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# Identification of BRDF parameters with spectral measurements in the visible light spectrum towards solar irradiation evaluation in urban environment for photovoltaic technologies

Blaise Raybaud<sup>1,2</sup>, Etienne Vergnault<sup>2</sup>, Angela Disdier<sup>1</sup>, Philippe Thony<sup>1</sup>,  
Jean-Jacques Roux<sup>2</sup>

<sup>1</sup>Université Grenoble Alpes, CEA/LITEN, INES, F-38000 Grenoble,  
France

<sup>2</sup>Université de Lyon, INSA-Lyon, CETHIL UMR5008, F-69621  
Villeurbanne, France

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## Abstract

In order to incorporate real construction material reflectance properties in a Monte-Carlo ray tracing algorithm, a representative cladding material is characterised with a spectrophotometer to obtain reflectance angular distribution. These measurements are used to design a BRDF model with a reduced number of parameters that are further identified, along with their confidence intervals. Obtained results are compared to reflectance coefficients measured with an integrating sphere. The prospects of such a model is to allow for a more accurate modelling of the optical properties of materials in reflection. This knowledge can then allow a better integration of photovoltaic technologies, both in terms of energy and visual acceptance. For the particular HPL cladding material tested and despite a highly specular behavior at grazing incidence angles, we found that the specular term only amounts to 4% of reflection.

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characterization of building materials

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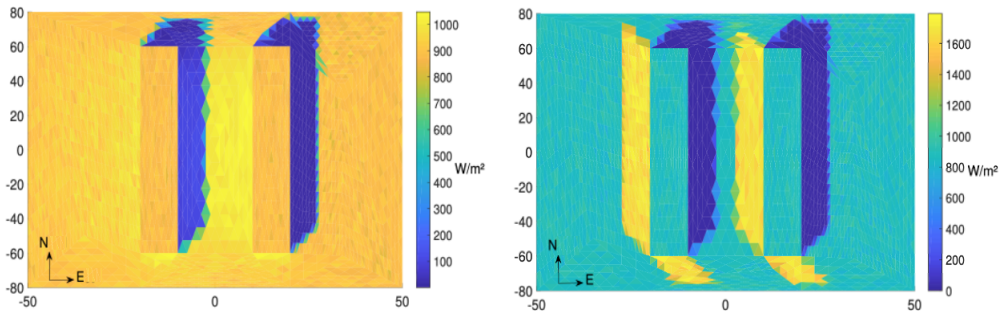
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## 1. Nomenclature

- $\theta_o$ : Angle of measurement.
- $\theta_i$ : Angle of incidence.
- $\theta_{mes,ini}$ : First angle for which a measurement in reflection is carried out (at fixed angle of incidence).
- $\theta_{mes,fin}$ : Last angle for which a measurement in reflection is carried out (at fixed angle of incidence).
- $\theta_{step}$ : Angular measurement pitch with the ARTA.
- $N$ : number of measurements performed (at fixed angle of incidence).
- $\varphi_o$ : Angle in which the observer is located in the plane of the sample, in relation to the axis  $\vec{y}$ .
- $\varphi_i$ : Angle of the incidence ray in the plane of the sample, in relation to the axis  $\vec{y}$ .
- $L_o$ : Radiance (power by unit area of reflecting material per unit solid angle of observer) [ $Wsr^{-1}m^{-2}$ ].
- $L_i$ : Radiance (power by unit area of reflecting material per unit solid angle of light source) [ $Wsr^{-1}m^{-2}$ ].
- $E_i$ : Irradiance (power per unit area at reflecting material surface) [ $Wm^{-2}$ ].
- $E_{i0}$ : Irradiance (power per unit area of incident beam cross-section) [ $Wm^{-2}$ ].
- $I_o$ : Reflected radiant intensity [ $W/sr$ ].
- $I_o^s$ : Specular component of the reflected radiant intensity [ $W/sr$ ].
- $I_o^d$ : Diffuse component of the reflected radiant intensity [ $W/sr$ ].
- $M_o$ : Outgoing power of reflecting material [ $W$ ].
- $M_i$ : Incident power on reflecting material [ $W$ ].
- $R_x$ : reflection coefficients for the  $x^{th}$  polarization from the Fresnel's law.
- $f_r(\theta_i, \theta_o, x)$ : BRDF function [ $sr^{-1}$ ].
- $Z_0$ : Impedance in vacuum.
- $Z_x$ : Impedance of the  $x^{th}$  medium.
- $n_x$ : Refractive index of the  $x^{th}$  medium.
- $\rho_x$ : reflection index of the BRDF model, with  $x$  the corresponding component (specular or diffused).
- $\alpha$ : Power coefficient of the specular component of the BRDF function.
- $k_x$ : Final reflection indexes obtained from the BRDF model, with  $x$  the corresponding component.
- $k_g$ : Global final reflection.
- $S_i$ : Illuminated surface of the sample [ $m^2$ ].
- $\cos_+(a) = \frac{\cos(a) + |\cos(a)|}{2}$  with  $a \in [0 : 2\pi]$

## 29 2. Introduction

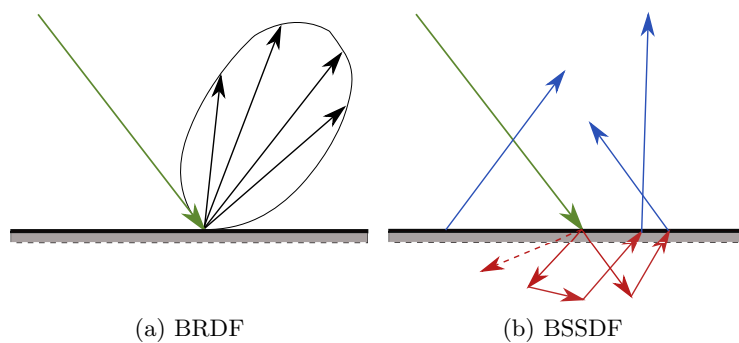
30 Studies of solar potential in urban environments are today mostly useful for  
31 numerous applications in various domain, as local production of solar energy  
32 in urban areas [1], access to natural light towards better life experience of  
33 citizen [2] or simulation of radiative transfer in visible and thermal spectral  
34 ranges to avoid overheating in districts (radiative trapping at district scale,  
35 redistribution of energy). These numerical studies have different objectives:  
36 to assess the solar potential for photovoltaic energy conversion [3, 4], to  
37 validate access to natural light in buildings for human comfort and health [5],  
38 but also to evaluate the exterior facade temperatures of the various buildings  
39 of a dense district [6, 7, 8]. Current simulations of solar light propagation in  
40 urban environment are generally based on visual rendering strategies used in  
41 graphics rendering. Generally, back reflected light on walls [9] is considered  
42 as scattered light without considering the spectral or polarization properties.  
43 We also have to distinguish light behaviour and surface properties in at least  
44 two spectral ranges: visible light (visual comfort, aesthetics, PV conversion)  
45 and thermal radiation (thermal comfort and emissivity, urban heat islands  
46 and greenhouse effects). The objective of this study is, firstly, to evaluate  
47 the impact of these properties of light for precise energy estimation and,  
48 secondly, to take into account the exact nature of light reflections in the  
49 visible spectral range (as opposed to thermal spectral range) in order to  
50 verify the hypothesis of diffuse light reflection currently assumed in most  
51 studies [10]. Raybaud's thesis [11] shows the impact that such patterns  
52 can have on the energy distribution within a street. Figure 1 shows the  
53 extreme cases, where the reflections are either only diffuse (left figure) or  
54 only specular (right figure).



*Figure 1: Comparison of energy distributions as a function of reflection mode (diffuse in the left figure; specular in the right figure) (from Raybaud [11])*

55 The study of optical properties leads to select a set of materials and opti-  
56 cal behaviour as reflection, transmission, absorption and diffusion. These

57 properties are described through mathematical functions, whose name of-  
 58 ten appears in BxDF format for Bidirectional Distribution "x" Function, x  
 59 standing for a specific optical behaviour. BxDF distribution functions are  
 60 mainly used in computer graphics rendering [12]. However, these methods  
 61 are being studied for a better assessment of the impact of the sun on heat  
 62 transfer in urban areas, in particular for "glass" materials [13] or High  
 63 Reflective Materials (HMRs) used to reduce the Urban Heat Island (UHI)  
 64 effect [14]. The visual rendering of virtual scenes is similar to solar light  
 65 and natural lighting studies in urban environments, especially when surface  
 66 properties are considered. It is then interesting to look at the functions used  
 67 in the simulation of visual rendering. These functions include in particular:  
 68 BTDF (transmittance), BRDF (reflectance), BSSRDF (surface scattering  
 69 in reflection), BSSTDF (surface scattering in transmission) or BSDF (BSS-  
 70 RDF and BSSTDF), as shown on Figure 2. These surface properties depend  
 71 on incidence and viewing angles, wavelength, polarization, and also local  
 72 space coordinates. When considering the simulation of light propagation  
 73 in urban scenes, functions related to the BRDF family are commonly used  
 74 [15, 13]. Radiance software is a priori the most adapted to take into ac-  
 75 count such models but the handling is complex [16]. Measuring reflectance  
 76 data to obtain a realistic BRDF of outdoor environments is also a chal-  
 77 lenge today [17]. The light reflected on the opaque materials is generally  
 78 divided into two components: Lambertian retrodiffusion and specular reflec-  
 79 tion [18]. These distribution functions are also used in yield calculations of  
 80 bifacial photovoltaic modules [19], with the aim of studying the impact of  
 81 local environment on the electricity production of the PV modules. Trans-  
 82 mission functions (BTDF) are often used to evaluate the effects of glazing  
 83 on interior lighting [20, 16].



84 In addition, Collin et al. emphasize the effects of polarization in reflections  
 85 [21] for such application. This is particularly true if incident light is de-  
 86 scribed with polarisation vector decomposition in the simulation process,

87 even if the human eye cannot distinguish light polarization.  
88 Initial work from Phong results in a widely used method for computer gen-  
89 erated pictures and visual rendering modelling [22]. This approach is based  
90 on the reconstruction of the scene in the direction of the viewer, with con-  
91 tributions from specular and diffused light, as is the Lafortune model [23].  
92 The Blinn-Phong model [24] proposes a different formulation by including  
93 the consideration of the local nature of the surface with micro-facets. This  
94 model therefore corresponds well to a BRDF and thus ensures energy con-  
95 servation. However, the specular term can become difficult to standardize.  
96 This depends in particular on the function of the chosen specular term [24].  
97 Other models have thus been progressively developed in order to better take  
98 into account the specular reflection. The surface roughness of the materials  
99 and then the polarization of light were also integrated into the models. The  
100 model from Cook-Torrance-Sparrow [25, 26] is the first physical rendering  
101 BRDF model. The surface is assimilated to a set of elementary mirror sur-  
102 faces respecting Fresnel's laws [27]. Under these assumptions, the model  
103 corresponds to a statistical distribution of the orientation of the set of sur-  
104 faces. Several other models are based on a theory of micro facets, like [28].  
105 Studies of solar potential in urban areas often use strong assumptions for  
106 reflection models while BRDF can encompass much more details but are  
107 sometimes impractical for ray tracing algorithms. We aim at designing a  
108 BRDF for solar potential evaluation that is suitable for ray tracing and at  
109 identifying its coefficients from spectrophotometry measurements.

110

111 The first part of this article presents the global methodology used for solar  
112 energy evaluation and more precisely, for the integration of measured optical  
113 surface properties. Then, Section 2 presents some classical BRDF models,  
114 their limitations regarding our objectives and the hypotheses, parameters  
115 and model used in this article, as long as the measurement methodology  
116 and model fitting strategy. The third section displays reflection's results  
117 obtained with the spectrophotometer on a selected cladding material sam-  
118 ple (a white manufactured High Pressure Laminate (HPL)) panel with color  
119 coating: the identified BRDF is compared to reference reflection coefficients.  
120 Then, Section 4 discusses the use of the proposed BRDF in a Ray-Tracing  
121 solar irradiation evaluation code and compares simulation results with ex-  
122 perimental data on the same setup. Lastly, this study is concluded and some  
123 directions for future research are proposed.

### 124 **3. Methodology**

125 *Overall methodology for solar irradiation evaluation.* The aim of this work  
126 is to implement real properties of light reflection in solar energy evaluations  
127 but without complex light electromagnetic description. This approach was



128 motivated by the optical reflection characterisation of several real construc-  
 129 tion materials. These measurements are analysed through models as simple  
 130 and realistic as possible, and then used to feed a radiative transfer simu-  
 131 lation tool. Analogous to existing irradiance models, a ray tracing method  
 132 based on Monte Carlo statistical sampling allows us to calculate the incident  
 133 flux in the visible wavelength range (380nm to 740nm). To get an efficient  
 134 implementation of reflection properties in the ray tracing calculations, an  
 135 integration of the reflections using Bidirectional Reflectance Distribution  
 136 Functions (BRDF) is desired. However, for easy integration, it is necessary  
 137 to reduce the number of parameters describing BRDF model. This will en-  
 138 sure rapid calculations when we will consider wide and dense urban areas for  
 139 precise solar energy evaluation. Mainly used for computer graphic render-  
 140 ing, usual BRDF models are sometimes not strictly compliant with optical  
 141 laws or are not physically sound for calculating heat transfer in the mid and  
 142 far infrared spectral range. We choose here to build the BRDF function in  
 143 the visible range combining specular and diffuse optical behaviors, making  
 144 sure that light energy is conserved during reflection and propagation.

145 The BRDF model proposed in this article is intended to be integrated into  
 146 the chain of calculation of solar radiation in urban areas (Figure 3). When  
 147 simulating solar radiation, the optical properties of the materials are often  
 148 neglected and only one coefficient like the albedo is taken into account.  
 149 The BRDF integration should allow a better evaluation of the geometric  
 150 distribution of reflections.

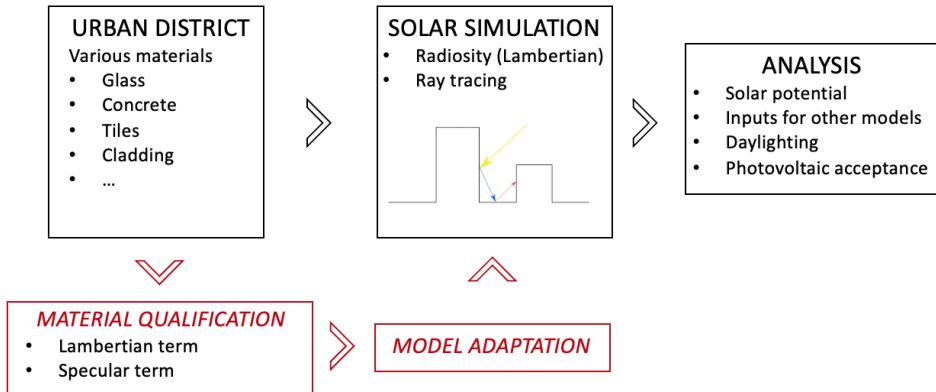


Figure 3: Evaluation strategy for simulating improved urban solar radiation taking into account the BRDF.

151 *Methodology for BRDF Identification.* The measurements are carried out  
 152 with a Lambda 950 UV-VIS spectrophotometer from Perkin Elmer, with a  
 153 motor driven goniometer allowing full angular reflectance analysis (Auto-  
 154 mated Reflectance Transmittance Analyzer - ARTA) [29]. The measured  
 155 spectral range goes from 300nm to 1500nm, wider than the visible spectral

156 range. Despite using only the visible portion of the spectrum to compute re-  
 157 flection coefficients, the whole available range of wavelength is displayed on  
 158 the graphics. The identification of a physical model, and mainly of the char-  
 159 acteristics of the specular lobe, is complicated by the relatively large opening  
 160 window of the optical sensors used in the ARTA device. This is dealt with  
 161 later, see Equation 22. We integrate as many characteristics of the sensors  
 162 as possible in our calculations when identifying the parameters. However,  
 163 some assumptions remain: the incident beam in the spectrophotometer is  
 164 considered as perfectly parallel and the spectral response of the sensors is  
 165 assumed perfect, which may induce a bias in spectral measurements, as will  
 166 be discussed in 5.

167 The following workflow (Figure 4) summarises the approach proposed in this  
 168 article:

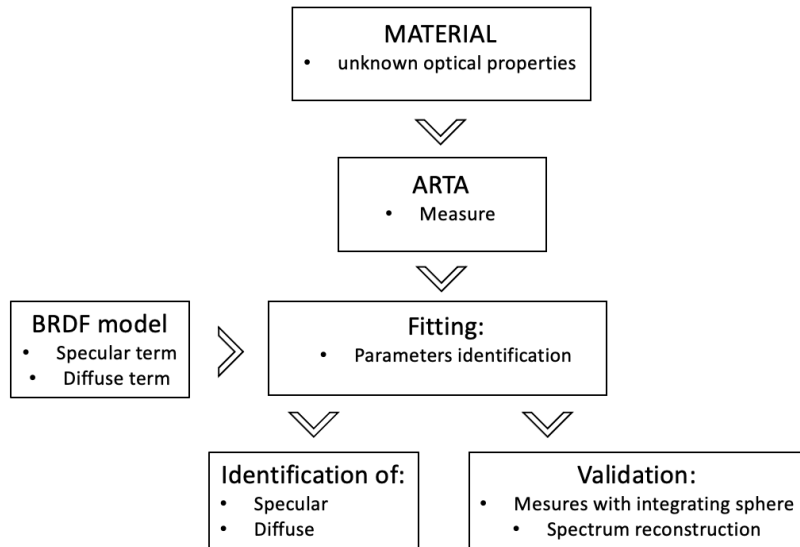


Figure 4: Workflow of the approach proposed in this article.

169 *3.1. Requirements for a BRDF to be used in urban solar irradiation simu-*  
 170 *lations*

171 Due to the diversity of models proposed in the literature, and taking into ac-  
 172 count the constraint of a simple model that can be easily integrated into ur-  
 173 ban solar potential simulations, the following section proposes a new BRDF  
 174 model inspired by the Blinn-Phong model. Fresnel coefficients are integrated  
 175 to take into account the effects of light polarization, like what is done, at a  
 176 much larger scale in [30].

177 *3.1.1. Definition of a BRDF*

178 In the following, a BRDF is defined as a ratio of reflected radiance  $dL_o$   
 179 ( $\text{Wm}^{-2}\text{sr}^{-1}$  [10, 31, 32], power per unit area of reflective material per unit  
 180 solid angle of observer) observed over an infinitesimal solid angle to inci-  
 181 dent irradiance  $dE_i$  on the sample ( $\text{Wm}^{-2}$ , power per unit area of reflective  
 182 material) :

$$f_r = \frac{dL_o}{dE_i}. \quad (1)$$

Other sources use an equivalent different definition, but starting from the emitted radiance  $dL_i$  by a light source:

$$f_r = \frac{dL_o}{dL_i \cos \theta_i d\omega_i}, \quad (2)$$

183 as the received irradiance is linked to the light source radiance by:

$$dE_i = dL_i \cos \theta_i d\omega_i \quad (3)$$

184 However, since our goal is a reconstruction of a BRDF from measurements  
 185 made with a goniospectrophotometer, the incident irradiance  $dE_{i_0}$  (along the  
 186 direction of the incident beam) is also constant and should not be confused  
 187 with  $dE_i$ , see Fig. 5:

$$dE_{i_0} = dL_i d\omega_i \quad (4)$$

188 A BRDF function is expressed in  $\text{sr}^{-1}$  and describes the angular distribution  
 189 resulting from the reflection of light reflected on a surface. As detailed  
 190 above, the optical properties depend on the position of the reflection point  
 191 and the angular position of the source and the observer. Only spatially  
 192 uniform functions (uniform surface) are considered here and the dependence  
 193 is limited to the zenithal incidence angle,  $\theta_i$ , the azimuthal incidence angle  
 194  $\varphi_i$ , the reflected zenithal angle  $\theta_o$  and the reflected azimuthal angle  $\varphi_o$ . The  
 195 light polarization  $x$  can also be considered:

$$f_r(\theta_i, \varphi_i, \theta_o, \varphi_o, x) = \frac{dL_o(\theta_o, \varphi_o, x)}{dE_i(\theta_i, \varphi_i, x)} \quad (5)$$

196 The goniospectrophotometer only performs measurements in the plane of  
 197 incidence (containing both the normal to the plane of the sample and the  
 198 direction of the incident light, see Figure 5). Using a spherical coordinate  
 199 system with vertical direction normal to the sample, the dependency of  
 200 the BRDF on  $\varphi_i$  is not relevant. The dependency on  $\varphi_o$  can be measured

201 partially, as  $\varphi_o$  is either 0 or  $\pi$ . The BRDF can then be written (omitting  
 202 the polarization):

$$f_r(\theta_i, \theta_o, \varphi_o) = \frac{dL_o(\theta_o, \varphi_o)}{dE_i(\theta_i)} \quad (6)$$

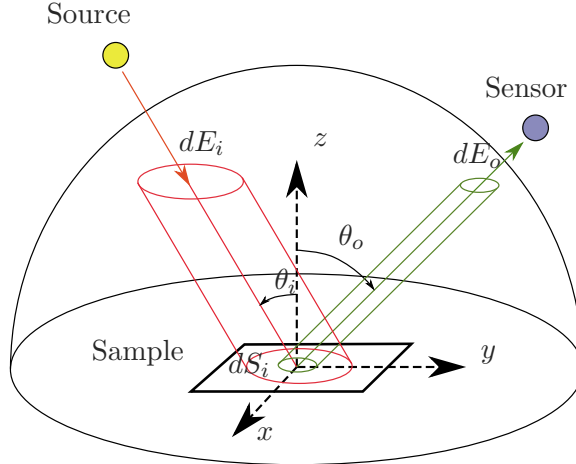


Figure 5: Illustration of the spatial variables used in the article.

203 According to the definition, a BRDF has to check the two different proper-  
 204 ties:

- Helmholtz reciprocity:

$$f_r(\theta_i, \varphi_i, \theta_o, \varphi_o) = f_r(\theta_o, \varphi_o, \theta_i, \varphi_i) \quad (7)$$

- Compatibility with energy conservation:

$$\int_{2\pi} f_r(\theta_i, \varphi_i, \theta_o, \varphi_o) \cos \theta_o d\omega_o \leq 1. \quad (8)$$

### 206 3.2. BRDF measurements

#### 207 3.2.1. Measurement protocol

208 The study presented in this article focuses on a sample of a white exterior  
 209 cladding material from a recognized brand. In the following, the sample will  
 210 be called "the sample".

211 Four different angles of incidence were used:  $\theta_{inc} = 20^\circ, 30^\circ, 45^\circ, 55^\circ$ . The  
 212 angle of incidence is noted  $\theta_{inc}$ .

213 For each angle of incidence, different angles of measure ( $\theta_o$ ) are used, stag-  
 214 gered in steps of  $\theta_{step} = 3^\circ$ . Because of the physical dimensions of the  
 215 apparatus, the measurement angle cannot be closer than  $15^\circ$  to the angle of

216 incidence. Table 6 indicates the number of measurements made for each con-  
 217 figuration. The first column titled  $\theta_{inc}$  corresponds to the angle of incidence  
 218 of the sample. The columns  $\theta_{mes,ini}$  and  $\theta_{mes,fin}$  correspond respectively to  
 219 the minimum and maximum angles (with respect to the direction of inci-  
 220 dence) for which a reflectance measurement was made.  $\theta_{step}$  indicates the  
 221 measurement angle increment.

$\theta_{inc}$	$\theta_{mes,ini}$	$\theta_{mes,fin}$	$\theta_{step}$
20	15	105	3
30	15	114	3
45	15	126	3
55	15	141	3

Figure 6: Table of different measurement configurations.

### 222 3.2.2. Raw measures

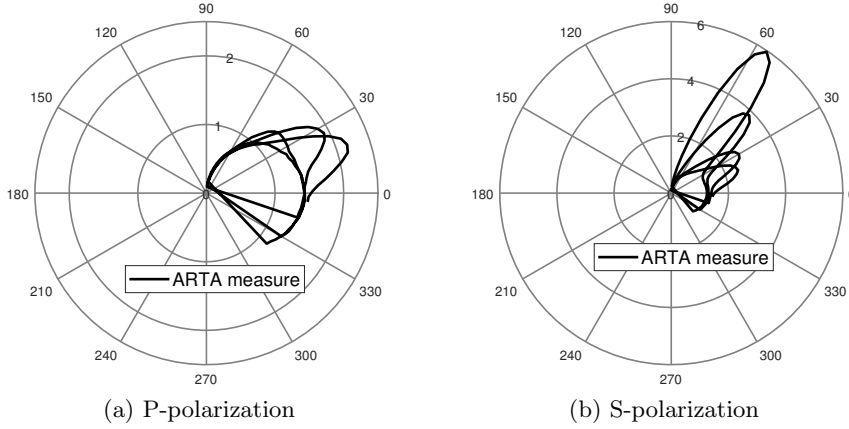
223 In this article, only results obtained with one white cladding material are  
 224 presented, see Figure 7. A behavior similar to what is discussed below has  
 225 been observed for other samples, with varying proportions of the diffuse  
 226 and specular components. On the measurements carried out, the energy  
 227 distribution measured in reflection appears to be dependent on the S or P  
 228 polarization of the incident light.

229 For both polarizations, a diffuse behaviour (the circular part of the curve),  
 230 mixed with a specular-like one is observed. The intensity of the peaks de-  
 231 creases with the angle of incidence for the P-polarization and increases for  
 232 the S-polarization. The position of the peak appears to match the theoret-  
 233 ical reflection angle given by the Snell–Descartes laws, revealing a geometric  
 234 effect in the energy distribution.

235 The amplitude of this reflection presents different attenuation phenomena  
 236 depending on the polarization. For an increasing angle of incidence, the  
 237 reflection of the P-polarized light decreases while that of the S-polarized light  
 238 increases. This observation recalls the laws of geometric optics described  
 239 by Fresnel. However, this specular effect is not geometrically perfect but  
 240 appears to be smeared around the theoretical angle of reflection.

### 241 3.2.3. Measurements reading

242 The ARTA measures the fraction  $r$  of the energy reflected by the sample  
 243 that is intercepted by the sensor. This is found (at least for the cladding  
 244 sample) to depend on the angle on incidence  $\theta_i$ , the angle of observation  $\theta_o$   
 245 and the polarization of light  $x$ . With  $dE_i$  the incident energy ( $\text{W}/\text{m}^2$ ) and  
 246  $dE_o$  the energy intercepted by the sensor that has an angle of aperture  $d\omega_c$



(a) P-polarization (b) S-polarization  
 Figure 7: Measurements from ARTA: percentage of incident energy reflected by the sample

247 this is written:

$$r(\theta_i, \theta_o, x) = \frac{dE_o}{dE_i} = \frac{dL_o d\Omega}{dE_i} = f_r(\theta_i, \theta_o, x) d\omega_c. \quad (9)$$

248 In the following, the link between the BRDF and ARTA measurements is  
 249 established, then the a model BRDF is built, adapted from the Fresnel  
 250 equations for the specular part.

### 251 3.3. BRDF model construction

252 In computer graphics, as Kumar [33] points out in his article, it is usual to  
 253 consider that the light reflection on a given surface is broken down into three  
 254 modes (Figure (8)). These three terms are a specular term, a uniform term  
 255 and a bright term. If the reflecting surface is itself a light source, a fourth  
 256 term is considered. It represents the energy emitted by the surface itself.  
 257 The uniform component describes the set of non-directional reflections, while  
 258 the specular and bright components describe the set of reflections with a  
 259 direction-dependent probability. The term specular describes the reflection  
 260 resulting from Snell-Descartes laws. This mode of reflection is similar to  
 261 a Dirac function in the theoretical direction of reflection. The term glossy  
 262 represents a modulation around this theoretical direction.

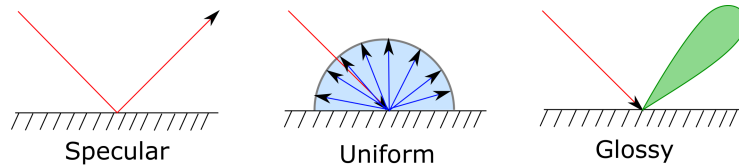


Figure 8: Schematic representation of the light reflection mode.

From the observed behavior, our BRDF model will have a diffuse part and a smeared-specular term, similar to the Blinn-Phong model [24], with the reflected radiant intensity written:

$$I_o = I_o^s + I_o^d. \quad (10)$$

### 263 3.3.1. Fresnel equations

Fresnel equations are used to describe the energy distribution of an electromagnetic radiation at the interface of two optical materials. These equations detail the difference in polarization for reflected and transmitted energy fluxes. These equations are verified under the following assumptions: the interface between the two media is flat and both media are homogeneous and isotropic. Here, as both media are non-magnetic, their impedances only depend on the refractive indices of the two media ( $Z_i = \frac{Z_0}{n_i}$ ) and the refractive index of air is  $n_1 = 1$ . Eventually, only the reflectance behaviour is of interest here and writes:

$$R_s(n_2, \theta_i, \theta_o) = \left| \frac{\cos \theta_i - n_2 \sqrt{1 - \left(\frac{1}{n_2} \sin \theta_i\right)^2}}{\cos \theta_i + n_2 \sqrt{1 - \left(\frac{1}{n_2} \sin \theta_i\right)^2}} \right|^2 \quad (11)$$

$$R_p(n_2, \theta_i, \theta_o) = \left| \frac{\sqrt{1 - \left(\frac{1}{n_2} \sin \theta_i\right)^2} - n_2 \cos \theta_i}{\sqrt{1 - \left(\frac{1}{n_2} \sin \theta_i\right)^2} + n_2 \cos \theta_i} \right|^2 \quad (12)$$

264 Although it is not transparent, the sample has an extinction angle : for a  
 265 given angle of incidence, the P-polarized light is not reflected. This angle,  
 266 noted  $\theta_B$  is called the Brewster angle:  $\theta_B = \text{atan}(n_2)$ . However, in the  
 267 absence of fine increments in the angle of incidence around the Brewster  
 268 angle, this relationship is impractical for determining the refractive index  
 269  $n_2$  of the sample and in the following,  $n_2$  will be kept as a parameter of our  
 270 BRDF model.

### 271 3.3.2. Model for the smeared-specular term

The specular part is supposed to follow Fresnel's law hence:  $\Phi_o^s = k_s R_x \Phi_i$ , with  $R_x$  the Fresnel reflection coefficient for polarization  $x$ . Furthermore, the radiant intensity (W/sr) is supposed be a lobe centered on the specular direction:

$$dI_o^s(\theta_o, \theta_i, x) = \rho_s R_x \frac{\alpha + 1}{2\pi} \left( \cos_+(\theta_s) \right)^\alpha E_i S_i, \quad (13)$$

with  $\theta_s$  the azimuthal angle defined from the specular direction. For values of  $\theta_s$  larger than  $\pi/2$ , there is no contribution of specular reflection, as noted by the use of the positive part of the cosine. As in the Lafortune model, the specular term is described by two coefficients  $\rho_s$  and  $\alpha$ . The coefficient  $\rho_s$  corresponds to the magnitude of the specular reflection whereas the coefficient  $\alpha$  corresponds to the glossy effect of the specular reflectance and governs the width of the glossy lobe while  $\frac{\alpha+1}{2\pi}$  is a ponderation factor. This specular model results in the following BRDF:

$$f_s(\theta_i, \theta_o, x) = \rho_s R_x \frac{\alpha + 1}{2\pi} \left( \cos_+ \left( \theta_s(\theta_i, \theta_o) \right) \right)^\alpha \quad (14)$$

### 272 3.3.3. Model for the diffuse term

273 The diffuse term corresponds to the incident light being scattered in all  
 274 directions, with the same probability and independently of the polarization.  
 275 However, the radiant intensity depends on the observation angle. This is  
 276 translated into Lambert's cosine law. A part  $\rho_d$  of the incident flux is re-  
 277 emitted uniformly in all directions:

$$dI_o^d(\theta_o) = \rho_d \frac{1}{\pi} \cos(\theta_o) E_i S_i, \quad (15)$$

associated with a diffuse part for the BRDF:

$$f_d(\theta_i, \theta_o) = \rho_d \frac{1}{\pi} \cos(\theta_o). \quad (16)$$

### 278 3.3.4. BRDF model and interpretation of the ARTA outputs

Both modes of reflection are brought together in the following BRDF model:

$$f_r(\theta_i, \theta_o, x) = \rho_d \frac{1}{\pi} \cos(\theta_o) + \rho_s R_x \frac{\alpha + 1}{2\pi} \left( \cos_+ \left( \theta_s(\theta_i, \theta_o) \right) \right)^\alpha. \quad (17)$$

The ARTA output is a ratio of energy, that is  $r = \Phi_o/\Phi_i$ . Assuming an infinitesimal angle of aperture  $d\omega_c$ ,  $\Phi_o = I_o d\omega_c$ . The ARTA measurements are performed with the receiver in the plane of incidence, that implies  $\theta_s = -\theta_o \cos(\phi_o) - \theta_i$  when  $\phi_o = \pi$  and from there:

$$r(\theta_i, \theta_o, \phi_o, x) = \left( \rho_d \frac{1}{\pi} \cos(\theta_o) + \rho_s R_x \frac{\alpha + 1}{2\pi} \left( \cos_+(\theta_o - \theta_i) \right)^\alpha \right) d\omega_c. \quad (18)$$

### 279 3.3.5. reflection coefficient calculation

280 For the two BRDF components and for each polarization, the reflection co-  
 281 efficients are obtained by integrating the BRDF on the upper half-space of



282 the sample. They depend on the wavelength. Their dependence in polar-  
 283 ization and angle of incidence are taken into account through the Fresnel  
 284 coefficient (Equation 19) :

$$\begin{cases} k_d(x) = \int_{\theta_o=0}^{\pi/2} \int_{\varphi_i=0}^{2\pi} f_d(\theta_i, \theta_o, \phi_o, x) \sin \theta_o d\theta_o d\phi_o = \rho_d(x) \\ k_s(\theta_i, x) = \int_{\theta_o=0}^{\pi/2} \int_{\varphi_i=0}^{2\pi} f_s(\theta_i, \theta_o, \phi_o, x) \sin \theta_o d\theta_o d\phi_o = \rho_s R_x(\theta_i) \end{cases} \quad (19)$$

285 The global reflection coefficient is obtained by adding the diffuse and spec-  
 286 ular reflection coefficients (Equation 20) :

$$k_g(\theta_i, x) = k_d(x) + k_s(\theta_i, x). \quad (20)$$

287 in order to take both polarizations into account in the global reflection co-  
 288 efficient, the contribution of each polarization is averaged (Equation 21):

$$k_g(\theta_i) = \frac{k_g(\theta_i, S) + k_g(\theta_i, P)}{2}. \quad (21)$$

289 Those reflection coefficients still depend on the wavelength. When used in  
 290 Section 5, they are averaged over the visible spectral range.

### 291 3.4. Model fitting

#### 292 3.4.1. Model fitting for a small aperture sensor

293 To fit the parameters of the BRDF model, a least squares method is used  
 294 between Eq (18) and the ARTA measures. The model is fitted over the entire  
 295 set of measurements, i.e. all reflection and incidence angles. Concerning  
 296 the polarization, the model is approximated either on both polarizations  
 297 at the same time, or one at a time. The approximation of the model is  
 298 performed using the Matlab function `lsqcurvefit`. This procedure lead  
 299 to the identification of the parameters in our BRDF model. However, the  
 300 smeared-specular term being quite narrow in width and the ARTA sensor  
 301 having a rather large aperture are in contradiction with Eq (18) therefore it  
 302 is substituted with a model taking into account the variability of the BRDF  
 303 over the width of the ARTA sensor.

#### 304 3.4.2. Correction of the model fitting

305 In this study, the sensor used by the ARTA has an aperture of  $26mm$  in  
 306 the plane of incidence and the height of this aperture is  $18mm$ . In this  
 307 configuration, this corresponds to an aperture ranging from  $-13^\circ$  to  $+13^\circ$   
 308 with respect to the defined measurement angle. Along to the axis orthogo-  
 309 nal to the measurement plane, the opening angle is from  $-9^\circ$  to  $+9^\circ$  with  
 310 respect to the fixed measurement angle. This gives an opening angle of the

311 solid-state sensor  $d\omega_c = 0.0516$  sr. The variation of the modelled BRDF  
 312 over the aperture angle can be taken into account. By minimizing the dif-  
 313 ference between the theoretical and actual measurements, the calibration of  
 314 the BRDF model coefficients can be refined. Taking into account the aper-  
 315 ture according to the azimuth angle only, the following relation is obtained  
 316 (Equation 22) :

$$r(\theta_i, \theta_o, \phi_o, x) = \frac{d\omega_c}{2\Delta\theta} \int_{\theta_o-\Delta\theta}^{\theta_o+\Delta\theta} f_r(\theta_i, \theta, \phi_o, x) d\theta, \quad (22)$$

317 This new relationship (22) is used in the model fitting procedure, instead of  
 318 (18).

### 319 *3.4.3. Estimating the uncertainties*

320 By performing a non-linear regression, it is possible to evaluate the uncer-  
 321 tainties of the parameters of the fitted model, i.e.  $\rho_s$ ,  $\rho_p$  and  $\alpha$ , as well as  
 322 possibly  $n_2$ . The use of the function `nlparcy` proposed by Matlab<sup>®</sup> pro-  
 323 vides the confidence intervals 95% for each coefficient. By injecting these  
 324 uncertainties into the coefficient calculations, we can evaluate the uncer-  
 325 tainties for the coefficients of the final Lambertian and specular reflections.  
 326 However, the function `nlparcy` Matlab<sup>®</sup> requires the covariance matrix or  
 327 the Jacobian matrix from the BRDF data approximation. A use of the func-  
 328 tion `nlinfit` (also from Matlab<sup>®</sup>) is then necessary. For each wavelength,  
 329 the uncertainties obtained on the different coefficients are propagated in the  
 330 calculation of the final reflection coefficients.

## 331 **4. Results: BRDF of a white cladding HPL pannel**

### 332 *4.1. Reference reflection coefficients*

333 In order to verify the results obtained thanks to the BRDF model fitting, the  
 334 samples are also tested using an integrating sphere mounted in a Lambda  
 335 950 Perking Elmer<sup>®</sup>. This device measures the overall reflection coefficient  
 336 (diffused plus specular) depending on the wavelength, under an incidence  
 337  $\theta_i = 8^\circ$ . The integrating sphere has an uncertainty of 1%. These measure-  
 338 ments are then compared to the global reflection coefficient obtained from  
 339 the BRDF fitting, as reconstituted using the ARTA measures, for different  
 340 wavelengths, under the same incidence.

### 341 *4.2. Results of model fitting*

342 *Results by polarization.* The obtained BRDF, after model fitting, is plot-  
 343 ted for each polarization (Figure 9). However, for the cladding sample, the

344 differences between corrected and uncorrected models, remain tiny and im-  
 345 perceptible. Concerning the P polarization, and for an angle of incidence  
 346 of  $45^\circ$ , the model presents difficulties in correctly fitting the measurements.  
 347 The specular reflection seems to be too important in this configuration only.

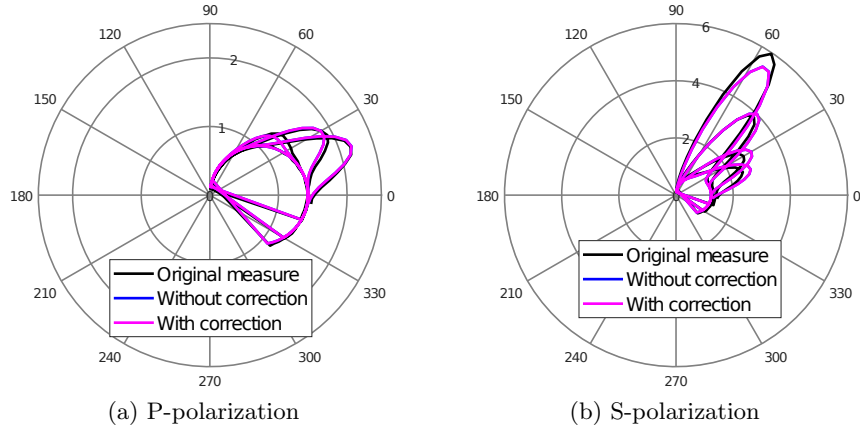


Figure 9: Fitted model from the ARTA measurements

348 The analysis of the reflection coefficients reconstructed from the measure-  
 349 ments and the model reveals a sharp change around 860nm. This wavelength  
 350 corresponds to a sensor change in the ARTA. Depending of the sensor, the  
 351 global reflection coefficients obtained are either above or below the reflec-  
 352 tion coefficient obtained with the integrating sphere, especially for the P-  
 353 polarization. Similar fluctuations depending of the wavelength are observed.  
 354 These fluctuations are essentially driven by the diffuse coefficient, the spec-  
 355 ular coefficient being constant over the wavelengths (Figure 10).

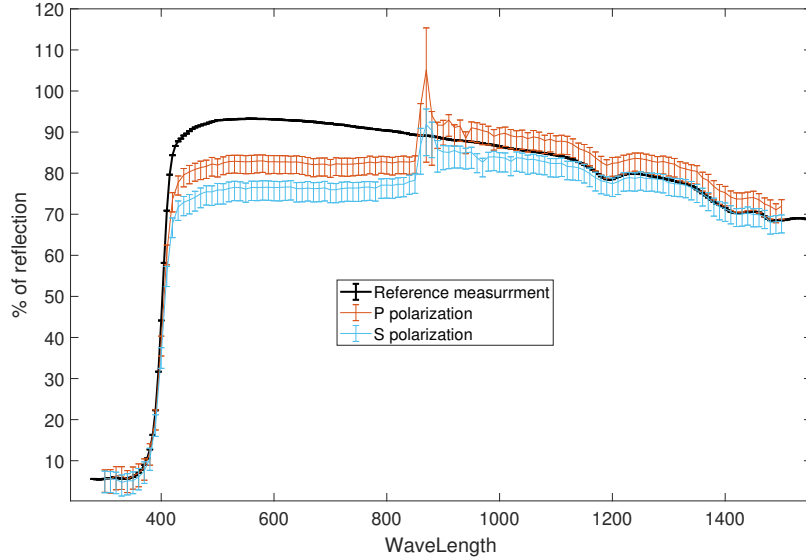


Figure 10: Global reflection coefficients for both polarization depending of the wavelength obtained with the BRDF model, for non corrected and corrected parameters, compared to the Integrating Sphere measures.

356 *Final coefficients.* For each wavelength, the model coefficients are approxi-  
 357 mated. Then, for both non corrected and corrected parameters, the model  
 358 allows us to obtain the reflection coefficients as a function of the wavelength.  
 359 reflection coefficients can be calculated as a function of polarization (Figure  
 360 10), or averaged over the polarizations as shown Figure 11. The reflection  
 361 coefficient appears to be largely underestimated before the 860nm gap, with  
 362 10% of reflection less than the reference measurement. But the estimation  
 363 is inside the uncertainty margin after this jump. The jump is due to the  
 364 impact of the diffused reflection coefficient which is the principal influencer  
 365 for the global reflection coefficient, whereas the specular reflection coefficient  
 366 is relatively constant (around 4) over all wave lengths, except at 870nm and  
 367 880nm, near to the sensor change.

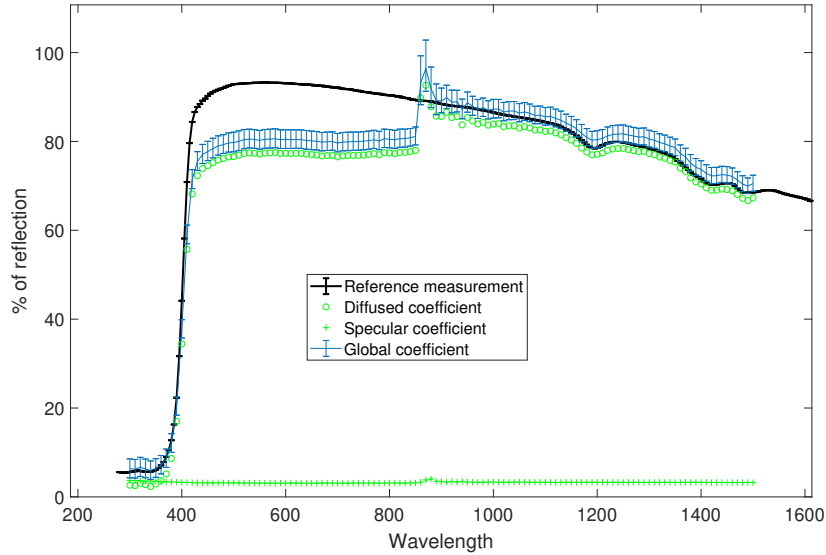
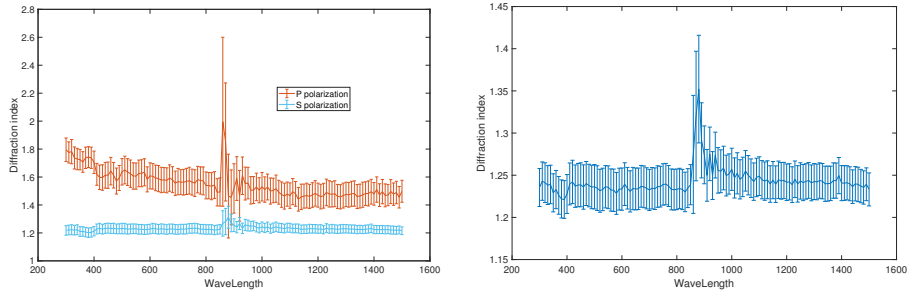


Figure 11: Global reflection coefficients for both polarization, depending of the wavelength, obtained with the BRDF model and compared to the Integrating Sphere measures.

368 Regarding the refractive index of the sample surface material, the trends  
 369 are varied between the two polarizations. If the refractive index is almost  
 370 constant for the S polarization, the refractive index is decreasing with longer  
 371 wavelength for the P polarization (Figure 12). This behavior (having a  
 372 different refraction index for both polarizations) is observed for birefringing  
 373 materials. However, here, the two different values come from identifying a  
 374 parameter inside a model and are too far apart for the sample material to be  
 375 considered as birefringing. The refractive index in the model can be forced  
 376 to the same value for both polarizations and this yields a value close to 1.24  
 377 (Figure 12). Again, sharp changes around the 860nm wavelength due to  
 378 sensor change are observed.



(a) Diffraction index for P and S polarizations (b) Diffraction index obtained for both polarizations.

Figure 12: Diffraction index fitted from the ARTA measurements.

### 379 5. Discussion: using the BRDF in a solar irradiation simulation

380 Raybaud's thesis [11] proposes a simulation of incident irradiance with re-  
 381 flection models mixing specular and diffuse. As mentioned in Section 4,  
 382 the reflection coefficients used in the solar irradiation models at this stage  
 383 are integrated over all wavelengths. Both reflections are simulated using a  
 384 Monte Carlo model coded in Matlab<sup>®</sup>. This simplified simulation assumes  
 385 that the facade surfaces are lambertian (reflection coefficient of 72%) and  
 386 specular (reflection coefficient of 4%). These values correspond to the values  
 387 identified for the white cladding. The energy gains from specular reflections  
 388 are therefore evaluated from a ray-tracing simulation taking into account  
 389 only the specular part of the reflections. These contributions are visible but  
 390 low (about 30W/m<sup>2</sup>, Figure 13).

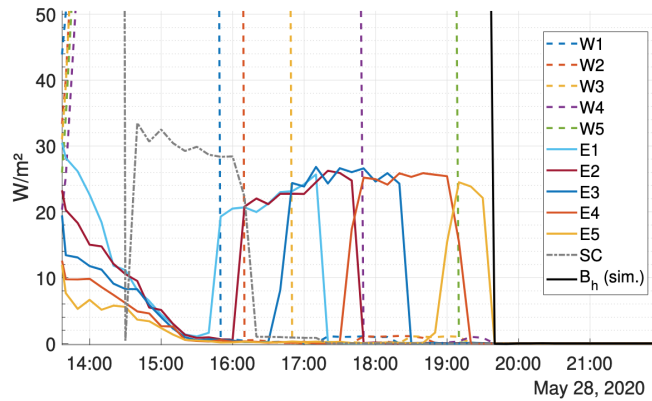


Figure 13: Energy contribution of specular reflections evaluated with a reflection coefficient of 4% corresponding to the identified value in the case of white cladding (from Raybaud [11])

391 The interest in making reflection models more complex in order to take  
 392 into account different reflection modes seems limited, especially for large-

393 scale simulations. Windows can generate significant reflections at certain  
394 angles of incidence and a better understanding of the reflection patterns of  
395 materials in the urban environment could provide more information on this  
396 point. On this point, these models can also be of interest for glare and  
397 visual comfort issues. They can also help to better simulate the spectrum  
398 incident on photovoltaic modules in order to better assess their integration  
399 and camouflage into the urban landscape.

## 400 6. Conclusion

401 This study considers the influence of light wavelength and polarization, angle  
402 of incidence and angle of observation on the reflectance of a cladding material  
403 used for building construction. This material appears highly reflective (to  
404 the eye) at grazing angles of incidence, rising the need for a BRDF model  
405 that is able to reproduce such a behavior. Inspired by the Fresnel equations,  
406 the specular term combined with a usual diffuse term in our BRDF model  
407 are able to match the observations carried out on the ARTA device. The  
408 largest discrepancy between the reconstructed BRDF and the measurements  
409 with the integrating sphere lies in the 400 – 860nm range of wavelength. Our  
410 best bet is that our calibration for this particular sensor is not accurate.

411 The results confirm the need to consider polarization and angular depen-  
412 dency when considering physical modeling of reflections, especially for angu-  
413 lar energy distribution. This is usually done through ray-tracing algorithms,  
414 where a BRDF model is required. A new BRDF model, integrating the po-  
415 larization dependence based on the Fresnel coefficients, is then proposed  
416 and validated on different samples. This model has only four parameters  
417 for each wavelength and polarization. Considering the usual split between  
418 diffuse and specular reflectance in light simulation, these four parameters  
419 can be reduced to two (diffuse and specular global reflection coefficients)  
420 keeping the same energy ratios.

421 With only two parameters (governing importance sampling in RT algo-  
422 rithms), our BRDF model can be used to verify the influence of the material  
423 properties on the energy distribution in an urban district, when the effect  
424 of such materials (sharp angular reflectance variations) is investigated.

425 One obvious question for further research is the wavelength and polarisation  
426 dependency of the BRDF. While it might not be of importance for energy  
427 simulations, visual integration of PV panels in the buildings' facades should  
428 strongly rely on color aspects. Then, our material sample has been tested as  
429 new. Exposure to the outdoor weather and solar rays will induce aging and  
430 maybe dirtiness, modifying the optical properties. Some further research is  
431 necessary to confirm that the specular part of the BRDF does not fade out  
432 with time.

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437 **References**

- 438 [1] G. Lobaccaro, F. Frontini, Solar energy in urban envi-  
439 ronment: How urban densification affects existing build-  
440 ings, *Energy Procedia* 48 (2014) 1559–1569. URL: <https://www.sciencedirect.com/science/article/pii/S187661021400438X>.  
441 doi:<https://doi.org/10.1016/j.egypro.2014.02.176>, proceedings  
442 of the 2nd International Conference on Solar Heating and Cooling for  
443 Buildings and Industry (SHC 2013).  
444
- 445 [2] C. Balocco, R. Calzolari, Natural light design for an an-  
446 cient building: A case study, *Journal of Cultural Heritage* 9  
447 (2008) 172–178. URL: <http://www.sciencedirect.com/science/article/pii/S129620740800023X>. doi:10.1016/j.culher.2007.07.007.  
448
- 449 [3] M. C. Brito, S. Freitas, S. Guimarães, C. Catita, P. Red-  
450 weik, The importance of facades for the solar PV potential of  
451 a mediterranean city using LiDAR data, *Renewable Energy* 111  
452 (2017) 85–94. URL: <http://www.sciencedirect.com/science/article/pii/S0960148117302768>. doi:10.1016/j.renene.2017.03.085.  
453
- 454 [4] E. Saretta, P. Bonomo, F. Frontini, A calculation method for the bipy  
455 potential of swiss façades at lod2.5 in urban areas: A case from ti-  
456 cino region, *Solar Energy* 195 (2020) 150–165. URL: <https://www.sciencedirect.com/science/article/pii/S0038092X19311624>. doi:<https://doi.org/10.1016/j.solener.2019.11.062>.  
457  
458
- 459 [5] G. Lobaccaro, S. Carlucci, S. Croce, R. Paparella, L. Finocchiaro,  
460 Boosting solar accessibility and potential of urban districts in the  
461 nordic climate: A case study in trondheim, *Solar Energy* 149  
462 (2017) 347–369. URL: <https://linkinghub.elsevier.com/retrieve/pii/S0038092X17302992>. doi:10.1016/j.solener.2017.04.015.  
463
- 464 [6] Z.-H. Wang, Monte carlo simulations of radiative heat ex-  
465 change in a street canyon with trees, *Solar Energy* 110  
466 (2014) 704–713. URL: <http://www.sciencedirect.com/science/article/pii/S0038092X14004988>. doi:10.1016/j.solener.2014.10.012.  
467
- 468 [7] P. Bijl, A. Heikkilä, S. Syrjälä, A. Aarva, A. Poikonen, Mod-  
469 elling of sample surface temperature in an outdoor weath-  
470 ering test, *Polymer Testing* 30 (2011) 485–492. URL:  
471 <http://linkinghub.elsevier.com/retrieve/pii/S014294181100047X>.  
472 doi:10.1016/j.polymertesting.2011.03.009.
- 473 [8] M. Manni, G. Lobaccaro, F. Goia, A. Nicolini, F. Rossi, Exploit-  
474 ing selective angular properties of retro-reflective coatings to miti-  
475 gate solar irradiation within the urban canyon, *Solar Energy* 189

- 476 (2019) 74–85. URL: <http://www.sciencedirect.com/science/article/pii/S0038092X19307078>. doi:10.1016/j.solener.2019.07.045.
- 477
- 478 [9] A. Vallati, L. Mauri, C. Colucci, P. Ocloń, Effects of radiative  
479 exchange in an urban canyon on building surfaces’ loads and tem-  
480 peratures, *Energy and Buildings* 149 (2017) 260–271. URL: <http://www.sciencedirect.com/science/article/pii/S0378778817311076>.  
481 doi:10.1016/j.enbuild.2017.05.072.  
482
- 483 [10] M. Langovoy, G. Wubbelier, Empirical BRDF models for standard ref-  
484 erence materials, *Euramet JRP-i21 Deliverable 4.2.1* (2013) 9.
- 485 [11] B. Raybaud, Evaluation de l’impact des propriétés optiques large-  
486 bande de l’environnement sur le productible (énergie incidente) en mi-  
487 lieu urbain, phdthesis, Université de Lyon, 2020. URL: <https://tel.archives-ouvertes.fr/tel-03178836>.  
488
- 489 [12] E. A. Yafei, V. Badler, N. Badler, J. T. K. Jr, Accurate graphics BRDF  
490 material properties and illumination for dirt and mudbrick structures,  
491 in: *International Conference on Cultural Heritage and New Technolo-  
492 gies — Vienna — 2019*, 2019.
- 493 [13] B. Wang, W. S. Koh, H. Liu, J. Yik, V. P. Bui, Simulation  
494 and validation of solar heat gain in real urban environments,  
495 *Building and Environment* 123 (2017) 261–276. URL: <https://www.sciencedirect.com/science/article/pii/S0360132317302925>.  
496 doi:10.1016/j.buildenv.2017.07.006.  
497
- 498 [14] J. Yuan, K. Emura, C. Farnham, Geometrical-optics analysis of reflec-  
499 tive glass beads applied to building coatings, *Solar Energy* 122 (2015)  
500 997–1010. URL: <https://www.sciencedirect.com/science/article/pii/S0038092X15005666>. doi:10.1016/j.solener.2015.10.015.  
501
- 502 [15] Y. Wu, J. H. Kämpf, J.-L. Scartezzini, Performance assessment of  
503 the BTDF data compression based on wavelet transforms in day-  
504 lighting simulation, *Solar Energy* 190 (2019) 329–336. URL: <http://www.sciencedirect.com/science/article/pii/S0038092X19307686>.  
505 doi:10.1016/j.solener.2019.07.096.  
506
- 507 [16] E. S. Lee, D. Geisler-Moroder, G. Ward, Modeling the direct sun  
508 component in buildings using matrix algebraic approaches: Meth-  
509 ods and validation, *Solar Energy* 160 (2018) 380–395. URL: <http://www.sciencedirect.com/science/article/pii/S0038092X17311118>.  
510 doi:10.1016/j.solener.2017.12.029.  
511
- 512 [17] J. M. Jurado, J. R. Jiménez-Pérez, L. Pádua, F. R. Feito,  
513 J. J. Sousa, An efficient method for acquisition of spec-

- 514 tral BRDFs in real-world scenarios, *Computers & Graph-*  
515 *ics* (2021). URL: [https://www.sciencedirect.com/science/article/pii/](https://www.sciencedirect.com/science/article/pii/S0097849321001850)  
516 [S0097849321001850](https://www.sciencedirect.com/science/article/pii/S0097849321001850). doi:10.1016/j.cag.2021.08.021.
- 517 [18] M. Yang, W. Xu, J. Li, Z. Zhou, Y. Lu, A modified version of  
518 BRDF model based on kubelka-munk theory for coating materials,  
519 *Optik* 193 (2019) 162982. URL: [http://www.sciencedirect.com/science/](http://www.sciencedirect.com/science/article/pii/S0030402619308599)  
520 [article/pii/S0030402619308599](http://www.sciencedirect.com/science/article/pii/S0030402619308599). doi:10.1016/j.ijleo.2019.162982.
- 521 [19] R. Guerrero-Lemus, R. Vega, T. Kim, A. Kimm, L. Shephard,  
522 Bifacial solar photovoltaics – a technology review, *Renewable*  
523 *and Sustainable Energy Reviews* 60 (2016) 1533–1549. URL:  
524 <https://linkinghub.elsevier.com/retrieve/pii/S1364032116002768>.  
525 doi:10.1016/j.rser.2016.03.041.
- 526 [20] M. Ayoub, A review on light transport algorithms and simula-  
527 tion tools to model daylighting inside buildings, *Solar Energy* 198  
528 (2020) 623–642. URL: [http://www.sciencedirect.com/science/article/](http://www.sciencedirect.com/science/article/pii/S0038092X20301250)  
529 [pii/S0038092X20301250](http://www.sciencedirect.com/science/article/pii/S0038092X20301250). doi:10.1016/j.solener.2020.02.018.
- 530 [21] C. Collin, S. Pattanaik, P. LiKamWa, K. Bouatouch, Dis-  
531 crete ordinate method for polarized light transport solution and  
532 subsurface BRDF computation, *Computers & Graphics* 45  
533 (2014) 17–27. URL: [http://www.sciencedirect.com/science/article/pii/](http://www.sciencedirect.com/science/article/pii/S009784931400082X)  
534 [S009784931400082X](http://www.sciencedirect.com/science/article/pii/S009784931400082X). doi:10.1016/j.cag.2014.09.002.
- 535 [22] B. T. Phong, Illumination for computer generated pictures, *Communi-*  
536 *cations of the ACM* 18 (1975) 311–317. URL: [http://portal.acm.org/](http://portal.acm.org/citation.cfm?doid=360825.360839)  
537 [citation.cfm?doid=360825.360839](http://portal.acm.org/citation.cfm?doid=360825.360839). doi:10.1145/360825.360839.
- 538 [23] E. P. F. Lafortune, S.-C. Foo, K. E. Torrance, D. P. Green-  
539 berg, Non-linear approximation of reflectance functions, in:  
540 *Proceedings of the 24th annual conference on Computer graphics*  
541 *and interactive techniques - SIGGRAPH '97*, ACM Press, 1997,  
542 pp. 117–126. URL: [http://portal.acm.org/citation.cfm?doid=258734.](http://portal.acm.org/citation.cfm?doid=258734.258801)  
543 [258801](http://portal.acm.org/citation.cfm?doid=258734.258801). doi:10.1145/258734.258801.
- 544 [24] J. F. Blinn, Models of light reflection for computer synthesized pictures,  
545 *SIGGRAPH Comput. Graph.* 11 (1977) 192–198. URL: [https://doi.](https://doi.org/10.1145/965141.563893)  
546 [org/10.1145/965141.563893](https://doi.org/10.1145/965141.563893). doi:10.1145/965141.563893.
- 547 [25] R. L. Cook, A reflectance model for computer graphics, *ACM Trans-*  
548 *actions on Graphics* 1 (1982) 18.
- 549 [26] K. E. Torrance, E. M. Sparrow, Theory for off-specular reflection from  
550 roughened surfaces\*, *Journal of the Optical Society of America* 57  
551 (1967) 1105. URL: [https://www.osapublishing.org/abstract.cfm?URI=](https://www.osapublishing.org/abstract.cfm?URI=josa-57-9-1105)  
552 [josa-57-9-1105](https://www.osapublishing.org/abstract.cfm?URI=josa-57-9-1105). doi:10.1364/JOSA.57.001105.

- 553 [27] M. Born, E. Wolf, A. B. Bhatia, P. C. Clemmow, D. Gabor, A. R.  
554 Stokes, A. M. Taylor, P. A. Wayman, W. L. Wilcock, Principles of Op-  
555 tics: Electromagnetic Theory of Propagation, Interference and Diffrac-  
556 tion of Light, 7 ed., Cambridge University Press, 1999. doi:10.1017/  
557 CBO9781139644181.
- 558 [28] C. Kelemen, L. Szirmay-Kalos, A microfacet based coupled specular-  
559 matte brdf model with importance sampling, In Eurographics Short  
560 Presentations 25 (2001).
- 561 [29] P. Nijnatten, Optical analysis of coatings by variable angle spec-  
562 trophotometry, Thin Solid Films 516 (2007) 4553–4557. URL: <https://www.sciencedirect.com/science/article/pii/S0040609007009145>.  
563 doi:10.1016/j.tsf.2007.06.027.  
564
- 565 [30] D. Xie, T. Cheng, Y. Wu, H. Fu, R. Zhong, J. Yu, Polarized reflectances  
566 of urban areas: Analysis and models, Remote Sensing of Environment  
567 193 (2017) 29–37. URL: [http://www.sciencedirect.com/science/article/  
568 pii/S0034425717300858](http://www.sciencedirect.com/science/article/pii/S0034425717300858). doi:10.1016/j.rse.2017.02.026.
- 569 [31] R. Montes, C. Ureña, An Overview of BRDF Models, Technical Re-  
570 port Technical Report LSI-2012-001, University of Granada, Granada,  
571 Spain, 2012.
- 572 [32] W. Shi, J. Zheng, Y. Li, X. Li, Q. An, Measurement and modeling  
573 of bidirectional reflectance distribution function (BRDF) on cutting  
574 surface based on the coaxial optical microscopic imaging, Optik 170  
575 (2018) 278–286. URL: [http://www.sciencedirect.com/science/article/  
576 pii/S0030402618306053](http://www.sciencedirect.com/science/article/pii/S0030402618306053). doi:10.1016/j.ijleo.2018.04.111.
- 577 [33] H. Kumar, J. Ramkumar, K. S. Venkatesh, Surface texture  
578 evaluation using 3d reconstruction from images by parametric  
579 anisotropic BRDF, Measurement 125 (2018) 612–633. URL: [http:  
580 //www.sciencedirect.com/science/article/pii/S0263224118303725](http://www.sciencedirect.com/science/article/pii/S0263224118303725).  
581 doi:10.1016/j.measurement.2018.04.090.