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Optimisation of the Pre-compensation Phase for GEO-feeder Optical Uplinks

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ABSTRACT

We propose a new MMSE method relying on phase and log-amplitude on-axis measurements and statistical priors to estimate the pre-compensation phase at point-ahead angle of a ground to geostationnary satellite telecom link suffering from anisoplanatism. This method shows to reduce the tip and tilt residual phase variance down to 49% and therefore brings a gain on the link margin up to 15 dB. It also shows to improve the fade statistics, reducing the number and mean duration of fades.

Keywords: Anisoplanatism, GEO-Feeder link, Adaptive Optics, Point ahead angle, Optical communications

1. INTRODUCTION

GEO-feeder optical uplink coupling is greatly impaired by atmospheric turbulence. A solution to improve the uplink signal statistics is to use adaptive optics pre-compensation to pre-distort the emitted laser beam wavefront. However, because of the point-ahead angle (PAA) inherent to the GEO-feeder link geometry, the downlink and uplink beams do not propagate through the same volume of turbulence. Therefore, the efficiency of the pre-compensation based on the downlink measurement is limited by anisoplanatism and the coupled flux aboard the satellite still experience long and deep fades.

While classic AO pre-compensation relying on the downlink phase measurement is the most mature technique, having led to several experimental demonstration on the field, ^{1–3} several concepts to address the anisoplanatism issue can be found in the literature. The most efficient method would be to access to a measurement at PAA, for instance from a laser guide star (LGS) as originally proposed by Tyson.⁴ This technique has been studied through numerical simulation showing the potential gain on the coupled flux brought by the LGS,⁵ and experimental set-up are currently implemented.⁶ However, the LGS technique suffers from tip tilt and focus indetermination,⁷ which are crucial modes to correct in order to improve the uplink signal statistics.⁸

As an alternative, we recently proposed a new method to predict the wavefront at PAA based on a minimum mean square error (MMSE) estimation relying on the downlink phase and log-amplitude measurements and statistical priors.⁸ This technique allows to rely only on a classical optical ground station (OGS) with no additional set-up and can renew interest in large aperture diameter (60 cm class). In this article, we propose to apply the theoretical estimator to end-to-end (E2E) numerical temporal data and to analyse the gain brought on phase, coupling and fade temporal statistics. It is shown to reduce the tip and tilt residual phase variance by 49% and to improve the link power margin up to 15dB. It also reduces the duration of the fades depending on the atmospheric scenario considered. We also discuss the aperture size trade-off.

Therefore, in this article, we present in Section 2 the optical GEO-Feeder uplink system and the estimation method. Section 3 describes the atmospheric and OGS parameters as well as the numerical framework used to evaluate the estimator gain. Finally, we evaluate in Section 4 the performance of the estimator with respect to key metrics such as residual phase variance, coupled flux statistics, temporal statistics of the signal and outage probability.

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2. SYSTEM AND METHOD

2.1 Reciprocal Geo-feeder Uplink

The considered GEO-Feeder system is a bidirectional optical link between the GEO satellite and the ground. An AO system at the OGS corrects the phase of the incident downlink field before single mode fiber (SMF) injection and pre-compensates the emitted uplink laser mode. Through all the modeling steps, we choose to exploit the reciprocity principle⁹ to express wavefront perturbations experienced in the point ahead direction in the OGS pupil and the associated coupled flux, as in.^{8,10–12} This principle states that the uplink coupling with the onboard receiver mode of the ground emitted mode after propagation to the satellite plane is equal to the coupling of this receiver mode back-propagated towards the OGS to the station emission mode. Using the reciprocal uplink, as represented in red in the Fig. 1 (a), we can therefore express the off-axis wavefront perturbations and thus analyse the AO pre-compensation performance.

We define the complex field after propagation from the satellite to the OGS pupil plane as:

$$\Psi(\mathbf{r}, t; \alpha) \triangleq A_0 \exp\left(\chi(\mathbf{r}, t; \alpha) + j\Phi(\mathbf{r}, t; \alpha)\right),\tag{1}$$

where χ are the log-amplitude fluctuations, A_0 is the intensity constant term, Φ is the perturbed phase, \mathbf{r} is the space coordinate vector defined over the telescope circular aperture of radius R_{tel} , associated to the aperture transmission function $P(\mathbf{r})$, t is the time and α is the angular direction of the considered optical path. The downlink beam axis is taken as the angular reference $\alpha=0$, the reciprocal uplink is thus located at $\alpha=PAA$ as shown in Fig. 1.



Figure 1. Scheme of the bidirectional reciprocal GEO-Feeder link. (a) Reciprocal system where the blue arrow represents the downlink path and resulting wavefront in the OGS aperture, and the red arrow represents the reciprocal uplink path and the resulting wavefront in the OGS aperture.(b) Reciprocal uplink coupled flux.

2.2 Uplink Channel Model

We consider a single input single output (SISO) telecom scheme and define the following channel model:

$$\mathbf{y} = \mathbf{f}(\mathbf{t})\mathbf{x} + \mathbf{n},\tag{2}$$

where y is the received symbol aboard the satellite, x is the sent symbol from the OGS, n is a white gaussian noise induced by the detector, and f(t) is the instantaneous channel coupling loss induced by the atmospheric turbulence. We do not consider instrumental pointing errors nor atmosphere molecular absorption in this analysis. We also do not consider geometrical losses except in section 4.5 in order to discuss the aperture size trade-off.

Thanks to the reciprocity principle we can express the losses induced by atmospheric turbulence as:

$$f(t) = f_{\text{pre-compensated,OGS} \rightarrow \text{satellite}}(t, \alpha_{\text{PAA}}) = f_{\text{compensated,satellite} \rightarrow \text{OGS}}(t, \alpha_{\text{PAA}}).$$
(3)

Therefore, the coupling losses can be expressed as the overlap integral between the corrected field in the pupil plane to the gaussian mode of the SMF at the OGS expressed in the pupil plane, and cropped by the pupil¹³:

$$f_{\text{compensated,satellite} \to \text{OGS}}(t, \alpha_{\text{PAA}}) = \left| \frac{\iint \Psi_{\text{res}}(\mathbf{r}, t, \alpha_{\text{PAA}}) \cdot M_0(\mathbf{r}) \cdot P(\mathbf{r}) d\mathbf{r}}{\sqrt{\iint |\Psi_{\text{res}}(\mathbf{r}, t, \alpha_{\text{PAA}})|^2 P(\mathbf{r}) d\mathbf{r} \cdot \iint |M_0(\mathbf{r})|^2 P(\mathbf{r}) d\mathbf{r}}} \right|^2,$$
(4)

where $\Psi_{\rm res}(\mathbf{r},t,\alpha_{\rm PAA})$ is the reciprocal residual field after correction expressed in the OGS pupil and equals to $\Psi_{\rm res}(\mathbf{r},t,\alpha_{\rm PAA}) = A_0 e^{\chi(\mathbf{r},t,\alpha_{\rm PAA})+j\Phi_{\rm res}(\mathbf{r},t,\alpha_{\rm PAA})}$. $\Phi_{\rm res}(\mathbf{r},t,\alpha_{\rm PAA})$ is the associated reciprocal residual phase after AO correction and equals to $\Phi_{\rm res}(\mathbf{r},t,\alpha_{\rm PAA}) = \Phi(\mathbf{r},t,\alpha_{\rm PAA}) - \Phi_{\rm corr}(\mathbf{r},t)$. For instance, in the classical pre-compensation scheme, the correction phase $\Phi_{\rm corr}(\mathbf{r},t)$ is equal to $\Phi(\mathbf{r},t,0)$.

2.3 Phase estimation at PAA

The idea of off-axis phase estimation was proposed in the literature for astronomy by Whiteley,¹⁴ only using the phase measurements and statistical priors. However, the efficiency of such an estimation is limited in the telecom scenario and geometry. We recently proposed a new method to estimate the pre-compensation phase at PAA.⁸ This method relies on a MMSE estimation using on-axis phase and log-amplitude measurements and statistical priors. As the log-amplitude results from atmospheric layers far from the pupil, it brings information about the turbulent layers inducing anisoplanatism.

The estimator is developed as following. We use a Zernike modal formalism to express the phases. We define the estimated phase as $\hat{\Phi}_{PAA}$ and consider an estimator that is linear with measurements. This formalism is detailed in.⁸ The estimated phase is therefore defined on the subset of modes measured and corrected by the AO system as:

$$\hat{\mathbf{\Phi}}_{\mathrm{AO,PAA}} = \mathbf{R}\mathbf{y}_{\mathrm{m}},\tag{5}$$

where $\mathbf{R} \in \mathbb{R}^{N_{AO} \times M}$ is a reconstructor whose M dimension depends on the size of the measurement vector $\mathbf{y}_m \in \mathbb{R}^{M \times 1}$. In this case, the measurement vector is defined as following:

$$\mathbf{y}_{\mathrm{m}} = \begin{bmatrix} \boldsymbol{\Phi}_{\mathrm{AO},0}^{\mathrm{T}} & \boldsymbol{\chi}_{\mathrm{AO},0}^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}},\tag{6}$$

where $\Phi_{AO,0} \in \mathbb{R}^{(N_{AO}-1)\times 1}$ is the on-axis measured phase represented on the Zernike modal basis that describes the correction subspace from modes 2 to N_{AO} and $\chi_{AO,0} \in \mathbb{R}^{N_{AO}\times 1}$ is the on-axis measured log-amplitude represented on the Zernike modal basis from modes 1 to N_{AO} . In this study, we discard the measurement noise since the telecom scenario leads to wavefront sensors in the high flux regime.

We define the phase error now as:

$$\Phi_{\rm AO,res} = \Phi_{\rm AO,PAA} - \dot{\Phi}_{\rm AO,PAA} = \Phi_{\rm AO,PAA} - \mathbf{R}\mathbf{y}_{\rm m}$$
(7)

By minimizing the mean square of this phase error, we obtain the MMSE estimator:

$$\mathbf{R}_{\mathrm{MMSE}_{\Phi_{\chi}}} = \operatorname{argmin}_{\mathbf{R}}(\mathbb{E}[\mathbf{\Phi}_{\mathrm{AO,res}}^{\mathrm{T}}\mathbf{\Phi}_{\mathrm{AO,res}}]) = \mathbf{\Gamma}_{\Phi_{\mathrm{y}_{\mathrm{m}}}}(\alpha_{\mathrm{PAA}}) \cdot \mathbf{\Gamma}_{\mathrm{y}_{\mathrm{m}}\mathrm{y}_{\mathrm{m}}}(0)^{-1}$$
(8)

where $\Gamma_{XY}(\Delta \alpha)$ is the covariance matrix between the physical quantity X and the physical quantity Y separated by an angle $\Delta \alpha$.

We can express the angular covariance matrix and autocovariance matrix as follows:

$$\boldsymbol{\Gamma}_{\Phi y_{m}}(\alpha_{PAA}) = \begin{bmatrix} \boldsymbol{\Gamma}_{\Phi \Phi}(\alpha_{PAA}) & \boldsymbol{\Gamma}_{\Phi \chi}(\alpha_{PAA}) \end{bmatrix}$$
(9)

and,

$$\mathbf{\Gamma}_{\mathbf{y}_{\mathrm{m}}\mathbf{y}_{\mathrm{m}}}(0) = \begin{bmatrix} \mathbf{\Gamma}_{\Phi\Phi}(0) & \mathbf{\Gamma}_{\Phi\chi}(0) \\ \mathbf{\Gamma}_{\Phi\chi}(0)^{\mathrm{T}} & \mathbf{\Gamma}_{\chi\chi}(0) \end{bmatrix}.$$
 (10)

The content of the different covariance matrices are detailed in.⁸ We highlight that this estimator is fully analytical and that it requires only the C_n^2 profile and system parameters knowledge to be computed.

In the following, we describe the OGS an AO parameters and atmospheric scenarii considered for the numerical analysis presented in this article. The aim is to study the gain on the coupled flux and fade statistics for different turbulence scenarii.

3. NUMERICAL FRAMEWORK

3.1 OGS and AO parameters

We consider the OGS and AO parameters presented in Tab. 1. This configuration is characterized by a large aperture that limits the geometrical losses induced by the beam divergence, but results in increased wavefront perturbations in the telescope aperture, and thus requires a good AO performance.¹¹ As a result, we set a 60 cm diameter which is incidently the value selected for the ONERA OGS Feelings¹⁵. In such a configuration, the anisoplanatic error due to the angular decorrelation of the turbulence is large, thus there is a greater interest in reducing PAA anisoplanatism. The trade-off between the geometrical losses and atmospheric losses will be discussed in section. 4.5. We consider a Feeder-link with a GEO satellite at 30° elevation corresponding to a point-ahead angle of 18.5 µrad. It is a plausible scenario for a link between a GEO satellite and an OGS in Europe. The considered wavelength is 1550 nm to benefit from off-the-shelf telecom components and of low atmospheric attenuation. Concerning the AO parameters, we consider a high quality correction with a number of correction modes as high as N_{AO} = 136 (nr=15) and a sampling frequency f_{samp} of 4.7 kHz, in accordance with current OGS AO systems.¹⁵

OGS Parameters		
Elevation	30°	
α_{PAA}	$18.5~\mu\mathrm{rad}$	
λ	$1550~\mathrm{nm}$	
$\mathrm{D}_{\mathrm{telescope}}$	$60~{\rm cm}$	
AO Parameters		
N _{AO}	136	
f_{samp}	$4.7~\mathrm{kHz}$	

Table 1. OGS and AO general parameters.

3.2 Atmospheric conditions

We consider statistically representative atmospheric C_n^2 profiles. These profiles, called MOSPAR-XY^{1,16,17}, are composite profiles constructed thanks to astronomical site measurement databases.^{18,19} X and Y are thresholds on the statistical distributions of the anisoplanatic angle θ_0 , describing the turbulence angular decorrelation, and the Fried parameter r_0 , corresponding to the strength of the phase perturbation. For example, the MOSPAR-9090 is a profile whose θ_0 is chosen such that $P(x > \theta_0) = 0.9$ and r_0 is chosen such that $P(y > r_0) = 0.9$ according to on-site measurements. To evaluate the estimator gain, we consider 3 profiles of turbulence with thresholds $(X, Y) \in \{(50, 50), (90, 90), (99, 99)\}$. Figure. 2 depicts the 3 C_n^2 profiles and table. 2 gives the associated integrated parameters. In addition, we consider a Bufton wind profile with a wind speed at low altitude $V_g = 10 \text{ m.s}^{-1}$ and wind speed at the tropopause $V_t = 30 \text{ m.s}^{-1}$. Finally, we consider a finite outer scale $L_0 = 5.12 \text{ m}$ and suppose that the inner scale $l_0 = 0$.

Profile	MOSPAR 5050	MOSPAR 9090	MOSPAR 9999
r ₀	$7.8~\mathrm{cm}$	4.0 cm	$2.6 \mathrm{~cm}$
θ_0	11.34 µrad	6.83 µrad	4.82 µrad
σ_{χ}^2	0.03	0.08	0.136

Table 2. Integrated parameter for 3 considered C_n^2 profiles with variable θ_0 and r_0 thresholds, at 30° elevation, computed at $\lambda = 1550$ nm.

3.3 Reciprocal E2E

Pseudo-analytical tools are available to evaluate the estimator gain on residual phase variance and coupled flux statistics. However, our tool⁸ do not currently allow to study temporal statistics of the signal. Therefore, we use



Figure 2. C_n^2 profiles f unction of the h eight. The zero r efference corresponds to the OGS p osition. 3 profiles are represented at the elevation 30°: MOSPAR 5050 (blue), MOSPAR 9090 (red), MOSPAR 9999 (black). Integrated parameters corresponding to these profiles are given in table. 2.

end-to-end (E2E) numerical simulation to generate optical fields and associated coupling t ime-series. This E2E numerical tool relies on a discretization of the turbulent volume and a Monte-Carlo method which consists in alternating steps of free space propagation and complex field perturbation by turbulent phase screens following Von Karman statistics. In order to evaluate the gain on the residual phase variance and coupled flux, we use the reciprocal approach and propagate two downlink beams located at $\alpha = 0$ and $\alpha = \alpha_{PAA}$ as described in Fig. 1 (a). We synthesize the reciprocal uplink corrected field by subtracting the chosen correction phase to the wavefront at PAA. We obtain the associated coupled flux by computing a numerical overlap integral with the SMF gaussian mode as described in Fig. 1 (b).

Concerning the AO error budget, we only consider anisoplanatic and fitting e rror. To e mulate the fitting error, the correction phase is represented on the limited number of Zernike mode that the AO system can correct. The classical pre-compensation is taken as a benchmark. In this case, the correction phase is the phase of the field at $\alpha = 0$ represented on N_{AO} modes. In the novel estimator case, we expand the phase and the log-amplitude of the on-axis downlink beam on the Zernike modal basis, apply the reconstructor as in Eq. 5 and reconstruct the spatial wavefront.

Finally, we consider 5x9400 samples for 5 different seeds with a time sampling frequency of 4.7 kHz, resulting in 10 s of data for each considered C_n^2 profile.

4. PERFORMANCE METRICS

The aim is to evaluate the gain brought by the estimator with respect to the classical pre-compensation case. This gain is first evaluated on the residual anisoplanatic phase v ariance. Secondly, we aim to relate the gain on the phase variance to the gain on the coupled flux and f ades temporal statistics. Indeed, it is at stake to study the signal statistics to determine the losses to take into account in the power link budget. Classically, this losses F_t are taken such that $P(f < F_t) = 10^{-3}$.²⁰ Furthermore, temporal statistics are key metrics to design interleaver duration ensuring reliability.²¹ Improving these temporal statistics would allow to reduce interleaver size, and thus, to reduce the latency of the link and the satellite onboard complexity.

4.1 Residual phase variance

The first relevant metric to evaluate the estimator gain is the residual phase variance that is the phase mean square error (MSE). The residual phase variance measures the distance between the estimated wavefront and the wavefront of interest $\Phi(\mathbf{r}, t, \alpha_{PAA})$. We choose to study the Zernike modal residual phase variance.

First, we compare the theoretical modal phase variances and the phase variance computed on the residual phase deduced from the simulated complex fields, for both classical pre-compensation and MMSE precompensation. The theoretical modal phase variance can be found in.⁸

Figure 3 (a,b,c) depicts the theoretical phase variance (dots) and the phase variance computed on the data (crosses) for the classical (black) and $\text{MMSE}_{\Phi\chi}$ pre-compensation cases (red) for the three considered atmospheric condition. The theoretical variances show good accordance with the variances computed on data in each case. We can observe a slight deviation concerning the high order modes variance in the tougher atmospheric case MOSPAR 9999. This could be explained by a higher log-amplitude variance σ_{χ}^2 as shown in table 2. In addition, we observe a systematic gain brought by the $\text{MMSE}_{\Phi\chi}$ estimator, especially on the low order modes which are the most energetic one, and the modes at the origin of the fades.

Figure 3 (e,f,g) quantify this gain. The reduction gain is defined as $100 \left(1 - \frac{\sigma_{\text{MMSE},i}^2}{\sigma_{\text{classic},i}^2}\right)$. The tip (mode number 2) is decreased by 50% from its initial value in the MOSPAR 5050 and 9090 cases, and of 39% in the MOSPAR 9999 case. The tilt (mode number 3) reduction is lower (between 20 and 30%).



Figure 3. Residual phase variance of the classical and MMSE pre-compensation case in the three turbulence cases. (a,b,c) Modal phase variance for the classical case (black, grey) and MMSE case (red, pink), from the theoretical phase variance (dots) and ensemble phase variance computed on data (crosses). (d,e,f) Theoretical (red line) and from data (black stars) modal MMSE phase variance reduction gain.

4.2 Coupled Flux Statistics

The second relevant metric concerns the coupled flux s tatistics. By studying the statistics of f for given atmospheric condition, it is possible to extrapolate the gain of the estimator on the power link margin. As there is no model yet to theoretically relate the residual phase variance to the coupled flux statistical moments for the anisoplanatic link, we compute the cumulative density function (CDF), also defined as the fading probability $P_{fade} = P(f \le f_t)$ on the numerical coupling samples, where f_t is the coupled flux threshold. These metrics highlight how the estimator impact the CDF shape.



Figure 4. Cumulative density function (CDF) of f function of the coupled flux threshold f_t (dB) for the three turbulence scenarii, and for the classical (black) and $MMSE_{\Phi\chi}$ (red) pre-compensation case.

In Fig. 4 (a,b,c), we plot the CDF function of the coupled flux threshold f_t in dBs for the classical (black) and the MMSE_{Φ_{χ}} (red) pre-compensation case for the three atmospheric scenarii. We observe for the three profiles that the MMSE_{Φ_{χ}} method decreases the slope of the CDF. We observe a gain at probability 10⁻³ of 5, 15 and 10 dBs for the profiles MOSPAR 5050, 9090, 9999, respectively. This means that deep fades are less likely to occur with the new pre-compensation method.

4.3 Temporal Statistics

Fade statistics embrace the fractional fade time P_{Fade} defined above, the fade frequency (to assess the expected frequency of synchronization loss and link re-acquisition) and the mean fade time (to design the interleaver size).²² As we do not consider a specific system with full link budget and detection threshold, we study the gain on these metrics for varying coupled flux thresholds $f_t \in [-35:10\log_{10}(\frac{\leq f>}{2})]$ dB, where < f > is the averaged coupled flux.

Figure. 5 depicts the coupled flux time-series on 4 s samples for the three atmospheric conditions. The fade reduction induced by the phase estimation can be visually highlighted on these three plots. We underline that



Figure 5. Example of 4 s coupled flux time-series for the 3 turbulence scenarii.



Figure 6. (a,b,c) Mean fade duration in ms function of the coupled flux threshold in dB for the three turbulence scenarii and for the classical (black) and MMSE (red) pre-compensation cases. (d,e,f) MMSE time reduction in % function of the coupled flux threshold in dB.

the estimator may sometimes create new fades because of the statistical nature of the estimator. However, the overall fade statistics are improved, as it will be shown on the macro-metrics in the following.

Figure. 6 (a,b,c) depicts the mean fade duration in ms function of the coupled flux threshold in dBs for the three atmospheric conditions. The associated time reduction is plot in Fig. 6 (d,e,f). We observe for the three turbulence scenarii a time reduction from 60% to 100% for the MOSPAR 5050 case, from 40% to 100% for the MOSPAR 9090 case and from 20% to 60% for the MOSPAR 9999 case. Additionally, we observe that the mean fade duration stays under 5 ms for every turbulence scenario. We can conclude that the estimator allows to decrease the mean fade duration of the signal.



Figure 7. (a,b,c) Fade frequency in Hz function of the coupled flux threshold in dB for the three turbulence scenarii and for the classical (black) and MMSE (red) pre-compensation cases. (d,e,f) MMSE fade frequency reduction in % function of the coupled flux threshold in dB.

Figure. 7 (a,b,c) depicts the number of fade per second function of the coupled flux threshold in dBs for the three atmospheric conditions. The associated fade frequency reduction is plot in Fig. 7 (d,e,f). We observe for the three turbulence scenarii a reduction of the fade frequency with a steeper slope of fade frequency related to the MMSE pre-compensation. We can conclude that the estimator allows to reduce the fade frequency.

As a conclusion, the estimator allows either to relax the power link margin at fixed interleaver size, or to decrease the interleaver size at fixed power margin with respect to the classical pre-compensation case. Therefore, it would allow more flexibility in the power-system complexity trade-off to make in the transmission chain design.

4.4 Channel Theoretical Limits

Classically, the theoretical limits of a channel are given by the capacity. In this case, the capacity is a random variable depending on the instantaneous SNR(t), depending itself on the instantaneous coupling losses f(t). Therefore, we consider the outage probability which is a relevant metric for fading channels. The outage probability is defined as the probability of the capacity to fall under a given rate R in bit per channel use:

$$P_{\text{outage}}(R) = P(C(\text{SNR}(f(t))) < R)$$
(11)

where the capacity C is the Shannon capacity $C(t) = \log_2(1 + SNR(f(t)))$.

In addition, we consider the impact of interleaving on the outage probability. Interleaving can be considered as transmission on N independent channels, where N is the size of the interleaver. The resulting outage probability is therefore expressed as the outage of the capacity averaged on these N channels:

$$P_{outage,int}(R) = P(C_i < R), \tag{12}$$

Where $C_i = \frac{1}{N} \sum_{j=1}^{N} C_j$ is the capacity averaged on the N parallel channel realizations. We compute both P_{Outage} and $P_{Outage,int}$ from the signal statistics.

Therefore, we study the gain of the estimator on the outage probability without interleaving as well as with an interleaver of a given duration. Figure. 8 plots the outage probability function of the SNR for the three atmospheric conditions and for interleaver durations of 0 (black), 20 (red), 50 (green) and 100 ms (blue) for a rate R=1 bit/channel use. Considering the outage without interleaving, we observe at probability 10^{-3} a gain between the classical and the MMSE method of 4, 15 and 10 dBs for MOSPAR 5050, 9090, 9999, respectively. With a 20 ms interleaver, we immediately observe a gain with respect the non-interleaved outage capacity for every turbulence scenario. The MMSE gain with respect to the classical method decreases to 3, 5 and 4 dBs. A similar behavior is observed for 50 and 100 ms. The reduction of the MMSE gain with respect to the increase of the interleaver duration is explained as following: the larger is the interleaver duration, the larger is the window over which the capacity is averaged, which benefits the most to signal with longer fades, at the cost of latency.

As a conclusion, with or without interleaving, the estimator always brings gain on the outage probability for every turbulence scenario with respect to the classical pre-compensation case.



Figure 8. Outage probability for the three turbulence scenarii and for the classical (lines) and MMSE (dashed lines) pre-compensation cases, for several interleaver sizes : 0 (black), 20 (red), 50 (green), 100 (blue) ms.

4.5 Aperture Size Trade-off

We showed that the estimator could relax the constraints on the link power budget and therefore, by reducing the losses induced by the atmospheric turbulence, renew interest on large telescope aperture for GEO-Feeder optical links that are of interest to reduce geometrical losses. We propose here to discuss this statement by comparing the outage probability for two telescope diameters (60 and 40 cm), with and without the MMSE pre-compensation method in both cases.

In this aim, in addition to the system hypothesis made in section. 3, we assume a satellite telescope diameter of 25 cm. The geometrical losses related to the system equals to -59.47 dB and -55.94 dB, for the 40 cm and 60 cm OGS systems, respectively. We also assume an emitted power of 50 W from the OGS. By adding these two static terms from the power link budget, it is possible to compute the outage probability and analyse the trade-off between the geometrical losses and turbulence induced losses.

Figure. 9 depicts the outage probability function of the SNR (E_n/N_0) for the three turbulence cases, for the diameters of 40 cm (dotted-lines) and 60 cm (lines) for the classical (black) and the MMSE (red) pre-compensation cases, for several interleaver durations.

Focusing on the MOSPAR 5050 case, we can see at outage probability 10^{-3} , the MMSE pre-compensation case at 60 cm is the best scenario with or without interleaving. Furthermore, we observe that with interleaving, the classical pre-compensation case perform better than classical and MMSE pre-compensated cases at 40 cm.

Concerning the MOSPAR 9090 case, the MMSE pre-compensation case at 60 cm is also the best scenario with or without interleaving at outage probability 10^{-3} . However, we notice in this case that with interleaving, the second best method is the 40 cm with MMSE pre-compensation.

Finally, in the MOSPAR 9999 case, the best method at outage probability 10^{-3} is the 40 cm with MMSE precompensation without interleaving or with short interleaving (20 ms), while the 60 cm with MMSE precompensation performs better with longer interleaver duration (50 ms and 100 ms). We can interpret that due to the smaller pre-compensation gain on the MOSPAR 9999 and still high tip tilt absolute phase variance, the geometrical losses gain on the 60 cm diameter is not sufficient to compensate the heavy losses induced by the atmospheric turbulence. Therefore, as the 40 cm telescope aperture limits these atmospheric induced losses, it is more favorable in these two cases (i,j).



Figure 9. Outage probability function of the SNR for a rate R=1 bit/channel use, for the three atmospheric cases. (a,b,c,d) is the outage for MOSPAR 5050, (e,f,g,h) for MOSPAR 9090 and (i,j,k,l) for MOSPAR 9999. Full lines depicts the 60 cm diameter and dotted lines the 40 cm, with the classical pre-compensation case in black and MMSE in red.

5. CONCLUSION

In this article, we proposed to evaluate the gain of our new phase estimator⁸ at point-ahead angle for precompensated optical GEO-Feeder uplink. We showed by applying the reconstructor to E2E data for several atmospheric turbulence case that both coupling and fade temporal statistics were improved by the phase estimation. We interpret this improvement by overall phase variance reduction thanks to the estimator, especially reducing the tip and tilt variance. Therefore, this method of phase estimation shows potential to relax the constraints on the power link budget as well as reducing satellite payload complexity by lowering down the interleaver duration required for telecom link reliability.

However, we notice in the tougher atmospheric turbulence regime that the estimator gain was lower and the tip and tilt absolute value was still high. The consequence is to lower the gain on the signal statistics. In future work, we will use additional measurements and priors in order to obtain a more reliable phase estimation with improved performance in tougher turbulence regimes.

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