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# Electrical Conductivity Tensor Modelling of Stratified Woven-Fabric Carbon Fiber Reinforced Polymer Composite Materials

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**Abstract**—In this paper, a new multi-scale approach to predict the effective electrical conductivity tensor of a woven-fabric composite layer for frequencies lower than 10MHz is presented. The result is then used to model the behaviour of a stratified composite material composed by a pileup of woven-fabric layers with different relative orientations. To take into account the orientation and the anisotropy of the material, a 3D multilayer hexahedral degenerated elements method is used to model the anisotropic thin plate. This approach enables to take into account random distribution of carbon fibers in the yarns, 3D effects and electric percolation threshold with an acceptable numerical cost. The developed model is then validated by results for one anisotropic thin layer.

**Index Terms**—Electrical conductivity tensor, multi-scale homogenization, percolation, woven-fabric material modelling.

## I. INTRODUCTION

Nowadays, the woven-fabric composites have a wide range of applications in industry especially in aeronautical and automobile sectors. A woven-fabric stratified composite material is a pileup of several woven-fabric layers. In each layers, yarns of carbon fibres are weaved with warp and weft yarns in orthogonal direction. The orientation of each layer of pile up can be different in order to obtain the good mechanical properties. However, these composites cannot be used to return electric current of the embedded equipments in aircraft. To improve the electrical conductivity of these composites, it is necessary to understand the electrical current circulation in order to obtain their electrical properties. These properties depend on weave style, electrical properties of the constituent materials, fiber filling rate, etc. Thus, the electrical conductivity along the fibers direction in the yarns is controlled by the continuous conduction mechanism along the fibers while electrical properties in the other directions are influenced by the electrical contacts between fibers (no capacitive current). Therefore, this conduction mechanism in transverse fibers direction is governed by the percolation phenomenon. Due to the complex architectural structure of these composites and the large differences in scale of the composites constituents (see Fig. 1), a direct numerical modelling leads to a too large numerical complexity.

In this work, the effective electrical conductivity tensor of stratified woven-fabric composite is evaluated using a complete methodology, based on multi-scale homogenization taking into account the percolation phenomenon (see Fig. 1).

## II. HOMOGENIZATION PROBLEM FORMULATION

Fig. 2 shows the multi-scale homogenization method problem formulation. The first transition, between carbon fiber and yarn, is done by a numerical generation of

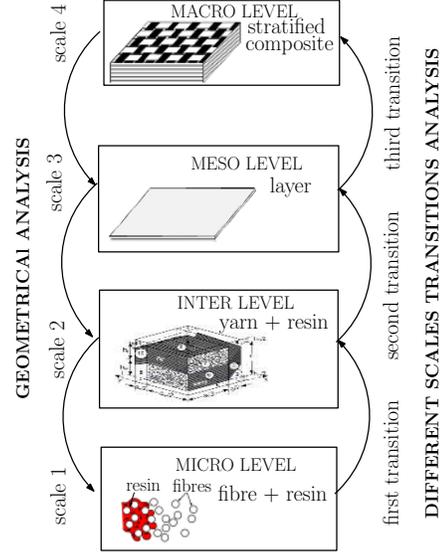


Fig. 1: Multi-scale homogenisation analysis of woven-fabric.

3D microstructure of a yarn. With this microstructure, the conductivities tensor is given by voltammetric simulation. With the second transition, the conductivities tensor of one layer is given based on weave style and electrical properties of yarns. In order to obtain the electrical behavior of the whole composite, a 3D multilayer hexahedral degenerated finite element is then used.

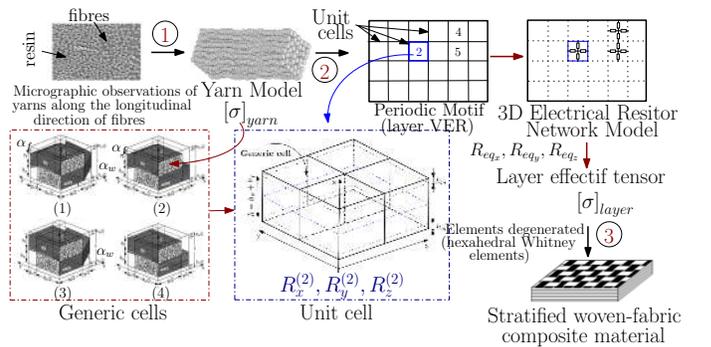


Fig. 2: Multi-scale homogenisation problem formulation.

### A. Scale Transition from Fibers to Yarns

1) *Generation of 3D microstructure Model of Yarn:* due to fabrication process, electrical contact between fiber can occur and create a percolation phenomenon along transverse

direction. This phenomenon depends on filling rate and geometrical pattern. According to micrographic observations, a numerical Representative Volume Element (RVE) of a yarn is generated taking into account the random position of fiber in yarn section [1]. The size of RVE has to be evaluate to ensure its statistical representativeness.

2) *3D Equivalent Electrical Network*: the electrical resistances  $R_c$  between fibers in contact is defined by a simplified analytical model based on Hertz [4] contact theory and Holm's formula [5]. With some geometry simplifications, the theory of contact between elastic bodies can be used to find contact areas and indentation depths  $\delta$  for contacting carbon fibers (Fig. 3). One consider two contact through points rather than line, due to undulation of fibers. The contacting part between two fibers is taken as two spheres with the same radius of the carbon fiber. The equivalent radius contact area  $a$  and the indentation depths  $\delta$  from Hertz's contact [4] are related to the applied local load  $F_c$  between neighbouring fibers during processing impacts on the microstructure by the equations :

$$a = \left( \frac{3}{8} \cdot \frac{RF_c}{E} \right)^{1/3} \quad \text{and} \quad \delta = \left( \frac{9}{8} \cdot \frac{F_c^2}{R} \cdot E^2 \right)^{1/3} \quad (1)$$

where  $E$  is the effective elastic modulus of the fiber and  $R$  the radius of carbon fibers.

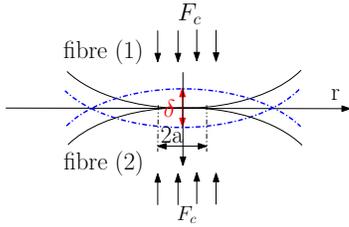


Fig. 3: Contact between two revolution fibers (Hertz's contact)).

Therefore, knowing the indentation depths (overlap setting) between each pair neighbouring fibers through our MATLAB® program generation of 3D microstructure, we deduce in  $\delta$  (1) the applied local load  $F_c$  between neighbouring fibers in contact. However, the contact area is directly related to the electrical contact resistance as described by Holm's formula [5] for constriction resistance (2), between two conductors connecting through a small area.

$$R_c = \frac{\rho_1 + \rho_2}{4a} \quad (2)$$

where  $\rho_1$  et  $\rho_2$  are the resistivity of the two conductors and  $a$  is the radius contact area. For the case of two carbon fibers with the same intrinsic resistivity  $\rho$  and radius  $R$ , the constriction resistance is defined in (3).

$$R_c = \frac{\rho}{\sqrt[3]{\left( \frac{3RF_c}{E} \right)}} \quad (3)$$

The fiber physicals proprieties used in these simulations are listed in TABLE I.

TABLE I: Physicals proprieties of HTA40 fiber.

Fiber diameter	Fiber modulus	Fiber resistivity
7 $\mu\text{m}$	238 Gpa	$1.6 \times 10^{-5} \Omega\text{m}$

3) *Results*: Fig. 4, shows the evolution of electrical conductivity according the fibers filling rate. For each filling rate, we generated 41 3D microstructure model of yarn are performed in order to ensure the statistical representativeness.

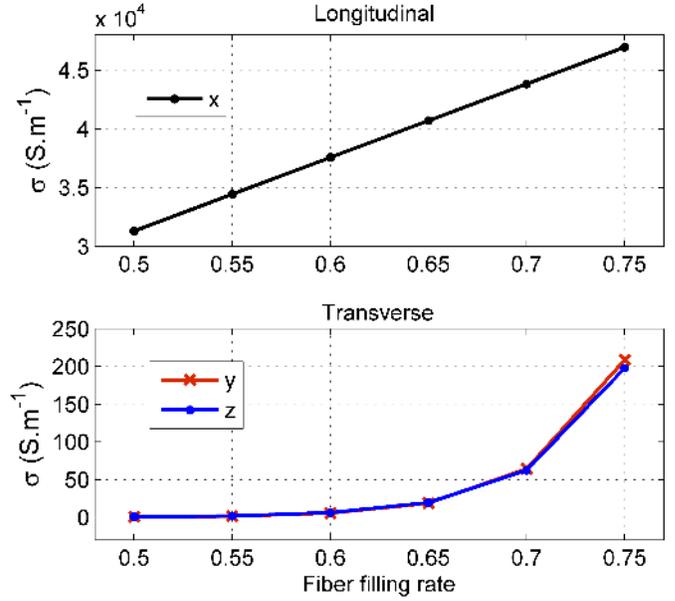


Fig. 4: Electrical conductivities, as function fiber filling rate.

The electrical conductivity in longitudinal (x-axis) direction increases proportionally with fiber filling rate in a linear fashion. That conductivity depends mainly on the intrinsic resistivity of the fibers and on fiber filling rate  $\phi_f$ . It can be calculated by the rule of mixture analytical model if one ignore the electrical conductivity of the insulating resin  $\sigma_r$  before to that of the fibers  $\sigma_f$ .

$$\sigma_L \simeq \phi_f \cdot \sigma_f \rightarrow \text{if } \sigma_f \gg \sigma_r \quad (4)$$

Transverse conductivities (in the width (y-axis) or thickness (z-axis) direction) increase as an exponential. This behaviour is directly related to percolation phenomenon. The quality of the transverse electrical conductivity estimation is directly related to a good evaluation on RVE size. This size is obtained by a statistical study based on student's law. This statistical study is done on the average of conductivities obtained for several RVE. If the confidence limit is not reached, larger RVE are generate. The thickness and the width of simulate sample are equal to 130  $\mu\text{m}$  and 130  $\mu\text{m}$ . Fig.5 shows the confidence interval limits of 90% obtained from Student's law, as a function of the length the 3D microstructure RVE. For each length, 41 simulations are done. For length RVE greater than 0,02 m the confidence limits are reached 90%.

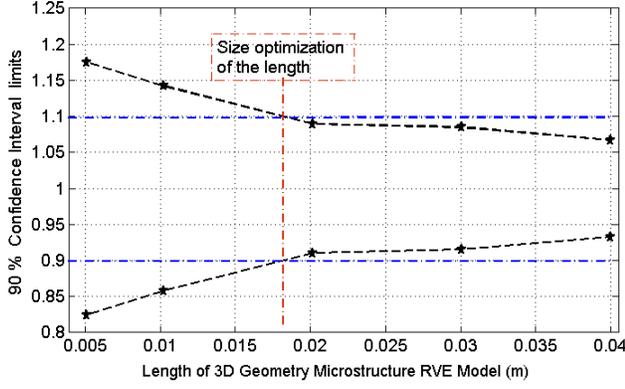


Fig. 5: Electrical conductivity confidence interval for several length of RVE with 60% fiber filling rate.

### B. Scale Transition from Yarn to layer

The scale transition is done using a 3D electrical network method applied to a periodic RVE of the layer. The structure of the layer of woven-fabric composite is assumed to be periodic and represented by the periodic motif (Representative Volume Equivalent (RVE)). This RVE can be partitioned into several tri-dimensional electrical conductive unit cells situated in parallel or series. All unit cells can be built from 4 generic cells by symmetrical and rotational operations. The generic cells electrical properties are determined from yarn effective electrical conductivity tensor (Fig. 2). For the entire set of scales transition, the formalism governing the electrical current transport remains actually the same. In this work, 5-harness satin woven-fabric layer is studied. This RVE (see Fig. 6) can be partitioned into several electrical conductive unit cells [6] (denoted by 1,2,...,5).

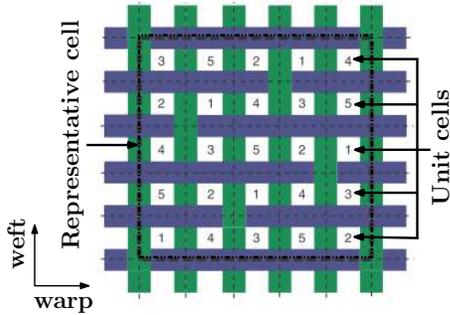


Fig. 6: Representative volume cell of 5-harness satin fabric layer [6].

However, this model depend mainly to the degree of accuracy of the geometrical description of the yarns and the layer architecture. The in-plane dimensions  $a$  and  $g$  of the unit cell define, respectively, the yarn width and the gap width between two adjacent yarns. The subscripts  $w$  and  $f$  denote warp and weft yarns, respectively. The out-of-plane dimensions  $h_m$ ,  $h_w$  and  $h_f$  stand for matrix layer, warp yarn, and weft yarn thickness. The thickness  $h$  of the unit cell is determined by that of the layer. The x-axis and y-axis are

chosen to coincide with the warp and weft yarns directions, respectively. One divide each generic cell into several elements to construct the corresponding 3D electrical resistance network and deduce its equivalent electrical resistance. The electrical resistance  $R_i$  of the  $i$ th conduction element in each generic unit cell is evaluated by :

$$R_i = \frac{L_i}{\sigma_i \cdot S_i} \quad (5)$$

where  $L_i$  and  $\sigma_i$  are, respectively, the length and effective electrical conductivity of the  $i$ th conduction element in the direction of current path, and  $S_i$  is the area of the conduction element cross-section perpendicular to the direction of current path. In each conduction element  $i$  of generic cell where the fibers are inclined, one take account the contribution of the impregnated yard with the fiber inclined at the mean angles  $\alpha_f$  and  $\alpha_w$  versus weft and warp direction, respectively. So, the electrical conduction of this element can be expressed as the following transformation equation [2] :

$$\begin{aligned} \sigma'_{d_l} &= \sigma_{d_l} \cos^2 \alpha_d + \sigma_{d_t} \sin^2 \alpha_d \\ \sigma'_{d_t} &= \sigma_{d_l} \sin^2 \alpha_d + \sigma_{d_t} \cos^2 \alpha_d \end{aligned} \quad (6)$$

where  $\sigma_{d_l}$  and  $\sigma_{d_t}$  ( $d=w,f$ ) are the longitudinal and transverse electrical conductivity, respectively, of the impregnated yarn. In the following, it is assumed that : (1) the yarn cross sectional is rectangular, (2) the warp and the weft yarns are orthotropic with an isotropic matrix, (3) current distribution is flowing along the direction where the electrical potential is imposed, (4) current lines are straight and parallel to one another. Consequently, using the works of G.N. Dul'nev [7], one can identified the equivalent resistances of the generic cells and deduce those of the unit cells in the warp, weft, and thickness directions. Moreover, basing on the partition of the representative volume cell in several unit cells, the corresponding electrical resistance network can be readily constructed and its equivalent resistances in the principal directions are expressed as :

$$R_{eq_x} = \frac{1}{5} \times \left( R_x^{(1)} + R_x^{(2)} + R_x^{(3)} + R_x^{(4)} + R_x^{(5)} \right) \quad (7)$$

$$\frac{1}{R_{eq_z}} = 5 \times \left( \frac{1}{R_z^{(1)}} + \frac{1}{R_z^{(2)}} + \frac{1}{R_z^{(3)}} + \frac{1}{R_z^{(4)}} + \frac{1}{R_z^{(5)}} \right) \quad (8)$$

with

$$R_{eq_x} = \frac{(a_f + g_w)}{\sigma_{ef_x} \cdot h \cdot (a_w + g_f)} \quad (9)$$

$$R_{eq_z} = \frac{h}{\sigma_{ef_z} \cdot 5(a_f + g_f) \cdot 5(a_w + g_w)} \quad (10)$$

where  $R_x^{(j)}$  and  $R_z^{(j)}$  are, the equivalent resistances of the  $j$ th unit cell in the warp and thickness directions, respectively.  $\sigma_{ef_x}$  and  $\sigma_{ef_z}$  are the corresponding, respectively, effective electrical conductivity component in the warp and thickness directions. The equivalent resistance as well as  $\sigma_{ef_y}$  in the weft direction can be obtained by interchanging the subscripts  $w$  and  $f$  ( $x$  and  $y$ ) in the all formulations of the equivalent resistances. Our model, may be used on a woven fabric unbalanced but in this study it is balanced in the warp and weft directions. Thus,  $a_w = a_f = a$ ,  $g_w = g_f = g$ ,  $h_w = h_f$ ,

$\sigma_{w_l} = \sigma_{f_l} = \sigma_L$  and  $\sigma_{w_t} = \sigma_{f_t} = \sigma_T$ . The principal material geometric parameters measured from SEM image (\*) or given by the maker (\*\*) are recapitulated in TABLE II.

TABLE II: Geometric parameters of Textile GO963 - 5-harness satin - Fiber HTA40 3k.

$h_f + h_w$ **	a *	g *	h *
0,29 mm	$1,7 \pm 0,08$ mm	$0,17 \pm 0,008$ mm	0,31 mm

### C. Simulation of stratified woven fabric

Multilayer hexahedral degenerated Whitney element [3] are used to obtain the current circulation in the whole composite knowing the conductivities tensor of each layer. This model allows to take into account phenomenon in the layers scale with a good balance between time simulation and precision. The conductivities tensor of each layer is given according the layer orientation and the tensor of periodic RVE in section II-B.

## III. EXPERIMENTAL VALIDATION OF ONE LAYER

### A. Description of experimental set-up

A two-probe method is used to identify the electrical conductivities in all directions. This method is based on DC volatampmetric measurement using Delta Method [8] supply mode to cancel thermoelectric voltage. The choice of current magnitude is very important because the resistance measurements in carbon fiber composites are sensitive to its magnitude [9].

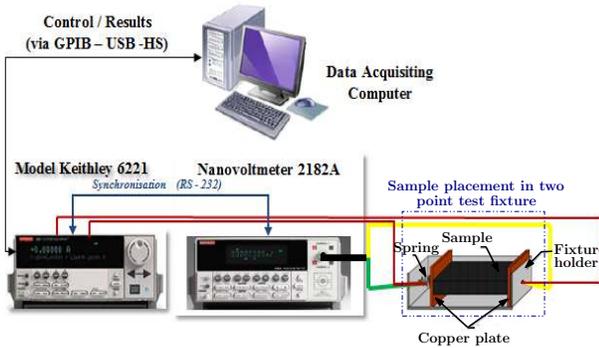


Fig. 7: Two-probe schematic illustration and Equipments used for electrical resistance measurement set-up.

### B. Results

To validate the model presented, the effective diagonal tensor electrical conductivity of 5-harness satin fabric stratified composite were calculated. This stratified composite was manufactured from stacking a five of woven-fabric layers of pre-impregnated, by interposing one or more polymer films doped carbon graphite between them (film stacking technique). The consolidation of this stack was made under a heating press into a RTM mould. The fiber filling rate in the yarn,  $\Phi_f$  is 50% (given by the maker), and the fiber filling rate measured in the stratified composite varied between 39% and 40%.

The electrical conductivity of the yarn is  $2.50 \times 10^{+4} S.m^{-1}$  along the fibers,  $0.30 S.m^{-1}$  in the transverse direction to the fiber axis, and  $1.1 \times 10^{-1} S.m^{-1}$  in the polymer films. The electrical conductivity tensor components calculated of our numerical model and those identified of stratified composite specimens are compared to the ones given in TABLE III. The calculated values were somewhat higher than the experimental results.

It shows a discrepancy of 3.6% average and less than 10% with estimated value in the directions  $x - y$  and  $z$ , respectively. The difference is mainly due to the incertitude of input parameters such as filling rate, fibers diameter, carbon conductivity, ...

TABLE III: Numerical results and comparison with experimental measurements.

Components	Num. model	Measured	Difference(%)
$\sigma_x (S.m^{-1})$	$1,06.10^{+4}$	$1,13.10^{+4}$	6.19
$\sigma_y (S.m^{-1})$	$1,06.10^{+4}$	$1,07.10^{+4}$	1
$\sigma_z (S.m^{-1})$	44,71	40,33	9,79

## IV. CONCLUSION

A complete methodology of electrical conductivity tensor evaluation of woven-fabric have been proposed. Based on 3D virtual material generation, a multi-scale homogenization taking into account the percolation phenomenon to the distinct scales of the composite material has been developed. The confrontation between the computed electrical conductivity tensor and the measured one shows a very good concordance.

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