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On the adaptation of the exergy definition in the field of aerodynamics

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ABSTRACT

The exergy concept originates from the field of static thermodynamics and expresses the maximum theoretically recoverable mechanical work from a system while it evolves towards its dead state (corresponding to the state in which the system is at a thermodynamic equilibrium with its environment). It accounts for both mechanical and thermal mechanisms, and allows to separate reversible and irreversible losses of potential for mechanical work recovery in the system's transformations. The physical insight provided by this concept motivated the development of an exergy-based performance evaluation method in the field of aerodynamics. The resulting formulation has the advantage of being independent of the feasibility of a drag/thrust breakdown (practical for the analysis of innovative aircraft architectures) and includes thermal effects in the performance metrics. It however relies on an adapted definition of exergy, in particular involving a dead state in motion (i.e. with non-negligible kinetic energy). This adapted definition is not trivial and raises theoretical concerns due to fundamental thermodynamic properties of exergy not being always satisfied. This paper aims at proposing a corrected version of this definition in order to ensure that the fundamental properties of exergy are respected. To this purpose, the exergy concept and the concerns raised by its original adaptation, which to the best of the authors' knowledge has been used in all exergy-based flowfield analyses in the field of applied aerodynamics, are first presented. At a second step, an unsteady exergy balance is derived in the geocentric reference frame (in which there are no ambiguities in the definition of exergy) and then transformed to one corresponding to a frame of reference in translation. The corrected adaptation of the exergy definition for aerodynamics applications is extracted from this transformation and the impact on the exergy balance is analysed.

NOMENCLATURE

Formulation

\mathcal{A}_Φ	Rate of viscous anergy generation
\mathcal{A}_w	Rate of anergy generation by shockwaves
$\mathcal{A}_{\nabla T}$	Rate of anergy generation by thermal mixing
\mathcal{X}_m	Rate of mechanical exergy outflow
\mathcal{X}_q	Rate of heat exergy supplied by conduction
\mathcal{X}_{tf}	Rate of exergy supplied by the propulsion system
\mathcal{X}_{th}	Rate of thermal exergy outflow
Φ	Dissipation rate per unit volume, = $(\bar{\tau} \cdot \nabla) \cdot \mathbf{V}$

\mathbf{q}	Heat flux by conduction, = $-k\nabla T$
S_b	Body surface
S_o	Outer boundary of the control volume
\mathbf{n}	Unit normal vector

Fluid and Flow Properties

χ	Mass-specific exergy
χ_f	Mass-specific flow exergy
ρ	Density
$\bar{\tau}$	Viscous stress tensor
E	Mass-specific total energy, = $e + \frac{1}{2}V^2$
e	Mass-specific internal energy, = $c_v T$
h	Mass-specific enthalpy, = $e + \frac{p}{\rho} = c_p T$
k	Thermal conductivity, = $c_p(\frac{\mu}{Pr} + \frac{\mu_t}{Pr_t})$
p	Static pressure
s	Mass-specific entropy
T	Static temperature
\mathbf{V}	Fluid velocity vector
\mathbf{W}	Shockwave velocity vector

Subscripts and superscripts

∞	Quantity at freestream conditions
$(\bar{\cdot})$	Quantity (\cdot) expressed in the geocentric reference frame \bar{R}

1. INTRODUCTION

The precise investigation of aircraft performance through relevant metrics has been a driving factor of research since the birth of the aviation sector. While the efficiency of various aircraft configurations was initially quantified experimentally, the development of numerical tools has progressively allowed to perform accurate simulations for precise performance evaluations. This improved precision, together with the lower costs associated to numerical computations compared to experiments, led to the incorporation of Computational Fluid Dynamics (CFD) to design processes. Consequently, post-processing tools were developed in parallel to enhance the accuracy of physical and numerical analyses of the results obtained from numerical simulations. The possibility to link any performance degradation to physical aspects of the flow field, including relevant efficiency metrics, quickly became a major stake of such post-processing tools.

The most mature approach developed and adopted in the case of aircraft consists in analysing aerodynamic performance with force-based methods. Such a method was developed with a near-field/far-field balance by van der Vooren and Destarac [23, 11], which allows to perform a phenomenological breakdown of drag components. By

quantifying the components of the generated drag, as well as providing a clear link to the physical phenomena in the flow, their formulation gives indications on the design modifications which could be carried out in order to achieve a better aircraft performance. Since then, a large amount of work at ONERA was dedicated to the refinement and extension of this drag decomposition method [8], such as an extension to rotating frames of reference [24, 14] or to unsteady flow analyses [18, 19, 20]. These force-based methods allow to consider the mechanical aspects of the performance analysis, yet are less adapted to configurations involving significant thermal exchanges or for which a clear thrust/drag separation is not possible.

Another analysis, based on a mechanical energy balance and referred to as power balance, was more recently developed by Drela [12]. This approach gives a clear physical breakdown of the flow of mechanical energy, and is not dependent on the feasibility of a thrust/drag separation. Sources of loss can be identified by a separation between reversible and irreversible losses (the latter being linked to viscous and shock phenomena). The power balance physical interpretation was since further studied [17] and used to evaluate the performance benefits from the Boundary Layer Ingestion (BLI) concept [21, 22, 17]. Whereas this method allows to study concepts involving strong engine-airframe interactions, it still does not account for thermal effects present in the flow field, which are non-negligible for a wide range of cases.

These considerations led to the development of another approach at ONERA based on the concept of exergy, which corresponds to the maximum mechanical work that can theoretically be extracted from a system while it evolves towards its dead state. The exergy concept is inherited from thermodynamics [10], where it is commonly used in order to perform static analyses. Arntz et al. [6, 4] proposed an adaptation of the exergy definition for aerodynamics applications, and developed an exergy balance adapted to such configurations. This allows to perform a separation between reversible and irreversible losses of exergy, with a link to the physical phenomena involved. This method has been applied to conventional and disruptive configurations of aircraft [4, 5, 7], as well as studied from a numerical point of view [15]. In recent years, further work was performed to use this exergy balance for various numerical and experimental applications [1, 2, 3], extend it to rotating frames of reference [13, 9] and to the analysis of unsteady flows [16]. These developments are all based on the exergy definition of Arntz et al., yet the latter raises questions as it does not guarantee that fundamental properties of exergy are always satisfied (such as the guaranteed positivity of exergy [10]).

The main objective of this paper is to present a clarification on the adaptation of the exergy definition for flowfield analyses in the context of aerodynamics studies. Sec. 2 details the exergy concept as defined in thermodynamics alongside its adaptation by Arntz et al. and the concerns it raises. Sec. 3 then presents the derivation of an unsteady exergy balance for an aircraft configuration in the geocentric reference frame, for which there is no ambiguity in the exergy definition, as the reference state used in the exergy definition possesses no kinetic energy. In Sec. 4, this balance is transformed in a frame of reference attached to an aircraft-type configuration, from

which the correct exergy definition is extracted and the impact on the exergy balance (derivation and final form) is evaluated.

2. ON THE DEFINITION OF EXERGY IN AERODYNAMICS APPLICATIONS

2.1 Definition of an exergy balance

Let us consider a volume of fluid surrounded by a thermodynamic reservoir with which it exchanges mechanical work (noted W), heat (noted Q) and mass, as illustrated in Fig. 1. An inertial reference frame \tilde{R} is defined so that the reservoir is considered to be at rest with no velocity. The dead state of the system under study (here the fluid) is defined as the thermodynamic state in which the fluid is at a thermodynamic equilibrium with its environment, i.e. there is no potential for work recovery as no thermodynamic exchanges take place between the system and its environment. In this particular case, the dead state of the fluid corresponds to the thermodynamic state of the reservoir surrounding it, since the latter is supposed to be large enough (in comparison to the system studied) for its thermodynamic state variables to be considered constant.

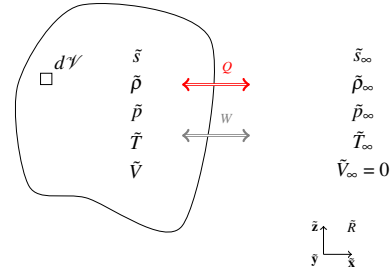


Figure 1: Volume of fluid exchanging work and heat with a thermodynamic reservoir at rest.

The goal is to quantify the maximum mechanical work that could be extracted from the mechanical and thermal thermodynamic processes that the fluid undergoes while it evolves towards its dead state. This is equivalent to quantifying the exergy of the fluid [10], which corresponds to the case where the mechanical work available is completely recovered and the thermal exchanges are converted into useful work by the use of a Carnot machine. When considering a perfect gas and neglecting the kinetic and gravitational potential energy, the specific exergy is defined as:

$$\tilde{\chi} = (\tilde{e} - \tilde{e}_\infty) + \tilde{p}_\infty \left(\frac{1}{\tilde{\rho}} - \frac{1}{\tilde{\rho}_\infty} \right) - \tilde{T}_\infty (\tilde{s} - \tilde{s}_\infty) \quad (1)$$

This quantity is always positive [10]. It means that physically, there is always a potential for work recovery as long as the fluid is not completely at a thermodynamic equilibrium with its environment. If the fluid is considered to be in motion with respect to its dead state, the kinetic energy cannot be neglected, but must be included the above exergy definition to get:

$$\tilde{\chi} = (\tilde{e} - \tilde{e}_\infty) + \tilde{p}_\infty \left(\frac{1}{\tilde{\rho}} - \frac{1}{\tilde{\rho}_\infty} \right) - \tilde{T}_\infty (\tilde{s} - \tilde{s}_\infty) + \frac{1}{2} \tilde{\mathbf{V}}^2 \quad (2)$$

where $\tilde{\mathbf{V}}$ is defined as the fluid velocity relative to its velocity in its dead state. As kinetic energy is also positive, specific exergy is mathematically always positive. This is physically correct as it represents the work that can be extracted from the thermodynamic transformations of the fluid between its considered state and its dead state. It cannot be negative mathematically or physically, as it would not be possible to extract a negative quantity of useful work. An exergy balance can be directly established from this definition as:

$$\frac{d}{dt} \int_{\mathcal{V}} \tilde{\rho} \tilde{\chi} d\mathcal{V} = \dot{W} + \left(1 - \frac{\tilde{T}_\infty}{\tilde{T}}\right) \dot{Q} - \dot{D}^* \quad (3)$$

where \dot{W} is the rate of mechanical work extracted, $(1 - \tilde{T}_\infty/\tilde{T})\dot{Q}$ is the rate of mechanical work extracted from thermal exchanges using a Carnot Machine and \dot{D}^* is the rate of exergy destruction in the volume. Additionally, \dot{W} can be further decomposed by introducing the boundary pressure mechanical work \dot{W}_p , as $\dot{W} = \dot{W}' + \dot{W}_p$. Thus, \dot{W}' represents the part of mechanical work exchanged which is not linked to boundary pressure work. When considering a steady-flow open system, another quantity referred to as *flow exergy* [10] can be introduced. It is defined by adding the transfer power (corresponding to the pressure forces driving the flow) to the exergy definition of Eq. (2). This gives:

$$\tilde{\chi}_f = \delta \tilde{e} + \tilde{p}_\infty \delta \left(\frac{1}{\tilde{\rho}} \right) - \tilde{T}_\infty \delta \tilde{s} + \frac{1}{2} \tilde{\mathbf{V}}^2 + \underbrace{\frac{\delta \tilde{p}}{\tilde{\rho}}}_{\text{Transfer power}} \quad (4)$$

$$= \delta \tilde{h} - \tilde{T}_\infty \delta \tilde{s} + \frac{1}{2} \tilde{\mathbf{V}}^2 \quad (5)$$

This quantity represents the exergy of a steady stream of matter and can be positive or negative. The exergy balance can then be reexpressed for a volume without the presence of a discontinuity in the flow field as:

$$\int_{\mathcal{V}} \frac{\partial \tilde{\rho} \tilde{\chi}}{\partial t} d\mathcal{V} + \int_{\partial \mathcal{V}} \tilde{\rho} \tilde{\chi}_f \tilde{\mathbf{V}} \cdot \tilde{\mathbf{n}} dS = \dot{W}' + \left(1 - \frac{\tilde{T}_\infty}{\tilde{T}}\right) \dot{Q} - \dot{D}^* \quad (6)$$

In this form of the exergy balance, the boundary pressure work is included in the exergy flux through surfaces in order to let the flow exergy appear. The equation established above shows that, in the case of a steady-flow hypothesis, the specific flow exergy is conserved. In non-steady flows, the conserved quantity is rather the specific exergy. The link between specific exergy and specific flow exergy is thus similar to the link between total energy and total enthalpy.

2.2 Previous work on exergy analysis in the field of aerodynamics

The use of an exergy balance for flow field analyses in aerodynamics applications was first introduced by Arntz et al. [4, 6]. The equations for a steady flow were derived in a reference frame R attached to a body in translation and the formulation was applied to several configurations with different levels of complexity [5, 7]. These derivations were based on the conservation equations, the entropy equation and a definition of exergy where the atmosphere is considered as the fluid's dead state (which is not at rest for a translating frame of reference). Due to the kinetic energy of the atmosphere being non-zero

in the reference frame in translation (attached to an object moving with velocity $-V_\infty$), the specific exergy was defined by Arntz et al. as:

$$\chi_f^A = (e - e_\infty) + \left(\frac{p}{\rho} - \frac{p_\infty}{\rho_\infty} \right) - T_\infty (s - s_\infty) + \frac{1}{2} \mathbf{V}^2 - \frac{1}{2} \mathbf{V}_\infty^2 \quad (7)$$

Note that this definition of exergy rather corresponds to a flow exergy, the use of which is still valid in those authors' derivation since it was made for a steady flow (cf. Eq (6)). Since then, multiple works have been based on this definition of exergy in order to use the same exergy-based formulation for experimental and numerical applications [1, 2, 3, 15] as well as unsteady flow analyses [16]. Other works have considered a reference state at rest, but have formulated an unsteady balance of flow exergy rather than exergy (cf. Sec. 2.1) [13]. Defining the exergy through Eq. (7) leads to several unclear considerations:

- As explained in Sec. 2.1, an energy balance leads to the conservation of total energy and not total enthalpy, for which a conservation equation can be written only under a steady flow assumption. Similarly, in the exergy balance, the quantity transported is exergy and not flow exergy. Hence, for unsteady (and steady to be fully rigorous) applications, the exergy definition used as a starting point when deriving the exergy balance must be Eq. (2).
- Exergy is always positive by definition (cf. Eq. (2)), with $\tilde{\mathbf{V}}$ considered as the velocity *relative to the dead state*. For external aerodynamics applications, the dead state is commonly taken as the atmosphere's thermodynamic state. Considering a fixed frame of reference with no fluid velocity in the atmosphere, this means that the flow under study has no kinetic energy in its dead state. This case is similar to the case detailed in Fig. 1, which means that Eq. (2) defines exergy.

Now let us consider the same dead state with a reference frame R moving at a velocity $-V_\infty$. In this case, Arntz et al. implicitly adapted the flow exergy definition by subtracting the kinetic energy of the flow in its dead state to the flow exergy (cf. Eq. (7)). Doing the same for the specific exergy definition would lead to:

$$\chi^A = \delta e + p_\infty \delta \left(\frac{1}{\rho} \right) - T_\infty \delta s + \frac{1}{2} (\mathbf{V}^2 - \mathbf{V}_\infty^2) \quad (8)$$

This definition was the one used as a starting point for the exergy balance extension to rotating reference frames investigated at ONERA [9]. This however is not trivial and leads to the loss of a fundamental property of exergy, as the specific exergy is not guaranteed to remain positive regardless of the reference state considered. As a result, the exactness of the above definition raises questions and requires careful investigation.

The following sections aim at clarifying these issues and at evaluating the potential impact of an error in the exergy definition used for the derivation of an exergy balance adapted to aerodynamics applications. To this purpose, the derivation of an unsteady exergy balance is first

presented in a reference frame for which the atmosphere, considered as the dead state of the gas under study, has no kinetic energy. In this case, it is clear that the exergy of the fluid is defined by Eq. (2) (where velocity is defined in the fixed frame of reference), excluding any ambiguity in the final balance. The final balance equation is then transformed to a reference frame in translation, which corresponds to the classical approach in applied aerodynamics studies. From this transformation, a new definition of exergy expressed in a translating frame of reference (thus different from Eq. (8)) is then introduced.

3. DERIVATION OF AN UNSTEADY EXERGY BALANCE IN THE GEOCENTRIC REFERENCE FRAME

3.1 System definition

The analysis is performed in a continuous control volume \mathcal{V} limited by an outer boundary S_o and the aircraft surface S_b , as shown in Fig. 2. This volume is closed and its limits can be permeable, with a vector \mathbf{n} normal to the surface and pointing outwards of the volume. A shockwave discontinuity surrounded by a surface S_w with a normal vector \mathbf{n} is also considered in the calculations. This last normal vector is pointing outwards of the control volume, i.e. towards the interior of the shockwave volume.

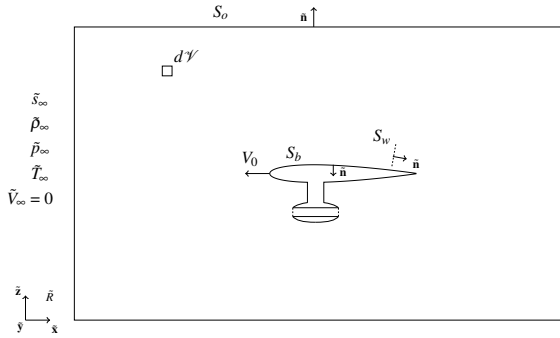


Figure 2: 2D cross section of a 3D control volume surrounding the aircraft.

The system is thermodynamically open as it exchanges mass, work and heat with the surrounding fluid across its boundaries. The atmosphere is considered as a thermal and mechanical reservoir which also corresponds to the dead state in the exergy definition. In this case, the analysis is carried out in a fixed reference frame \tilde{R} , assumed inertial. The control volume is therefore moving with the aircraft at a velocity $\mathbf{V}_0 = -V_0\tilde{\mathbf{x}}$. It is supposed that the shockwave discontinuity is attached to the aircraft and moves with a velocity $\tilde{\mathbf{W}}$ in \tilde{R} , which is equal to \mathbf{V}_0 in the case of a steady shockwave. As a consequence of this choice of reference frame, the atmosphere is considered to be at rest with $\tilde{\mathbf{V}}_\infty = \mathbf{0}$, and the flow is unsteady in \tilde{R} .

The divergence theorem and the definition of the material derivative in this case are written as:

$$\int_{\mathcal{V}} \nabla \cdot (\cdot) d\mathcal{V} = \int_{S_b} (\cdot) \cdot \mathbf{n} dS + \int_{S_o} (\cdot) \cdot \mathbf{n} dS - \int_{S_w} [\![\cdot]\!] \cdot \mathbf{n} dS \quad (9)$$

$$\frac{d}{dt} \int_{\mathcal{V}} (\cdot) d\mathcal{V} = \int_{\mathcal{V}} \frac{\partial (\cdot)}{\partial t} d\mathcal{V} + \int_{\partial \mathcal{V}} (\cdot) \mathbf{V} \cdot \mathbf{n} dS - \int_{S_w} [\![\cdot]\!] \mathbf{W} \cdot \mathbf{n} dS \quad (10)$$

where (\cdot) is a continuous tensor and $[\![\cdot]\!]$ is the jump across the discontinuous shockwave surface S_w .

3.2 Conservation equations

The Navier-Stokes conservation equations for the mass, momentum and total energy of the fluid expressed in the reference frame \tilde{R} are:

$$\begin{cases} \int_{\mathcal{V}} \frac{\partial \tilde{\rho}}{\partial \tilde{t}} d\mathcal{V} = - \int_{\partial \mathcal{V}} \tilde{\rho} \tilde{\mathbf{V}} \cdot \mathbf{n} dS + \int_{S_w} [\![\tilde{\rho} \tilde{\mathbf{W}}]\!] \cdot \mathbf{n} dS & (11) \\ \int_{\mathcal{V}} \frac{\partial \tilde{\rho} \tilde{\mathbf{V}}}{\partial \tilde{t}} d\mathcal{V} = - \int_{\partial \mathcal{V}} (\tilde{\rho} \tilde{\mathbf{V}} \otimes \tilde{\mathbf{V}} + (\tilde{p} - \tilde{p}_\infty) \tilde{\mathbf{I}} - \tilde{\boldsymbol{\tau}}) \cdot \mathbf{n} dS \\ \quad + \int_{S_w} [\![\tilde{\rho} \tilde{\mathbf{V}} \otimes \tilde{\mathbf{W}}]\!] \cdot \mathbf{n} dS & (12) \\ \int_{\mathcal{V}} \frac{\partial \tilde{\rho} \tilde{E}}{\partial \tilde{t}} d\mathcal{V} = - \int_{\partial \mathcal{V}} (\tilde{\rho} \tilde{E} \tilde{\mathbf{V}} + (\tilde{p} \tilde{\mathbf{I}} - \tilde{\boldsymbol{\tau}}) \cdot \tilde{\mathbf{V}} + \tilde{\mathbf{q}}) \cdot \mathbf{n} dS \\ \quad + \int_{S_w} [\![\tilde{\rho} \tilde{E} \tilde{\mathbf{W}}]\!] \cdot \mathbf{n} dS & (13) \end{cases}$$

With the corresponding compatibility equations for the jump across the shockwave:

$$[\![\tilde{\rho} (\tilde{\mathbf{V}} - \tilde{\mathbf{W}})]\!]_w \cdot \mathbf{n} = 0 \quad (14)$$

$$[\![\tilde{\rho} \tilde{\mathbf{V}} \otimes (\tilde{\mathbf{V}} - \tilde{\mathbf{W}}) + \tilde{p} \tilde{\mathbf{I}} - \tilde{\boldsymbol{\tau}}]\!]_w \cdot \mathbf{n} = 0 \quad (15)$$

$$[\![\tilde{\rho} \tilde{E} (\tilde{\mathbf{V}} - \tilde{\mathbf{W}}) + (\tilde{p} \tilde{\mathbf{I}} - \tilde{\boldsymbol{\tau}}) \cdot \tilde{\mathbf{V}} + \tilde{\mathbf{q}}]\!]_w \cdot \mathbf{n} = 0 \quad (16)$$

Due to Eq. (11), the energy equation can be rewritten as:

$$\int_{\mathcal{V}} \frac{\partial \tilde{\rho} \tilde{\delta E}}{\partial \tilde{t}} d\mathcal{V} = - \int_{\partial \mathcal{V}} (\tilde{\rho} \tilde{\delta E} \tilde{\mathbf{V}} + (\tilde{p} \tilde{\mathbf{I}} - \tilde{\boldsymbol{\tau}}) \cdot \tilde{\mathbf{V}} + \tilde{\mathbf{q}}) \cdot \mathbf{n} dS \\ + \int_{S_w} [\![\tilde{\rho} \tilde{\delta E} \tilde{\mathbf{W}}]\!] \cdot \mathbf{n} dS \quad (17)$$

where, under the assumptions made previously, $\tilde{\delta E} = \delta \tilde{e} + \frac{1}{2} \tilde{\mathbf{V}}^2$.

3.3 Entropy equation

In addition to the conservation laws, the equation for the entropy evolution inside the control volume is needed to quantify the energy production in the exergy balance. This equation is derived from the Gibbs equation applied to a fluid element during a time variation $d\tilde{t}$:

$$\tilde{T} \frac{d\tilde{s}}{d\tilde{t}} = \frac{d\tilde{e}}{d\tilde{t}} - \frac{\tilde{p}}{\tilde{\rho}^2} \frac{d\tilde{\rho}}{d\tilde{t}} \quad (18)$$

The local forms of the mass, momentum and energy equations are injected in this expression, and the equation is integrated over the control volume \mathcal{V} to obtain:

$$\int_{\mathcal{V}} \left(\frac{\partial \tilde{\rho} \tilde{\delta s}}{\partial \tilde{t}} + \tilde{\nabla} \cdot (\tilde{\rho} \tilde{\delta s} \tilde{\mathbf{V}}) \right) d\mathcal{V} = \int_{\mathcal{V}} -\frac{1}{\tilde{T}} \tilde{\nabla} \cdot \tilde{\mathbf{q}} d\mathcal{V} \\ + \int_{\mathcal{V}} \frac{1}{\tilde{T}} (\tilde{\boldsymbol{\tau}} \cdot \tilde{\mathbf{V}}) \cdot \tilde{\mathbf{V}} d\mathcal{V} \quad (19)$$

The next step is to introduce Fourier's law $\tilde{\mathbf{q}} = -k \tilde{\nabla} \tilde{T}$ where k is the thermal conductivity and $\tilde{\Phi} = (\tilde{\boldsymbol{\tau}} \cdot \tilde{\mathbf{V}}) \cdot \tilde{\mathbf{V}}$ as the dissipation rate per unit volume. Then, using the divergence theorem, Eq. (11) and multiplying Eq. (19) with \tilde{T}_∞ leads to:

$$\tilde{T}_\infty \int_{\mathcal{V}} \frac{\partial \tilde{\rho} \tilde{\delta s}}{\partial \tilde{t}} d\mathcal{V} = -T_\infty \int_{\partial \mathcal{V}} \tilde{\rho} \tilde{\delta s} \tilde{\mathbf{V}} \cdot \mathbf{n} dS - \int_{\partial \mathcal{V}} \frac{\tilde{T}_\infty}{\tilde{T}} \tilde{\mathbf{q}} \cdot \mathbf{n} dS \\ + \int_{\mathcal{V}} \frac{\tilde{T}_\infty}{\tilde{T}^2} k (\tilde{\nabla} \tilde{T})^2 d\mathcal{V} + \int_{\mathcal{V}} \frac{\tilde{T}_\infty}{\tilde{T}} \tilde{\Phi} d\mathcal{V} + \tilde{T}_\infty \int_{S_w} [\![\frac{1}{\tilde{T}} \tilde{\mathbf{q}} - \tilde{\rho} \tilde{\delta s} \tilde{\mathbf{W}}]\!] \cdot \mathbf{n} dS \quad (20)$$

3.4 Exergy balance

The goal of this section is to derive an exergy balance based on the straightforward definition of exergy for a reference state at rest. In \tilde{R} , the fluid is at rest in its dead state, meaning that specific exergy is defined as:

$$\tilde{\chi} = \delta\tilde{e} + \tilde{p}_\infty \delta \left(\frac{1}{\tilde{\rho}} \right) - \tilde{T}_\infty \delta\tilde{s} + \frac{1}{2} \tilde{\mathbf{V}}^2 \quad (21)$$

without any ambiguity regarding the kinetic energy at its reference state.

The exergy balance then takes the form:

$$\begin{aligned} & \int_{\mathcal{V}} \frac{\partial \tilde{\rho} \tilde{\chi}}{\partial \tilde{t}} d\mathcal{V} + \int_{\partial \mathcal{V}} \tilde{\rho} \tilde{\chi} \tilde{\mathbf{V}} \cdot \tilde{\mathbf{n}} dS - \int_{S_w} \llbracket \tilde{\rho} \tilde{\chi} \tilde{\mathbf{W}} \rrbracket \cdot \tilde{\mathbf{n}} dS \\ &= \int_{\mathcal{V}} \frac{\partial}{\partial \tilde{t}} \left(\tilde{\rho} \delta\tilde{e} - \tilde{\rho} \tilde{T}_\infty \delta\tilde{s} + \frac{1}{2} \tilde{\rho} \tilde{\mathbf{V}}^2 \right) d\mathcal{V} \\ &+ \int_{\partial \mathcal{V}} \left(\tilde{\rho} \delta\tilde{e} - \tilde{\rho} \tilde{T}_\infty \delta\tilde{s} + \frac{1}{2} \tilde{\rho} \tilde{\mathbf{V}}^2 \right) \tilde{\mathbf{V}} \cdot \tilde{\mathbf{n}} dS \\ &+ \int_{\mathcal{V}} \frac{\partial}{\partial \tilde{t}} \left(\tilde{\rho} \tilde{p}_\infty \delta \left(\frac{1}{\tilde{\rho}} \right) \right) d\mathcal{V} + \int_{\partial \mathcal{V}} \tilde{\rho} \tilde{p}_\infty \delta \left(\frac{1}{\tilde{\rho}} \right) \tilde{\mathbf{V}} \cdot \tilde{\mathbf{n}} dS \\ &- \int_{S_w} \llbracket \tilde{\rho} \left(\delta\tilde{e} - \tilde{T}_\infty \delta\tilde{s} + \frac{1}{2} \tilde{\mathbf{V}}^2 \right) \tilde{\mathbf{W}} \rrbracket \cdot \tilde{\mathbf{n}} dS - \int_{S_w} \llbracket \tilde{\rho} \tilde{p}_\infty \delta \left(\frac{1}{\tilde{\rho}} \right) \tilde{\mathbf{W}} \rrbracket \cdot \tilde{\mathbf{n}} dS \end{aligned} \quad (22)$$

Several mathematical manipulations and the injection of Eqs. (11), (17) and (20) into Eq. (22) lead to:

$$\begin{aligned} & \underbrace{\int_{\mathcal{V}} \frac{\partial \tilde{\rho} \tilde{\chi}}{\partial \tilde{t}} d\mathcal{V}}_{(I)} = - \underbrace{\int_{\partial \mathcal{V}} \tilde{\rho} \tilde{\chi} \tilde{\mathbf{V}} \cdot \tilde{\mathbf{n}} dS}_{(II)} + \underbrace{\int_{S_w} \llbracket \tilde{\rho} \tilde{\chi} \tilde{\mathbf{W}} \rrbracket \cdot \tilde{\mathbf{n}} dS}_{(III)} \\ & - \underbrace{\int_{\partial \mathcal{V}} ((\tilde{p}\tilde{\mathbf{I}} - \tilde{\boldsymbol{\tau}}) \cdot \tilde{\mathbf{V}}) \cdot \tilde{\mathbf{n}} dS}_{(IV)} + \underbrace{\int_{\partial \mathcal{V}} \tilde{p}_\infty \tilde{\mathbf{V}} \cdot \tilde{\mathbf{n}} dS}_{(V)} \\ & - \underbrace{\int_{\partial \mathcal{V}} \left(1 - \frac{\tilde{T}_\infty}{\tilde{T}} \right) \tilde{\mathbf{q}} \cdot \tilde{\mathbf{n}} dS}_{(VI)} - \underbrace{\int_{\mathcal{V}} \frac{\tilde{T}_\infty}{\tilde{T}} \tilde{\Phi} d\mathcal{V}}_{(VII)} - \underbrace{\int_{\mathcal{V}} \frac{\tilde{T}_\infty}{\tilde{T}^2} k(\tilde{\nabla}\tilde{T})^2 d\mathcal{V}}_{(VIII)} \\ & - \underbrace{\tilde{T}_\infty \int_{S_w} \llbracket \frac{1}{\tilde{T}} \tilde{\mathbf{q}} + \tilde{\rho} \delta\tilde{s}(\tilde{\mathbf{V}} - \tilde{\mathbf{W}}) \rrbracket \cdot \tilde{\mathbf{n}} dS}_{(VIII)} \quad (23) \end{aligned}$$

This equation is an exergy balance expressed in a fixed reference frame in which the atmosphere possesses no kinetic energy. (I) is the time rate of specific exergy change inside the control volume. (II) represents the exergy flux through the boundaries of the control volume, which can be an overall exergy inflow or outflow (depending on the system under study). (III) is the rate of work done by the pressure and viscous forces on the boundaries of the control volume. (IV) is the rate of isobaric work done at the boundary of the control volume. In the case where the work is performed in order to compress the gas in its transformation towards its dead state, this term represents a usable rate of work done by the atmospheric pressure. In the case where the gas is expanding against the atmospheric pressure, this term is an additional work that has to be provided for the transformation towards the dead state to take place. (V) is the thermal exergy provided or retrieved by thermal conduction through $\partial \mathcal{V}$. (VI), (VII) and (VIII) represent the rate of energy generated

(or equivalently the rate of exergy destroyed irreversibly) respectively due to viscous effects, thermal mixing and entropy creation by shockwaves inside the control volume.

In physical terms, this balance equation expresses that the change of exergy with respect to time inside the control volume is due to any inflow/outflow of exergy through the external surfaces, the rate of work done by external surface forces and irreversible exergy losses inside the control volume.

4. TRANSFORMATION OF THE EXERGY BALANCE FOR A REFERENCE FRAME IN TRANSLATION

4.1 System definition

Let us consider a frame of reference R in translation with a velocity \mathbf{V}_0 with respect to the fixed reference frame \tilde{R} defined in Fig. 2 (as shown in Fig. 3):



Figure 3: Moving frame R at a velocity \mathbf{V}_0 with respect to the fixed frame \tilde{R} .

This reference frame is commonly used in aerodynamic studies to simplify analyses. In this frame the aircraft is motionless, and the flow is steady. However, the fluid is *not* at rest in its dead state, but in motion with a velocity $\mathbf{V}_\infty = -\mathbf{V}_0$.

At $t = 0$, the two frames are considered to be coincident with a common origin. Under these conditions, the variables in the translating frame of reference R can be expressed as functions of the ones linked to \tilde{R} :

$$t = \tilde{t} \quad x = \tilde{x} - V_0 \tilde{t} \quad y = \tilde{y} \quad z = \tilde{z} \quad (24)$$

The time derivative must be transformed using:

$$\begin{aligned} \frac{\partial K}{\partial \tilde{t}} &= \frac{\partial K}{\partial t} \frac{\partial t}{\partial \tilde{t}} + \frac{\partial K}{\partial x} \frac{\partial x}{\partial \tilde{t}} + \frac{\partial K}{\partial y} \frac{\partial y}{\partial \tilde{t}} + \frac{\partial K}{\partial z} \frac{\partial z}{\partial \tilde{t}} \\ &= \frac{\partial K}{\partial t} - \nabla \cdot (K \mathbf{V}_0) + K \nabla \cdot \mathbf{V}_0 \end{aligned} \quad (25)$$

where K is any continuous scalar quantity and the second equality is obtained due to \mathbf{V}_0 being constant in space and time. When considering an integral expression, this gives (using the divergence theorem):

$$\int_{\mathcal{V}} \frac{\partial K}{\partial \tilde{t}} d\mathcal{V} = \int_{\mathcal{V}} \frac{\partial K}{\partial t} d\mathcal{V} - \int_{\partial \mathcal{V}} K \mathbf{V}_0 \cdot \mathbf{n} dS + \int_{S_w} \llbracket K \mathbf{V}_0 \rrbracket \cdot \mathbf{n} dS \quad (26)$$

The system presented in Fig. 2 is then considered with respect to R , as shown in Fig. 4.

The system is thermodynamically open as it exchanges mass, work and heat with the surrounding fluid across its boundaries. The atmosphere is considered as a thermal and mechanical reservoir which also corresponds to the dead state in the exergy definition. In this case the analysis is carried out in R , assumed inertial. The control volume is therefore fixed with the fluid flowing

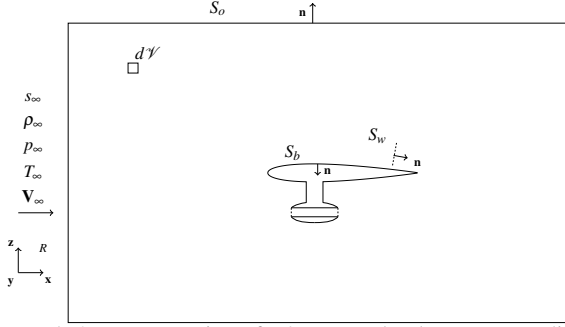


Figure 4: 2D cross section of a 3D control volume surrounding the aircraft as seen in R .

in and out of it. R is chosen so that the aircraft velocity is null, whereas the atmosphere is considered to have a velocity $\mathbf{V}_\infty = -\mathbf{V}_0$. In general, the fluid velocity expressed in the reference frame in translation is $\mathbf{V} = \tilde{\mathbf{V}} - \mathbf{V}_0 = \tilde{\mathbf{V}} + \mathbf{V}_\infty$, and similarly the shock velocity is expressed as $\mathbf{W} = \tilde{\mathbf{W}} + \mathbf{V}_\infty$. In the case where the shock is steady in R , this gives $\mathbf{W} = 0$.

The nabla operator is not affected when moving from \tilde{R} to R , nor is any quantity that is not directly dependent on $\tilde{\mathbf{V}}$ (i.e. $\tilde{\nabla} = \nabla$ and $\tilde{b} = b$ where b is any scalar quantity not dependent on the fluid velocity). As a consequence, the conservative variables are not affected by this change of reference frame, with the exception of the total energy which is expressed in R as:

$$E = e + \frac{1}{2}\mathbf{V}^2 = \tilde{E} + \mathbf{V} \cdot \mathbf{V}_\infty - \frac{1}{2}\mathbf{V}_\infty^2 \quad (27)$$

The above equation highlights that the value of the total energy changes between \tilde{R} and R (i.e. $\tilde{E} \neq E$). An additional component depending on V_∞ appears in the definition of E , which is linked to the kinetic energy difference that is perceived due to the translation of R . The forms of the conservation equation obtained separately for \tilde{E} in \tilde{R} and E in R are however identical, as the terms depending of V_∞ are cancelled due to the mass and momentum conservation equation being expressed in R .

4.2 Exergy balance

4.2.1 Exergy definition

As discussed in Sec. 2, exergy is a scalar quantity that represents an amount of recoverable mechanical work from a system as it evolves towards its dead state through thermodynamic processes. Thus, it is not dependent on the reference frame it is expressed in (i.e. $\chi = \tilde{\chi}$). The exergy definition must however be written with the fluid velocity expressed in R :

$$\chi = \tilde{\chi} = \delta e + p_\infty \delta \left(\frac{1}{\rho} \right) - T_\infty \delta s + \frac{1}{2}(\mathbf{V} - \mathbf{V}_\infty)^2 \quad (28)$$

$$= \chi^A - \mathbf{V} \cdot \mathbf{V}_\infty + \mathbf{V}_\infty^2 \quad (29)$$

In contrast to the definition of total energy in R , which is expressed as a function of \mathbf{V} (cf. Eq. (27)), the above expression is not further modified. This is because, as mentioned previously, the specific exergy quantifies the maximum amount of work that could be recovered as the

system evolves towards its dead state. Thus, the consideration of a relevant dead state for the calculations is essential, and the dead state must correspond to the atmosphere thermodynamic state in aerodynamics applications.

To further detail this explanation, let us consider a fluid system which would only be composed of the atmosphere at rest. In this case, Eq. (28) guarantees that the exergy of the system equals zero independently of the choice of reference frame between \tilde{R} and R . In the case where any additional thermodynamic perturbation is considered (such as a compression or an expansion), the exergy would still be independent on the choice of reference frame, and its thermodynamic properties (in particular the guaranteed positivity of exergy) would be respected.

Another option would have been to adapt the exergy definition to the reference frame considered, in which case the kinetic energy component would be expressed in R as $\frac{1}{2}V^2$ (similarly to what is done for the total energy conservation). In that case, an identical form of the exergy balance could have been derived for this new definition of exergy, as the terms depending of V_∞ would have been cancelled using the conservation equations. However, adapting the definition of the specific exergy to each reference frame (similarly to what is performed for the total energy) would lead to a modification of the system's exergy depending on the chosen frame of reference. Considering the case of a system only composed of the atmosphere as above, the exergy would still be equal to zero in \tilde{R} but would be equal to $\frac{1}{2}V_\infty^2$ in R , even though this would not correspond to a physical acceleration of the fluid. Thus, a fictive non-zero exergy resulting from the change of reference frame would have been interpreted as a potential for mechanical work recovery. A different option would have been to adopt the definition χ^A . This remedies the reference frame inconsistency, since the exergy computed in each reference frame correctly equals zero as the atmosphere kinetic energy is subtracted. Nevertheless, it leads to the loss of the guarantee that the specific exergy is always positive when considering more complex systems.

As a consequence, Eq. (28) constitutes the only definition of specific exergy that both guarantees the thermodynamic properties of $\tilde{\chi}$ (and in particular its positivity) and does not lead to an inconsistency when a change of reference frame is performed. This reasoning is essential, as it shows that the exergy definition introduced by Arntz et al. [4, 6] and used in following aerodynamics studies is not consistent in that aspect.

4.2.2 Transformed exergy balance

The time derivative in Eq. (23) is then transformed in the reference frame R in translation, and the fluid velocity expressed in R is injected to obtain:

$$\begin{aligned} & \int_{\mathcal{V}} \frac{\partial \rho \chi}{\partial t} d\mathcal{V} + \int_{\partial \mathcal{V}} \rho \chi \mathbf{V} \cdot \mathbf{n} dS - \int_{S_w} [\rho \chi \mathbf{W}] \cdot \mathbf{n} dS \\ &= - \int_{\partial \mathcal{V}} ((p\tilde{\mathbf{l}} - \tilde{\mathbf{t}}) \cdot (\mathbf{V} - \mathbf{V}_\infty)) \cdot \mathbf{n} dS + \int_{\partial \mathcal{V}} p_\infty (\mathbf{V} - \mathbf{V}_\infty) \cdot \mathbf{n} dS \\ & - \int_{\partial \mathcal{V}} \left(1 - \frac{T_\infty}{T} \right) \mathbf{q} \cdot \mathbf{n} dS - \int_{\mathcal{V}} \frac{T_\infty}{T} \Phi d\mathcal{V} - \int_{\mathcal{V}} \frac{T_\infty}{T^2} k(\nabla T)^2 d\mathcal{V} \\ & - T_\infty \int_{S_w} \left[\frac{1}{T} \mathbf{q} + \rho \delta s (\mathbf{V} - \mathbf{W}) \right] \cdot \mathbf{n} dS \quad (30) \end{aligned}$$

Note that since V_∞ is constant, $\bar{\bar{\tau}} = \bar{\tau}$ and $\bar{\Phi} = \Phi$. The exergy balance of Eq. (30) is obtained by projecting Eq. (23) in R . It can also be derived by starting from the conservation equations expressed in R and by deriving an exergy balance as was done for \bar{R} in Sec. 3.

4.3 Further decomposition of the exergy balance

4.3.1 Phenomenological decomposition

Whereas Eq. (30) allows a clear physical interpretation by itself, the decomposition is carried on in order to further refine the exergy balance interpretation. The exergy flux is decomposed using the exergy definition in R as:

$$\chi = \underbrace{\delta e + p_\infty \delta \left(\frac{1}{\rho} \right)}_{\chi_s} - T_\infty \delta s + \underbrace{\frac{1}{2} (\mathbf{V} - \mathbf{V}_\infty)^2}_{KE} \quad (31)$$

where χ_s is referred to as the static exergy and KE is the flow kinetic energy.

The static exergy is the exergy that would have been available if the gas had no kinetic energy and no flow-stream was considered. This corresponds to the useful work that could be retrieved from a gas compressed by (or expanding against) a piston in a closed container by recovering the mechanical work (equal to exergy) and transforming the thermal transfers into useful work (by a Carnot machine).

The flow kinetic energy can also be rewritten as $\frac{1}{2} (\mathbf{V} - \mathbf{V}_\infty)^2 = \frac{1}{2} \tilde{\mathbf{V}}^2$. This highlights that the actual flow kinetic energy, as seen in the fixed reference frame \bar{R} , is perceived as a perturbation kinetic energy when seen from the reference frame in translation R . This observation is useful for the following comparisons between this derivation and previous work done on the exergy balance.

The kinetic energy is directly expressed using $\tilde{\mathbf{V}} = \delta \mathbf{V} = (u, v, w)^T$, giving $\tilde{\mathbf{V}}^2/2 = (u^2 + v^2 + w^2)/2$. The exergy flux appearing in Eq. (30) can thus be decomposed to get:

$$\begin{aligned} \int_{\mathcal{V}} \frac{\partial \rho (\chi_s + \frac{1}{2}(u^2 + v^2 + w^2))}{\partial t} d\mathcal{V} &= - \int_{\partial \mathcal{V}} \rho \chi_s \mathbf{V} \cdot \mathbf{n} dS \\ &+ \int_{S_w} [\rho \chi_s \mathbf{W}] \cdot \mathbf{n} dS - \int_{\partial \mathcal{V}} \frac{1}{2} \rho (u^2 + v^2 + w^2) \mathbf{V} \cdot \mathbf{n} dS \\ &+ \int_{S_w} [\frac{1}{2} \rho (u^2 + v^2 + w^2) \mathbf{W}] \cdot \mathbf{n} dS - \int_{\partial \mathcal{V}} \left(1 - \frac{T_\infty}{T} \right) \mathbf{q} \cdot \mathbf{n} dS \\ &- \int_{\partial \mathcal{V}} (((p - p_\infty) \bar{\bar{I}} - \bar{\tau}) \cdot (\mathbf{V} - \mathbf{V}_\infty)) \cdot \mathbf{n} dS - \int_{\mathcal{V}} \frac{T_\infty}{T} \Phi d\mathcal{V} \\ &- \int_{\mathcal{V}} \frac{T_\infty}{T^2} k (\nabla T)^2 d\mathcal{V} - T_\infty \int_{S_w} [\frac{1}{T} \mathbf{q} + \rho \delta s (\mathbf{V} - \mathbf{W})] \cdot \mathbf{n} dS \end{aligned} \quad (32)$$

Several notations are introduced from this equation:

- $\dot{\mathcal{X}}_v(t) := \int_{\mathcal{V}} \frac{\partial \rho \chi_s}{\partial t} d\mathcal{V} + \int_{\mathcal{V}} \frac{\partial \frac{1}{2} \rho (u^2 + v^2 + w^2)}{\partial t} d\mathcal{V}$ is the unsteady rate of change of exergy inside the control volume. It is composed of two terms respectively representing a static and a kinetic contribution.
- $\dot{\mathcal{X}}_{th} := - \int_{\partial \mathcal{V}} \rho \chi_s \mathbf{V} \cdot \mathbf{n} dS + \int_{S_w} [\rho \chi_s \mathbf{W}] \cdot \mathbf{n} dS$ is the

static exergy flux. This term is usually referred to as thermal or thermocompressible exergy flux.

- $\dot{\mathcal{X}}_m := - \int_{\partial \mathcal{V}} \rho \frac{1}{2} (u^2 + v^2 + w^2) \mathbf{V} \cdot \mathbf{n} dS + \int_{S_w} [\frac{1}{2} \rho (u^2 + v^2 + w^2) \mathbf{W}] \cdot \mathbf{n} dS - \int_{\partial \mathcal{V}} ((p - p_\infty) \bar{\bar{I}} - \bar{\tau}) \cdot (\mathbf{V} - \mathbf{V}_\infty) \cdot \mathbf{n} dS$ is the mechanical exergy flux, composed of the flow of kinetic perturbation exergy and the transfer power linked to these velocity perturbations. It is purely dependent on the velocity as perceived in the fixed reference frame \bar{R} , which is considered to be a perturbation velocity when analysing the configuration in R .
- $\dot{\mathcal{X}}_q := - \int_{\partial \mathcal{V}} (1 - \frac{T_\infty}{T}) \mathbf{q} \cdot \mathbf{n} dS$ is the exergy transferred by thermal conduction through non-adiabatic surfaces.
- $\dot{\mathcal{A}}_\phi := \int_{\mathcal{V}} \frac{T_\infty}{T} \Phi d\mathcal{V}$ is the viscous anergy, corresponding to the rate of irreversible exergy destruction due to viscous effects.
- $\dot{\mathcal{A}}_{\nabla T} := \int_{\mathcal{V}} \frac{T_\infty}{T^2} k (\nabla T)^2 d\mathcal{V}$ is the thermal anergy, corresponding to the rate of irreversible exergy destruction due to thermal mixing.
- $\dot{\mathcal{A}}_w := T_\infty \int_{S_w} [\frac{1}{T} \mathbf{q} + \rho \delta s (\mathbf{V} - \mathbf{W})] \cdot \mathbf{n} dS$ is the wave anergy, corresponding to the rate of irreversible exergy destruction due to the presence of shockwaves.

Using these notations, the exergy balance is rewritten in a compact form as:

$$\underbrace{\dot{\mathcal{X}}_v(t)}_{\text{Unsteady exergy variation}} = \underbrace{\dot{\mathcal{X}}_m + \dot{\mathcal{X}}_{th} + \dot{\mathcal{X}}_q}_{\text{Flow exergy flux}} - \underbrace{(\dot{\mathcal{A}}_\phi + \dot{\mathcal{A}}_{\nabla T} + \dot{\mathcal{A}}_w)}_{\text{Exergy destruction}} \quad (33)$$

The above equation corresponds to a classical balance of a physical quantity in which any unsteady variation comes from its fluxes at the domain borders and its volume production or destruction within the volume. It is a more detailed equivalent of Eq. (6) in R .

4.3.2 Separation of the contributions on S_b and S_o

The above analysis can be refined by separating the contributions on S_b (referred to as near field) and S_o (referred to as far field) for the exergy flux terms. For example, the mechanical exergy is decomposed as $\dot{\mathcal{X}}_m = \dot{\mathcal{X}}_{mb} - \dot{\mathcal{X}}_{mo}$ where:

$$\dot{\mathcal{X}}_{mb} = - \int_{S_b} \left(\rho \frac{1}{2} (\mathbf{V} - \mathbf{V}_\infty)^2 \mathbf{V} + ((p - p_\infty) \bar{\bar{I}} - \bar{\tau}) \cdot (\mathbf{V} - \mathbf{V}_\infty) \right) \cdot \mathbf{n} dS \quad (34)$$

$$\begin{aligned} \dot{\mathcal{X}}_{mo} &= \int_{S_o} \left(\rho \frac{1}{2} (\mathbf{V} - \mathbf{V}_\infty)^2 \mathbf{V} + ((p - p_\infty) \bar{\bar{I}} - \bar{\tau}) \cdot (\mathbf{V} - \mathbf{V}_\infty) \right) \cdot \mathbf{n} dS \\ &- \int_{S_w} [\frac{1}{2} \rho (u^2 + v^2 + w^2) \mathbf{W}] \cdot \mathbf{n} dS \end{aligned} \quad (35)$$

The sign convention is chosen so that any near-field contribution is considered as an exergy inflow while any far-field contribution is considered as an exergy outflow. The exergy balance then becomes:

$$\begin{aligned} \dot{\mathcal{X}}_{mb} + \dot{\mathcal{X}}_{thb} + \dot{\mathcal{X}}_{qb} &= \dot{\mathcal{X}}_v(t) + \dot{\mathcal{X}}_{mo} + \dot{\mathcal{X}}_{tho} \\ &+ \dot{\mathcal{X}}_{qo} + \dot{\mathcal{A}}_\phi + \dot{\mathcal{A}}_{\nabla T} + \dot{\mathcal{A}}_w \end{aligned} \quad (36)$$

where the far-field terms are placed on the same side as the unsteady exergy variation. This choice is arbitrary in the present case, unlike formulations such as the unsteady drag breakdown developed by Toubin et al. [18, 19, 20], where this was required to account for delay effects.

4.3.3 Decomposition of the near-field mechanical exergy

While the exergy balance derived above is complete, the analysis can be further refined. This is done by decomposing the mechanical contribution to the near-field exergy inflow (defined as $\dot{\mathcal{X}}_b = \dot{\mathcal{X}}_{mb} + \dot{\mathcal{X}}_{thb}$) as:

$$\begin{aligned} \dot{\mathcal{X}}_b = & - \int_{S_b} \left(\rho \left(\chi_s + \frac{1}{2} V^2 \right) \mathbf{V} + ((p - p_\infty) \bar{\mathbf{I}} - \bar{\boldsymbol{\tau}}) \cdot \mathbf{V} \right) \cdot \mathbf{n} dS \\ & + \int_{S_b} (\rho \mathbf{V}_\infty \cdot \mathbf{V}) \mathbf{V} \cdot \mathbf{n} dS - \int_{S_b} \frac{1}{2} V_\infty^2 \rho \mathbf{V} \cdot \mathbf{n} dS \\ & + \mathbf{V}_\infty \cdot \int_{S_b} ((p - p_\infty) \bar{\mathbf{I}} - \bar{\boldsymbol{\tau}}) \cdot \mathbf{n} dS \end{aligned} \quad (37)$$

This decomposition consists in separating the contributions depending on \mathbf{V}_∞ , i.e. related to the change of reference frame, and the ones depending on V . The second and third terms of the above equation are obtained by expanding the kinetic contribution to specific exergy as $\chi = \chi_s + V^2/2 - \mathbf{V} \cdot \mathbf{V}_\infty + V_\infty^2/2$. They represent fluxes of the terms responsible for the kinetic energy difference (associated to a reference frame transformation) through permeable parts of S_b . The second term of Eq. (37) represents a kinetic energy difference associated to a variation of momentum, while the third one is representative of a kinetic energy difference associated to a non-zero mass-flow variation along S_b (e.g. unbalanced massflow rate across the propulsion system permeable boundaries). The last term of Eq. (37) is finally representative of the variation of work rate of surface forces on S_b associated to the reference frame transformation. From this decomposition and by separating the contributions on permeable and non-permeable surfaces of S_b (respectively noted S_{bp} and S_{bnp}), two quantities are introduced:

$$\dot{\mathcal{X}}_{if} := - \int_{S_{bp}} \left(\rho \left(\chi_s + \frac{1}{2} V^2 \right) \mathbf{V} + ((p - p_\infty) \bar{\mathbf{I}} - \bar{\boldsymbol{\tau}}) \cdot \mathbf{V} \right) \cdot \mathbf{n} dS \quad (38)$$

$$\begin{aligned} \dot{\mathcal{X}}_{trb} := & - \mathbf{V}_\infty \cdot \int_{S_{bp}} (\rho \mathbf{V} \otimes \mathbf{V} + (p - p_\infty) \bar{\mathbf{I}} - \bar{\boldsymbol{\tau}}) \cdot \mathbf{n} dS \\ & - \mathbf{V}_\infty \cdot \int_{S_{bnp}} ((p - p_\infty) \bar{\mathbf{I}} - \bar{\boldsymbol{\tau}}) \cdot \mathbf{n} dS + \int_{S_{bp}} \frac{1}{2} V_\infty^2 \rho \mathbf{V} \cdot \mathbf{n} dS \end{aligned} \quad (39)$$

$\dot{\mathcal{X}}_{if}$ is referred to as the throughflow exergy, and represents the flux of a flow exergy that is computed for a reference state at rest in R . As this is not the real flow exergy flux (which is defined with respect to a reference state at rest in \tilde{R}), an exergy difference $\dot{\mathcal{X}}_{trb}$ composed of three different terms which are linked to the momentum and mass conservation in R appears. This difference appears for the sole reason that the analysis is performed in a reference frame in translation, in which the atmosphere is not at rest. The first two terms of Eq. (39) represent the difference of the rate of work of surface forces on walls due to the reference frame transformation from \tilde{R} to R . Note that the first term is representative of the interior of the propulsion system (limited by permeable surfaces).

In the specific case where R translates at the same velocity as the body, the sum of these first two terms corresponds to the rate of work of aerodynamic forces applied to the body as perceived in \tilde{R} (no aerodynamic work is perceived R as the body is motionless). The rate of work of aerodynamic forces perceived in \tilde{R} is different from zero in cases where thrust and drag do not balance each other, leading to the need for a non-aerodynamic force to work for the body to be at equilibrium (as mentioned by Drela [12], Arntz [4], Sanders and Laskaridis [17]). Note that the third term in Eq. (39) is unchanged with respect to Eq. (37). Thus in the case where R translates at the same velocity than the body, the exergy difference associated to the reference frame transformation can be written as:

$$\dot{\mathcal{X}}_{trb} = - \mathbf{V}_\infty \cdot \mathbf{F}_{aero} - \frac{1}{2} V_\infty^2 \dot{m}_b \quad (40)$$

where \mathbf{F}_{aero} is the overall aerodynamic force acting on the body and $\dot{m}_b = - \int_{S_{bp}} \rho \mathbf{V} \cdot \mathbf{n} dS$ is the mass flow variation on S_{bp} .

$\dot{\mathcal{X}}_{trb}$ can vanish in different cases, the first being the trivial case where $\mathbf{V}_\infty = \mathbf{0}$, i.e. $\tilde{R} = R$ and thus no exergy difference arises from the reference frame transformation. Otherwise, it can vanish if $\dot{m}_b = 0$ and $\mathbf{F}_{aero} = \mathbf{0}$. The condition $\mathbf{F}_{aero} = \mathbf{0}$ corresponds to the case where the body is at equilibrium due to aerodynamic forces, i.e. thrust balances drag. In this case, there is no overall work exchange between the body and the fluid in \tilde{R} or in R , as aerodynamic forces are not modified between the two reference frames. It should however be noted that thrust and drag applied to the body separately lead to non-zero rates of work (modified when moving from \tilde{R} to R), but the two rates of work compensate each other in both reference frames. It is important to note that $\mathbf{F}_{aero} = \mathbf{0}$ does not lead to $\dot{\mathcal{X}}_{trb} = 0$ if $\dot{m}_b \neq 0$ (or vice versa), thus both effects should be considered in the evaluation of the exergy difference associated to the reference state not being at rest in R .

The cases where $\dot{\mathcal{X}}_{trb} = 0$ and $\dot{\mathcal{X}}_{trb} \neq 0$ should also be discussed in terms of near-field exergy inflow variation. If $\dot{\mathcal{X}}_{trb} = 0$, we obtain $\dot{\mathcal{X}}_b = \dot{\mathcal{X}}_{if}$, i.e. the near-field flow exergy flux is equal between choosing a reference state which is at rest in R or one which is at rest in \tilde{R} for the definition of exergy (even though both the exergy and flow exergy themselves are not the same, cf. Sec. 4.2.1). If $\dot{\mathcal{X}}_{trb} \neq 0$ and $\dot{m}_b = 0$, the body is not at an aerodynamic force equilibrium. This force is balanced by a non-aerodynamic force (e.g. its weight [12]), the work rate of which is equal in magnitude to the work rate of aerodynamic forces on the body. When the reference frame is moved from \tilde{R} to R , the near-field aerodynamic rate of work is no longer perceived (as the body appears motionless) but can be estimated using the exergy difference $\dot{\mathcal{X}}_{trb}$. $\dot{\mathcal{X}}_{if}$ is then representative of the near-field exergy inflow that would be perceived if the body was at an aerodynamic equilibrium. Although the decomposition into $\dot{\mathcal{X}}_{if}$ and $\dot{\mathcal{X}}_{trb}$ is practical because it allows to isolate the aerodynamic force contribution, it should be noted that the real near-field exergy inflow corresponds to $\dot{\mathcal{X}}_b$ rather than $\dot{\mathcal{X}}_{if}$. Finally, if $\dot{\mathcal{X}}_{trb} \neq 0$, $\mathbf{F}_{aero} \neq \mathbf{0}$ and $\dot{m}_b \neq 0$, an additional fictive (and thus unavailable) kinetic energy as-

sociated to a massflow variation across S_b is considered in $\dot{\mathcal{X}}_{trb}$. This exergy difference consideration is similar to the one discussed in Sec. 4.2.1.

Overall, this reasoning highlights a fundamental difference between the thermodynamic variables in the dead state and \mathbf{V}_∞ for aerodynamics applications. Whereas p_∞ , ρ_∞ , T_∞ and s_∞ have a direct physical link with the thermodynamic dead state of the gas, V_∞ will always be equal to zero in the reference frame in which exergy is correctly defined (i.e. \tilde{R}). The non-zero reference state velocity \mathbf{V}_∞ is thus representative of the change of reference frame that is performed to simplify aerodynamics analyses (e.g. R is usually chosen in order to carry out a steady analysis of the flow). As such, \mathbf{V}_∞ does not correspond to a physically different thermodynamic dead state.

The unsteady exergy balance is finally written as:

$$\dot{\mathcal{X}}_{if} + \dot{\mathcal{X}}_{qb} = \dot{\mathcal{X}}_v(t) + \dot{\mathcal{X}}_{trb} + \dot{\mathcal{X}}_{mo} + \dot{\mathcal{X}}_{tho} + \dot{\mathcal{X}}_{qo} + \dot{\mathcal{A}}_\phi + \dot{\mathcal{A}}_{\nabla T} + \dot{\mathcal{A}}_w \quad (41)$$

The unsteady exergy variation can also be distributed between $\dot{\mathcal{X}}_{mo}$ and $\dot{\mathcal{X}}_{tho}$ to get:

$$\dot{\mathcal{X}}_{if} + \dot{\mathcal{X}}_{qb} = \dot{\mathcal{X}}_{trb} + \dot{\mathcal{X}}_m(t) + \dot{\mathcal{X}}_{th}(t) + \dot{\mathcal{X}}_{qo} + \dot{\mathcal{A}}_\phi + \dot{\mathcal{A}}_{\nabla T} + \dot{\mathcal{A}}_w \quad (42)$$

where $\dot{\mathcal{X}}_m(t) := \int_{\mathcal{V}} \frac{\partial \frac{1}{2} \rho (u^2 + v^2 + w^2)}{\partial t} d\mathcal{V} + \dot{\mathcal{X}}_{mo}$ and $\dot{\mathcal{X}}_{th}(t) := \int_{\mathcal{V}} \frac{\partial \rho x_s}{\partial t} d\mathcal{V} + \dot{\mathcal{X}}_{tho}$.

4.3.4 Link with the exergy balance derived by Arntz et al. for steady flows

Eqs. (41) and (42) are not directly equivalent to the exergy balance derived by Arntz et al. [6, 4]. This equivalence could however be recovered by adding and subtracting $\frac{1}{2} V_\infty^2$ to the near-field mechanical exergy of Eq. (37) and performing a different separation of the contributions as:

$$\dot{\mathcal{X}}_{mb} = - \underbrace{\int_{S_b} \left(\frac{1}{2} \rho (V^2 - V_\infty^2) \mathbf{V} + ((p - p_\infty) \bar{\mathbf{I}} - \bar{\boldsymbol{\tau}}) \cdot \mathbf{V} \right) \cdot \mathbf{n} dS}_{\dot{\mathcal{X}}_{prop}} + \underbrace{\int_{S_b} (\rho (\mathbf{V}_\infty \cdot \mathbf{V} - V_\infty^2) \mathbf{V} + \mathbf{V}_\infty \cdot ((p - p_\infty) \bar{\mathbf{I}} - \bar{\boldsymbol{\tau}})) \cdot \mathbf{n} dS}_{-W\dot{\Gamma}} \quad (43)$$

where $\dot{\mathcal{X}}_{prop}$ is referred to as the propulsive exergy by Arntz et al. [4, 6] and $W\dot{\Gamma}$ is another definition of the exergy difference. This separation is consistent with the flow exergy definition χ_f^A introduced by Arntz et al. (cf. Eq. (7)) in which the reference state kinetic energy is subtracted from the flow exergy expressed in R , a choice that does not guarantee the positivity of exergy (as discussed in Section 4.2.1). Consequently, $\dot{\mathcal{X}}_{prop}$ corresponds to $\dot{\mathcal{X}}_{if}$ (i.e. the flow exergy flux perceived in R) from which the reference state kinetic energy is subtracted. $W\dot{\Gamma}$ then corresponds to the associated exergy difference linked to the reference frame translation, which was introduced as part of the far-field mechanical exergy outflow by Arntz et al.

In the case of a steady flow analysis in external aerodynamics, a hypothesis was made by Arntz et al. on the fact that the fuel mass flow rate is negligible with respect to the air mass flow rate, giving $\dot{m}_b \approx 0$. For studies in which this hypothesis is respected (as those performed by Arntz et al.), this gives $\dot{\mathcal{X}}_{if} = \dot{\mathcal{X}}_{prop}$ and $\dot{\mathcal{X}}_{trb} = W\dot{\Gamma}$. Thus, a direct equivalence can be retrieved between the results obtained by Arntz et al. and the exergy balance derived in this paper from the corrected exergy definition. Additionally, in this specific case, the expression of $\dot{\mathcal{X}}_{trb}$ is identical to that of the potential energy rate introduced by Drela in the power balance method [12]. Note however that the equivalence between $\dot{\mathcal{X}}_{trb}$ and $W\dot{\Gamma}$ is not respected for cases where $\dot{m}_b \neq 0$.

Finally, both Eqs. (41) and (42) allow to recover the exergy balance of Arntz et al. [6, 4] when considering a steady-state flow. The derivation is however more direct with the corrected exergy definition of Eq. (28), as the different contributions can be directly decomposed without the need to add and subtract any term to retrieve the $\dot{\mathcal{X}}_{trb}$ expression. Additionally, the $\dot{\mathcal{X}}_{trb}$ contribution appears naturally with its near-field expression as part of the mechanical exergy provided/consumed by the configuration under study, while $W\dot{\Gamma}$ appeared with its far-field expression in previous works (after specific operations to extract this term from a total enthalpy flux term). The corrected definition of exergy also provides a clearer and more general physical interpretation of the term $\dot{\mathcal{X}}_{trb}$, which is different from the one suggested in previous works.

It should finally be noted that similarly to the work of Drela and Arntz et al., the mechanical far-field exergy outflow can be decomposed as:

$$\dot{\mathcal{X}}_{mo} = \underbrace{\int_{S_o} \rho \frac{1}{2} u^2 \mathbf{V} \cdot \mathbf{n} dS}_{\dot{E}_u} + \underbrace{\int_{S_o} \rho \frac{1}{2} (v^2 + w^2) \mathbf{V} \cdot \mathbf{n} dS}_{\dot{E}_{vw}} + \underbrace{\int_{S_o} ((p - p_\infty) \bar{\mathbf{I}} - \bar{\boldsymbol{\tau}}) \cdot (\mathbf{V} - \mathbf{V}_\infty) \cdot \mathbf{n} dS}_{\dot{E}_{p\tau}} \quad (44)$$

where \dot{E}_u and \dot{E}_{vw} are respectively the axial and transversal perturbation kinetic energy, while $\dot{E}_{p\tau}$ is the associated surface forces rate of work. Note that, in contrast to previous works, it has been chosen to define the last term as an overall surface force rate of work, including the viscous contribution.

5. CONCLUSION AND PERSPECTIVES

This paper presents a clarification of the notion of exergy and its difference with the notion of flow exergy, with a particular focus on aerodynamics applications. The theoretical derivation of an unsteady exergy balance is first carried out in the geocentric reference frame, in which exergy is defined without an ambiguity. At a second step, the definition of exergy is transformed in the case of a translating frame of reference, resulting in a corrected definition with respect to previous exergy-based flowfield analyses in aerodynamics (in particular respecting the positivity of exergy). The corrected definition also highlights that the freestream fluid velocity is directly linked to the choice of a translating reference frame different from the one in which exergy is defined (in which the at-

mosphere is always at rest), whereas the rest of the dead state variables are representative of the thermodynamic state of the atmosphere. The impact of this modification is then assessed on the exergy balance expressed in a reference frame in translation, showing that the derivation, decomposition and interpretation of the different terms is more direct compared to previous works. In particular, the term \dot{X}_{irb} is introduced as a part of the near-field mechanical exergy, and is interpreted in physical terms as an exergy difference linked to the analysis being performed in the reference frame attached to the body in translation. The relation between the newly derived exergy definition and previous works is also investigated, showing that the same form of exergy balance is obtained in the case of steady flow. For future works, the exergy balance derived in this paper should be numerically investigated on complex unsteady cases.

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