and targeted energy transfer were achieved. The study will help to understand energy transfer mechanisms in material systems with embedded granular media and passive energy redirection characteristics [379].

Weiss et al. [380] investigated analytically multi-scale vibratory energy exchange between a primary oscillator with a Saint-Venant term and a NES. Lamarque and Savadkoobi [381] explored targeted energy transfer between a primary oscillator with a set of parallel Saint-Venant elements and a NES around 1:1 resonance. The energy exchange process was studied at different timescales. Periodic and/or strongly modulated response regimes were predicted by the system behavior modulations at a slow time-scale around its invariant manifold. These can be used to design NESs for passive control and/or energy harvesting of vibration of structures with Saint-Venant elements. These vibratory energy exchange studies provide with analytical design tools for tuning NES parameters according to passive control of linear/nonlinear primary structures by means of NESs [382].

Saeed and Al-Shudeifat [383] proposed a NES to suppress two-dimensional vibrations of a primary oscillator caused by a two-dimensional impulsive

loading. They found that the NES could engage in efficient targeted energy transfer for various combinations of longitudinal and lateral primary oscillator vibration.

6 Concluding remarks

Nonlinear energy sinks have attracted much attention because of their ability to achieve energy-targeted transfer under certain conditions. The introduction of essential nonlinearities may effectively and passively suppress vibrations of systems, but it may complicate dynamic responses of the systems or even deteriorate the system vibrations. This review summarizes the development process of NESs from 2001 to the present, including nonlinear dynamics mechanisms, implementations and configuration design, various optimizations, and applications in a wide range of engineering fields. The main focus is on the implementation of essential nonlinear stiffness, the effects of passive damping, and the improvements to compensate for the deficiencies in engineering vibration problems. This review shows that investigations on NES are increasingly active. There are the following promising topics for the future work.

- (1) Designing and constructing novel NESs with their complex dynamic analysis will dominate the field. In particular, the challenges are to reduce vibrations in multiple directions rather than a single direction and to integrate new materials into NESs to achieve desired nonlinear stiffness and damping.
- (2) Nonlinearities beyond a cubic term in NES may enhance vibration reduction. Investigations will focus on their mechanical or material design, vibration suppression performance, and resulting complex dynamic behaviors.
- (3) Enlargement of admitting excitation variation ranges is a demanding topic for NES investigations, as a NES works efficiently only for certain excitations and otherwise it may even deteriorate dynamic behaviors.
- (4) It is always a key issue to reduce the cost of NESs for passive vibration controls, especially to reduce the additional mass in a NES to maintain satisfactory vibration suppression effects.

- (5) Suppression of low-frequency and ultra-low-frequency vibrations of primary structures is a challenging issue in NES designs, as passive vibration absorbers usually fail to play the role of vibration reduction in those cases.
- (6) Some semi-active or active control designs may be introduced into a NES in order to realize real-time variations of NES parameters with the vibration environment to achieve optimal vibration reductions.

In summary, this review attempts to summarize the known achievements in the field of the configuration design, nonlinear dynamics, and application of nonlinear energy sinks with complete references. In addition, future promising topics are suggested. Hopefully, the review will serve as a collection of ideas and a source of inspirations for NES investigations and applications.

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Compliance with ethical standards

Conflict of interest The author declares that he has no conflict of interest.

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