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Stress–adaptive design of 2D contact interfaces with uniform pressure: a bio–inspired approach

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Abstract

In this article, a bio–mimetic approach to the generative design of contact interfaces with uniform pressure distribution is explored. During the morphogenesis process, biological joints grow depending on the states of stress generating a shape adapted to the mechanical loads. This adaptation is driven by two main rules: shear stress inhibits growth, whereas cyclic hydrostatic compressive stress promotes it. In this work, we demonstrate that the stress–dependent growth rules of synovial joint morphogenesis can be applied to generate contact interfaces with uniform pressure in engineering applications. For that, we present a mathematical model that comprises a contact formulation and a bio–inspired growth function; the model is solved numerically using finite element methods. We analyse the impact of the growth rules of synovial joint development on the contact pressure distribution of two–dimensional contact interfaces. We study the parameter space of the bio–inspired growth function to fine–tune the model parameters. The model is tested in several cases with different boundary conditions and material properties. The results show that the proposed generative design process leads to contact interfaces that provide uniform contact pressure. A quantitative metric of the uniformity of the contact pressure is also defined. This metric indicates that the bio–inspired process generates geometries as good as those obtained with other methods present in the literature without the need of an initial shape close to the final one. Thus, this work demonstrates that synovial joint morphogenesis can be adapted for generative design in engineering.

Keywords: bio–inspired design, generative design, joint morphogenesis, contact interface, stress adaptation

28 **1. Introduction**

29 Synovial joints are characterised by remarkable wear resistance. This performance is attributed
30 to a complex structure that comprises tribological, material, and geometric properties [1]. During
31 morphogenesis, such a structure is formed by a growth process which is influenced not only by
32 chemical but also by mechanical cues [2–4]. For example, in chondrocytes and other cells, me-
33 chanical loads can activate integrins—a class of transmembrane proteins that attach to the extra-
34 cellular matrix—which later activate intracellular signalling pathways that modify the cell activity
35 [5]. In fact, it has been shown that the absence of foetal movement leads to bone and cartilage
36 abnormalities [3, 4]. Hence, synovial joints adapt to mechanical loads as they grow.

37 This load adaptation is driven by two main rules. Cycles of compressive stress promote chon-
38 drocyte proliferation and cartilage repair and maintenance [6, 7]. In contrast, shear stress promotes
39 chondrocyte ossification and cartilage degradation [8, 9]. This means that cyclic compressive
40 stress promotes growth, while shear stress inhibits it and promotes material hardening. The accu-
41 racy of these rules has been proven computationally in several works [10–12] where the authors
42 reproduce synovial joint formation.

43 In mechanical engineering, joints are also common mechanisms for load transfer. They usually
44 undergo dynamic contact loads which expose them to wear. In consequence, material is constantly
45 lost on the surface and the lifetime of these mechanisms is shortened [13]. Further, if the wear rate
46 is not uniformly distributed, certain regions end up being overexposed to material lost and the
47 failure occurs prematurely [14]. Hence, it is convenient to ensure a uniform wear rate.

48 Wear rate depends on the contact pressure, the material properties and the sliding speed of
49 the bodies in contact [13, 15]. At imposed speeds, contact pressure uniformity is therefore a key
50 factor. Nonetheless, the contact analysis is highly non-linear and presents unexpected phenomena.

Declarations of interest: none

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51 For example, the contact between finite cylinders leads to infinite contact pressure at the edges—a
52 phenomenon called edge effect.

53 In theory, this phenomenon can be reduced by crowning the cylinder along its axial direction.
54 Lundberg [16] investigated this phenomenon and found an analytical expression that leads to uni-
55 form contact pressure. Although it is inaccurate close to the edges [17], it has been widely used
56 in engineering and is an important reference profile. For example, it has been used in the shape
57 design of bearing rollers [18, 19] and gear teeth [20]. Nonetheless, since Lundberg’s profile is not
58 adequate for many applications—for example when the contact interface does not comprise any
59 flat surface—other strategies have been developed to address this problem, as described below.

60 With the aim of improving contact pressure distribution several approaches have been inves-
61 tigated. Among them, we can find developments in material properties [21–23], on topology
62 [24–26], on surface texture [27–29] and on shape [30–35]. These works are based on optimisation
63 algorithms following either gradient-based or heuristic strategies. The former requires rigorous
64 mathematical formulations and the computation of derivatives [36]. Although they converge fast,
65 the computation of the gradient may constrain the problem. For example, in [30] the shape of the
66 contour in contact must remain constant, and the optimisation is achieved by modifying the shape
67 of the contours that are not in contact. In addition, gradient-based algorithms are particularly
68 sensitive to the initial guess since they can end up trapped in a local minimum [36]. Concerning
69 heuristic approaches, they can provide almost global solutions, but they usually require a more
70 extensive evaluation of the design space [36]. In addition, for the optimisation of the contact pres-
71 sure distribution, it is sometimes necessary to impose the general shape of the profile, as done in
72 [19, 34]. Further, when the design variable is the contact profile, the modifications are restricted
73 to local variations [19, 32–35]. In consequence, the designer needs to have a certain knowledge
74 of the form of the target shape. Another approach was proposed in [37, 38] where it was shown
75 the feasibility of applying cartilage growth rules in engineering design. Nonetheless, they did not
76 evaluate the uniformity of the contact pressure distribution and they focused on a particular case.

77 In this work, we demonstrate that the stress-dependent growth rules of synovial joint mor-
78 phogenesis can be applied to generate contact interfaces with uniform pressure in engineering
79 applications. We present a mathematical model that comprises a contact formulation and a bio-

80 inspired growth function. We define a quantitative measure for pressure uniformity to evaluate the
 81 obtained contact interfaces. We study the impact of the growth rules of synovial joint develop-
 82 ment on the contact pressure distribution of two-dimensional contact interfaces. Then, we adapt
 83 the growth function for the generative design of contact interfaces and analyse the parameter space
 84 to fine-tune the model parameters. The model is tested in several cases with different boundary
 85 conditions and material properties.

86 The rest of the document is organised as follows. The bio-inspired growth model is presented
 87 in Section 2 and its numerical approximation in Section 3. In Section 4, we establish an analysis of
 88 the parameters to study the behaviour of the growth function and in Section 5 we set up examples
 89 to demonstrate the versatility of the methodology. Finally, in Section 6, we examine and discuss
 90 the results and in Section 7, we provide the conclusions of the study.

91 **2. Bio-inspired stress-dependent growth function**

92 *2.1. Force balance equilibrium for bodies in contact*

93 Synovial joints are formed in a growth process highly dependent on chemical and mechanical
 94 stimuli [2]. As a result, they are adapted to both internal and external stimuli. To use this process
 95 for industrial applications, we first need to explore the growth forces that generate this adaptation.
 96 In particular, the growth forces that are dependent on mechanical cues.

97 We shall begin with the properties of the desired design. Let us consider a set of $k \in \mathbb{N}$
 98 continuous domains: $\Omega_i \in \mathbb{R}^3$ (where $1 \leq i \leq k$) with closed boundaries: $\Gamma_i = \partial\Omega_i \in \mathbb{R}^3$.
 99 Let us also consider that these domains might be: in contact with each other, under external
 100 forces and constrained at given points. Thus, each $\Gamma_i = \Gamma_i^c \cup \Gamma_i^f \cup \Gamma_i^d$ where Γ_i^c , Γ_i^f and Γ_i^d are
 101 respectively: the contact boundaries, the boundaries where external forces are applied and the
 102 constrained boundaries. The force balance equation is then given by [39]:

$$\nabla \cdot \boldsymbol{\sigma}_i(\mathbf{x}) = \mathbf{0} \quad \forall \mathbf{x} \in \Omega_i, \quad (1)$$

$$\boldsymbol{\sigma}_i(\mathbf{x}) \cdot \mathbf{n}_i(\mathbf{x}) = \mathbf{f}_i(\mathbf{x}) \quad \forall \mathbf{x} \in \Gamma_i^f, \quad (2)$$

$$\boldsymbol{\sigma}_i(\mathbf{x}) \cdot \mathbf{n}_i(\mathbf{x}) = \mathbf{p}_i(\mathbf{x}) \quad \forall \mathbf{x} \in \Gamma_i^c, \quad (3)$$

$$\mathbf{u}_i(\mathbf{x}) = \mathbf{u}_i^0(\mathbf{x}) \quad \forall \mathbf{x} \in \Gamma_i^d, \quad (4)$$

103 for $1 < i < k$. $\boldsymbol{\sigma}_i$ are stress tensors; \mathbf{n}_i are outward unit vectors; \mathbf{f}_i and \mathbf{p}_i are traction stress
 104 vectors related to external forces and contact pressures, respectively; and \mathbf{u}_i^0 are fields of fixed
 105 displacements.

106 Our objective is to define Ω_i such that each \mathbf{p}_i has a uniform distribution—in other words,
 107 $\partial \mathbf{p}_i / \partial \mathbf{t}_i = 0$ along Γ_i^c , where \mathbf{t}_i are tangent unit vectors to the surfaces Γ_i : $\mathbf{t}_i \cdot \mathbf{n}_i = 0$. To solve this
 108 problem, we will use the theory of synovial joint morphogenesis to obtain Ω_i by a process of load
 109 adaptation.

110 Before continuing, let us establish some useful notation. The hydrostatic stress is defined as:
 111 $\sigma_{\text{hyd}} = \text{tr}(\boldsymbol{\sigma})/3$. For the shear stress let us use the von Mises stress $\sigma_v = \sqrt{3J_2}$ —where J_2 is the
 112 second invariant of the deviatoric stress tensor. It will also be useful to write the compressive stress
 113 as: σ_{hyc} which is 0 when $\sigma_{\text{hyd}} > 0$ and $-\sigma_{\text{hyd}}$ when $\sigma_{\text{hyd}} < 0$.

114 2.2. The growth function

115 As we will use the theory of synovial joint morphogenesis to define Ω_i , let us see the biolog-
 116 ical growth model. For cellular processes of growth, we can assume that the inertial forces are
 117 negligible with respect to viscous and elastic forces [40]. Then, the equilibrium of forces can be
 118 written as [40–42]:

$$\nabla \cdot (\boldsymbol{\sigma}_i(\mathbf{x}) + \boldsymbol{\sigma}_g(\mathbf{x})) = \mathbf{0} \quad \forall \mathbf{x} \in \Omega_i, \quad (5)$$

119 where $\boldsymbol{\sigma}_g$ is a stress tensor of growth forces. This tensor is then in charge of the shape adaptation
 120 to external loads. In synovial joint formation, this tensor is likely to promote oriented growth
 121 as chondrocytes and collagen fibres have different properties and orientations depending on the
 122 proximity to the contact region [43]. Near the surface, they are oriented tangentially to the sur-
 123 face. Farther from the surface, they are oriented perpendicularly to the surface [43]. In fact, it
 124 has been suggested that such preferential orientation might be related to the direction of principal
 125 strains [44]. Thus, growth orientation not only influences the shape but clearly also the mechan-
 126 ical properties of the cartilage, particularly, its anisotropic nature. Nonetheless, in biology, some
 127 authors have considered a hydrostatic nature for the growth tensor in computational models and
 128 have obtained shapes similar to those seen in nature [10]. Therefore, for this work, we will also

129 assume hydrostatic growth stress for simplicity. We can then let $\sigma_g = \sigma_g \mathbf{I}$ where \mathbf{I} is the identity
130 matrix.

131 Research in joint formation indicates that, apart from biochemical cues, growth is controlled
132 by compressive and shear stresses following the next two laws [6–9]: (i) proliferation takes places
133 when chondrocytes are subjected to cyclic compressive stress and (ii) ossification is triggered by
134 shear stress. If we neglect the difference in material properties before and after ossification, we
135 can translate proliferation into growth and ossification into growth inhibition. In short, we can say
136 that cyclic compressive stress prompts growth, while shear stress inhibits it.

137 To obtain the desired growth function, let us define different combinations of these two basic
138 rules:

- 139 • **Scenario I: expansion proportional to compressive stress.** In this situation, we can study
140 the effect of the first biological law in the absence of the second.
- 141 • **Scenario II: contraction proportional to compressive stress.** In this situation, we can see
142 the behaviour of the growth process in opposition to the first biological law. This will shed
143 light on how the contact interfaces driven by compressive stress stimuli can be.
- 144 • **Scenario III: expansion proportional to compressive stress and inhibited by high shear
145 stress.** In this situation, we can test the evolution of the growth process following the two
146 biological laws.
- 147 • **Scenario IV: expansion proportional to compressive stress and inhibited by high shear
148 or high compressive stresses.** In this situation, we propose to enhance the inhibition of the
149 growth. Inhibition will take place not only in the regions with high shear stress but also in
150 regions with high compressive stress. As the distributions of shear and compressive stresses
151 are not the same, this scenario will help us understand which regions should be discouraged
152 from growing.

153 Since in biology, inhibition is usually imposed by threshold parameters, the following equation
154 describes the proposed scenarios:

$$\sigma_g(\mathbf{x}) = \alpha_g \sigma_{\text{hyc}}(\mathbf{x}) H(\tau_{\text{lim}} - \sigma_v(\mathbf{x})) H(\sigma_{\text{lim}} - \sigma_{\text{hyc}}(\mathbf{x})), \quad (6)$$

155 where α_g is a parameter referring to the strength of the growth force, τ_{lim} and σ_{lim} are respectively
 156 the shear and the compressive stress thresholds, and $H(\varphi)$ is the Heaviside function defined as:

$$H(\varphi) = \begin{cases} 1 & \text{for } \varphi \geq 0, \\ 0 & \text{for } \varphi < 0. \end{cases} \quad (7)$$

157 Although this type of equation has been successfully used in biology [10], in [37, 38] it was
 158 indicated that a discontinuous σ_g as that in Eq. (6) leads to discontinuous growth. Further, this
 159 discontinuity also affects the contact pressure distribution. To make Eq. (6) continuous, instead of
 160 the Heaviside function, let us use the Sigmoid function which is defined as:

$$S(\varphi) = \frac{1}{1 + \exp(-\nu\varphi)}, \quad (8)$$

161 where ν is a positive parameter. Fig. 1 shows the behaviour of the $S(\varphi_{\text{ref}} - \varphi)$ varying ν and an
 162 offset along the φ -axis, φ_{ref} , for $\varphi \in [0, 1]$. As it can be seen in Fig. 1a, as ν approaches zero, S
 163 becomes the constant function 0.5. In contrast, as ν approaches infinity, S becomes the Heaviside
 164 function. Further, Fig. 1b shows that: for a given ν , there exists a minimum value γ such that for
 165 all $\varphi_{\text{ref}} \geq \gamma$, $S(\varphi_{\text{ref}} - \varphi) \approx 1$ for all $\varphi \in [0, 1]$. Similarly, there also exists a maximum value λ such
 166 that for all $\varphi_{\text{ref}} \leq \lambda$, $S(\varphi_{\text{ref}} - \varphi) \approx 0$ for all $\varphi \in [0, 1]$. Hence, changing the $H(\cdot)$ by $S(\cdot)$ in Eq. (6)
 167 yields:

$$\sigma_g(\mathbf{x}) = \alpha_g \sigma_{\text{hyc}}(\mathbf{x}) S(\tau_{\text{lim}} - \sigma_v(\mathbf{x})) S(\sigma_{\text{lim}} - \sigma_{\text{hyc}}(\mathbf{x})). \quad (9)$$

168 This function is now continuous; however, the stress magnitude is highly dependent on the
 169 magnitude of the external loads. As we want to define a system able to modify the contact shape
 170 independent of the stress magnitude, it is necessary to define the growth function in terms of
 171 normalised hydrostatic and shear stresses. We propose a normalisation based on the maximum
 172 stress values: let $\sigma_{\text{hyc}}^*(\mathbf{x})$ and $\sigma_v^*(\mathbf{x})$ be normalised compressive and shear stresses:

$$\sigma_{\text{hyc}}^*(\mathbf{x}) = \frac{\sigma_{\text{hyc}}(\mathbf{x})}{\max(\sigma_{\text{hyc}})}, \quad \sigma_v^*(\mathbf{x}) = \frac{\sigma_v(\mathbf{x})}{\max(\sigma_v)}.$$

173 As $\sigma_{\text{hyc}}^*(\mathbf{x}), \sigma_v^*(\mathbf{x}) \in [0, 1]$, the adaptation becomes independent of the stress magnitude—in other
 174 words, it only depends on the stress distribution. Using the normalised stresses, the growth force

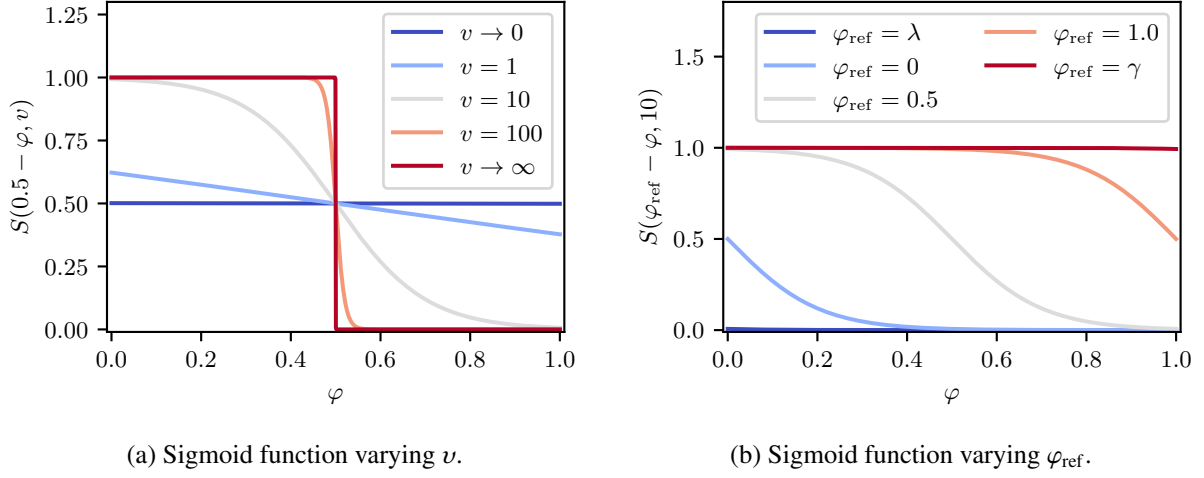


Figure 1: Sigmoid function response. As v approaches zero, S becomes the constant function 0.5. In contrast, as v approaches infinity, S becomes the Heaviside function. Further, for a given v , there exists a minimum value γ such that for all $\varphi_{\text{ref}} \geq \gamma$, $S(\varphi_{\text{ref}} - \varphi) \approx 1$ for all $\varphi \in [0, 1]$. Similarly, there also exists a maximum value λ such that for all $\varphi_{\text{ref}} \leq \lambda$, $S(\varphi_{\text{ref}} - \varphi) \approx 0$ for all $\varphi \in [0, 1]$.

175 equation can then be rewritten as:

$$\sigma_g(\mathbf{x}) = \left(\alpha_g \sigma_{\text{hyc}}^*(\mathbf{x}) S(\tau_{\text{lim}}^* - \sigma_v^*(\mathbf{x})) S(\sigma_{\text{lim}}^* - \sigma_{\text{hyc}}^*(\mathbf{x})) \right) \beta_g, \quad (10)$$

176 where τ_{lim}^* and σ_{lim}^* are now dimensionless parameters. β_g has units of pressure to make Eq. (10)
 177 compatible with Eq. (5), ~~for this work $\beta_g = 1\text{GPa}$.~~ The strength of σ_g is now given by the product
 178 of β_g and α_g . Thus, we can let α_g be a dimensionless parameter that provides the magnitude and
 179 β_g the parameter that contains the units, in this case $\beta_g = 1\text{GPa}$. The parameter space for each
 180 scenario is shown in Table 1.

Table 1: Parameter space for each scenario of study. In scenario I, we consider uninhibited expansion proportional to hydrostatic compressive stress; in scenario II, uninhibited contraction proportional to hydrostatic compressive stress; in scenario III, expansion proportional to hydrostatic compressive stress and inhibited by high shear stress; and in scenario IV, expansion proportional to hydrostatic compressive stress and inhibited by both high shear and high hydrostatic compressive stresses.

Scenario	α_g	τ_{lim}^*	σ_{lim}^*
I	$(0, +\infty)$	$[\gamma, +\infty)$	$[\gamma, +\infty)$
II	$(-\infty, 0)$	$[\gamma, +\infty)$	$[\gamma, +\infty)$
III	$(0, +\infty)$	$(0, \gamma)$	$[\gamma, +\infty)$
IV	$(0, +\infty)$	$(0, \gamma)$	$(0, \gamma)$

181 *2.3. The material constitutive law*

182 As the purpose of this work is to study adaptation in an engineering context, it is reasonable to
 183 consider a linear elastic model. Therefore, the stress–strain relation is given by:

$$\boldsymbol{\sigma}(\mathbf{x}) = \frac{E}{1 + \nu} \left(\boldsymbol{\varepsilon}(\mathbf{x}) + \frac{\nu}{1 - 2\nu} \text{tr}\boldsymbol{\varepsilon}(\mathbf{x}) \mathbf{I} \right), \quad (11)$$

184 where E and ν are Young’s modulus and Poisson’s ratio, respectively, and $\boldsymbol{\varepsilon}(\mathbf{x}) = 0.5 (\nabla \mathbf{u}(\mathbf{x}) + (\nabla \mathbf{u}(\mathbf{x}))^\top)$
 185 is the linear strain tensor.

186 It would be possible to use the properties of biological materials. Although this would be
 187 closer to synovial joint development, the adaptation might be biased by stress fields that do not
 188 represent the engineering context. In addition, even in biology, linear elastic models have been
 189 used to reproduce synovial joint and bone formation [10–12].

190 **3. A bio–inspired shape–design methodology**

191 We propose an iterative design process that allows us to replicate cyclic compressive stresses.
 192 Each cycle is divided into a stress computation step and a growth step. For the stress computation,
 193 we need to solve Eqs. (1) to (4) and, for the growth step, Eqs. (4) and (5) together with the following
 194 boundary condition:

$$\mathbf{u}_i(\mathbf{x}) \cdot \mathbf{n}_i(\mathbf{x}) = 0, \quad \forall \mathbf{x} \in \Gamma_i^f. \quad (12)$$

195 In other words, during the growth step, the displacement along Γ_i^f is purely tangential. Then, we
 196 redefine Ω_i by applying the computed displacements.

197 To objectively compare the response of the algorithm using different values of the parameters
 198 in Table 1, let us consider a time–like variable, \hat{t}_m . As it can be seen in Eq. (10), σ_g is proportional
 199 to α_g . This means that the velocity of growth is also proportional to α_g . Therefore, we consider:

$$\hat{t}_m = m|\alpha_g|, \quad (13)$$

200 where $m \in \mathbb{N}$ is the number of a given iteration, as a time–like measure. For example, if we have
 201 two cases, one with $\alpha_g = 1$ and the other with $\alpha_g = 10$, one iteration of the latter is equivalent to
 202 ten iterations of the former.

Algorithm 1 Bio–inspired shape–design algorithm.

Input: Geometry: $\Omega_i(0)$ for $1 \leq i \leq k$.

Input: Boundary conditions: \mathbf{f}_i and \mathbf{u}_i^0 .

Input: Material properties: E_i and ν_i .

Input: Growth function parameters: α_g , τ_{lim}^* and σ_{lim}^* .

Input: Total simulation time: \hat{t}_f .

Output: Geometry adapted to the loading conditions: $\Omega_i(t_f)$.

```
1:  $m \leftarrow 1$ 
2:  $\hat{t}_m \leftarrow m|\alpha_g|$ 
3: while  $\hat{t}_m < \hat{t}_f$  do
4:   Compute the stress state that stimulates growth solving Eqs. (1) to (4).
5:   Compute the growth stress using Eq. (10).
6:   Find the displacement field  $\mathbf{u}(\mathbf{x}(\hat{t}_m))$  that satisfies Eqs. (4), (5) and (12).
7:    $\mathbf{x}(\hat{t}_{m+1}) \leftarrow \mathbf{x}(\hat{t}_m) + \mathbf{u}(\mathbf{x}(\hat{t}_m))$ 
8:    $m \leftarrow m + 1$ 
9:    $\hat{t}_m \leftarrow m|\alpha_g|$ 
10: end while
```

203 Additionally, Eq. (10) also depends on the parameters of inhibition, ν , τ_{lim}^* and σ_{lim}^* . There-
204 fore, if the inhibition region is larger with one set of parameters than with another, the geometry
205 will change more slowly with the former set than with the latter. In order to compare the model
206 response with different parameters, we will make the L_2 –norm of the growth function equal to $|\alpha_g|$
207 at each iteration. In other words, we will use σ_α instead of σ_g defined as:

$$\sigma_\alpha = \frac{|\alpha_g|}{\|\sigma_g\|_{L_2(\cup_i \Omega_i)}} \sigma_g \quad (14)$$

208 where $\|\cdot\|_{L_2(\cup_i \Omega_i)}$ refers to the L_2 –norm in $\cup_i \Omega_i$. The use of Eqs. (13) and (14) allows us to
209 objectively compare the response of the algorithm using different parameters.

210 Algorithm 1 presents the algorithm to mimic the biological growth process. As can be seen,
211 computing the displacement field (steps 4–6) is quite simple compared to gradient–based algorithms—
212 there are no derivatives to calculate nor surface–dependent variables. The implementation was
213 done using Code_Aster [45] and GMSH [46]. The former was used to solve the equations by
214 finite element methods and the latter was used to control the evolution of the mesh. The compu-
215 tational implementation is available in: [https://github.com/sarroyavet/BioDesign_ANR/](https://github.com/sarroyavet/BioDesign_ANR/tree/main/StressAdaptiveDesignOf2D)
216 [tree/main/StressAdaptiveDesignOf2D](https://github.com/sarroyavet/BioDesign_ANR/tree/main/StressAdaptiveDesignOf2D)

217 **4. Impact of the parameters on the contact interface**

218 In this section, we will set up a reference case of study to analyse the impacts of the model
219 parameters on the uniformity of the contact pressure. In addition, we will define metrics to measure
220 the performance of the algorithm and the generated contact interfaces. As we will explore different
221 growth scenarios and parameters, we will consider a two-dimensional framework and frictionless
222 contact. In the context of contact analysis, the theory of semi-infinite elastic bodies is widely
223 used [17]. In consequence, we will consider the contact between a half-space and an infinite
224 cylinder that satisfies the state of plane-strain. Further, the consideration of frictional effects
225 significantly augments the computational time. For example, in [47], the authors showed that wear
226 simulations considering friction were up to three times longer than their frictionless counterparts.
227 Even though we are not modelling wear, the process is similar: a calculation of the contact pressure
228 followed by a geometry update. Thus, using a two-dimensional frictionless contact framework
229 will significantly reduce computational time.

230 We will also limit the growth to only one domain. Thus, the degrees of freedom of the design
231 problem will be reduced to one domain, facilitating the analysis of the adaptation process. This
232 assumption is also reasonable in the engineering context as it would be appropriate to define an
233 easy-to-manufacture geometry and only generate an adapted and complex shape. For example,
234 in the case of roller bearings, it is common to crown the roller profile while letting flat that of
235 the race. Nonetheless, it would be interesting to explore the advantages of allowing both domains
236 grow in future works.

237 To sum up, in this section, the model and the algorithm will be simplified by the following
238 assumptions: (i) two-dimensional framework where plane-strain conditions are applied, (ii) fric-
239 tionless contact and (iii) growth applied to only one domain.

240 *4.1. Description of the case of study*

241 Let us consider the contact between two elastic semi-infinite bodies: a cylinder (Ω_1) and a
242 half-space (Ω_2). Let us also apply a load on the cylinder in the normal direction to the surface of
243 the half-space. Since this case satisfies the plane-strain conditions, it can be reduced to a two-
244 dimensional framework, a schematic view is depicted in Fig. 2a. To analyse the adaptation of one

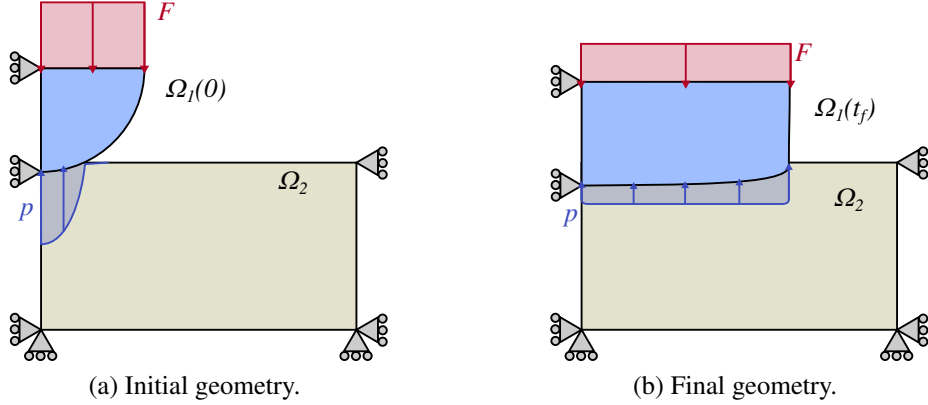


Figure 2: Two-dimensional setting of the case of study. Here p refers to the contact pressure along the contact interface and F to the applied force.

Table 2: Geometric parameters of the initial geometries. R, l_i are illustrated in Fig. 3 for both types of geometry.

Geometry	R [mm]	l_1 [mm]	l_2 [mm]	l_3 [mm]	l_4 [mm]
Circle	4.5	50	18	—	—
Rectangle	0.45	50	18	4.5	4.5

245 domain keeping the external stimuli untouched, let now the growth be only applied to the cylinder.

246 In other words, let $\sigma_g(\mathbf{x}) = 0$ for all $\mathbf{x} \in \Omega_2$.

247 The goal of the growth function is to yield a shape to obtain a constant contact pressure. Thus,
 248 after the design process, we expect to have a geometry adapted to the load that generates a uniform
 249 contact pressure as depicted in Fig. 2b.

250 4.2. Geometry, material properties and loading conditions

251 First, let Young's modulus and Poisson's ratio be 210GPa and 0.3, respectively, and let $F =$
 252 10kN. Regarding the initial cross section of Ω_1 , we can define two types depending on the dis-
 253 tribution of the initial contact pressure: (i) a higher pressure in the middle of the contact area or
 254 (ii) a higher contact pressure at the edges of the contact area. We can use a circle for the former
 255 and a rectangle with rounded edges for the latter. Fig. 3 shows a schematic representation of the
 256 geometry and Table 2 establishes the values of the dimensions. The load, the material properties
 257 and the initial size of the geometries were adapted from the case of study presented in [19].

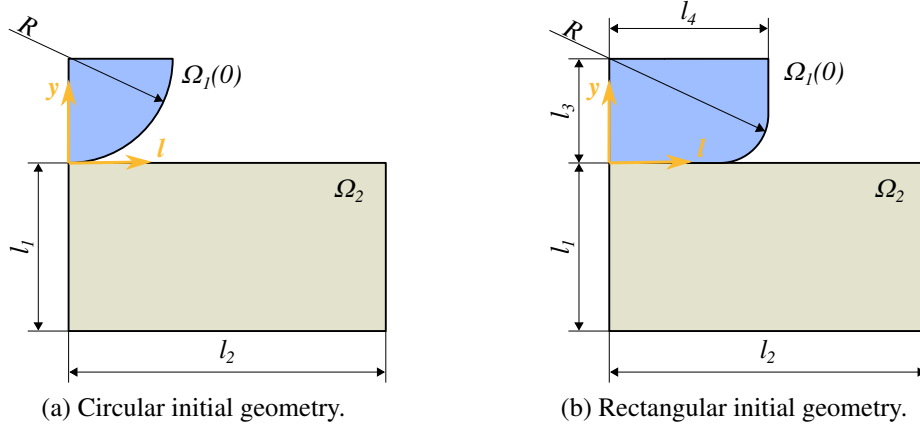


Figure 3: Geometry of the case of study.

258 4.3. Performance evaluation

259 To evaluate the performance of the procedure, let us define a measure to qualify the contact
 260 pressure distribution. In the best situation, the contact pressure is uniformly distributed and, there-
 261 fore, all the contact area is properly exploited. Thus, let us define a measure to compare the
 262 pressure distribution of a given contact area with an equivalent uniform pressure. Such a measure
 263 should be able to tell how close a given pressure distribution is to a uniform distribution. For each
 264 \hat{t}_m , let $a_c(\hat{t}_m)$ be the contact area and let $p_e(a_c(\hat{t}_m))$ be an equivalent constant pressure given by:

$$p_e(a_c(\hat{t}_m)) = \frac{F}{a_c(\hat{t}_m)}. \quad (15)$$

265 Now, let us define the pressure distribution quality as:

$$Q_p(\hat{t}_m) = 1 - \frac{\|p_e(a_c(\hat{t}_m)) - p(\hat{t}_m)\|_{L_2(\Gamma_1^c(\hat{t}_m))}}{\|p_e(a_c(\hat{t}_m))\|_{L_2(\Gamma_1^c(\hat{t}_m))} + \|p(\hat{t}_m)\|_{L_2(\Gamma_1^c(\hat{t}_m))}}, \quad (16)$$

266 where $\|\cdot\|_{L_2(\Gamma_1^c(\hat{t}_m))}$ refers to the L_2 -norm in $\Gamma_1^c(\hat{t}_m)$. By means of the triangle inequality [48], we
 267 know that $0 \leq Q_p \leq 1$. Additionally, $Q_p \rightarrow 1$ as $p \rightarrow p_e$ and $Q_p \rightarrow 0$ as p concentrates at a
 268 specific point. We then look for high values of Q_p . Table 3 indicates the value of Q_p based on the
 269 reconstruction of results presented in other works.

270 Apart from this evaluation at each step, we can also compute the average of Q_p as well as the

Table 3: Evaluation of the pressure distribution quality present in other works based on the reported graphs. We used a plot digitiser tool to compute Q_p .

	Reference	Q_p	Strategy
Contact profile as design variable.	[18] Fig. 4	0.93	Optimisation and approximation of a logarithmic profile.
	[34] Fig. 11	0.95	Optimisation of a super-elliptic profile.
	[32] Fig. 15	0.95	Optimisation of the contact profile.
	The present work	0.96	Bio-inspired generative design.
	[19] Fig. 7	0.99	Optimisation of a logarithmic profile.
Other design variables.	[30] Fig. 6	0.86	Optimisation of the non-in-contact contour.
	[24] Fig. 9	0.94	Optimisation of a predefined topology.
	[23] Fig. 5a	0.94	Optimisation of the material properties.
	[26] Fig. 6b	0.99	Topology optimisation.

271 average of the maximum contact pressure along the simulation as:

$$\bar{Q}_p = \sum_{m=1}^{m=m_f} \frac{Q_p(\hat{t}_m)}{m_f},$$

$$\bar{p}_{\max} = \sum_{m=1}^{m=m_f} \frac{p_{\max}(\hat{t}_m)}{m_f},$$

272 with $t_f = |\alpha_g| m_f$, where m_f refers to the last iteration and t_f to the total time. Thus, we can evaluate
 273 the consistency of the design process in a general manner.

274 4.4. Exploration of the design process

275 As we aim to understand how the growth rules of joint formation can be applied to our case
 276 of study, we propose the following pathway. First, we will see the impact of α_g in terms of
 277 convergence in both scenarios I and II. From this study, we will define an appropriate value of α_g .
 278 Then, we will explore the response of the design process in scenarios III and IV and qualify the
 279 contact pressure distribution varying τ_{\lim}^* , σ_{\lim}^* and ν . Following this exploration, we will select an
 280 appropriate combination of parameters and examine the evolution of the design process utilising a
 281 rectangle as the initial geometry.

282 5. Examples of application

283 In this section, we will present other examples to evaluate the applicability of the bio-inspired
 284 growth model after the definition of the parameters. We will consider plane-stress and axisymmet-

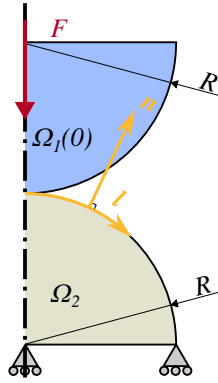


Figure 4: Initial geometry of the axisymmetric case of study. The coordinate system l and n will be used later to define the surface of Ω_1 relative to the surface of Ω_2 .

285 ric conditions and we will change the material properties, the magnitude of the load and the fixed
 286 geometry (Ω_2). Further, we will also consider an additional rule to satisfy an additional design
 287 requirement.

288 5.1. Plane–stress and softer material

289 For this test, let us consider the contact between two thin plates. We shall consider the analo-
 290 gous plane–stress case to the plane–strain case presented in Section 4.1 (see Fig. 3). Nonetheless,
 291 let us consider a softer material (for both Ω_1 and Ω_2) with $E = 2\text{GPa}$ and $\nu = 0.4$ —these elastic
 292 properties are close to those of some engineering polymers—, $R = 10\text{mm}$ and $F = 100\text{N}$.

293 5.2. Axial symmetry and geometries with distinct material properties

294 In this case, let us consider the contact between a soft material for Ω_1 ($E = 2\text{GPa}$ and $\nu = 0.4$)
 295 and a rigid material for Ω_2 ($E = 250\text{GPa}$ and $\nu = 0.2$) and the initial geometry presented in
 296 Fig. 4 with $R = 14\text{mm}$ and $F = 3000\text{N}$. This case represents the loading conditions and material
 297 properties of hip–joint prostheses under static analyses [49, 50].

298 5.3. Additional design requirement: maximum von Mises stress

299 So far, the design process makes the domain grow indefinitely. This means that we need to
 300 include an additional requirement to constrain the growth. It could be a geometric restriction
 301 (such as a maximum contact area) or a structural constraint (such as a maximum shear stress).
 302 We can combine the growth function in Eq. (10)—whose main objective is to provide a uniform

303 contact pressure—with a restriction function. Let us then rewrite Eq. (5) as:

$$\nabla \cdot (\boldsymbol{\sigma}(\mathbf{x}) + \eta_g \boldsymbol{\sigma}_g(\mathbf{x}) + \eta_r \boldsymbol{\sigma}_r(\mathbf{x})) = \mathbf{0}, \quad (17)$$

304 where η_g and η_r are the strengths of the contact pressure improvement and of the restriction,
 305 respectively. We can use Q_p to define η_g as:

$$\eta_g = 1 - Q_p. \quad (18)$$

306 For the definition of $\boldsymbol{\sigma}_r$, let us impose a maximum von Mises stress. First, we can define a von
 307 Mises stress quality as:

$$Q_v = 1 - \frac{\max(\sigma_v(\mathbf{x})) - \tau_{\text{ref}}}{\max(\sigma_v(\mathbf{x})) + \tau_{\text{ref}}}, \quad (19)$$

308 where τ_{ref} is the target von Mises stress of the restriction. As it can be seen, $Q_v \rightarrow 1$ as $\max(\sigma_v) \rightarrow$
 309 τ_{ref} , $Q_v < 1$ when $\max(\sigma_v) > \tau_{\text{ref}}$ and $Q_v > 1$ when $\max(\sigma_v) < \tau_{\text{ref}}$. We can also consider $\boldsymbol{\sigma}_r = \mathbf{I}$
 310 (hydrostatic) and

$$\eta_r = \alpha_r(1 - Q_v), \quad (20)$$

311 where α_r is a positive parameter. As we can see, $\boldsymbol{\sigma}_r$ is space-independent and produces global
 312 expansion or contraction. For this case, we will consider the plane-strain case presented in Sec-
 313 tion 4.

314 6. Results and discussion

315 6.1. Convergence

316 To ensure numerical precision, the contact algorithm was tested with different contact element
 317 lengths. For that, the contact pressure was calculated using the circular geometry described above.
 318 This case can be analytically solved using Hertz theory [17] in cases where the cylinder diameter
 319 is significantly larger than the contact area width. In consequence, we applied a load of 100N so
 320 that, according to Hertz theory, the maximum contact pressure is 903.4MPa, and the contact area
 321 width is 0.141[mm]. Thus $R \gg a_c$ and the conditions for Hertz theory are satisfied.

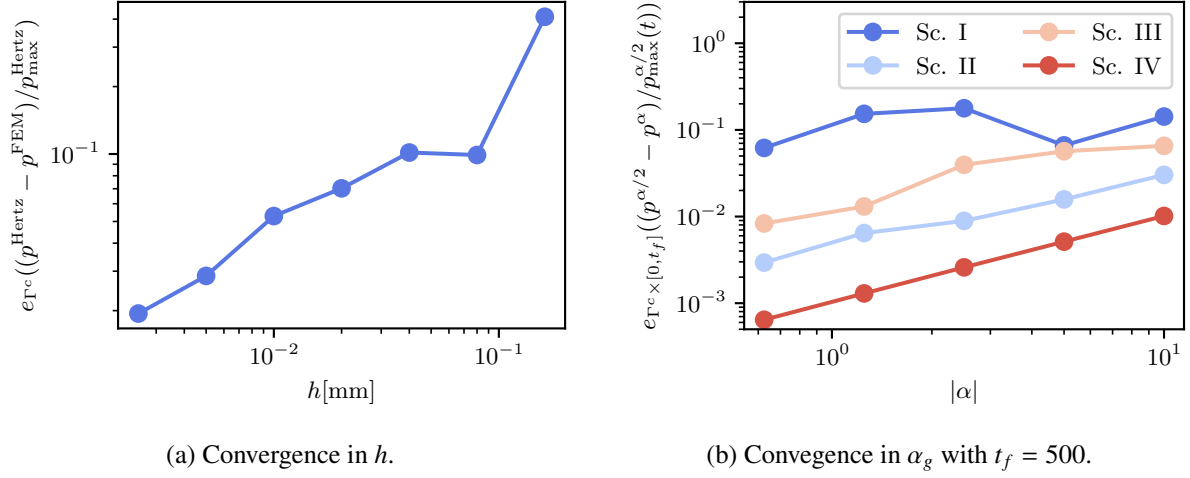


Figure 5: Convergence of the model in mesh size and equivalent time step. In (a) $e_{\Gamma^c}(f) = \sqrt{\int_{\Gamma^c} f(\mathbf{x})^2 d\Gamma^c / \int_{\Gamma^c} d\Gamma^c}$ and in (b) $e_{\Gamma^c \times [0, t_f]}(f) = \sqrt{\int_0^{t_f} \int_{\Gamma^c} f(\mathbf{x}, t)^2 d\Gamma^c dt / \int_0^{t_f} \int_{\Gamma^c} d\Gamma^c dt}$ are the L_2 -norms of a given function f . In (a) $f = (p^{\text{Hertz}} - p^{\text{FEM}})/p_{\text{max}}^{\text{FEM}}$ which is the normalised error between the contact pressure calculated by Hertz theory and by the finite element method. In (b) $f = (p^{\alpha/2} - p^\alpha)/p_{\text{max}}^{\alpha/2}(t)$ which is the normalised contact pressure error using as reference a simulation using $\alpha/2$.

322 Fig. 5a shows the L_2 -norm of the normalised difference between the results using Hertz theory
 323 and finite elements varying the element length. The normalisation was made with respect to the
 324 maximum contact pressure calculated by Hertz theory. It can be seen that as the mesh is refined,
 325 the numerical approximation approaches the analytical solution. In addition, for this case, an
 326 element length of 0.01mm is sufficient to make the error norm lower than 0.05. Thus, this value
 327 was selected for the other simulations.

328 As indicated in Eq. (13), α_g works as a time step variable. Therefore, the convergence of
 329 the algorithm with α_g was also tested. For that, t_f was set to 500—which is large enough to see
 330 the behaviour of the design process and to test the convergence. The L_2 -norm of the normalised
 331 contact pressure between simulations was computed. The normalisation was made by means of
 332 the maximum contact pressure at each step. Fig. 5b shows the relative difference among several
 333 simulations varying α_g from 0.3125 to 10 for each scenario. We can see that for scenarios II, III
 334 and IV, as α_g decreases, such a difference also decreases. Thus, for these scenarios the algorithm is
 335 also convergent in terms of α_g —we will take a closer look for scenario I in the following section.
 336 We selected $\alpha_g = 1.25$ to study the response of the design process in scenarios III and IV as, for

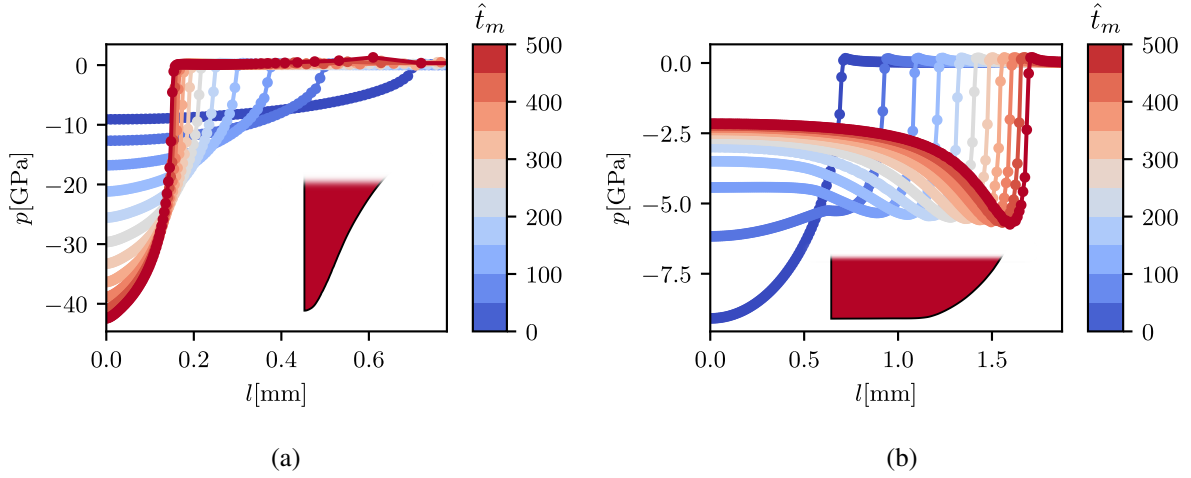


Figure 6: Evolution (from blue to red lines) of the contact pressure along the contact line in scenarios I (a) and II (b) with $|\alpha_g| = 0.3125$. l refers to arc length and the red shapes refer to the geometry of last iteration.

337 such a value, the error norm is lower than 0.05. For scenarios I and II, we used the simulations
 338 with $\alpha_g = 0.3125$, the lowest value used in the convergence analysis.

339 6.2. The response of the design process in scenarios I and II

340 Fig. 6a shows the evolution of the contact pressure distribution (from blue to red lines) for
 341 scenario I. For this case, it was necessary to add a condition to the mesh such that the number of
 342 nodes in contact was always above a given threshold—we used a minimum of 50 nodes. This addi-
 343 tional condition was necessary since the contact area was reduced during the simulation. We can
 344 see that growing (from blue to red distributions) proportionally to the compressive stress makes a
 345 pressure distribution highly concentrated on the initial point of maximum contact pressure. This is
 346 a reasonable result given the fact that the maximum compressive stress is located on the surface—
 347 more precisely, at the point of maximum contact pressure. Thus, this region grows faster than its
 348 neighbours enlarging the curvature. In addition, this curvature increase reduces the contact area
 349 and, in consequence, the maximum contact pressure rises. We can say that in the scenario I, the
 350 shape tends to have an infinite contact pressure and a zero contact area. This singularity explains
 351 the behaviour of the convergence seen in Fig. 5b.

352 On the other hand, Fig. 6b shows that contraction (from blue to red distributions) in proportion
 353 to compressive stress makes the pressure concentrate at the edges. As the pressure changes

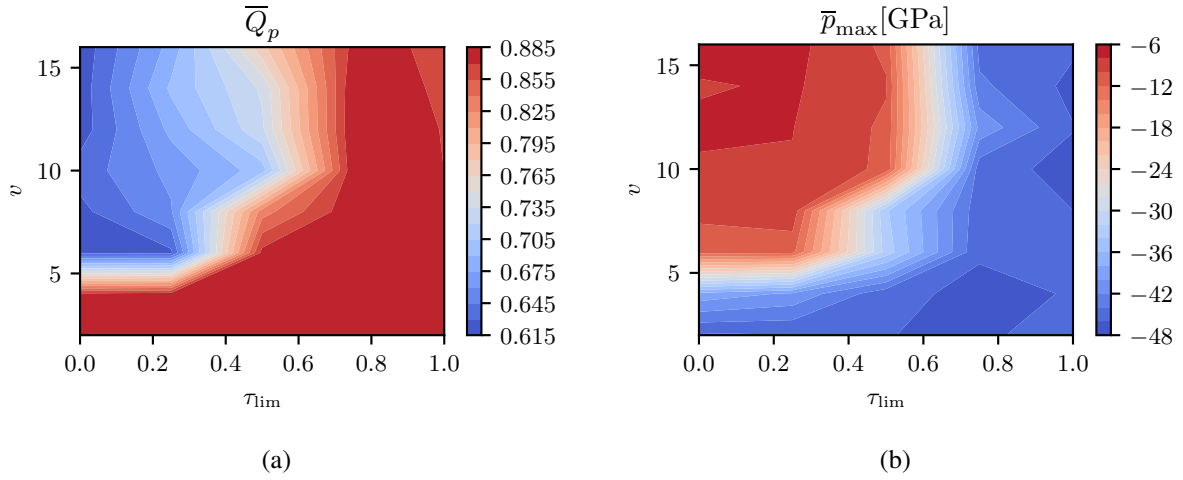


Figure 7: Response of the design process in scenario III with $t_f = 2000$ in terms of the average of the pressure quality (a) and the average of the maximum contact pressure (b) along the simulation.

354 from being concentrated in the middle to the edges, the profile flattens and the maximum pressure
 355 lowers. However, once the edge effect is formed, the maximum pressure rises ~~and~~ **Additionally,**
 356 ~~it can also be seen~~ the pressure distribution tends to become singular at the edge. Yet, in the
 357 used timescale, this tendency does not affect the convergence with respect to α_g , as can be seen in
 358 Fig. 5b. Lastly, we can also see that this contraction also enlarges the contact area.

359 6.3. Response of the design process in scenarios III and IV

360 We now know that while growing proportionally to compressive stress decreases the contact
 361 area and concentrates the pressure at the middle, contraction proportional to compressive stress
 362 increases the contact area and concentrates the pressure at the edges. The former generates geome-
 363 tries that tend to have an infinite curvature at the contact zone, while the latter makes geometries
 364 which tend to be flat (zero curvature). Since both scenarios are not desirable, we continued the
 365 exploration of the growth rules.

366 Fig. 7 shows the response of the design process in scenario III. Fig. 7a indicates the average of
 367 the pressure quality and Fig. 7b the average of the maximum contact pressure along the simulation.
 368 It can be seen that the pressure quality is higher (up to 0.855) at lower values of ν or higher values
 369 of τ_{lim}^* . However, such a combination almost eliminates the inhibition. As seen in Fig. 1, low
 370 values of ν and high values of τ_{lim}^* cause S to be almost a constant function. If S becomes a

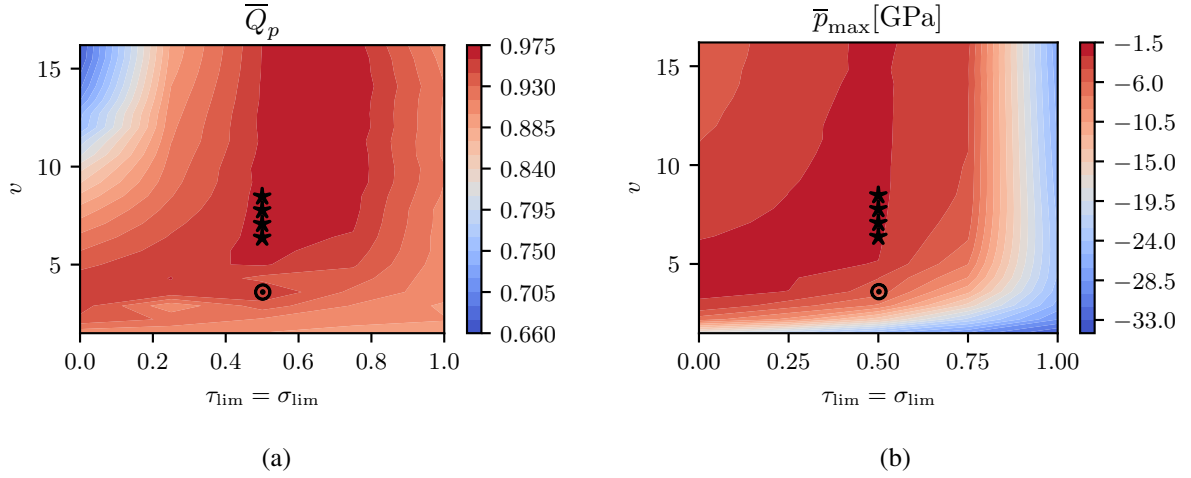


Figure 8: Response of the design process in scenario IV with $t_f = 2000$ in terms of the average pressure quality (a) and the average of the maximum contact pressure (b) along these simulation. The \star symbols refer to the best cases. In such cases $\nu = 6.4, 7.1, 7.8, 8.5$. The \odot symbol refers to the case with $\nu = 3.6$.

371 constant function, Eq. (10) reduces to $\sigma_g(\mathbf{x}) = c\sigma_{\text{hyc}}^*(\mathbf{x})$, where c is a constant, and no inhibition
 372 is present. In fact, we can see in Fig. 7b that the region of higher \bar{Q}_p is also the region of higher
 373 contact pressure. This is exactly what happened in the uninhibited growth scenario.

374 We can also see that, in scenario III, we can not obtain an appropriate result. In both graphs of
 375 Fig. 7, the red regions specify the desired results—high pressure quality and low contact pressure.
 376 As can be seen, these regions exclude each other—the red region in Fig. 7a intersects with the blue
 377 region in Fig. 7b and the other way around—; this justifies the need to modify the growth function.

378 To address this situation, an additional inhibition was added. The results in scenario I indi-
 379 cate that unrestricted growth proportional to the compressive stress yields extremely high contact
 380 pressure. Since the maximum shear stress is not located in the same region as the maximum
 381 compressive stress, the region of maximum compressive stress is not always necessarily inhibited.
 382 Therefore, the growth force keeps enlarging the curvature around the point of maximum contact
 383 pressure. Thus, in scenario IV, the growth force was inhibited by high compressive stress. As a
 384 consequence, two new parameters appear σ_{lim}^* and another ν . Yet, we let $\sigma_{\text{lim}}^* = \tau_{\text{lim}}^*$ and let the
 385 same ν for both Sigmoid functions of Eq. (10). Fig. 8 shows the response of the design process
 386 with the new growth function modifying ν and the stress limits. In Fig. 8a, we see that \bar{Q}_p is

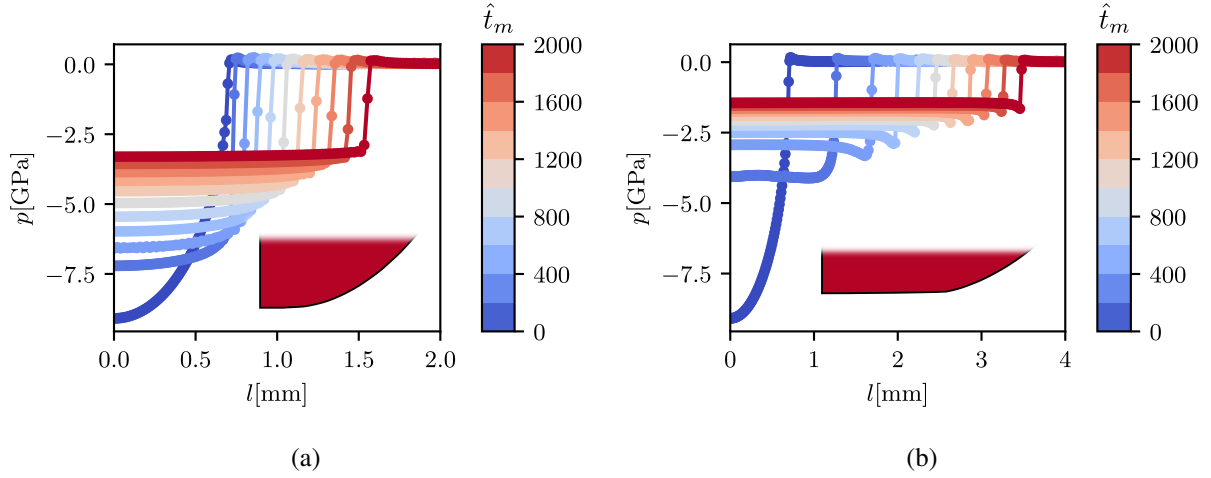


Figure 9: Evolution (from blue to red lines) of the contact pressure along the contact line in scenario IV with $\tau_{\text{lim}}^* = \sigma_{\text{lim}}^* = 0.5$ and de circular initial geometry. $\nu = 3.6$ in (a) and $\nu = 7.1$ in (b). l refers to arc length and the red shapes refer to the geometry of the last iteration.

387 significantly greater (up to 0.975) than in the previous scenario. In addition, the zone of higher
 388 pressure quality intersects the zone of lower maximum contact pressure—for $\sigma_{\text{lim}}^* = \tau_{\text{lim}}^* = 0.5$ and
 389 $\nu > 5$. Hence, we see that the added inhibition leads to the desired contact pressure distribution.

390 Fig. 8 also indicates the best cases by \star (they correspond to $\nu = 6.4, 7.1, 7.8, 8.5$). In addition,
 391 in Fig. 8a, there is a second local maximum value around $\nu = 3.6$ indicated by \odot . In order to
 392 better understand this situation, Fig. 9 shows the evolution of the contact pressure distribution with
 393 $\tau_{\text{lim}}^* = \sigma_{\text{lim}}^* = 0.5$ and $\nu = 3.6$ and $\nu = 7.1$. In both cases, as the iterations go on, the contact area
 394 augments, the maximum contact pressure decreases and the contact pressure distribution flattens.
 395 In addition, the pressure quality is high—above 0.96 for $\nu = 3.6$ and above 0.97 for $\nu = 7.1$.
 396 Nonetheless, we can notice an important difference. For $\nu = 3.6$, the maximum pressure remains
 397 in the centre, while for $\nu = 7.1$, it goes to the edges. Thus, the combined inhibition of shear and
 398 compressive stresses balances the expansion and contraction present in scenarios I and II. With
 399 $\nu = 3.6$, the effects in scenario I slightly dominate, while with $\nu = 7.1$ the effects in scenario II
 400 are somewhat more present. This type of behaviour can also be seen in logarithmic profiles. For
 401 example, Cui and He [19] presented a modified Lundberg profile with similar contact pressure
 402 distributions. Depending on the combination of parameters, the pressure tends to be concentrated

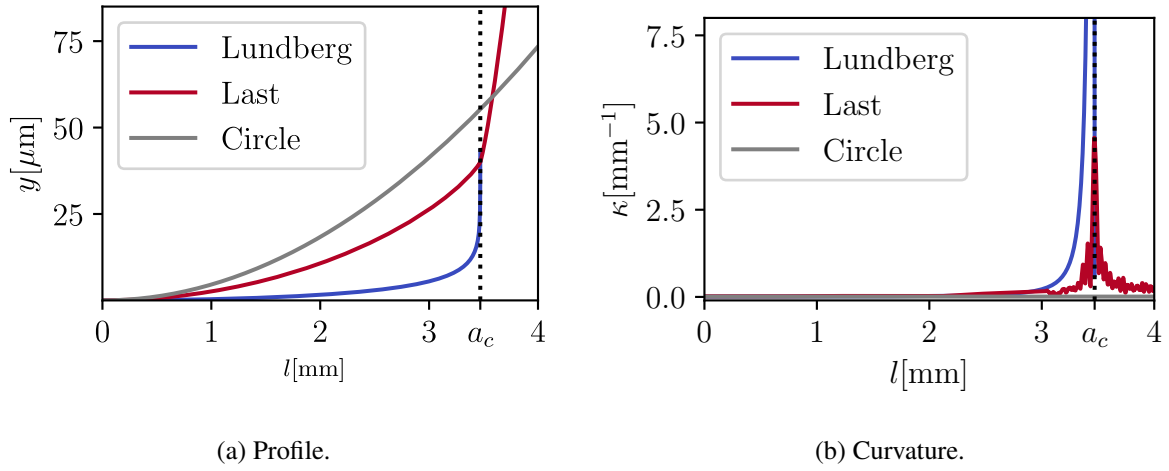


Figure 10: Geometric characteristics of the last profile obtained with $\nu = 7.1$.

403 either in the middle or at the edges.

404 6.4. Geometric properties of the contact interface

405 In terms of geometry, we can see both the profile and the curvature of the last shape obtained
 406 with $\nu = 7.1$ in Fig. 10. Fig. 10a shows that the algorithm generates a profile whose slope increases
 407 slowly from the centre to a region close to the end of the contact area. Then, it rises rapidly. This
 408 can be more easily seen in Fig. 10b where the curvature is plotted. There is a peak of maximum
 409 curvature around the end of the contact area—the edge of the contact interface—that suggests the
 410 presence of a sharp edge. This can be related to the fact that the junction between the adapted
 411 profile and a residual of the initial profile is at a_c .

412 This figure also shows the profiles and curvatures of logarithmic profile (constructed based on
 413 Lundberg’s equations [18–20, 51, 52]) and of a circular profile. Again, we can see similarities
 414 between our profile and logarithmic profiles. As we can see, the Lundberg profile curvature is also
 415 low at the middle and with a sharp peak at the edges. The contact interface using $\nu = 3.6$ also
 416 presents this affinity.

417 6.5. Influence of the initial geometry

418 To further investigate the response of the design process, the initial geometry was changed.
 419 More precisely, the initial contact pressure distribution. Fig. 11 shows the evolution of the contact

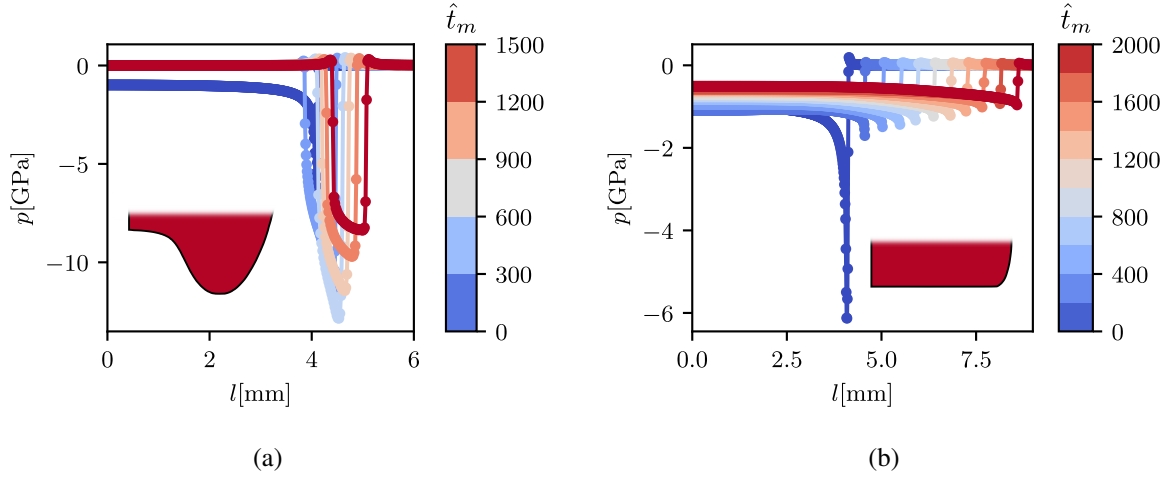


Figure 11: Evolution (from blue to red lines) of the contact pressure along the contact line in scenario IV with $\tau_{\text{lim}}^* = \sigma_{\text{lim}}^* = 0.5$ and the rectangular initial geometry. $\nu = 3.6$ in (a) and $\nu = 7.1$ in (b). l refers to arc length and the red shapes refer to the geometry of the last iteration.

420 pressure for a rectangular initial geometry and $\tau_{\text{lim}}^* = \sigma_{\text{lim}}^* = 0.5$. In Fig. 11a $\nu = 3.6$ and in
 421 Fig. 11b $\nu = 7.1$. Although the results seem different from what happened with the circular initial
 422 geometry, in reality, the influence of the growth force is the same. For $\nu = 3.6$, the point of
 423 initial maximum pressure defines the region where the pressure will be concentrated. Thus, a
 424 protuberance around this region appears. In contrast, since for $\nu = 7.1$ the edge effect slightly
 425 dominates, the domain grows as in Fig. 9b. Therefore, for applications where a uniform contact
 426 pressure is desirable, it is better to set the initial geometry such that the initial contact pressure
 427 distribution has its maximum centred.

428 6.6. Change of material properties and deformation model

429 To study the applicability of the growth function, we defined other cases of study modifying the
 430 material properties, the shape of Ω_2 , the magnitude of the load, and the two-dimensional model.
 431 We tested such modifications using the growth function with $\sigma_{\text{lim}}^* = \tau_{\text{lim}}^* = 0.5$ and $\nu = 7.1$.

432 For the plane-stress case, in Fig. 12, we can see the evolution of the contact pressure with
 433 $\sigma_{\text{lim}}^* = \tau_{\text{lim}}^* = 0.5$ and $\nu = 7.1$. As we can see, the contact area is enlarged while the contact pressure
 434 becomes uniform, reaching a value of pressure distribution quality of $Q_p = 0.98$. Similarly to the
 435 previous case (Fig. 9b), the maximum pressure is located at the edge.

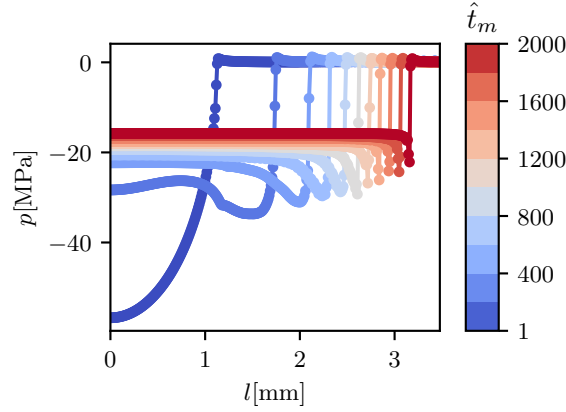


Figure 12: Evolution (from blue to red lines) of the contact pressure along the contact line for the plane–stress case with $\nu = 7.1$. l refers to arc length.

436 With respect to the axisymmetric case, in Fig. 13, we can see the results of the growth process
 437 with $\sigma_{\text{lim}}^* = \tau_{\text{lim}}^* = 0.5$ and $\nu = 7.1$. Fig. 13a shows the shape of Ω_1 in the coordinate system
 438 defined by l and n drawn in Fig. 4. Interestingly, the profile is also similar to Lundberg’s profile
 439 as can be seen in Fig. 10a. In addition, as seen in Fig. 13b, the contact area was also enlarged, and
 440 the contact pressure distribution was improved. In addition, we obtained a high value of pressure
 441 distribution quality at the last iteration, $Q_p = 0.97$.

442 Although the number of examples presented is not enough to definitely conclude about the
 443 response of the model, they show a tendency to generate contact interfaces with uniform pressure.
 444 This is an important insight since we consider that the efficacy of the bio–inspired generative
 445 design cannot be sensitive to the parameters of the growth function. In other words, we consider
 446 that the response of the design process seen in Fig. 8 is representative for situations different to the
 447 one described in Section 4.

448 In terms of material properties, the model can be extended to improve the growth phenomenon
 449 or to use more complex materials. For the former, it is possible to add a viscous stress tensor
 450 similar to the one present in [42] to model cell migration. Such a tensor would directly introduce
 451 time dependence. Nonetheless, it could represent an unnecessary cost for a methodology to gener-
 452 ate shapes adapted to their loading environments. Further, additional parameters—shear and bulk
 453 viscosities—would be introduced. Regarding more complex materials, it would be interesting to

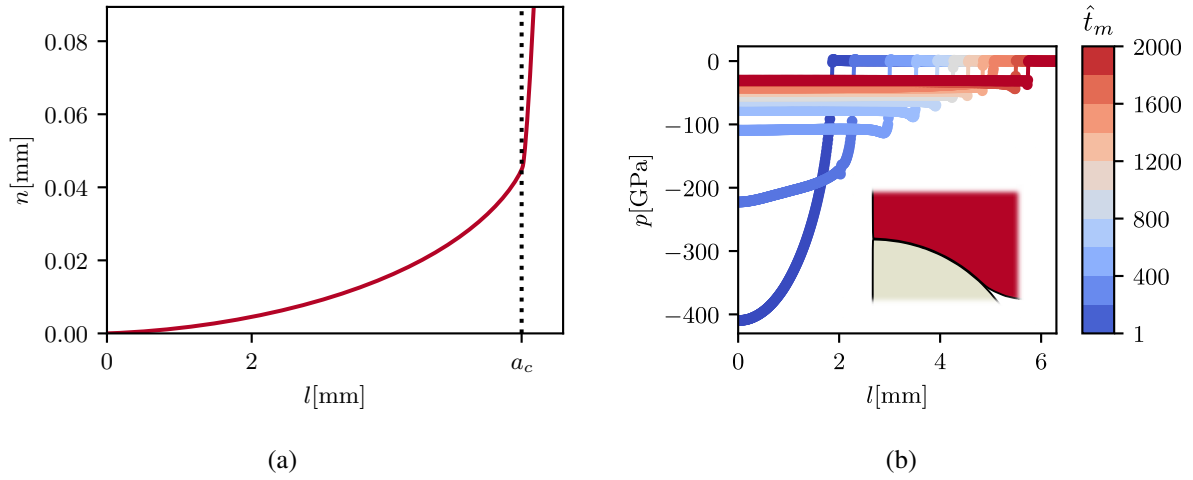


Figure 13: Results of the axisymmetric case with $\nu = 3.6$ after the growth process. (a) shows the contact interface of the last geometry in the coordinate system l and n seen in Fig. 4 and (b) shows the evolution (from blue to red) of the contact pressure (l refers to arc length). The red shape refers to the geometry of the last iteration.

454 study the process response adding plasticity. In addition, in line with bio–inspiration from synovial
 455 joints and considering the increasing development of cartilage–inspired materials with poroelastic
 456 properties [53], the use of poroelastic constitutive laws may be of interest.

457 6.7. Response of the design process after adding another design requirement

458 In the previous results, since there were no restrictions for the design process, only the evolu-
 459 tion of the contact pressure distribution was evaluated regardless of the profile length. In contrast,
 460 in this section a restriction to the maximum shear stress is also present; thus, the **contact length is**
 461 **fixed.** length of the contact profile reaches a stable value.

462 The design algorithm was set with $\tau_{\text{lim}}^* = \sigma_{\text{lim}}^* = 0.5$, $\nu = 7.1$, $\tau_{\text{ref}} = 1.0\text{MPa}$, $\alpha_g = \alpha_r =$
 463 1.25 , $F = 10\text{kN}$ and the circular initial geometry. Figs. 14a to 14c show the evolution of the
 464 performance variables along the design procedure—the maximum contact pressure and von Mises
 465 shear stress, the contact area and the contact pressure and von Mises shear stress qualities—and
 466 Fig. 14d indicates the distribution of the contact pressure along the design iterations.

467 As we can see, the contact interface changes rapidly at the beginning and, once Q_p and Q_v are
 468 close to 1, the evolution slows down. For this shape: $\max(|p_c|) = 1.749\text{GPa}$, $\max(\sigma_v) = 0.999\text{GPa}$,
 469 $a_c = 3.369\text{mm}$, $Q_p = 0.96$ and $Q_v = 1.001$. Fig. 14 shows that the design process is able to satisfy

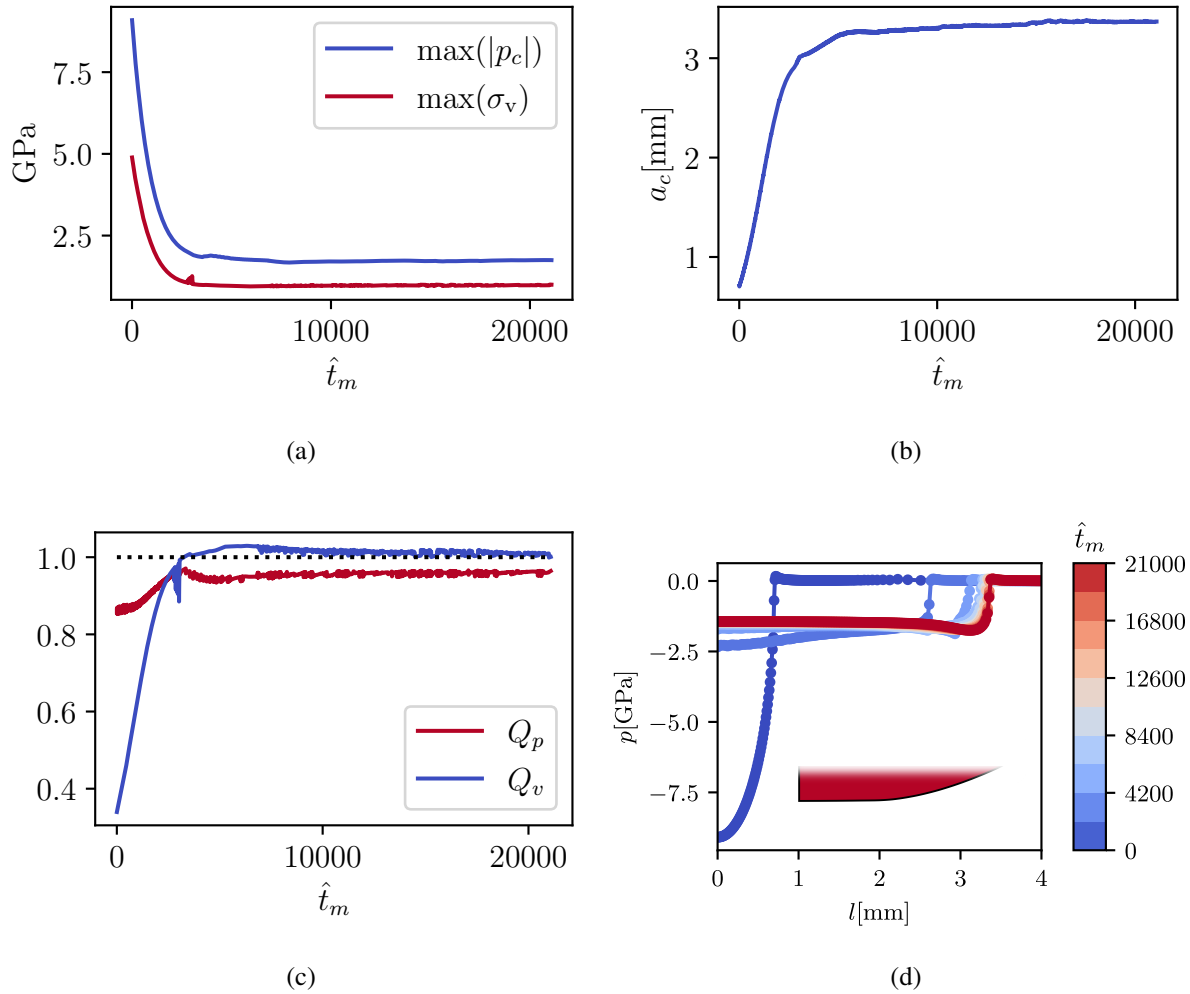


Figure 14: Performance of the contact interface along the design procedure. (a) Evolution of maximum contact pressure and von Mises stress. (b) Evolution of the contact area. (c) Evolution of the contact pressure and von Mises stress qualities. (d) Evolution (from blue to red lines) of the contact pressure along the contact line for the design case. l refers to arc length and the red shape refers to the last iteration.

470 the restriction and makes the contact pressure uniform. This result demonstrates that the process
471 of synovial joint morphogenesis can be adapted to engineering applications

472 Nonetheless, the shape of the contact pressure is slightly different (see Figs. 9b and 14d).
473 The edge effect is rounded by applying the restriction term. This means that σ_r also affects the
474 distribution of the contact pressure. In addition, σ_g also modifies the contact area. Thus, it would
475 be interesting to separate the role of each term. In other words, it would be convenient to define
476 σ_g and σ_r such that the former would only affect the contact pressure distribution while the latter
477 would only modify the contact area.

478 6.8. Comparison with other works

479 The results presented show that the proposed model generates high quality contact interfaces
480 (in terms of contact pressure distribution). We obtained values of Q_p between 0.96 and 0.98. As
481 can be seen in Table 3, only the works presented in [19, 26] are superior in terms of Q_p (see
482 Table 3). Nonetheless, in [19], the methodology is defined only for contact against flat geometries.
483 The work presented in [26] can be applied to curved geometries. However, its design variable is
484 not the shape of the contact interface but the material distribution; hence, the potential engineering
485 applications are not the same than those of the present work.

486 7. Conclusion

487 In this work, we studied the impact of applying the rules of synovial joint morphogenesis on
488 the generative design of contact interfaces. For that, we implemented them in an algorithm that
489 mimics synovial joint growth under engineering conditions in an iterative manner. The output of
490 the generative design process is a contact interface adapted to the loading conditions.

491 The obtained contact interfaces provided a uniform pressure distribution. Such interfaces were
492 achieved following this stress–adaptive rule: growth is proportional to compressive stress, but it
493 is inhibited at regions with high shear stress or high compressive stress. The results showed that
494 the rule works well under different two–dimensional situations (plane–strain, plane–stress and ax-
495 isymmetric conditions), with different material properties and under different loading conditions.
496 Further, the results also showed that the adaptation process can be applied against curved surfaces.
497 Furthermore, it is possible to control the size of the contact interface by an additional rule that

498 promotes a global expansion (contraction) when the maximum shear stress is higher (lower) than
499 a given requirement.

500 The proposed model differs from those present in the literature since it does not require the
501 initial geometry to be close to the final solution, which is an important characteristic in terms of
502 generative design.

503 Nonetheless, we acknowledge that our work has some limitations. The present work focuses
504 on the contact interface and does not deal with the shape of the free boundary. In addition, as, so
505 far, the current algorithm has only been implemented for frictionless and linear elastic conditions,
506 we cannot conclude about the direct applicability of the model outside these conditions.

507 Future work is required to continue developing the proposed generative design process. It is
508 necessary to determine appropriate criteria to define the shape of the free boundaries. It would be
509 interesting to study the behaviour of the bio-inspired growth process in applications with dynamic
510 loads or with friction and lubrication. Additionally, more biological processes could be mimicked.
511 For example, it might be interesting to explore the advantages of mimicking the hardening effect
512 during the ossification process or the definition of an anisotropic growth tensor—for instance, one
513 taking into account the direction of the principal strains.

514 Finally, the results presented here demonstrate that synovial joint morphogenesis can be adapted
515 to generative design in engineering.

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