

Robust inversion under uncertainty for flooding risk analysis

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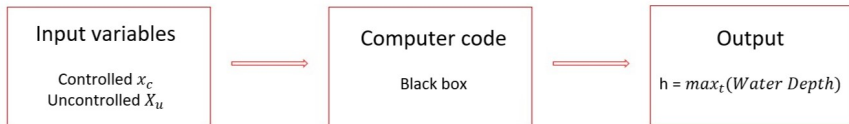
1 Introduction

2 Method

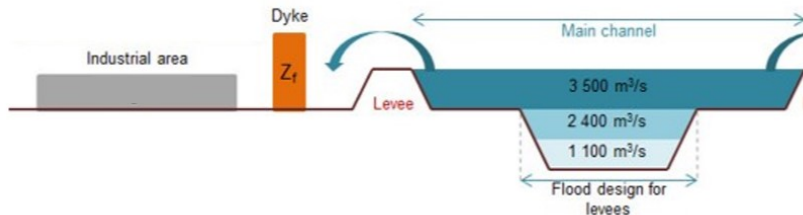
3 Application

Context

- Hydraulic simulators



- River case



Objective

Target variable : h water depth at a specific location

Controlled variable : Height of the dyke z_f

Uncontrolled variable : Peak discharge Q_{max}

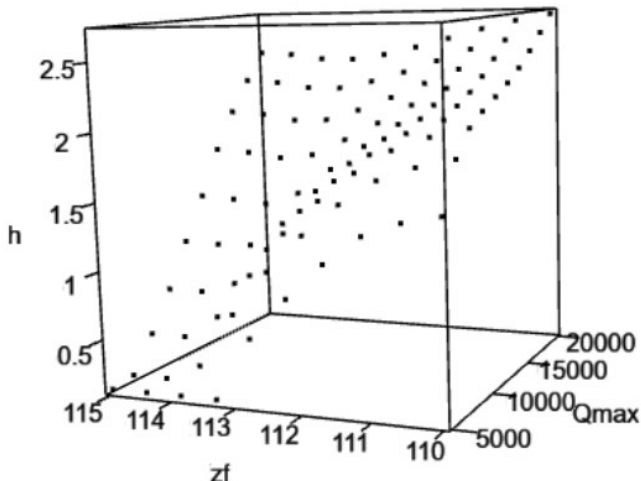
Objective :

$$S_\alpha = \{z_f, \mathbb{P}(h(Q_{max}, z_f) > t) > \alpha\} = \{z_f, \mathbb{Q}_{1-\alpha}(h(Q_{max}, z_f)) > t\}$$

Challenge : Every run of hydraulic simulators is very long \implies metamodel

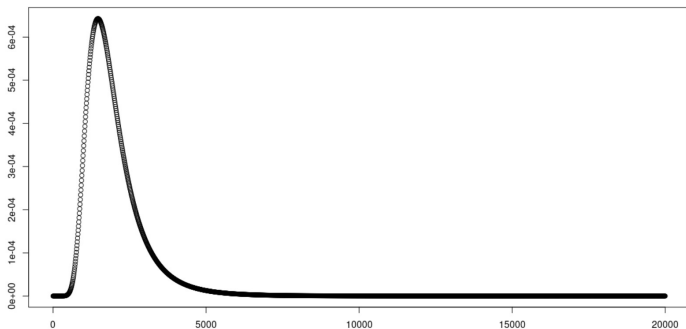
Creation of toy problem

We run 100 hydraulic simulations in a grid $[5000, 20000] \times [110, 115]$ and create an interpolating test function : Reference function (package Akima)



Probabilistic view

$$Q_{max} \sim GEV(loc = 1548.32, scale = 578.45, shape = 0.15)$$



Adapted sampling

$$\alpha = 10^{-4}$$

Simulations are done with $q_{max} \in [5000, 20000]$. and

$$\mathbb{P}(Q_{max} \leq 5000) = 0.986 = p_1$$

We suppose $\forall z_f, \forall q < 5000, h(q, z_f) \leq Q_{1-\alpha}(h(Q_{max}, z_f))$

With \tilde{Q} a random variable with a density function the truncated GEV from 5000 to ∞ , we have :

$$\forall z_f, Q_{1-\alpha}(h(Q_{max}, z_f)) = Q_{1-\tilde{\alpha}}(h(\tilde{Q}, z_f)) \text{ with } \tilde{\alpha} = \frac{\alpha}{1-p_1}$$

$$\implies S_\alpha = \{z_f, Q_{1-\tilde{\alpha}}(h(\tilde{Q}, z_f)) > t\}$$

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General algorithm

Objective : Start with a small design and add points cleverly chosen

1: Create $X = \text{LHS design (n points)}$

2: Compute $Y = h(X)$

3: Fit Metamodel on (X, Y)

4: **while** not stopping criterion **do**

Minimize over $x_{new} = (q, z)$ the criterion $RSUR$

Add $x_{new}^* = \underset{x_{new}}{\operatorname{argmin}} RSUR(x_{new})$ to the design : $X = X \cup x_{new}$

Compute $y_{new} = h(x_{new}^*)$ and add it to Y : $Y = Y \cup y_{new}$

Fit Metamodel on (X, Y)

5: **end while**

RSUR criterion

Objective : The objective is to add points that minimise the uncertainty on the estimation of S_α

Uncertainty around S_α for metamodel M_X :

$$U_{S_\alpha}(M_X) = \int_{z_f} p_{\tilde{\alpha}}^{M_X}(z_f)(1 - p_{\tilde{\alpha}}^{M_X}(z_f))$$

with $p_{\tilde{\alpha}}^{M_X}(z_f) = \mathbb{P} \left[\mathbb{Q}_{1-\tilde{\alpha}}(M_X(\tilde{Q}, z_f)) > t \mid M_X \right]$

RSUR criterion :

$$RSUR(q, z) = \mathbb{E} \left[U_{S_\alpha}(M_{X \cup (q, z)}) \mid M_X(q, z) \right]$$

Computation of $RSUR(x_{new})$

We have $x_{new} = (q, z)$

Require: G a grid in 2D with $(q^k)_{k=1, \dots, N_q}$ i.i.d samples of \tilde{Q} , and N_z equally spaced z_f

- 1: Get $(y_{new}^{(i)})_{i=1, \dots, N_{y_{new}}}$ samples of $M(x_{new})$
- 2: **for** $i = 1 : N_{y_{new}}$ **do**
- 3: Add $(x_{new}, y_{new}^{(i)})$ to the design : $\tilde{X} = X \cup x_{new}$, $\tilde{Y} = Y \cup y_{new}^{(i)}$
- 4: Fit M^i on (\tilde{X}, \tilde{Y})
- 5: Simulate n_{traj} trajectories of the metamodel on G
- 6: $\forall z_f, \forall$ trajectory, compute $\mathbb{Q}_{1-\tilde{\alpha}}(M_X(\tilde{Q}, z_f))$
- 7: $\forall z_f$, compute $p_{\tilde{\alpha}}^{M^i}(z_f)$
- 8: Compute $U_{S_\alpha}(M^i)$
- 9: **end for**
- 10: Compute $RSUR(x_{new}) = \frac{1}{N_{y_{new}}} \sum_{i=1}^{N_{y_{new}}} U_{S_\alpha}(M^i)$

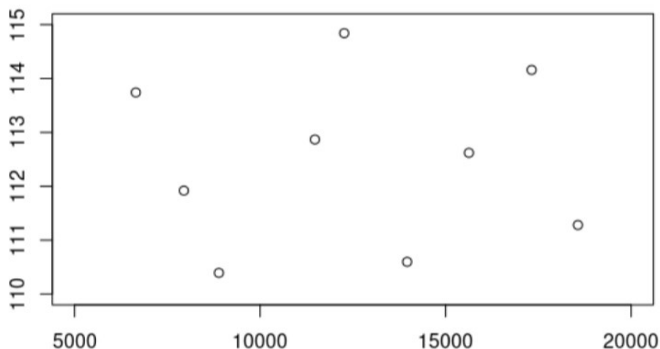
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Initial LHS

We take $\alpha = 10^{-4}$ and $t = 2$

Start with 9 points LHS design :



Initial model

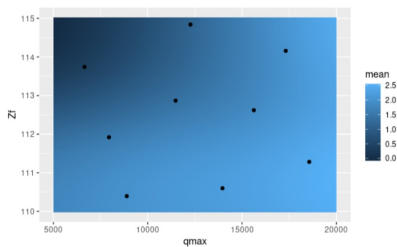


Figure: Kriging mean

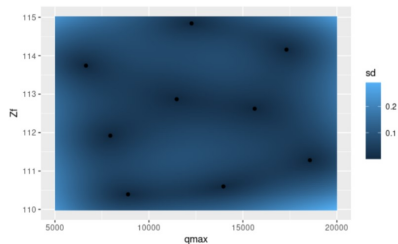
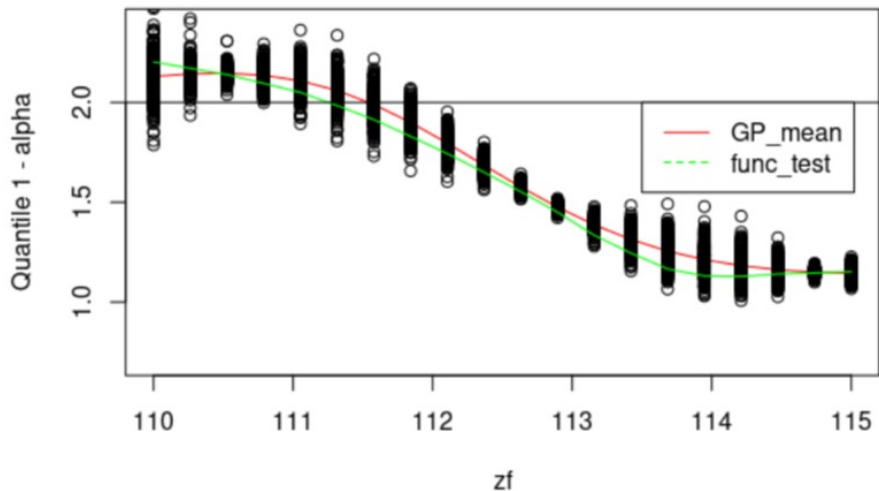
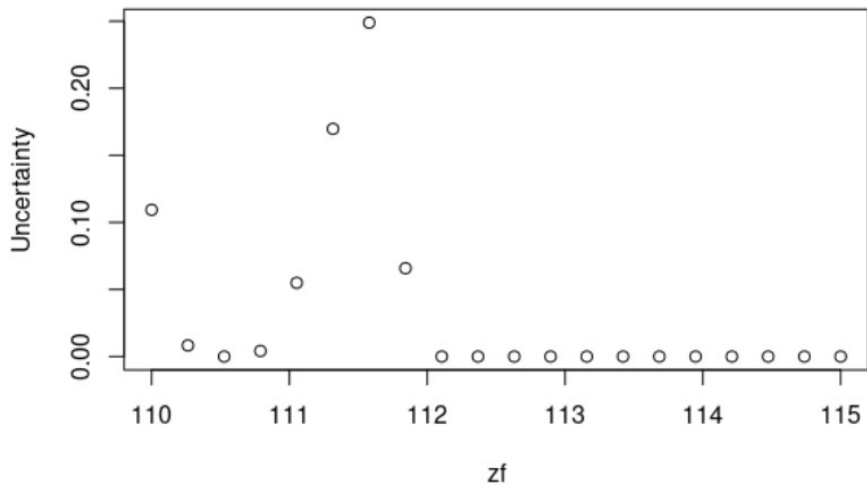


Figure: Kriging std error

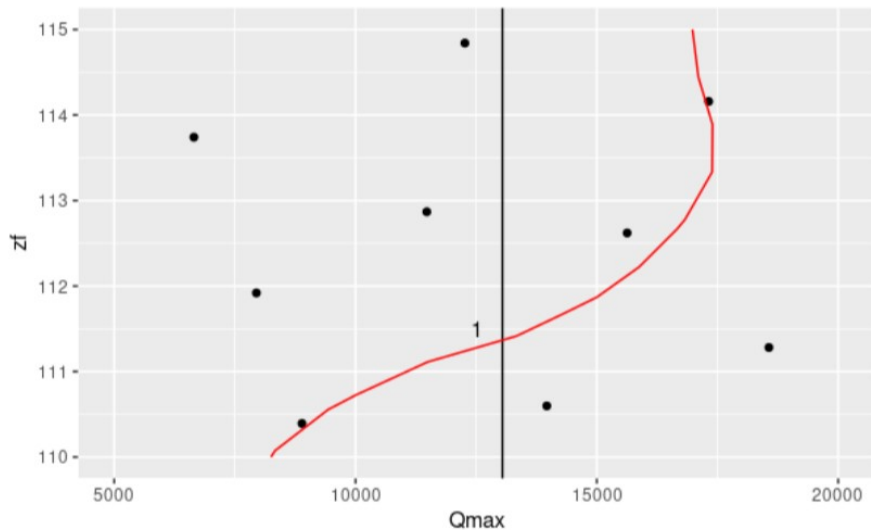
Initial quantiles estimation



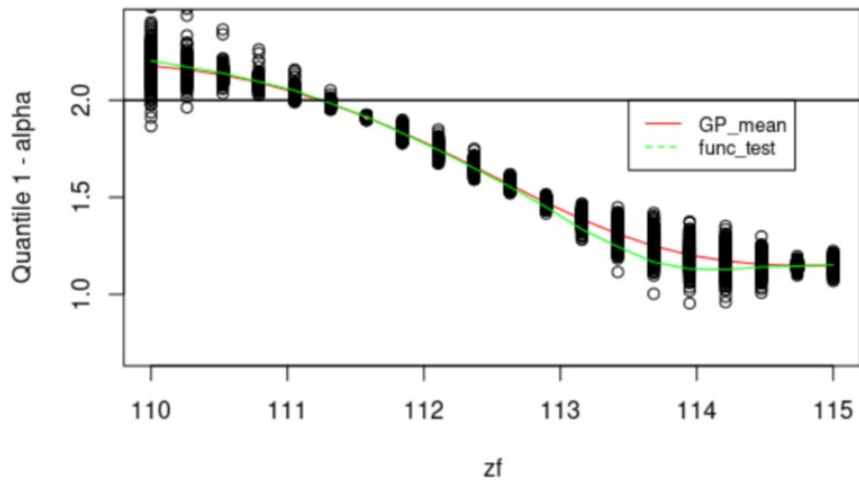
Initial Uncertainty



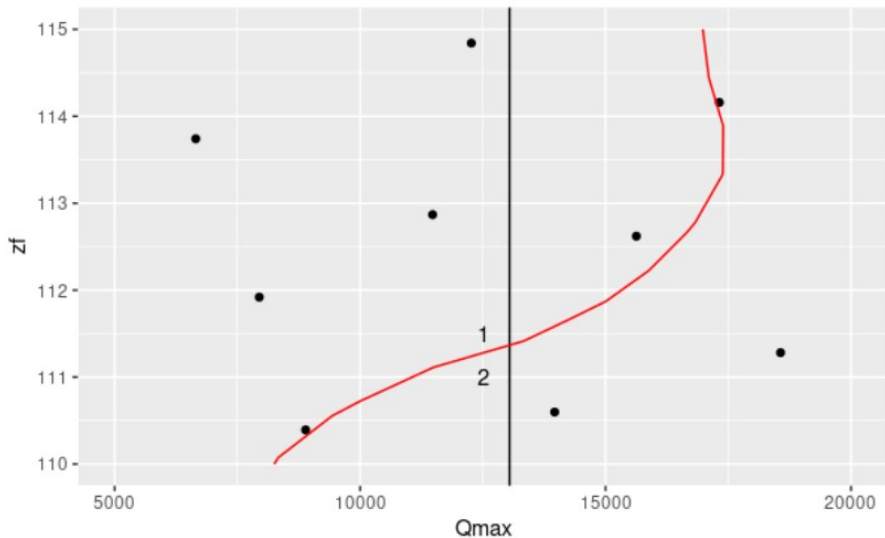
First iteration I



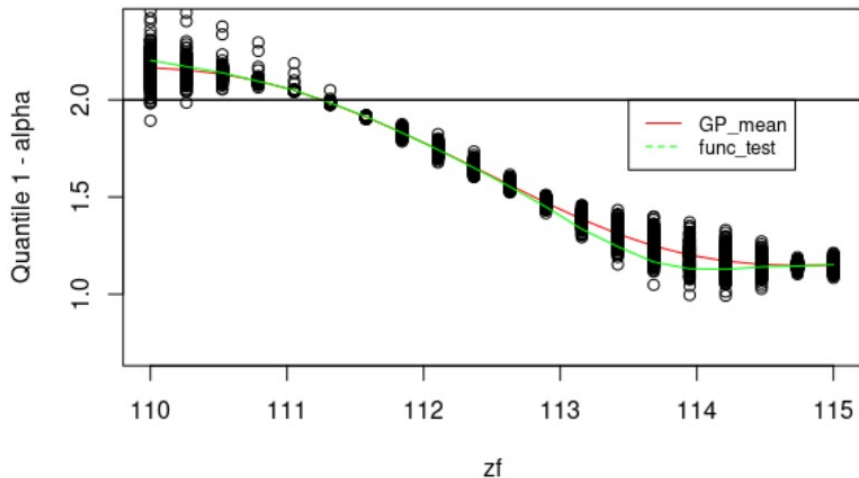
First iteration II



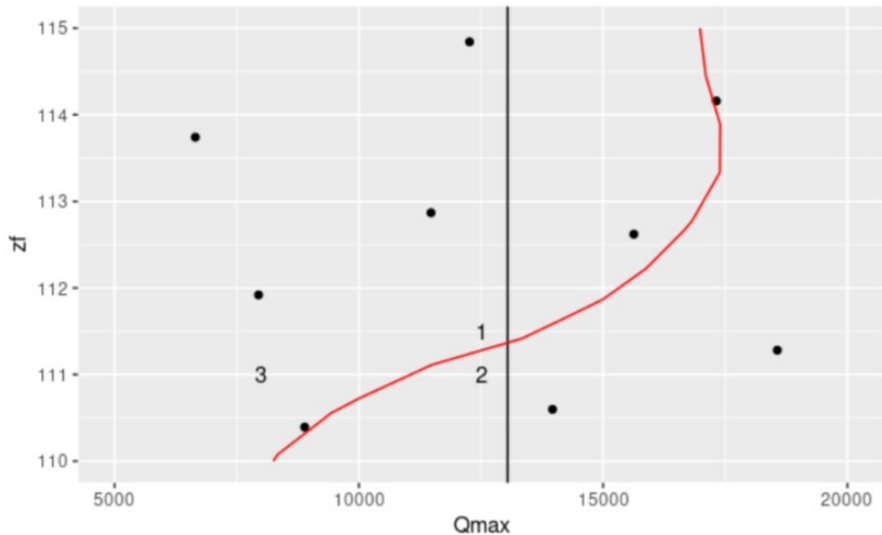
Second iteration I



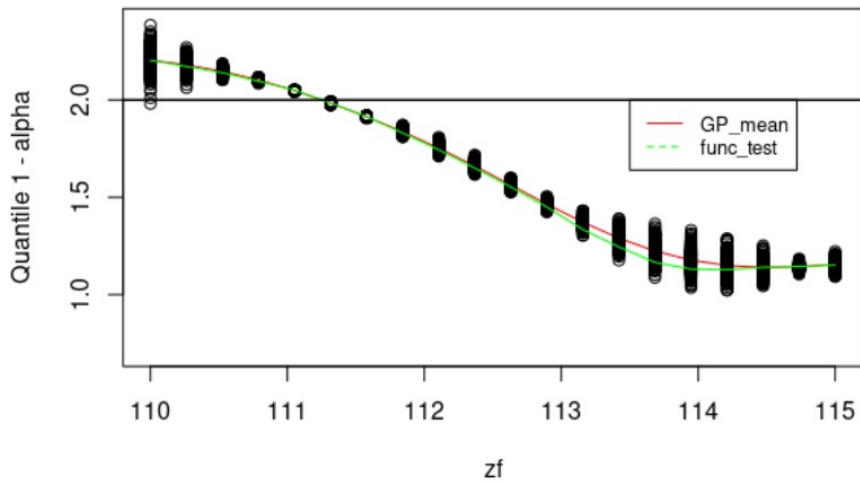
Second iteration II



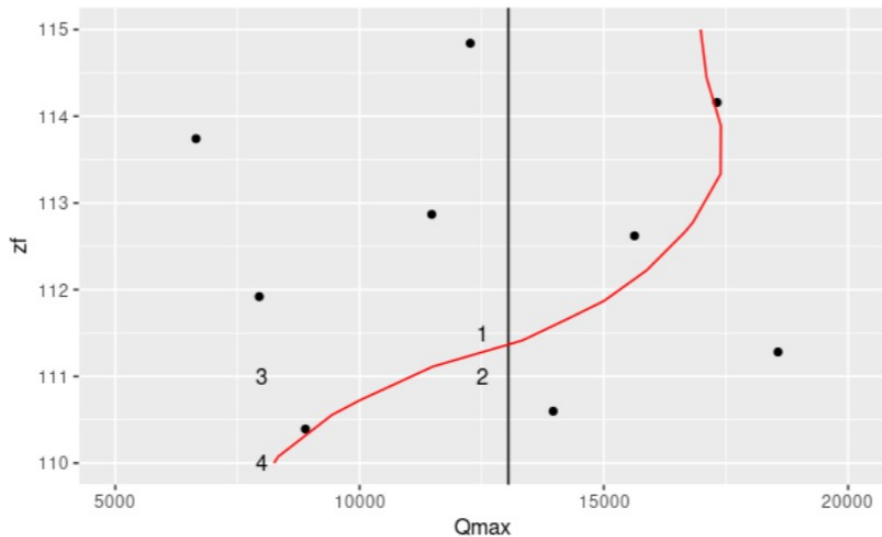
Third iteration I



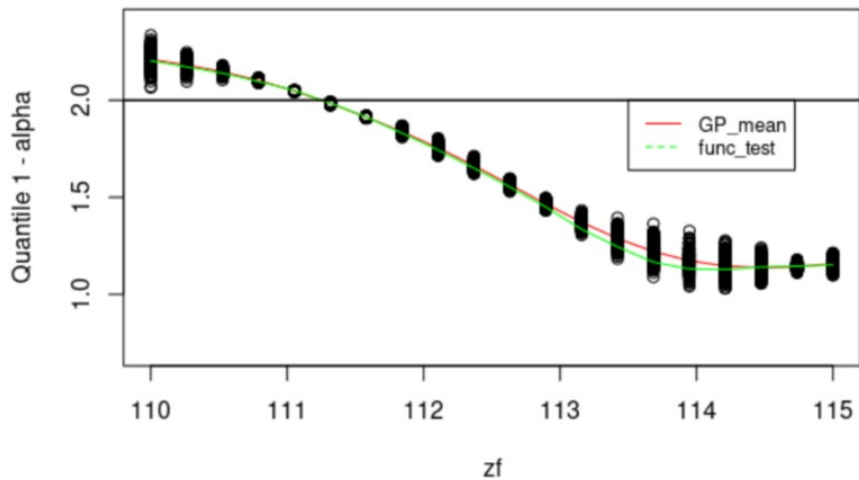
Third iteration II



Fourth iteration I



Fourth iteration II



Future work

- Extend this work to a problem in higher dimension
- Perform an approach based on quantization