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# Scattering by a dielectric cylinder with arbitrary cross section using Pseudo-spectral Modal Method

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## Abstract

We present a new semi-analytical formulation for diffraction by structured cylinders. A pseudo-spectral method is used to solve the Maxwell equations written in curvilinear coordinates. The program is compared with the numerical results obtained using Comsol Multiphysics.

## 1. Introduction

The last two decades have been marked by the study and fabrication of structured materials to control light. To solve complex electromagnetic problems, numerical methods are widely used. We propose a semi-analytical model based on the pseudo-spectral method using the Chebyshev differentiation matrix [1, 2, 3, 4]. In this method, Maxwell's equations are written in their covariant forms [5]. For this purpose, the curvilinear coordinate method is used to render an arbitrary-walled cylinder into a cylinder of circular cross-section. We are working in physical space, writing the boundary conditions is very easy.

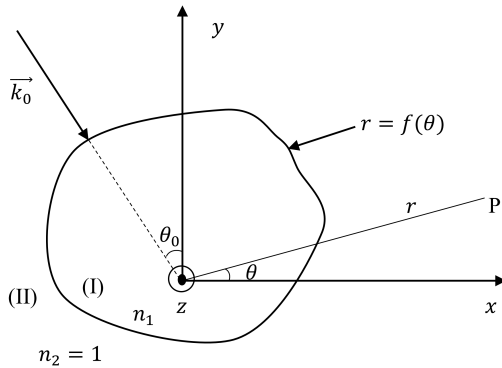


Figure 1: Geometry of the problem.

Since the structure has cylindrical symmetry and  $2\pi$  periodic with  $\theta$ , in the usual cylindrical coordinate the incident field is given by :

$$E_z^i(r, \theta) = E_0 \exp[ik_0 r \sin(\theta - \theta_0)] \quad (1)$$

## 2. resolution

### 2.1. Coordinate change

The change of coordinates transforms a complex section into a circular section. We start from a circular cylinder of radius  $u_1 = R_1$  through the profile  $u = u_2$  to a circular cylinder of radius  $u_3 = R_3$

$$a(u, \theta) = \begin{cases} R_1 + [f(\theta) - R_1] \frac{u - u_1}{u_2 - u_1} & \text{for } u_1 \leq u \leq u_2 \\ f(\theta) + [R_3 - f(\theta)] \frac{u - u_2}{u_3 - u_2} & \text{for } u_2 \leq u \leq u_3 \end{cases} \quad (2)$$

The fields at the bottom of radius R1 and at the top of radius R3 are developed in Fourier-Bessel and Hankel series.

$$E_z^{(1)}(r, \theta) = \sum_{m=-M}^M A1_m J_m(k_0 n_1 r) \exp(im\theta) \quad (3)$$

$$E_z^{(3)}(r, \theta) = \sum_{m=-M}^M \{ [(-i)^m J_m(k_0 r) + A3_m H_m^{(2)}(k_0 r)] \exp(im\theta) \} \quad (4)$$

### 2.2. Maxwell's Covariant Equations

In a homogeneous medium without source, considering the angular frequency  $\omega$  in the harmonic regime  $\exp(-i\omega t)$  and with Einstein's convention, the Maxwell-Minkowski equations are written.

$$\begin{cases} \xi^{ijk} \partial_j E_k = +i\omega B^i \\ \xi^{ijk} \partial_j H_k = -i\omega D^i \end{cases} \quad (5)$$

where  $i, j, k = 1, 2, 3$ .  $\xi^{ijk}$  is the Levi-Civita indicator defined by :

$$\xi^{ijk} = \begin{cases} 0 & \text{if two superscripts are equal} \\ +1 & \text{if } (i, j, k) \text{ is an even permutation of } (1, 2, 3) \\ -1 & \text{if } (i, j, k) \text{ is an odd permutation of } (1, 2, 3) \end{cases}$$

The constitutive equations of the middle are given by :

$$\begin{cases} B^i = \mu_0 \mu_r^{ij} H_j \\ D^i = \epsilon_0 \epsilon_r^{ij} E_j \end{cases} \quad (6)$$

### 2.3. Boundary conditions

For the completeness of the mathematical problem, we need to specify the boundary conditions. The Continuity of the tangential components of the fields are satisfied.

The first boundary condition is the continuity of the field at the frontier of the fictitious inside cylinder.

$$\begin{cases} E_0^- = J_1 A_1 \\ H_0^- = \tilde{J}_1 A_1 \end{cases} \quad (7)$$

The subscript 0 is relative to  $u_0$ . The second boundary condition is the continuity of field in the surface defined by  $u = u_2$ :

$$\begin{cases} E_{N-1}^- = E_0^+ \\ H_{N-1}^- = H_0^+ \end{cases} \quad (8)$$

$N - 1$  corresponds to  $u_{N-1}$ . The third boundary condition is the continuity of the field at the frontier of the fictitious outside cylinder.

$$\begin{cases} E_{N-1}^+ = J_3 + H^{(2)} A_3 \\ H_{N-1}^+ = \tilde{J}_3 + \tilde{H}^{(2)} A_3 \end{cases} \quad (9)$$

### 3. Results

By expanding the equation (5) we obtain a differential matrix. After incorporating the boundary conditions, we obtain a system to be inverted by keeping only the  $J_3$  in the second member.

The radar cross section, which is the relationship of square modules of diffracted and incident fields, is defined as.

$$\sigma = \lim_{r \rightarrow \infty} 2\pi r \frac{|E^{diff}|^2}{|E^{inc}|^2} \quad (10)$$

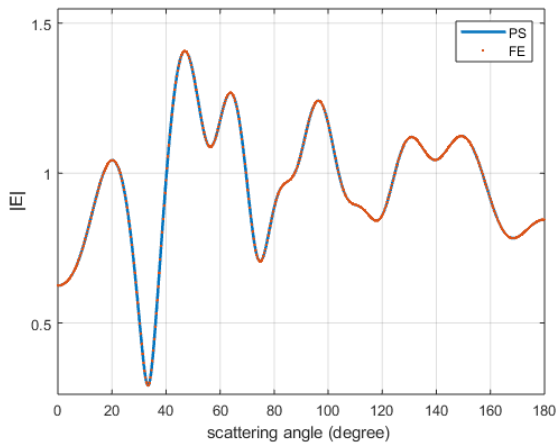


Figure 2: Comparison of the total field surrounding the elliptic cylinder, with large radius  $a = 1\lambda$  and small radius  $b = 0.8\lambda$ , at a distance of  $r = 2.5\lambda$ . Refractive index is  $n_1 = 1.5$ .

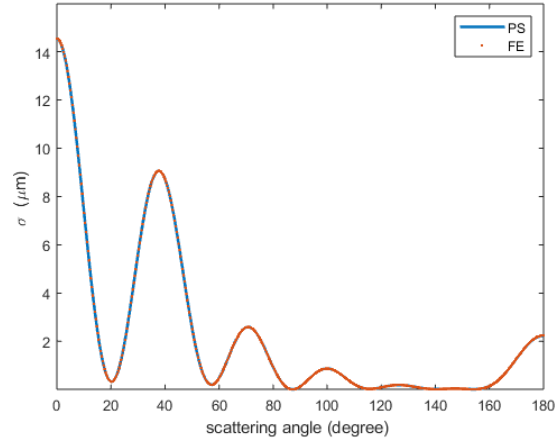


Figure 3: Radar Cross Section of the elliptic cylinder, with large radius  $a = 1\lambda$  and small radius  $b = 0.8\lambda$ , with refractive index  $n_1 = 1.5$

### 4. Conclusions

We have proposed a pseudo-spectral modal method for electromagnetic diffraction. The method gives accurate solutions with the cylinder of arbitrary section. The speed of convergence for pseudo-spectral is greatly than different technical calculation. The specification of this method lies in the boundary condition can be written analytically without requiring any mesh.

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