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# A new strong-field type Schwarzschild metric closer to observation in gravity theory

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## Abstract

The Schwarzschild metric is the most commonly used metric solution that can be derived from Einstein's field equation. It is used to predict most gravitational phenomena that can be checked with observation, especially for weak field phenomena. However, in one of the final steps of deriving the final metric, one relies on Newton's theory, which suggests that the standard and well-known Schwarzschild metric is valid for weak fields only. Unfortunately, few sources make this clear, and the physics community does not seem to be fully aware of it. As a result, the weak field solution has been used to predict strong gravitational fields as well, for example predictions about black holes, both large black holes and micro black holes.

We present a new metric that provides the same predictions as the weak field Schwarzschild metric in weak gravitational fields, but gives significant new predictions in strong gravitational fields in particular for black holes, but also for wormholes. Observational evidence related to strong fields is more consistent with this new metric than the weak field approximation metric, which in some cases might have been used improperly to predict results linked to strong gravitational fields.

**Key Words:** general relativity theory, Schwarzschild metric, weak field, strong field, black holes.

## 1 The Standard Schwarzschild Metric

Einstein's [1] field equation is given by:

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu} \quad (1)$$

Based on the assumption of a non-spinning spherical mass with no electrical charge or angular momentum, as well as setting the cosmological constant to zero, Schwarzschild [2, 3] derived the following metric for spherical polar coordinates:

$$c^2 d\tau^2 = \left(1 - \frac{\alpha}{R}\right) dt^2 - \left(1 - \frac{\alpha}{R}\right)^{-1} dR^2 - R^2(d\theta^2 + \sin^2\theta d\phi^2) \quad (2)$$

where  $\alpha$  has to be determined before the metric can be used for any practical purposes. The standard methodology in the literature to determine the value of the  $\alpha$  is to rely on Newton's [4] theory, in short one has

$$\begin{aligned} \frac{1}{2}mv_e^2 &= \frac{GMm}{R} \iff \left(\frac{v_e^2}{c^2}\right)_{Newton} = \frac{2GM}{Rc^2} \\ \frac{v_e^2}{c^2} &= 1 \longrightarrow \alpha = R_s = \frac{2GM}{c^2} \end{aligned}$$

This yields the standard well known (textbook) Schwarzschild metric:

$$c^2 d\tau^2 = \left(1 - \frac{2GM}{c^2 R}\right) c^2 dt^2 - \left(1 - \frac{2GM}{c^2 R}\right)^{-1} dR^2 - R^2(d\theta^2 + \sin^2 \theta d\phi^2) \quad (3)$$

We note that only a few books on general relativity, such as Hobson, Efstathiou, and Lasenby [5], specifically mention that this is a weak field limit. Other books, such as Weinberg's [6], Poisson and Will's [7], and the major work by Misner, Thorne, and Wheeler [8], do not point out specifically that the last step in deriving their final Schwarzschild metric is linked to calibrating it to Newton's weak field limit and that the solution is therefore likely valid for weak fields only.

If this is the case, the weak field Schwarzschild metric is not appropriate to make predictions related to strong gravitational fields, including studies of black holes at the centers of galaxies and quasars. For instance, it has been used to predict the escape velocity and the Schwarzschild radius in general relativity theory. We argue that such weak-field metric predictions are likely not valid in strong gravitational fields, which we define as fields where the escape velocity  $v_e$  is significant relative to  $c$ . We do not contest the existence of black holes, but as we will soon see, a new strong-field metric solution gives different and more logical predictions that even seem to be in line with observations.

To illustrate the weakness of the weak field limit, consider the escape velocity in Newton's theory, which is determined by solving the equation for  $v$ :

$$\frac{1}{2}mv^2 - G\frac{Mm}{r} = 0 \quad (4)$$

which gives  $v = \sqrt{\frac{2GM}{r}}$ . It is evident that this formula cannot be valid when  $v$  is close to the speed of light because the kinetic energy formula used in the derivation,  $E_k \approx \frac{1}{2}mv^2$ , is only the first term of a Taylor series expansion from Einstein's relativistic kinetic energy,  $E_k = mc^2\gamma - mc^2$ . However, it is well known that general relativity theory predicts the same escape velocity as Newton, as shown in [9]. For years, it has been a mystery to us why the escape velocity derived from general relativity theory is the same as that derived from Newton's theory, where the one derived from Newton theory is clearly valid only when  $v \ll c$ . We think we have found the reason: in general relativity theory, the final Schwarzschild metric is mistakenly assumed to be valid in a strong gravitational field, despite being "calibrated" to Newton for weak field and then used also for strong gravitational field predictions. We claim that the general relativity escape velocity, which is identical to the one we get from pure Newton, is actually valid for a weak gravitational field only, because derived from the Schwarzschild metric, which, in its last step, is calibrated to the weak field Newtonian gravity theory.

So, nothing is wrong with the Schwarzschild metric itself; it is just that one needs to be aware that the last step used to turn it into a practical and useful formula relies on Newton and that the whole solution therefore likely is only valid for weak gravitational fields. The issue, therefore, is how to have the last step calibrated in such a way as to take into account relativistic effects and obtain a Schwarzschild metric solution that holds also for the strongest gravitational fields. We show and discuss this in the next sections.

## 2 The new strong field metric

We will here suggest how to get a new strong field type of Schwarzschild metric, but now based on taking into account strong gravitational field. The Newton limit is given by

$$\begin{aligned} \frac{1}{2}mv_e^2 &= \frac{GMm}{R} \iff \left(\frac{v_e^2}{c^2}\right)_{Newton} = \frac{2GM}{Rc^2} \\ \frac{v_e^2}{c^2} &= 1 \longrightarrow \alpha = R_s = \frac{2GM}{c^2} \end{aligned} \quad (5)$$

The relativistic limit is given by <sup>1)</sup>

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<sup>1</sup>See the appendix for a discussion on validity of inserting the  $\gamma$  factor here.

$$\begin{aligned}
\gamma mc^2 - mc^2 &= \frac{\gamma m GM}{R} \\
\frac{1}{\gamma^2} &= 1 - \frac{v_e^2}{c^2} = \left(1 - \frac{GM}{Rc^2}\right)^2 = 1 - 2\frac{GM}{Rc^2} + \frac{G^2 M^2}{R^2 c^2} \\
\left(\frac{v_e^2}{c^2}\right)_{relativity} &= 2\frac{GM}{Rc^2} - \frac{G^2 M^2}{R^2 c^2}
\end{aligned} \tag{6}$$

where in equation (6) the factor gamma appears consistently in the expression of the kinetic energy and the interaction potential energy (a discussion about the "relativistic mass" is given in the Appendix). In the limit when  $v = v_{esc} \rightarrow c$  and Newton becomes relativistic<sup>2</sup> we have the correspondence,

$$\begin{aligned}
\left(\frac{v_e^2}{c^2}\right)_{Newton} &\rightarrow \left(\frac{v_e^2}{c^2}\right)_{relativity} \\
\frac{2GM}{Rc^2} &\rightarrow \frac{2GM}{Rc^2} - \frac{G^2 M^2}{R^2 c^2}
\end{aligned}$$

Then, the Swartzschild component becomes

$$\left(1 - \frac{\alpha}{R}\right) = \left(1 - \frac{2GM}{Rc^2}\right) \rightarrow \left(1 - \frac{2GM}{Rc^2} + \frac{G^2 M^2}{R^2 c^2}\right) = \left(1 - \frac{2GM}{Rc^2} + \frac{U}{R^2 c^2}\right)$$

where  $U$  is a physical quantity related to the gravitation potential in the metric of the Reiser Nordström type [10, 11]. Our full strong field metric is therefore given by

$$c^2 d\tau^2 = \left(1 - \frac{2GM}{c^2 R} + \frac{G^2 M^2}{c^4 R^2}\right) c^2 dt^2 - \left(1 - \frac{2GM}{c^2 R} + \frac{G^2 M^2}{c^4 R^2}\right)^{-1} dR^2 - R^2 (d\theta^2 + \sin^2 \theta d\phi^2) \tag{7}$$

For all weak field phenomena, it gives predictions that are virtually identical to those of the standard (weak field) Schwarzschild metric, with any differences between the two being too small to be observable in weak field experiments. The first term is identical to the weak field Schwarzschild metric approximation, which is the cornerstone of much of gravitational physics. For large values of  $R$  (weak field), the second term is negligible with respect to the first. Most of the very solid tests of general relativity have been done in the weak field, such as gravitational time dilation, gravitational red-shift and light deflection. In many weak field experiments, one can set up experiments where one has direct observations in all parts of the experiment. For example, for gravitational time dilation, all one needs is basically two atomic clocks at different altitudes. For gravitational red-shift, one can send a light beam down from a tower and thereby have full control of observations of the frequency sent out and received, as first published by Pound and Rebka [12] in 1959. There is no doubt that the weak field Schwarzschild metric approximation gives very accurate predictions for long series of gravity phenomena observed in a weak gravitational field.

Observations from a strong gravitational field are much more indirect, as there is no strong gravitational field on Earth or even in our solar system. We define a strong gravitational field as one where the escape velocity is significant compared to that of light, which means it's close to the radius of black holes as well as in very dense stars such as Neutron stars. In this case,  $R$  is small and the second term is no longer negligible and becomes relevant in the observation of black holes, which are found at the centers of galaxies, including quasars only indirectly. The only observations we have from these objects are incoming photons. In other words, we have no observable information about the exact wavelength of these photons when emitted from just outside these black holes. As a result, much of the interpretation of black holes is done through a mathematical lens, which has been the weak field Schwarzschild metric. An interesting question is what our strong field solution predicts about strong gravitational fields and how this compares to observations. This will be the topic of the next sections.

### 3 The Implications of Our New Strong Field Metric

The new strong field metric has a series of implications and predictions for gravity in strong gravitational fields. We will discuss some of them here.

<sup>2</sup>See the appendix for the validity of inserting the Lorentz factor in Newton.

### 3.1 Black Holes

The main difference in prediction between the well-known weak field Schwarzschild metric and our strong field metric is that the radius where the escape velocity is equal to the speed of light is now  $R_{s,2} = \frac{GM}{c^2}$  instead of  $R_s = \frac{2GM}{c^2}$ . Additionally, the escape velocity that applies to both weak and strong fields is given by  $v_e = \sqrt{\frac{2GM}{R} - \frac{G^2M^2}{c^2R^2}}$  instead of the Newton weak field limit  $v_e = \sqrt{\frac{2GM}{R}}$ , in weak gravitational fields the two escape velocity predictions are indistinguishable. Even if this cannot be measured directly as we would need to hover just outside a black hole, there is as we will see in the sections below indirect evidence from for example high-z quasars.

### 3.2 The strong field limit gives a perfect fit between the Planck scale and the Planck mass micro black hole

In a recent paper [13], we showed that the weak field Schwarzschild metric, which has been used to make predictions about micro black holes, cannot match more than one or two aspects of the Planck scale with any mass candidate for a micro black hole. However, when we take into account that the black hole radius is given by  $R_{s,2} = \frac{GM}{c^2}$  (the radius where  $v_e = c$ ) in the strong field metric, then a Planck mass micro black hole fits all aspects of the Planck scale.

We mistakenly claimed that micro black holes in general relativity cannot match the aspects of the Planck scale. However, if our new strong field metric is consistent with Einstein's field equation, then this is all fully consistent with general relativity theory, it is just that one mistakenly has done predictions about strong fields using a weak field approximation metric. To give a few examples, let us assume the micro black holes to have Planck mass size. Then the Schwarzschild radius from the weak field metric is

$$R_s = \frac{2Gm_p}{c^2} = 2l_p \quad (8)$$

One may ask why it is twice the Planck length and not just the Planck length. After all, the Planck length is considered the smallest meaningful radius or length in quantum gravity (see, for example, [14–16]). Furthermore, the escape velocity from the Planck length is now greater than the speed of light, with  $v_e = \sqrt{\frac{2Gm_p}{l_p}} = c\sqrt{2}$ . However, one could argue that it is meaningless to calculate an escape velocity within the Schwarzschild radius, and we would agree on that, but what if this is not the correct radius where the escape velocity is  $c$ , due to having calibrated the Schwarzschild metric to Newton. Additionally, the density of the Planck mass micro black hole would be only 1/8 of the Planck mass density. Why is that? Alternatively, one could assume that the micro black hole is half the Planck mass, in which case its Schwarzschild radius would be the Planck length. However, the reduced Compton wavelength of the mass would now be twice the Planck length, and it is often assumed that a particle can be compressed at most to its reduced Compton wavelength. Moreover, the surface acceleration would be only half the Planck acceleration. We discuss this topic in much more detail in [13].

The crucial point is that as soon as we switch to the strong Schwarzschild metric, a Planck mass size black hole fits all aspects of the Planck scale. Its Schwarzschild radius would be the Planck length, the acceleration at the Schwarzschild radius would be the Planck acceleration, and if the Planck acceleration lasts for the Planck time, one would reach the speed of light. The density of the black hole would be the Planck density, and so on. The strong field Schwarzschild metric seems to offer much better logic and a perfect match between the Planck mass black hole and all aspects of the Planck scale. This could be important to better understand the Planck scale and to get to a quantum gravity theory.

### 3.3 Wormholes seems forbidden in the strong field limit

In 1916, Flamm [17] hinted at the existence of wormholes. In 1935, Einstein and Rosen [18] were the first to mathematically predict wormholes using the Schwarzschild metric. They used the standard Schwarzschild metric, which is described in Section 1 of this paper, and as is often done when working with the Schwarzschild metric, they set  $c = 1$  and  $R_s = \frac{2GM}{c^2} = 2m$ .

Next, Einstein and Rosen defined a new variable,  $u^2 = R - 2m$ , and replaced  $R$  with  $u^2 + 2m$  in the Schwarzschild metric to obtain:

$$\begin{aligned}
ds^2 &= \left(1 - \frac{R_s}{R}\right) c^2 dt^2 - \frac{dR^2}{1 - \frac{R_s}{R}} - R^2(d\theta^2 + \sin^2 \theta d\phi^2) \\
ds^2 &= -4(u^2 + 2m)du^2 - (u^2 + 2m)^2(d\theta^2 + \sin^2 \theta d\phi^2) + \frac{u^2}{u^2 + 2m} dt^2
\end{aligned} \tag{9}$$

They then discussed the special case when  $u = 0$ . In this case, the term  $g_{1,1} = \left(1 - \frac{R_s}{R}\right) c^2 dt^2 = \frac{u^2}{u^2 + 2m} dt^2$  vanishes, while the other terms in the Schwarzschild metric remain well-defined. This was interpreted to mean that one could move in space without moving in time, essentially providing a passage from one part of the universe to another through a wormhole.

Wormhole physics has been investigated by a series of physicists [19–35], and it remains a hot topic to this day despite the lack of observable findings of wormholes.

However, we will claim that the interpretations of wormholes could be incorrect as they are predicted by a weak field approximation of the Schwarzschild metric, whereas wormholes are related to strong gravitational fields. Also, for simplicity, in our strong field metric, we set  $c = 1$  and  $R_{s,2} = \frac{GM}{c^2}$  when  $v_e = c$ . For notation purposes, we can set  $m = \frac{GM}{c^2}$ . Similar to Einstein and Rosen, we set  $u^2 = R - m$ . The choice of  $u$  is just as in the Einstein-Rosen solution, such that we end up with the  $dt^2$  term vanishing. We can now replace  $R$  with  $R = u^2 + m$ . We end up with

$$\begin{aligned}
ds^2 &= \left(1 - \frac{2GM}{Rc^2} + \frac{G^2M^2}{R^2c^4}\right) c^2 dt^2 - \frac{dR^2}{1 - \frac{2GM}{Rc^2} - \frac{G^2M^2}{R^2c^4}} - R^2(d\theta^2 + \sin^2 \theta d\phi^2) \\
ds^2 &= \left(\frac{u^2}{u^2 + m}\right)^2 dt^2 - \frac{4(u^2 + m)^2}{u^2} du^2 - (u^2 + m)^2(d\theta^2 + \sin^2 \theta d\phi^2)
\end{aligned} \tag{10}$$

If we now set  $u = 0$ , the  $dt^2$  term vanishes, as expected, similarly to how it did in the Einstein-Rosen modified Schwarzschild weak field metric. However, in our strong field metric, the  $du^2$  term now becomes infinite, meaning that it is mathematically undefined or we can say it goes to a singularity. The fact that the  $du^2$  term is no longer mathematically valid should be interpreted as implying that no valid solution for the metric exists when the  $dt^2$  element disappears. That is, unlike in the weak field Schwarzschild metric, nothing can move in the strong field metric without time passing by. In other words, wormholes are not allowed in this metric. This should naturally be investigated by more researchers, but the conclusion so far is that wormholes do not exist.

### 3.4 Explaining the lack of time dilation in quasars with the strong field metric

In a groundbreaking study of high- $z$  quasars, Hawkins [36, 37] surprised the scientific community by reporting that these objects showed no evidence of velocity time dilation, contrary to what had been expected. In the words of Hawkins:

*”There does not appear to be a satisfactory explanation for the absence of a time dilation effect in quasar power spectra...”*

Recently, we have shown in a paper [38] that if we use the strong field metric with  $R_{s,2} = \frac{GM}{c^2}$  instead of the weak field metric with  $R_s = \frac{2GM}{c^2}$ , then light can be sent out from much closer to the center of the black hole than previously assumed, still outside the event horizon that is now only half the distance predicted by the weak field Schwarzschild geometry. This leads to a stronger gravitational redshift for strong field objects than expected from the weak field metric, with a larger portion of the redshift being due to gravitational redshift rather than velocity (Doppler) redshift. As a result, the velocity time dilation is much lower than expected. Once again, the observations seem to support our new metric.

### 3.5 The maximum mass of Neutron stars corresponds with observations in the strong field limit

In a recent paper [39], we have shown that the maximum mass of Neutron stars, when taking into account relativistic mass, is fully consistent with our new strong field metric. Our metric predicts a maximum mass of Neutron stars to be 2.147 solar masses, while the largest observed Neutron star is  $2.14 \pm 0.1 M_\odot$ , as reported by Cromartie et al. [40].

In contrast, the weak field Schwarzschild metric, using the same logic, predicts a maximum mass of only  $0.76M_{\odot}$ . While several different approaches to predict the maximum Neutron mass in the literature have been suggested, this again highlights the need for multiple researchers to investigate this further over time. However, we believe that this is a promising development in favor of the new strong field metric.

## 4 Conclusion

In this paper, we have presented a new metric solution, valid for weak and strong gravitational field, to the Schwarzschild solution. We believe that our solution is consistent with the general theory of relativity, although it is subject to further discussion and investigation over time. Our solution provides more logical predictions, such as for micro black holes, where the Planck mass black hole now fits all aspects of the Planck scale. This is not possible to achieve in the weak field Schwarzschild metric. Additionally, our solution predicts that wormholes cannot exist. The strong field metric also appears to explain the lack of observed velocity time dilations in high- $z$  quasars. Overall, we believe that our new metric represents an important step forward in understanding the behavior of strong gravitational fields, and we look forward to future research in this area.

## Conflicts of Interest:

The author declares no conflict of interest.

## Data Availability Statement:

No data has been used for this study.

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## Appendix: Relativistic effects before taking into account general relativity

As early as 1981 and 1986, Bagge [41] and Phipps [42] suggested modifying Newtonian mechanics to take into account special relativistic effects, resulting in a modified equation of the form:

$$F = G \frac{Mm\gamma}{R^2} \quad (11)$$

Where  $\gamma$  is the Lorentz factor  $\gamma = 1/\sqrt{1 - v^2/c^2}$ . That is, since the small mass  $m$  moves relative to the large gravitational mass  $M$ , they suggested it should be relativistic. Actually, as mentioned below, the meaning of the term “relativistic mass” has been evolving through the years and sometimes misinterpreted by some authors. Nevertheless, Phipps had in the same paper claimed that this relativistic correction alone gave the correct prediction of Mercury precession. However, it was shortly afterwards shown by Peters [43] that Phipps had made a small error in his calculation of Mercury precession, and that the Mercury precession with this method still seemed to be off compared to the observed Mercury precession. Phipps [44] agreed on the mistake made in his calculation but still thought the relativistic adjustment of the Newton formula could be of interest and needed further investigation. Recently, Corda [45] has shown that if, besides the “relativistic mass”, one also takes into account that the mass of Mercury is not insignificant relative to that of the Sun, then one gets the correct prediction of Mercury precession. Haug [46] has looked closer at this problem and has indicated that when one properly takes into account that both  $M$  and  $m$  are moving relative to a third observer, the Earth observer, then one could get again the correct prediction of Mercury precession. We will not conclude on this issue here, but just point out that this is likely not a fully settled question yet; see also [47, 48].

In general, relativistic modifications to Newton’s theory have been dismissed and not looked at mostly for another reason. The main reason for so little interest in adding relativistic effects to Newton has been that parts of the general relativity research community has typically rejected relativistic mass. To understand the pros and cons of relativistic mass, we should go back and look at the history of the debate on relativistic mass.

Lorentz [49] already in 1899 was the first to introduce the relativistic mass in the form we find it in many textbooks [50–53] and some papers that we soon will get back to. The Lorentz relativistic mass in today’s notation is given by  $m\gamma$ , where  $m$  is the rest mass. In his most famous paper on special relativity theory, Einstein [54] suggested two types of relativistic mass, namely transverse mass  $m\gamma^2$  and longitudinal mass  $m\gamma^3$ , neither of which are used in any standard theory today. In 1906, Einstein [55] tried to come up with experiments that could distinguish between the different suggested relativistic masses, but with little or no success.

Max Planck [56] in 1906 was the first to introduce the relativistic momentum  $p = mv\gamma$ . Tolman and Lewis [57] in 1909, as well as Tolman [58] in 1912, concluded based on conservation of momentum that had been experimentally observed that it was the Lorentz relativistic mass of the form  $m\gamma$  the only one consistent with this. In other words, they rejected the relativistic mass formulas that had been suggested by Einstein. This was a view soon held by a series of physicists, for example, Veride [59] and later even by prominent physicists like Pauli [60]. Pauli concludes, “*This leads to a complete confirmation of the relativistic (mass) formula, which can thus be considered as experimentally verified. It has not been possible up until now to establish this variability for masses other than that of the electron experimentally, because of the smallness of the effect, even for fast alpha particles.*”

In his 1965 book titled ‘*Einstein’s Theory of Relativity*’, Born [61] describes the relativistic mass of the form  $m = m_0\gamma$  and attributes it to Lorentz. Born explains that the Lorentz relativistic mass formula as introduced by Lorentz was only for an electron, but he argues that the formula is correct for any moving mass. Born concludes that to have conservation of momentum, *the mass is to have different values according to the system of reference, according to the velocity of the moving body. Based on conservation of momentum in the case of inelastic collision*”, Max Born concludes that it is impossible to *retain the axiom of classical mechanics that mass is a constant quantity peculiar to each body*”. The book [62] was first published in 1920 but was updated in 1962 and 1965.

However, it should also be mentioned that other prominent physicists like Wien [63] mentioned Einstein’s relativistic mass as late as 1921 without even mentioning the Lorentz relativistic mass.

Einstein did not seem to have been successful with his relativistic mass prediction formulas, despite the enormous success of the rest of his paper on special relativity theory. We must remember that the Lorentz relativity theory

still was somewhat a competitor to Einstein even at this point, even if the two theories strongly overlap. Einstein was certainly the first to suggest relativistic energy of the form  $E = mc^2\gamma$ , so he could have arrived at the relativistic mass simply by dividing by  $c^2$ , but he had not done this and had instead derived other formulas for relativistic mass. We will also come back to dividing the relativistic mass by  $c^2$ , as there are also discussions on pros and cons in the literature to this day. Whether it was Lorentz who invented the relativistic mass, which now seemed preferable by most physicists and played a role in Einstein becoming negative towards relativistic mass and abandoning it altogether, or if it mostly had to do with his work on general relativity theory, will likely be unknown.

In deriving his general relativity theory, Einstein relies on relativistic mass given by Max Planck, but now incorporates relativistic mass in fourth-momentum, as well as in the stress-energy tensor. In other words, Einstein does not rely anywhere directly on relativistic mass. Einstein wrote in a letter to Lincoln Barnett, an American journalist, dated 19 June 1948:

*It is not good to introduce the concept of the mass of a moving body for which no clear definition can be given. It is better to introduce no other mass  $M = m/\sqrt{1 - v^2/c^2}$  concept than the "rest mass",  $m$ . Instead of introducing  $M$ , it is better to mention the expression for the momentum and energy of a body in motion.*

This letter has been actively used among physicists who are negative towards relativistic mass, for example, Hecht [64]. Other prominent experts in general relativity, such as Adler [65] in 1987 and Taylor and Wheeler [66], have also been negative towards relativistic mass. One of their main arguments against relativistic mass is, in their own words, "In reality, increases of energy with velocity originate in the geometric properties of spacetime itself." They also claim that relativistic mass "implies the name "mass" – belonging to the magnitude of a 4-vector – to a very different concept, the time component of a 4-vector." Okun [67] has been a strong opponent of the concept of relativistic mass in his 1989 paper. Rindler [68, 69], a defender of special and general relativity theory, however, went out against Okun's arguments and defended the use of the concept of relativistic mass.

Sadnín [70] defended relativistic mass in a 1991 paper and went through many of the arguments against relativistic mass put forward by Okun and Taylor and Wheeler. He concluded that their arguments against it are rather weak and stated, "Relativistic mass paints a picture of nature that is beautiful in its simplicity. We should continue to use relativistic mass along with consistent interpretations of Newton's second law and  $E = mc^2$  in introductory courses. Insisting on its removal as a useful tool from all textbooks, as Okun does, is a form of unnecessary censorship."

Jammer [71], in his detailed study of mass published in the year 2000, has dedicated a whole chapter to relativistic mass and the pros and cons arguments that have been presented in the literature. He mentions that this has been a debate going on for decades and that it is not over and much about preferences. The debate continues to this day. For example,

Atkin [72] in 2015 defends relativistic mass and states in his conclusion, "The term mass should mean relativistic mass rather than the currently popular meaning of rest mass." However, Sullivan [73] comments on the paper and refers to the critics of relativistic mass given by Okun, Adler, and Hecht, but also to the defense of relativistic mass by Sadnín [70]. Sullivan states, "Of course, there is nothing wrong with introducing relativistic mass, and some physicists have made a case for its retention [7, 8]; ultimately, it comes down to an individual pedagogical decision." Dai and Dai [74], in a recent 2018 paper, rely on relativistic mass when deriving mass-energy equivalence and the mass-velocity relation without relying on light. They state, "In relativity, the inertial mass of a moving particle is not a constant quantity but a function of the particle's velocity."

Field [75] in a 2020 paper criticizes several of the arguments made by Okun, Adler, and Wheeler on relativistic mass. For example, Field points out that "Okun is erroneously conflating the 'rest mass of a photon' and the entirely different 'mass equivalent of the energy of a photon.'"

Oas [76] in 2008 went through over 600 books and papers in physics and showed that a majority of books and papers mentioning relativistic mass are positive about it. However, many of these books and papers only touch upon the topic, so the fact that a large number of physicists (if not the majority) are still positive about relativistic mass is not a good argument in itself.

Of the recent books that really go into a detailed discussion of relativistic mass, Jammer's seems to be neutral on the topic and only concludes that it is a long ongoing debate with many interesting arguments pro and con. Petkov [77] in his 2009 book "Relativity and The Nature of Spacetime" criticizes the critics of relativistic mass. He discusses relativistic mass in quite some detail and concludes, "So, if we cannot talk about relativistic mass, by the same argument we should only talk about proper time, which is invariant, and deny the name 'time' to the coordinate time." as he claims that some of the same arguments used against relativistic mass can be used against relativistic time."

Another interesting point is if we even need relativistic mass to incorporate relativistic effects in the Newton formula. Adler and Bazin [78] in a section of their book derives gravitational red-shift in a weak gravitational field based on replacing the small mass with  $E/c^2$ , so in that case we could argue for that one could re-write  $F = G \frac{Mm\gamma}{R^2} = G \frac{ME\gamma/c^2}{R^2}$  when dealing with relativistic effects in Newton theory and that we simply are dealing only with relativistic energy and not relativistic mass.

All we can conclude is that the physics community does not agree on whether relativistic mass is a valid concept or not. The debate has been ongoing for decades, and based on a series of recent papers discussing it, it is clear that the debate is not over. To pretend that the debate is over and come to a conclusion without exploration is seldom the right way to go. The appropriate approach is to fully explore what incorporating relativistic mass leads to in terms of predictions.