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# Coupled Numerical and Experimental Identification of Geometrical Parameter for Predicting the Electrical Conductivity of CFRP layers

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The analysis of the electrical behavior of a unidirectional (UD) composite material requires a lot of resources and measurement techniques. To overcome this constraint, the real composite material is replaced by a virtual material with same overall behavior. In this paper, an approach to determine setting parameters of the virtual material model is presented in order to accurately reconstruct the electrical behavior of a real UD material in the three spatial directions.

**Index Terms**—Electrical conductivity tensor, Homogeneous material, Unidirectional composite material.

## I. INTRODUCTION

CARBON FIBER REINFORCED POLYMER (CFRP) structural materials are composed of several layers of unidirectional (UD) plies. They replace metallic materials in various applications, mainly in aeronautics and space industry. These materials have great mechanical characteristics for a light weight. Composites with UD conductive reinforcements can be represented through three different scales: microscopic (scale of a fiber  $\approx 10\mu\text{m}$ ), mesoscopic (layer's scale  $\approx 185\mu\text{m}$ ) and macroscopic (composite part scale  $\approx$  up to several meters). The mesoscopic and macroscopic scales are the most widely used scales for modeling and analyzing the physical behavior of CFRP. Indeed, the direct simulation of the physical behavior of such structure with local considerations (microscopic scale) is impossible with the current capacity of computation because of the too large number of unknowns. To overcome these constraints, each layer of the composite is replaced by a homogeneous material having the same average behavior in macroscopic scale. Knowing the electrical conductivity tensor of a UD ply oriented at  $0^\circ$ , one can deduce the conductivity tensor of the ply oriented according to another angle by simple mathematical transformation [1]. In mesoscopic scale, geometrical characteristics such as the random distribution of fibers, scale factor, heterogeneity, etc. make difficult the modeling of the behavior of these composites. As shown in Fig. 1, the composite material is then cut into regular n-REV (Representative Elementary Volume) cells with several fibers to obtain a good statistical representation. The geometric construction of the elementary cell must, of course, take into account the aforementioned complexities.

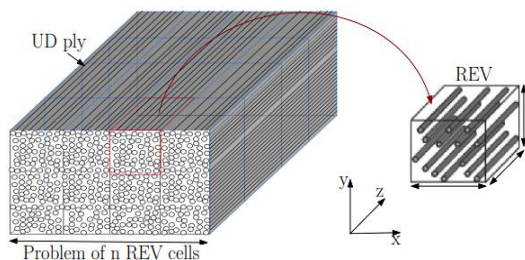


Fig. 1 Geometric decomposition in n REV cells of a UD ply of composite

In this paper, authors propose an identification of model parameters representing the REV cell through experimental measurement of conductivity and micrographic cross sections. Due to the random fibers position, statistical distribution will be also studied in Section V.

## II. GENERATING THE GEOMETRY OF A REPRESENTATIVE CELL

A REV cell with a random distribution of fibers in its volume must be constructed. Previous works [2], [3] illustrate significant structural variability due to the random distribution of fibers in the matrix. Indeed, fibers are not perfectly parallel but wavy. This can lead to contacts and conductive bridges allowing the current circulation between fibers (electric percolation phenomenon). To generate the virtual material, the REV is sliced along the fiber direction as shown in Fig. 2. To take into account the variability of the microstructure of these composites, one uses a method inspired by molecular dynamics to define fibers position in a slice to accurately reconstruct real electrical behavior. The REV is then constructed slice by slice. The position of the center of a fiber in a slice  $n+1$  depends on the placement of the center of this fiber in previous slice  $n$ .

For the first slice, fibers are randomly positioned until an imposed fiber volume fraction is obtained. This process is done initially without worrying about the possible overlap between the fibers. Each fiber overlapping with others applies a repulsive force. These forces induce a displacement of the fibers to the point of equilibrium where all the fibers no longer overlap beyond a maximum allowed value [1]. If the centers of two fibers are combined, the applied repulsive force is very large. Then, it decreases as the two fibers move away, then vanishes in order to keep the electrical contact. In addition, the applied

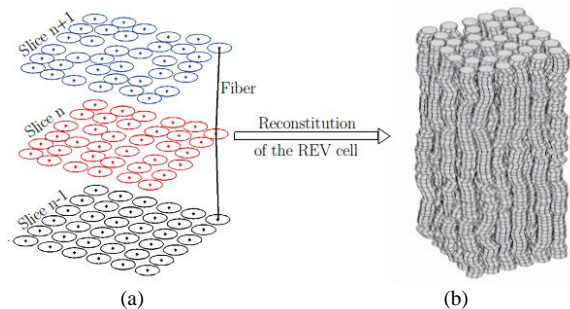


Fig. 2 (a) Composite division in the fiber direction, (b) REV cell

forces to the fibers in contact with the boundary of the REV are canceled in the direction that push them outside. Fig. 3 illustrates the steps to generate the first slice of a cell representative of the microstructure of a UD composite. For the second slice, each fiber center is randomly placed in a disk whose center is the center of the same fiber in the first slice and the radius is the image of the maximum displacement allowed for the fiber. This radial displacement is defined to avoid fibers crossing. In case of fibers overlap, the displacement operation (Fig. 3 (b)) is performed again until there is no more overlap above the permitted limit. The remaining slices are built in the same way as the second slice.

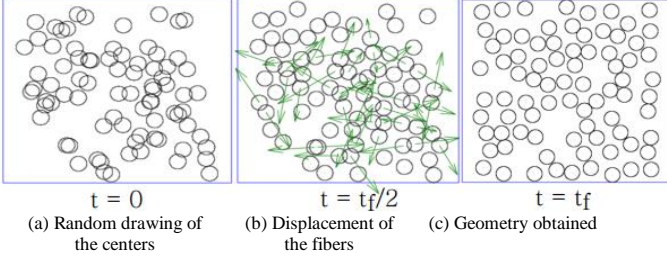


Fig. 3 Placing fibers in a slice of the composite

Once the geometry in 3 dimensions is obtained, an equivalent 3D electrical impedance network simulation is deduced in order to perform a volt-ampereometric test. For our working frequencies (<1MHz), the impedances due to capacitive and inductive effects are less than 1% of the resistive part, they are therefore negligible [4], [5]. Each node of the network is connected to its neighbors by a resistance, as shown in Fig. 4. It is then possible to perform a volt-ampereometric test to obtain the conductivity along the three directions of the REV by applying voltage difference in all points in contact with border.

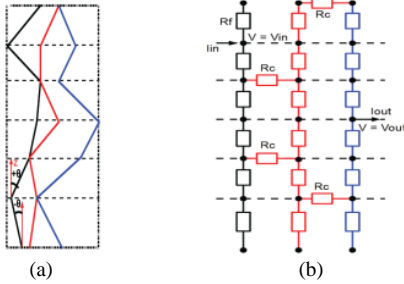


Fig. 4 (a) Arrangement of fibers. (b) Network of impedances

#### Resistance of fiber $R_f$

The resistance of fiber  $R_f$  between two slices is calculated as a function of the conductivity of fibers  $\sigma$ , the diameter of fibers  $\phi$  and the length of the fiber between two layers  $df$ . As the angle of inclination of the fiber between two layers is very low, it will be taken equal to the distance  $dz$  between two layers and  $R_f$  writes:

$$R_f = dz \cdot 4 / (\sigma \cdot \pi \cdot \phi) \quad (1)$$

#### Contact resistance $R_c$

The fibers in contact in the same slice are connected by a contact resistance  $R_c$ . A 2D electrokinetic model solved using finite element (FE) method is then developed to estimate this

resistance. It is calculated using the following vector formulation of the Maxwell equations:

$$\text{div}(J) = \text{div}(\sigma \cdot E) = \text{div}(\sigma \cdot \text{grad}(V)) = 0 \quad (2)$$

$$\int_{\Omega} \overrightarrow{\text{grad}}(V) \cdot \sigma \cdot \overrightarrow{\text{grad}}(w) \cdot d\Omega - \int_{\Gamma} \frac{\partial V}{\partial n} w d\Gamma = 0 \quad (3)$$

where  $V$  is the electrical potential,  $\sigma$  the electrical conductivity of fibers and  $w$  the shape function.

The FEM code gives a contact resistance for 1m of fiber. This allows the resistance value to be plotted as a function of the overlap between two fibers with a contact length of 1  $\mu\text{m}$  as shown in Fig. 5. When generating the 3D geometry, the contact length between layers is evaluated and the value of the 3D contact resistance is therefore deduced.

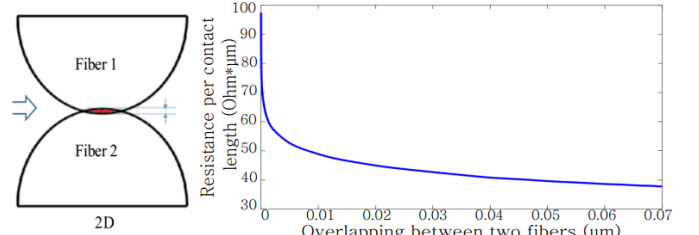


Fig. 5 Variation of the contact resistance

The electrical behavior of the network (Fig. 4(b)) can be formulated using a Kirchhoff Node Law method. As a result, the voltage at each node  $i$  of the layer  $k$  on a multi-layer network can be expressed by the following equation:

$$V_i^k = \frac{\sum_j V_j^k \cdot g_{ij}^k + \sum_{r \in (k+1, k-1)} V_i^r \cdot g_{il}^{k,r}}{\sum_j g_{ij}^k + \sum_{r \in (k+1, k-1)} g_{il}^{k,r}} \quad (4)$$

where  $k+1$  and  $k-1$  respectively represent the upper and lower layers of the layer  $k$ ,  $j$  represents the neighboring nodes at the node  $i$ ,  $g_{ij}^k$  represents the conductance between the nodes  $i$  and  $j$  and  $g_{il}^{k,r}$  represents the conductance connecting the nodes  $i$  of two layers. One can thus write the problem in matrix form:

$$[K][V] = [F] \quad (5)$$

where  $[K]$  is the matrix of conductance,  $[V]$  that of electrical potentials and  $[F]$  the source term. To perform a volt-ampereometric test, the tension at the nodes in contact with the bottom and top surfaces of the REV is imposed by the Dirichlet conditions. Equation (5) is thus solved and the outgoing and incoming currents are then deduced. A verification is made to ensure that these currents are equal.

### III. GEOMETRY PARAMETERS

Many parameters are involved in the generation of the 3D REV cell such as: the dimensions of the REV cell (width  $x$ , thickness  $y$  and length  $z$ ), the distance between slices ( $dz$ ) representing the periodicity of potential contacts between fibers, the characteristics of the fibers (conductivity  $\sigma$ , diameter  $\phi$ ) and the volume filling ratio ( $\tau$ ) of fibers.

A too large REV cell induces a system numerically too complex to be solved but in the other hand the cell must be large enough to obtain statistically identical behaviors. The thickness  $y$  of the REV-cell can be taken equal to the thickness of a classical ply of composite material ( $\approx 185\mu\text{m}$ ). The dimension  $x$  in the transverse direction to the fibers and the number  $nz$  of slices of the cell will be studied statistically in order to choose the optimal ones with less dispersion.

The volume filling rate is a crucial parameter which is strongly connected to the possible appearance of electrical percolation path in transverse directions.

The conductivity  $\sigma$  of a fiber can be easily obtained through conductivity measurement campaigns.

These different parameters are the key elements of the virtual material. Some are more sensitive than others. Fig 6 shows the high sensitivity of the transverse conductivity vs. the average contact periodicity length  $dz$ . This key parameter is very difficult to identify while some other parameters are more easily accessible with micrographic sections and conductivity measurement campaigns. An inverse model is used to determine this non-accessible parameter  $dz$ .

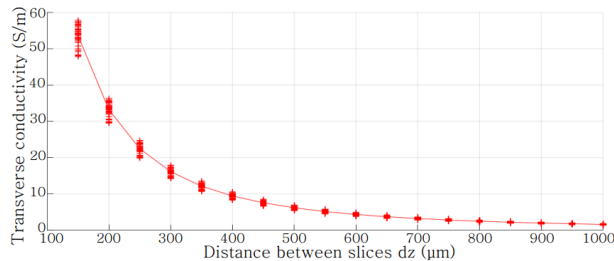


Fig. 6 Transverse electrical conductivity as a function of distance  $dz$ . 41 simulations for each length.

#### IV. CHARACTERIZATION

##### A. Micrographic Observation

Micrographic sections were made on different samples as shown in Fig. 7. It illustrates the view in the section at the interface between a ply oriented at  $0^\circ$  and another at  $90^\circ$ . It can be clearly seen that the fibers are not straight but wavy which creates contacts between fibers.

The Hough algorithm is used to determine the diameter of the fibers [6]. Fig. 8 shows the use of the algorithm and the histogram of the diameters found. It shows that the diameter  $\phi$  is around  $6.7 \pm 0.28\mu\text{m}$ .

The volume filling ratio can also be evaluated from the micrographic sections. Indeed, knowing the radii of the different fibers in an observation window, one can calculate the area occupied by all the fibers and then divide it by the total surface, which gives an idea of the volume filling ratio.

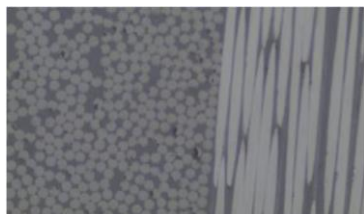


Fig. 7 Micrographic section in the section of a composite [0/90]

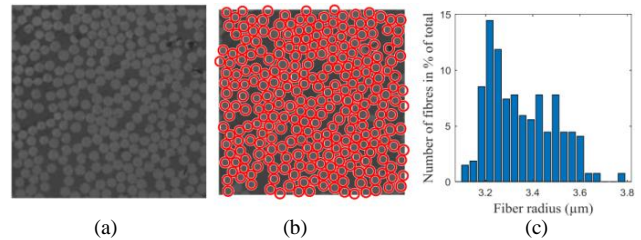


Fig. 8 (a) Micrographic section. (b) Use of Hough algorithm. (c) Distribution of fiber radii

With fibers radii and volume ratio, slices can be created with the method described in Fig. 3. The distribution of the fibers in a given slice and the distribution obtained from the micrographic slices were compared. As shown in Fig. 9, it can be seen that the distribution of fibers remains fairly uniform in both cases.

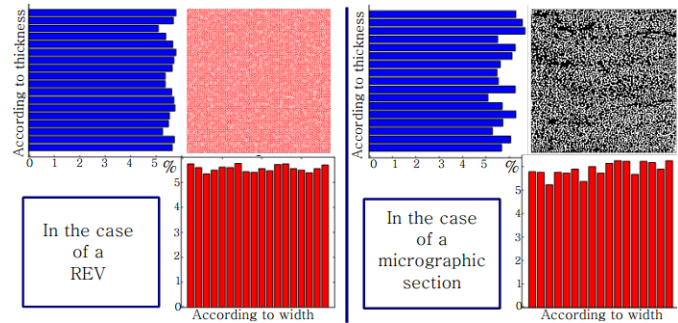


Fig. 9 Fiber distribution

##### B. Conductivity Measurement

To determine the last parameters, it is necessary to make conductivity measurements of our different samples of unidirectional laminated composite materials.

The electrical measurements of the samples are made using the two-probe method volt-ampereometric in delta mode to overcome thermo-electric effect (Fig. 10).

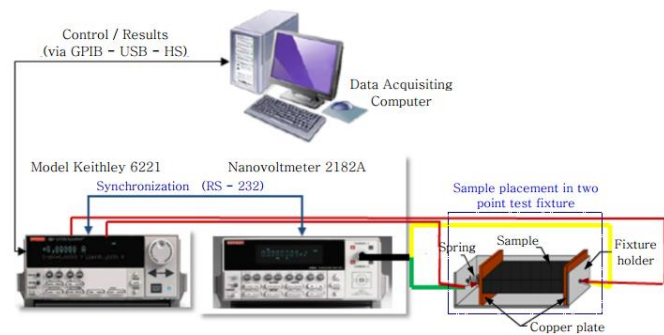


Fig. 10 Structure of the electrical characterization test bench for composites [2]

To characterize the fibers, conductivity measurements were performed on Torayca T700 carbon fibers with characteristics in Table I. The measurements give an average conductivity of  $58857 \pm 272 \text{ S/m}$ .

TABLE I  
CHARACTERISTICS OF TORAYCA T700 CARBON FIBERS

Filament Diameter	Electrical Conductivity	Tensile Modulus	Poisson's Ratio
7 $\mu\text{m}$	62500 S/m	230 GPa	0.26 - 0.28



Conductivity measurements in the fiber direction and in the transverse direction are made on unidirectional laminated composite materials. The results are presented in Table 2 for a composite of 15 plies and 19 plies. With the fiber conductivity measurement results ( $\sigma$ ) and the identified volume ratio  $\tau=60\%$ , the theoretical conductivity ( $\sigma_f$ ) in the fiber direction can be calculated with the formula  $\sigma_f = \sigma \cdot \tau$ . In this case, a conductivity of  $35280 \pm 163$  S/m is obtained. This result agrees well with recorded data in Table 2, which allows us to confirm the choice of volume ratio, fiber conductivity and fiber radius.

TABLE II  
CONDUCTIVITY MEASUREMENTS

Sample	Conductivity	
	Fiber direction	Transverse direction
[0] <sub>15</sub>	33242.16 $\pm$ 966.53	7.58 $\pm$ 0.14
[0] <sub>19</sub>	34848.77 $\pm$ 1115.24	7.28 $\pm$ 0.36

## V. DIMENSIONS OF THE REV

With the micrographic sections and electrical tests performed with Torayca T700 fibers one was able to set some simulation parameters according to Table 3. With this hypothesis, one can study the size REV and determine the distance  $dz$  using experimental measurements.

TABLE III  
SIMULATION PARAMETERS

Fibre radius (r)	Fiber Conductivity	REV thickness	Volume filling ratio	Maximum overlap
3.5 $\mu$ m	58800 S/m	185 $\mu$ m	60%	r/50 = 70 nm

- REV size

For the study of the REV size to ensure a good statistical representativeness, according Gaussian distribution, one generates 41 geometries. First, based on the work done in [2], [3], one set the width of the REV greater than 1mm to study the choice of  $nz$ . Fig. 11(a) shows the confidence limits of normalized conductivities to 90% by the normal distribution as a function of the number of layers in the length direction. From  $nz=10$  the dispersion on the transverse conductivity of 95% samples do not exceed 10%.

Secondly, one sets  $nz=10$  to be in the critical situation, then studies the optimal width of the REV. Fig. 10(b) shows the confidence limits of the conductivities normalized to 90% by the normal law as a function of the width of the REV cell. For REV with widths greater than 1mm, the transverse conductivity dispersion of 90% of the samples does not exceed 10%.

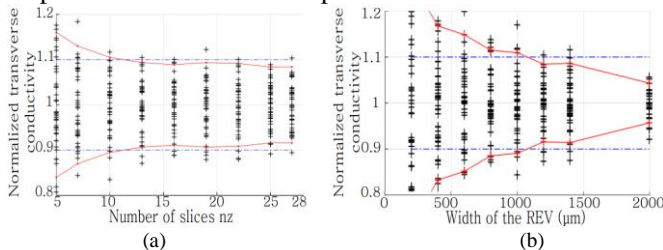


Fig 11 REV size study

- Distance between two slices ( $dz$ )

Now, one can adjust the only remaining parameter, i.e. the distance  $dz$  between two slices. To do this, 41 geometries are made for different  $dz$  values with a REV width of 1.5mm and a

number of slices of 15. In the literature the distance  $dz$  is about 320 $\mu$ m [7]. For this identification procedure, the distance  $dz$  is chosen between 150  $\mu$ m and 950  $\mu$ m. Fig. 12 shows the evolution of the transverse conductivity as a function of distance  $dz$ . The different measures in Table 3 were also represented. It can be seen that a distance  $dz = 450\mu$ m gives the expected results.

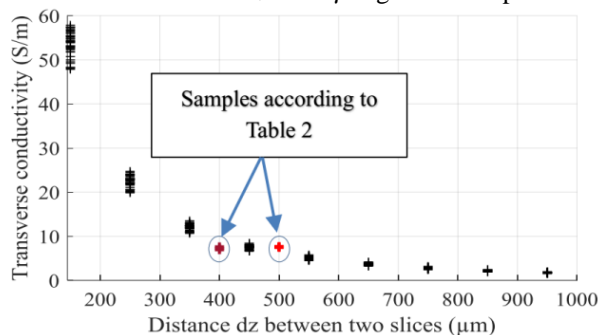


Fig. 12 Choice of the distance  $dz$  between two slices

## CONCLUSION

Composite materials electrical behavior strongly depends on the physical properties of the material. It is thus necessary to understand and analyze the physical behavior of composite materials with the help of different models. These models depend mainly on the intrinsic physical properties of these components and the volume fraction of reinforcements. In this paper, thanks to micrographic sections and conductivity measurements, one was able to determine the different parameters involved in homogenization by virtual material.

## ACKNOWLEDGMENT

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