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Towards quantifying the effect of pump wave amplitude on cracks in the Nonlinear Coda Wave Interferometry method

Shilin QU^{a,*}, Benoît HILLOULIN^b, Olivier CHUPIN^c, Jean-Michel PIAU^c, Odile ABRAHAM^a, Vincent TOURNAT^d

^a*GERS-GeoEND, Université Gustave Eiffel, IFSTTAR, CS5004, F-44344 Bouguenais, France*

^b*Nantes Université, Ecole Centrale Nantes, CNRS, GeM, UMR 6183, 1 rue de la Noë, 44321 Nantes, France*

^c*LAMES-MAST, Université Gustave Eiffel, IFSTTAR, CS5004, F-44344 Bouguenais Cedex, France*

^d*Laboratoire d'Acoustique de l'Université du Mans (LAUM), UMR 6613, Institut d'Acoustique - Graduate School (IA-GS), CNRS, Le Mans Université, France*

Abstract

In Non-Destructive Testing and Evaluation (NDT&E), an ultrasonic method called Non-linear Coda Wave Interferometry (NCWI) has recently been developed to detect cracks in heterogeneous materials such as concrete. The underlying principle of NCWI is that a pump wave is used to activate the crack breathing which interact with the source probe signal. The resulting signal is then measured at receiver probes. In this work, a static finite element model (FEM) is used to simulate the pump wave/crack interaction in order to quantify the average effect of the pump waves on a crack. By considering both crack opening and closure phases during the dynamic pump wave excitation, this static model aims to determine the pump stress amplitude for a given relative crack length variation due to the dynamic pump wave excitation at different amplitudes. Numerical results show, after reaching a certain stress amplitude, a linear relationship between the relative crack length variation and the equivalent static load when considering a partially closed crack at its tips. Then, numerical NCWI outputs, e.g., the relative velocity change θ and the decorrelation coefficient K_d , have been calculated using a spectral element model (SEM). These results agree with previously published experimental NCWI results derived for a slightly damaged 2D glass plate.

Keywords: concrete, coda, nonlinear acoustics, crack breathing, ultrasound, finite element method, spectral element method, NCWI

1. Introduction

2. Background

Cracks are often considered as one of the most impactful features in civil structures, as they induce a strength loss and reduce durability. Cracks can be generated by fatigue loadings [1], chemical attacks [2, 3] or shrinkage [4–6]. In the field of the NDT&E of heterogeneous

*Corresponding author

Email addresses: shilin.qu@univ-eiffel.fr (Shilin QU), benoit.hilloulin@ec-nantes.fr (Benoît HILLOULIN)

6 materials like concrete, elastic waves with wavelengths comparable to the size of hetero-
7 geneities are scattered multiple times by the heterogeneities, resulting in the generation of
8 the coda wave [7–11]. These coda waves analysis constitutes a powerful tool for detecting
9 cracks thanks to its high sensitivity (temperature) [12], minor mechanical evolution (e.g.
10 cracks) [13–15]. This is because the trajectory of coda waves is much longer than ballistic
11 waves due to multiple scattering. The cumulative effect during the traveling time makes
12 coda waves highly sensitive to media changes even when they are weak. A linear ultrasonic
13 NDT&E technique, called Coda Wave Interferometry (CWI) has been developed in order
14 to detect micro damage, e.g. cracks formed between aggregates and mortar in concrete
15 [13, 16, 17] or other complex media [18–21], or to monitor the health of structures over the
16 long term [12, 15, 22–25].

17 *Principle of CWI*

18 In CWI, the principle consists of comparing coda signals in different material states.
19 A sweep sine signal, whose wavelengths are comparable to the size of the heterogeneities,
20 serves as a source to generate coda signals. A reference coda is recorded at an initial state
21 (e.g. the intact state), and a perturbed coda is recorded at another state (e.g. damaged
22 state). The appearance of cracks leads to two major effects: 1) a local decrease in the
23 Young’s modulus around the cracks, hence a small decrease in wave velocity; and 2) a
24 modification of the scattering property of elastic waves. These two effects can be quantified
25 by two CWI observables θ and K_d , retrieved from the reference and perturbed coda signals
26 by applying the stretching method [26, 27]. θ represents the relative velocity change of
27 the elastic medium while K_d represents the decorrelation coefficient between reference and
28 perturbed coda signals. However, CWI has several drawbacks. First, it requires a reference
29 state (usually expected on an intact sample), which is not always realistic. Second, CWI
30 cannot detect defects, including closed cracks, that may also exist in concrete structures [28].
31 In order to improve CWI, a nonlinear ultrasonic NDT&E technique, called NCWI, has been
32 developed based on CWI.

33 *Principle of NCWI*

34 In NCWI, a series of high-amplitude low-frequency (LF) dynamic pump waves $\sigma_{dyn}(t)$,
35 typically a sine sweep with varying maximum amplitude, denoted $\sigma_{dyn}^{max} = |max(\sigma_{dyn}(t))|$ in
36 the following, is used to generate local nonlinearities around the cracks, especially closed
37 cracks, via the crack breathing effect. The frequency range of the pump waves usually covers
38 the first 50 modal frequencies of the inspected sample in order to achieve an equipartition of
39 energy of pump waves in the media. During pump wave excitation, multiple desynchronized
40 low-amplitude high-frequency (HF) probe waves, whose wavelength is close to the size of the
41 heterogeneities found in the medium, are used to generate coda waves. The application of LF
42 pump waves and HF probe waves over a crack induces two semi-coupled dynamic phenomena:
43 1) interaction between pump wave and cracks (e.g. crack breathing), 2) interaction between
44 probe wave and breathing of cracks. Similarly to CWI, NCWI also needs a reference state
45 and a perturbed state. In NCWI, the reference state is when the pump is switched off, the
46 reference coda is then compared with a perturbed coda, whose pump amplitude is nonzero.
47 Thus, NCWI is free of the need for an intact sample to serve as the reference state. In
48 addition, the positive and negative pump wave amplitudes will close and open the crack

tips, thus producing the crack breathing effect. This effect makes NCWI sensitive to closed cracks [29, 30]. Experimental observations have shown that θ and K_d follow respectively a linear and quadratic relation with respect to pump stress amplitude, which yields two NCWI observables, namely α_θ and α_{K_d} . α_θ and α_{K_d} represent respectively the coefficient of linear and quadratic terms of θ and K_d with respect to pump stress amplitude. These terms reflect the level of nonlinearity of the inspected sample and can be used as criteria of the global damage level. Despite these improvements, NCWI also displays some drawbacks. For instance, in a large structure, it is difficult to obtain an equipartition of pump wave energy inside the structure, which means that the nonlinearity induced by certain defects is less than expected or even incapable of being excited by pump waves.

59 *Crack imaging*

Over the past few years, numerous works have focused on both the numerical [31–33] and the experimental [29, 30, 34–40] aspects of CWI and NCWI. However, both are only able to detect and evaluate the damage level of a material at the global level in a comparative manner. It can not localize the defects. Therefore, the need for imaging techniques based on NCWI has become more acute. Various techniques exist to image defects in complex media, e.g. X-ray tomography [41], 2D/3D microscopy [42], air-coupled time reversal mirror [43]. However, these techniques are unable to image weak changes such as microcracking, especially in very heterogeneous media. The localization of weak changes using diffuse waves (LocaDiff) [44–47] with CWI has been under study for the past ten years. LocaDiff is an imaging algorithm founded on a diffusive propagation model and a maximum likelihood approach. It requires a reference state and uses an analytical solution for the scattering cross section. A combination of CWI and Time Reversal (TR), called CWI-TR, which eliminates the need for intact samples and allows for the detection of closed cracks, has recently been investigated in [48]. According to this method, TR is employed with multiple pump wave generators to refocus pump waves at specific points so as to activate crack nonlinearity. CWI is performed at the same time in order to form a map of CWI observables. However, the refocused pump wave has a limited detection zone, i.e. at or around just the shallow surface, which means only surface-breaking cracks or shallow subsurface cracks can be detected. It is expected that a combination of the LocaDiff methodology and NCWI will enable the 3D imaging of closed cracks and/or cracks without requiring a reference sample.

The imaging process is founded on (1) a model space and (2) a measurement space. The model space contains model parameters that can be estimated theoretically. The measurement space contains measurement parameters that are given by experiments. The inverse problem consists in recovering the model parameters from the measurement parameters. In the case of LocaDiff with CWI, the measurement parameters are the experimental relative velocity variation θ and the decorrelation coefficient K_d and the corresponding model parameters are the intrinsic relative velocity variation $\delta v/v$ and the scattering cross section of the crack σ_D [47], where σ_D represents the scattering ability of a defect. This term is equal to the ratio of scattered energy flux to the incident energy flux [49–52]. Although LocaDiff can be performed with both measurement parameters [47], the majority of existing works relies on K_d for the inversion [44–46]. Therefore, the use of K_d and σ_D will be preferred in the following during this study.

92 *Static models: pump wave/crack interaction*

93 In an attempt to combine LocaDiff with NCWI, it is anticipated to introduce a depen-
94 dency of the pump stress amplitude on the scattering cross section of a crack σ_D . The aim
95 of the forward problem therefore is to relate σ_D to the pump stress amplitude σ_{dyn}^{max} . Re-
96 cent experimental works have shown that θ and K_d obeys a linear relation and a quadratic
97 relation with the pump stress amplitude [30, 34, 35]. According to the numerical study of
98 [31], these relations can be reproduced numerically by considering cracks at different relative
99 crack length variation \widetilde{dL}/L , where dL represents the algebraic crack length variation at
100 crack tip that is equivalent to the average effect of dynamic pump stress waves during the
101 crack breathing on the crack tips. Therefore, different \widetilde{dL}/L represent the average effect of
102 different pump stress amplitudes. The relation between the σ_{dyn}^{max} and σ_D can be found by
103 firstly getting the relation between σ_{dyn}^{max} and \widetilde{dL}/L then replace \widetilde{dL}/L with its corresponding
104 σ_D . However, the real pump stress wave excitation involves the interaction of elastic waves
105 with cracks which is a dynamic process. Furthermore, the resulting crack breathing effect
106 involves an unilateral contact problem which is relatively complicated to model. Different
107 models have been proposed in order to model the crack/elastic wave interaction such as:
108 realistic morphology model [53], pillar model [54] and mass-spring model [55]. These models
109 contain numerous parameters which can be difficult to recover in an inverse problem and
110 some of them are difficult to measure experimentally. Due to this reason, in order to develop
111 a robust inverse method for closed crack detection in very heterogeneous materials, a model
112 which contains as less parameters as possible is to be preferred.

113 In this paper, a simplified Finite Element (FE) static model is proposed which brings
114 the dynamic problem involved in the crack breathing activated by the dynamic pump wave
115 down to a quasi-static problem.

116 This article is presented in three parts. In the first part, the motivation of the study
117 and the main steps of the models will be presented. In the second part, the methodology
118 of the static model and the numerical NCWI model (via SEM2D [56, 57]) are introduced
119 first. Then, a comparison between numerical and experimental NCWI results is proposed
120 in order to validate the protocol of the static model. In the last part, the results of the
121 improved static model and the results of NCWI model are presented in order to show firstly
122 the relation between σ_{dyn}^{max} and \widetilde{dL}/L and then the relations of θ and K_d with respect to
123 σ_{dyn}^{max} .

124 **2. Motivation of the study**

In NCWI, the LF pump waves can activate the crack breathing effect, whereby crack tips
open and close alternatively. To detect the crack breathing effect, multiple desynchronized
HF probe waves are sent into the medium at the same time as the dynamic pump waves
in order to generate coda waves. The coda waves at a zero pump amplitude are set as the
reference coda, while the coda waves at different nonzero pump amplitudes are taken as
the perturbed coda. Then, θ can be calculated at each pump amplitude according to the

stretching method [26, 27] (Eq. 1):

$$CC(\theta_n) = \frac{\int_{t_1}^{t_2} u_i(t(1 + \theta_n))u_p(t)dt}{\sqrt{\int_{t_1}^{t_2} u_i^2(t(1 + \theta_n))dt \int_{t_1}^{t_2} u_p^2(t)dt}}, \quad (1)$$

125 where $u_i(t)$ and $u_p(t)$ denote the reference and perturbed codas, respectively and t_1, t_2 are
 126 the beginning and the end of the time integration window. The final value of θ is the one
 127 that maximizes $CC(\theta_n)$. The decorrelation coefficient K_d is calculated using the following
 128 equation:

$$K_d = 100(1 - CC(\theta)). \quad (2)$$

129 Recent research has shown that the change in CWI observables θ and K_d due to the
 130 crack breathing effect resulting from the dynamic stress pump wave can be reproduced by a
 131 simple change in crack length ratio dL/L [31]. Hence, a numerical model which can give the
 132 relation between the relative crack length variation dL/L and the maximum dynamic stress
 133 pump wave amplitude σ_{dyn}^{max} in NCWI is the goal of this study. The layout of this paper that
 134 demonstrates the step-by-step procedure to achieve this goal is represented in Fig.1:

- 135 1. static modeling using FEM that models both the opening and closure of a crack under
 136 a far field stress σ_{yy}^{ff} corresponding to the pump effect, which gives the relationship
 137 between dL^\pm/L and σ_{dyn}^{max} where dL^-/L and dL^+/L represent the relative crack length
 138 variation in closure and opening cases respectively,
- 139 2. use of the relation between σ_{yy}^{ff} and dL^\pm/L to convert the dynamic pump stress waves
 140 signal $\sigma_{dyn}(t)$ into transient relative crack length variation $[dL/L]_{dyn}(t)$,
- 141 3. time average of $[dL/L]_{dyn}(t)$ and establishment of the relation between σ_{dyn}^{max} and
 142 $\widetilde{dL}/L = \frac{1}{L\Delta_t} \int_0^{\Delta_t} dL(\sigma_{dyn}(t))dt$ where Δ_t is the duration of the coda acquisition,
- 143 4. application of cracks at different \widetilde{dL}/L in numerical simulation of NCWI via SEM2D,
 144 and qualitative comparison of the relations between θ and K_d vs. \widetilde{dL}/L with experi-
 145 mental NCWI results.

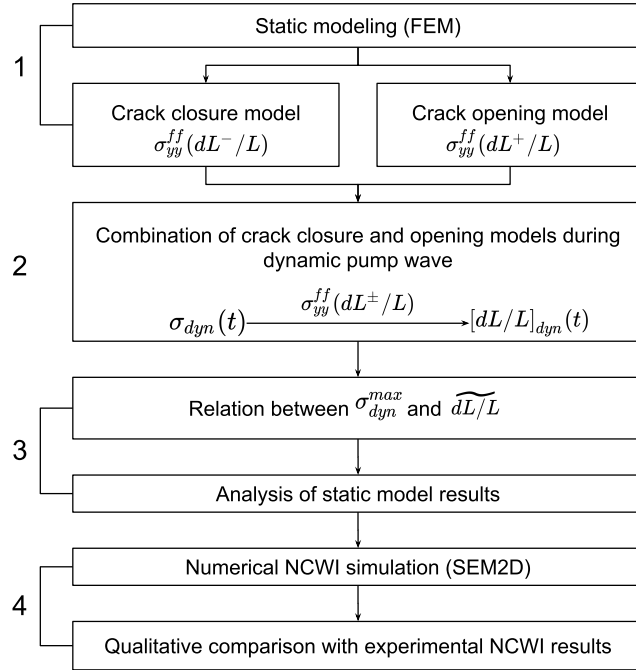


Figure 1: Workflow of the study.

3. Modeling methodology

3.1. Static model

The physical description of crack breathing is the alternation between open and close states of crack tips due to external excitation. Therefore, the crack displacement for a given far field stress equivalent to the pump effect is looked for in the static modeling. The crack displacement is calculated numerically using a simplified 2D model (plane strain) of a single crack embedded into an infinite homogeneous elastic medium. The crack geometry is considered with an initial opening under no stress condition and verifies the following equation: $y = \pm y_{geo}(x) = \pm \frac{h}{4}(1 + \cos(\pi \frac{2x}{L}))$ with $0 \leq x \leq L/2$, $0 \leq y_{geo} \ll D$ and $y_{geo}(-x) = y_{geo}(x)$ where h is the aperture at the crack center and L the length of the crack. (see Fig.2). This FE static model entails several imposed hypotheses, namely:

1. symmetry with x and y of the whole problem (geometry, loading, boundary conditions),
2. small strains and displacements,
3. the equivalent static load is biaxial,
4. the crack is considered to have some initial opening, but with tangent crack tips at $x = \pm L/2$,
5. the dimension of the meshed domain is large enough to be equivalent to an infinite medium,
6. the medium under the stress pump wave at each time step can be considered as a static problem.

166 Under the above hypothesis, a FE model will be constructed using the software *FreeFEM++*
 167 [58] to establish the relation between σ_{dyn}^{max} and \widetilde{dL}/L which will subsequently be related to a
 168 given dynamic pump waves amplitude. A detailed description of the static model is provided
 169 in Appendix B.

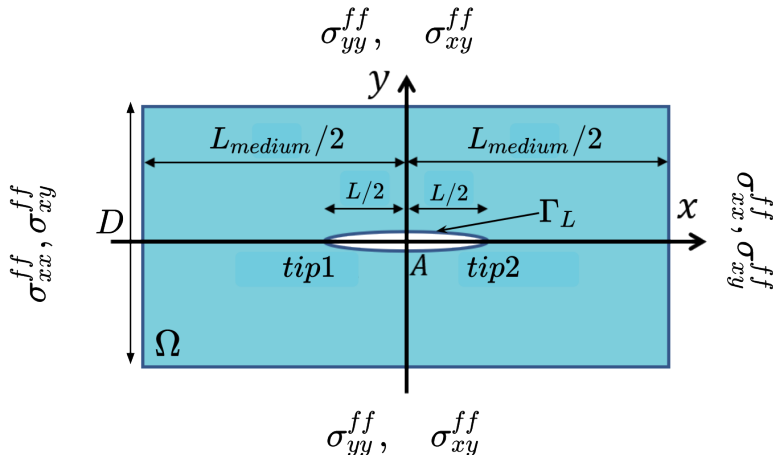


Figure 2: Schematics of the finite 2D rectangular domain Γ of dimension $L_{medium} \times D$ with a partially closed crack Γ_L centered at the center point A, the 2D domain is subjected to a far field stress σ^{ff} on the boundaries.

170 3.2. Crack closure and crack opening

In order to simulate the interaction between the pump wave and the crack, both crack closure and crack opening need to be considered, as illustrated in Fig.3, which correspond to the crack under compression and tension respectively. The crack closure is modeled via a sub-problem decomposition method detailed in Appendix C. The crack closure process is used to obtain the relation between the far field compressive stress σ_{yy}^{ff} and the crack closure length dL^- , whereas the crack opening problem is directly solved by applying a far field tensile stress in the numerical model. The crack has a curved shape ending with a straight line with zero aperture. The junction between both parts is smooth with a curvature radius R defined as:

$$\frac{1}{R} = \left. \frac{d^2y}{dx^2} \right|_{x=L/2} \quad (3)$$

171 Given the geometry of the crack expressed in the previous section, $R = L^2/(\pi^2h)$. Thus
 172 h will be later used as a main parameter to assess the influence of the crack geometry on
 173 the results. A geometrical parameter named the limit aperture of the crack denoted ω is
 174 introduced to determine the crack opening length dL^+ that deals with a pre-close crack
 175 segment denoted dL_{pre} .

176 3.3. Implementation of the static model

177 The modeling is applied to an experimental NCWI configuration made with glass, described
 178 in [59]. The elastic characteristics of the glass are presented in Table 1. As per the
 179 theoretical description, the size of the finite 2D model is considered infinite relative to the

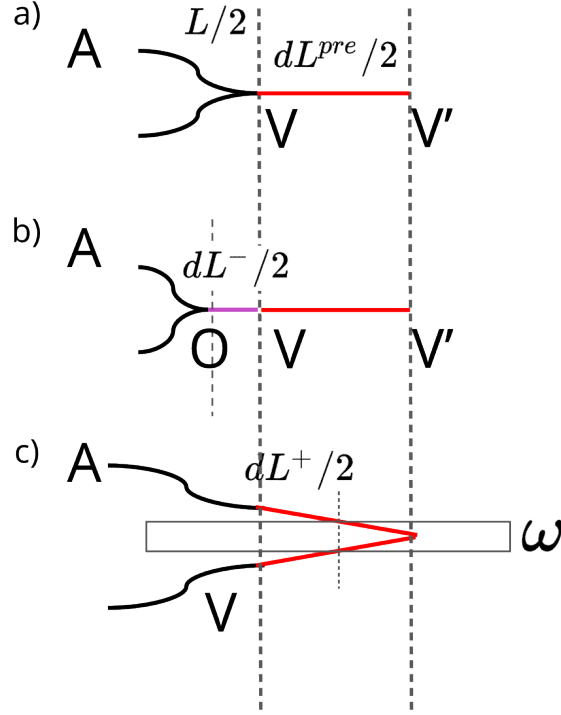


Figure 3: Crack states: a) initial , b) closed , c) open.

180 crack length. Therefore, a minimum sample size needs to be determined such that σ_{yy}^{ff} will
 181 be independent of this size. Various theoretical sizes were investigated. As illustrated in
 182 Fig.4, σ_{yy}^{ff} can be considered stable from $L_{medium} = 100$ mm. Therefore, L_{medium} is fixed at
 183 a secured value of 300 mm in the following.

$\rho(kg/m^3)$	E(GPa)	ν	$V_p(m/s)$	$V_s(m/s)$
2500	69	0.25	5755	3323

Table 1: Elastic characteristics of glass

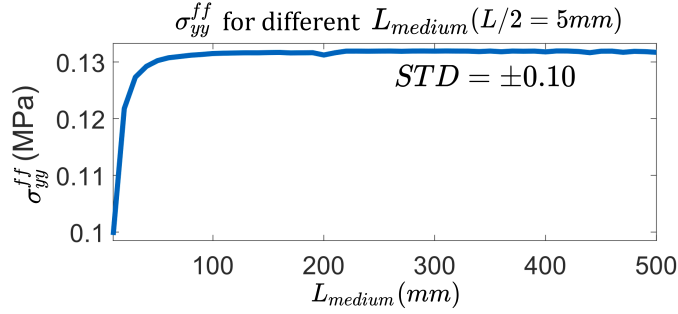


Figure 4: External normal stress σ_{yy}^{ff} applied on top of the medium (at segment $A'B'$) with respect to the medium size L_{medium} , with $L/2 = 5mm$, $dL = 0.1mm$, $h/2 = 5\mu m$ and $D = L$, STD stands for standard deviation

184 Since the half crack's aperture $h/2$ is $5\mu m$, the order of magnitude of the vertical dis-
 185 placement along the crack surface due to σ_{dyn}^{max} will not be greater than $10^{-6}m$, which is much
 186 smaller compared to the medium's size L_{medium} 300mm. In order to avoid abrupt change in
 187 the size of element of the mesh around crack tip, a uniformly refined rectangle submesh of
 188 size $7.5mm \times 25\mu m$ covering the crack segment VA is added, while the segments' size along
 189 adjacent borders follow a logarithmic growth which guarantees a smooth transition. The
 190 final mesh employed is shown in Fig.5.

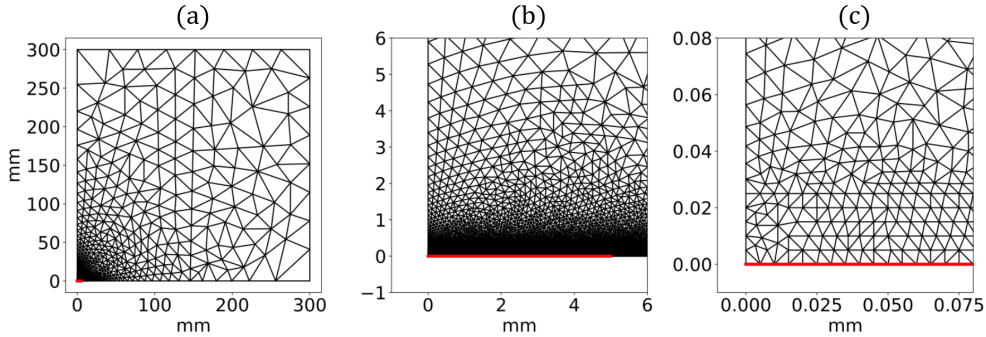


Figure 5: Schematic of the $300mm \times 300mm$ mesh used in the FE simulation, with $L/2 = 5mm$ and $h/2 = 5\mu m$: a) total mesh, the crack segment is marked by the red line, b) zoom on the crack segment (red line), c) zoom on the left corner of the rectangular refined mesh.

191 3.4. Tools for numerical modeling of NCWI and validation process

192 The numerical modeling of NCWI is performed with the software SEM2D [60]. The
 193 Spectral Element Method (SEM) is often considered to be a derivation of FEM, which is
 194 also based on a weak formulation. Yet SEM displays advantages in solving Partial Differential
 195 Equations (PDE) via higher-order polynomial interpolation functions at a low numerical cost
 196 [61, 62]. At high-order interpolations, SEM can exhibit a fast spectral convergence and low
 197 numerical dispersion [63, 64] which is suitable for simulating HF wave propagation. In our
 198 case, the minimum element size needed to obtain spectral convergence lies between 4 mm

199 and 8mm. In order to validate the static model, a comparison between numerical NCWI
 200 results (θ and K_d versus pump amplitude) by use of the deformed crack at different \widetilde{dL}/L
 201 values obtained from the static model with experimental ones has been undertaken. θ can
 202 be obtained from Eq.1 and Eq.3. The configuration of the numerical model with crack
 203 geometry at different \widetilde{dL}/L obtained thanks to the static model is presented in Fig.6. The
 204 values of \widetilde{dL}/L need to be related to given pump amplitudes. A $200\text{mm} \times 200\text{mm}$ squared 2D
 205 homogeneous glass plate is used as the medium of propagation. A crack at 9 different \widetilde{dL}/L
 206 (from 0.0% to 8.0% with a step of 1.0 %, $L=10\text{mm}$, $h=10\mu\text{m}$ at unclosed state) is placed in
 207 a circle whose radius is 25mm centered at $x = 155\text{mm}$ and $y = 140\text{mm}$. A high-frequency
 208 sweep sine from 200kHz to 800kHz during 0.2ms is used as probe wave source which is located
 209 at $x = 50\text{mm}$ and $y = 0\text{mm}$. Each boundary has 9 receivers with a 4-ms acquisition duration.
 210 In order to demonstrate the robustness of the numerical model with respect to both crack
 211 location and orientation, 5 crack locations and orientations are randomly chosen inside the
 212 orange circle, as presented in Fig.7. The final NCWI results will be calculated as the average
 213 over these 5 random selections with corresponding errors at each pump amplitude.

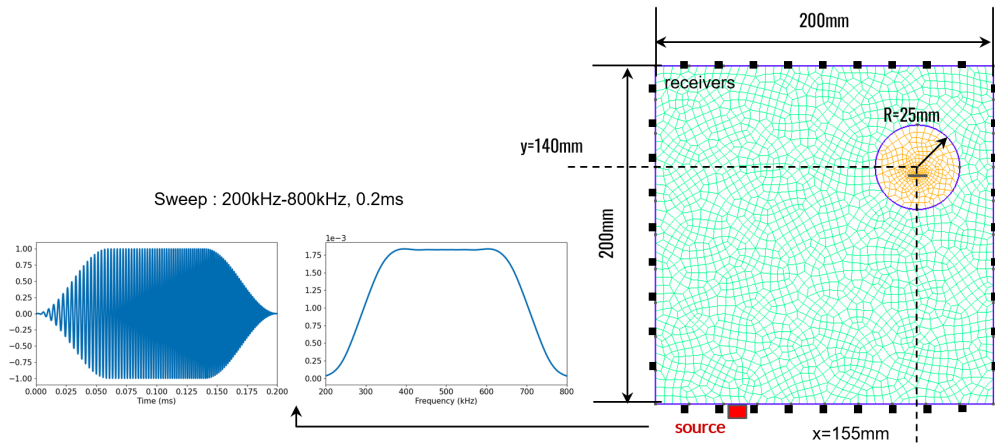


Figure 6: Schematics of the numerical model of NCWI: homogeneous 2D glass plate ($200\text{mm} \times 200\text{mm}$), a sweep type source ranging from 200kHz to 500kHz during 0.2ms located at $x=50\text{mm}$ and $y=0\text{mm}$, 9 receivers are placed at each border with 20mm spacing between each of them. The orange circular zone located at $x=155\text{mm}$ and $y=140\text{mm}$ represents the zone which limits the position of the crack during the random selection process.

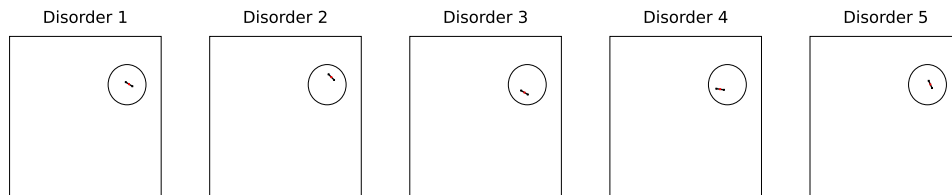


Figure 7: Schematic of the geometry of homogeneous 2D glass plate ($200\text{mm} \times 200\text{mm}$) containing one crack of 10mm in length and of $10\mu\text{m}$ in aperture over 5 random selections of the crack position and its orientation.

214 **4. Results and discussion**

215 *4.1. Static model results*

216 The results of the parametric study on adimensionnal analysis of crack closure are pre-
 217 sented in Fig.8. As shown in Appendix B.1, the relative crack length variation depends only
 218 on σ_{yy}^{ff} . Therefore, in the following, σ^{ff} is replaced by σ_{yy}^{ff} and is inferred as the instant
 219 stress amplitude of the dynamic pump wave at moment t_i ($\sigma_{dyn}(t_i)$). It can be noticed that
 220 σ_{yy}^{ff}/E obeys a quadratic relation with respect to dL^-/L and a linear relation with respect to
 221 h/L , which means that the dependency on h/L could be characterized by a single constant
 222 coefficient thanks to its linearity with respect to σ_{yy}^{ff}/E .

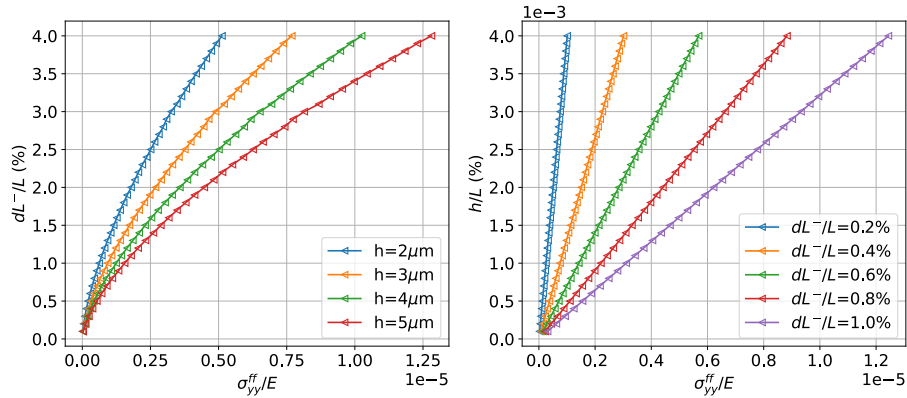


Figure 8: Variation of the normalized far field normal stress σ_{yy}^{ff}/E with respect to a) crack closure dL^-/L and, b) normalized crack's aperture h/L .

223 In Fig.9, it can be noticed that σ_{yy}^{ff}/E obeys an irregular relation with respect to crack
 224 opening dL^+/L , which is related to the deformation of the crack. Four dL^{pre}/L are chosen in
 225 order to perform the parametric study on ω/h . In the crack opening case, $\omega/2$ is not allowed
 226 to be greater than the crack aperture at point V , $u_y(V)$. Therefore, for each dL^{pre}/L , the
 227 value of ω/h varies between 0 and the maximum value $\omega/h = 2u_y(V)/h$.

228 It is worth noting that for each value of dL^{pre}/L , the maximum of ω/h chosen is different.
 229 This is due to different normal displacement at point V , $u_y(V)$, for a given σ_{dyn}^{max} at different
 230 dL^{pre}/L . When $\omega/h = 0$, the pre-close part is considered completely open once $\sigma_{dyn}(t) >$
 231 0. When $\omega/h > 0$, the value of dL^+/L stays at 0 at the beginning because the normal
 232 displacement at point V (or the crack aperture at point V) under weak tension is still
 233 smaller than the given ω . Once this criteria is reached, dL^+/L becomes positive and the
 234 crack is considered opened. The greater the value of ω/h is, the higher the tensile stress is
 235 required to reach crack opening. The relation between ω/h and the required tensile stress
 236 for crack opening is linear.

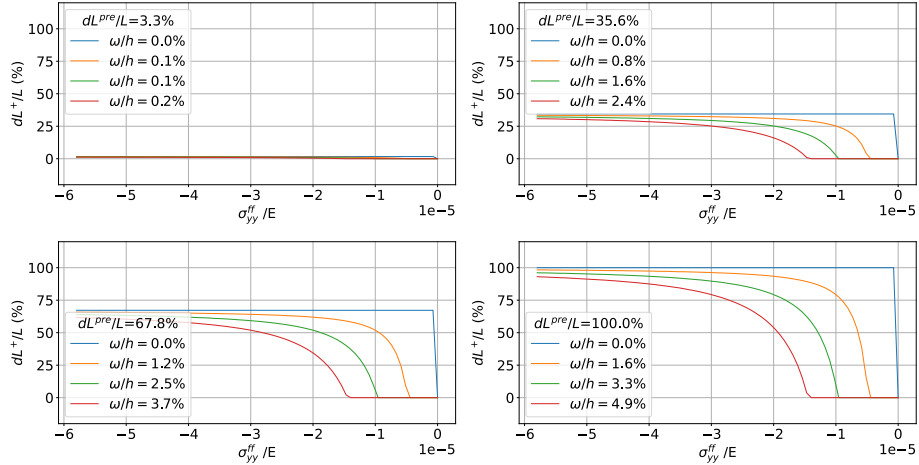


Figure 9: Variation of dL^+/L with respect to the far field normalized normal stress σ_{yy}^{ff}/E at different preclosure length dL^{pre} and limit aperture ω .

237 Using the relation between σ_{yy}^{ff}/E and dL^\pm/L found in Figures 8 and 9, the dynamic
 238 amplitude of pump waves (Fig. 10 left) can be converted to $[dL/L]_{dyn}$ (Fig. 10 right) which
 239 is time-dependent. An example of this conversion is presented in Fig.10.

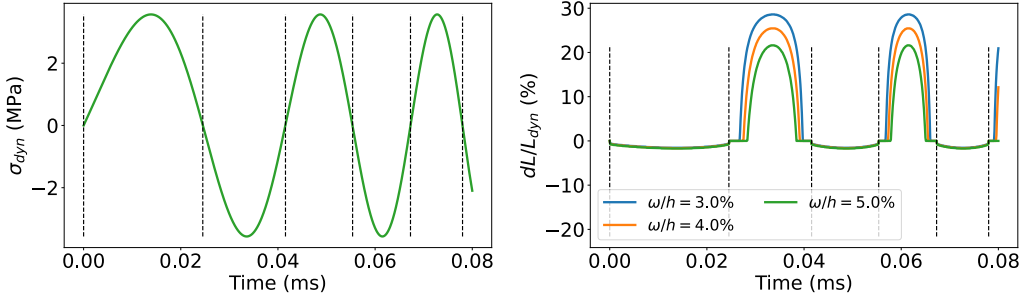


Figure 10: Left: zoomed chirp signal (0.08ms over 11ms) of the dynamic stress pump wave at $\sigma_{dyn}^{max} = 3.56\text{MPa}$ (pump wave is a sine sweep that varies in time, the positive part represents the compressive state where the crack is closed, and the negative part represents the tensile state where the crack is open). Right: calculation of the instant relative crack length variation $[dL/L]_{dyn}$ of dynamic stress pump wave at different limit aperture ω at $\sigma_{dyn}^{max} = 3.56\text{MPa}$, $dL^{pre}/L = 30\%$.

240 After converting the dynamic pump wave's amplitude into $[dL/L]_{dyn}(t)$, the time average
 241 of $[dL/L]_{dyn}$ during the coda acquisition time (4 ms), denoted \widetilde{dL}/L , can be calculated. Fig.11
 242 shows the results of \widetilde{dL}/L at different ω/h . It is observed that the overall averaged effect of
 243 the dynamic pump wave excitation on a pre-close crack is equivalent to crack opening. This
 244 result is in agreement with the numerical NCWI results in [31]. Besides, a linear relation
 245 between \widetilde{dL}/L and σ_{dyn}^{max} is found, which satisfies the last hypothesis imposed in section 3.1.
 246 This relation will be used later on to perform the conversion from \widetilde{dL}/L to σ_{dyn}^{max} during the
 247 numerical NCWI modeling. However, due to the application of ω , a certain initial stress
 248 amplitude is required in order to consider the crack as opened, such need of initial stress is

249 observed numerically and experimentally in wave/crack interaction [65, 66] in order to open
 250 a closed crack. The value of this initial stress depends on ω and dL^{pre} . The determination
 251 of ω and dL^{pre} is rather flexible regarding the fact that the linearity between \widetilde{dL}/L and σ_{dyn}^{max}
 252 is preserved whatever the values of ω and dL^{pre} are. Also, the non uniform relation between
 253 σ_{dyn}^{max} and \widetilde{dL}/L after the application of ω implies that the level of non-linearity depends on
 254 the initial crack aperture. It will be demonstrated in the next section that the values of ω
 255 and dL^{pre} can be determined as long as the numerical NCWI results are quantitatively in
 256 agreement with experimental results.

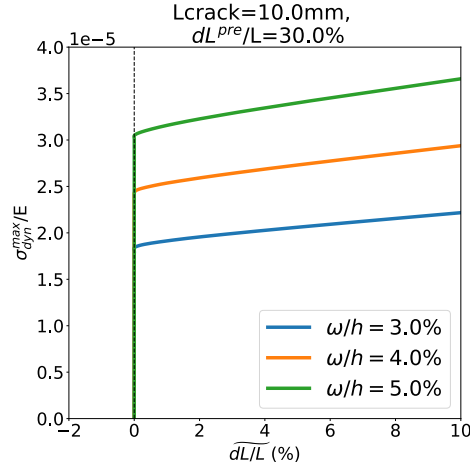


Figure 11: Results of σ_{dyn}^{max} with respect to \widetilde{dL}/L for a crack of $L = 10\text{mm}$ and $dL^{pre}/L = 30\%$ at 3 different ω/h (3.0%, 4.0% and 5.0%). A certain amplitude of σ_{dyn}^{max}/E is required to trigger the crack opening. This value equals to 1.8 (1.2MPa), 2.4 (1.7MPa) and 3.0 (2.1MPa) respectively to the 3 chosen ω/h .

257 4.2. Numerical NCWI results

258 The numerical coda signals at 5 different \widetilde{dL}/L values (0.0%, 1.0%, 3.0% 5.0%, 7.0%),
 259 presented in Fig.12 show that the greater the value of \widetilde{dL}/L , the greater the coda signal
 260 dilatation and decorrelation relative to the reference coda. Since the current results are
 261 applicable for just one crack, the modification is rather weak. However with NCWI, even
 262 these weak changes can be quantified with precision.

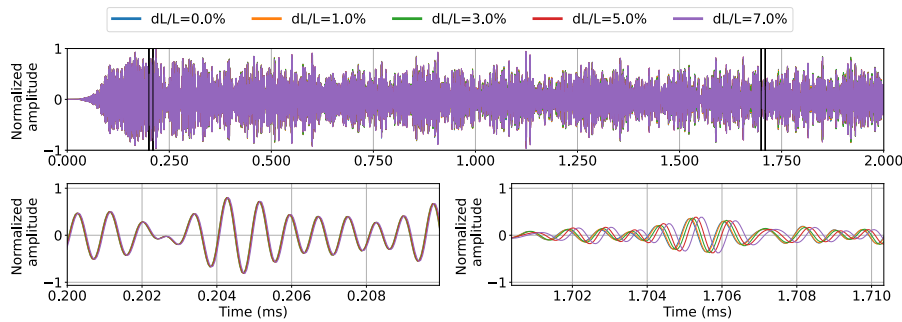


Figure 12: Numerical coda signals obtained at 5 different \widetilde{dL}/L (0.0%, 1.0%, 3.0% 5.0%, 7.0%) for a cosine-like crack of $L=10\text{mm}$, $h=10\mu\text{m}$ inside a $200\text{mm} \times 200\text{mm}$ 2D glass plate.

263 The numerical NCWI simulation results are presented in Fig.13. The values of θ and
 264 K_d are averaged over 5 disorders at each $d\widetilde{L}/L$ value. These results reveal a linear and
 265 a quadratic relation between θ , K_d , respectively. The orders of magnitude of θ and K_d
 266 are 10^{-3} and 10^0 , respectively. These observations are in agreement with the experimental
 267 results found for a weakly damaged glass plate [59]. Moreover, it is observed that the
 268 standard deviation of θ and K_d increase with $d\widetilde{L}/L$. This finding may be correlated with
 269 the sensitivity of wave/crack interactions to crack position and orientation: the modification
 270 of wave propagation trajectories is cumulative during propagation. The geometrical change
 271 in the crack can modify its surface inside as well as its scattering property. These two
 272 modifications will lead to changes in the effective coda wave velocity and waveform distortion,
 273 which ultimately change the values of θ and K_d . Therefore, as $d\widetilde{L}/L$ increases, the changes
 274 of θ and K_d become more pronounced.

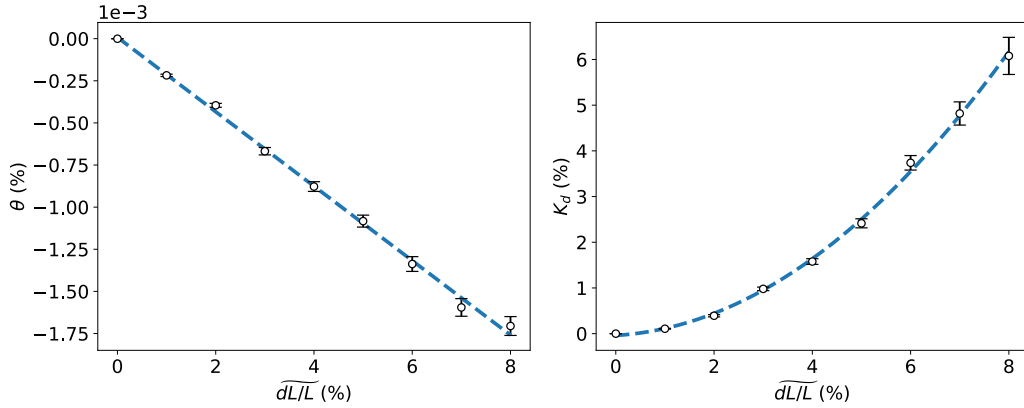


Figure 13: Numerical NCWI result: $\theta(d\widetilde{L}/L)$ et $K_d(d\widetilde{L}/L)$ for a crack at length of 10mm and at aperture of $10 \mu m$ in a glass plate of $200mm \times 200mm$ averaged over 5 disorders in crack's position.

275 Then, an estimation of σ_{dyn}^{max} by using the relation between $d\widetilde{L}/L$ and σ_{dyn}^{max} found in
 276 Fig.11 for a crack of 10mm of length and $dL^{pre} = 30\%$ is presented in this section. This
 277 single parameter set choice is due to the linearity observed previously in dL^{pre} and ω/h with
 278 respect to $d\widetilde{L}/L$. The NCWI result after conversion from $d\widetilde{L}/L$ to σ_{dyn}^{max} is shown in Fig.14.
 279 Since $d\widetilde{L}/L$ is linear to σ_{dyn}^{max} , the linear and quadratic relations of θ and K_d with respect to
 280 σ_{dyn}^{max} is preserved. Due to the requirement of an initial stress to reach the open crack state
 281 (Fig.11), σ_{dyn}^{max}/E does not start from zero. For the choice of $\omega/h = 5\%$ and $dL^{pre} = 30\%$,
 282 this initial stress is of $\sigma_{dyn}^{max}/E \approx 3.0$ (or 2.1MPa). In NCWI, the pump strain is at an order
 283 of magnitude around 10^{-5} , which corresponds to around 1MPa in a glass plate. This stress
 284 amplitude is smaller than 2.1MPa, but thanks to the static model, this value can be tuned
 285 to needed amplitude by varying ω/h and $dLpre/L$.

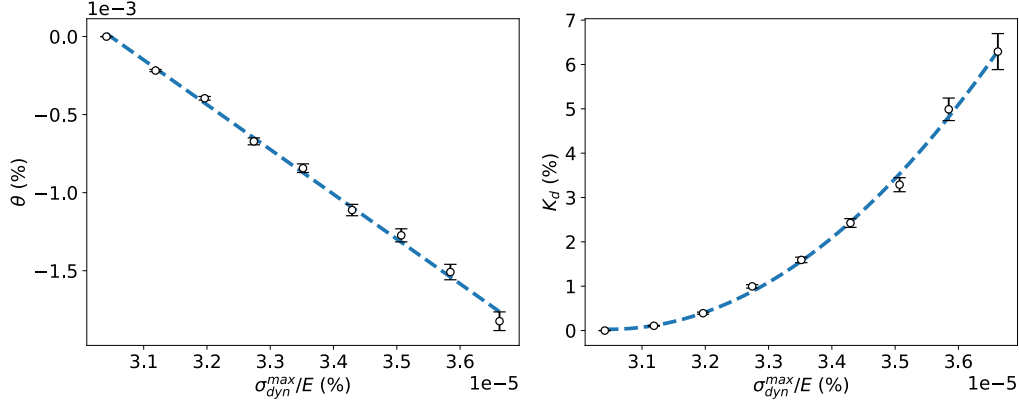


Figure 14: θ and K_d with respect to σ_{dyn}^{max}/E for a crack with $\omega/h = 10\%$ and $dL^{pre} = 100\%$.

286 5. Conclusion and perspectives

287 This study has investigated the average effect of dynamic pump waves on cracks in
 288 Nonlinear Code Wave Interferometry (NCWI) inside a finite 2D homogeneous medium using
 289 a simplified static finite element model. A time average of the relative crack length variation
 290 over the dynamic pump wave, with a given value of the initial external load, has been used
 291 due to the application of the limit aperture ω during both crack opening and closure. A
 292 2D Spectral Element model has been implemented to simulate the influence of a crack in a
 293 2D homogeneous reverberating medium on NCWI results. According to these results, the
 294 following main conclusions can be drawn:

- 295 • The static model contains five adimensional parameters: \widetilde{dL}/L , σ_{dyn}^{max}/E , h/L , dL^{pre}/L
 296 and ω/h . Since the relative crack length variation \widetilde{dL}/L is what is looked for and
 297 σ_{dyn}^{max}/E is imposed initially during NCWI, therefore, this static model can properly
 298 work by only tuning the other three parameters, which can be useful for purposes of
 299 inversion.
- 300 • Linear relations between the maximum dynamic pump stress wave amplitude σ_{dyn}^{max} and
 301 the relative crack length variation \widetilde{dL}/L have been found for a crack of $L=10\text{mm}$ and
 302 $h=10\mu\text{m}$ at different limit aperture ω/h by considering both crack closure and crack
 303 opening during the dynamic pump wave excitation.
- 304 • The numerical NCWI results were compared to experimental measurements in a 2D
 305 glass plate: the relative velocity variation of the medium θ is linear to σ_{dyn}^{max}/E and the
 306 coefficient of decorrelation K_d is quadratic to σ_{dyn}^{max}/E . The standard deviation of θ
 307 and K_d increases with the relative crack length variation \widetilde{dL}/L due to the accumulated
 308 change in wave propagation trajectories.

309 In conclusion, this study has provided the possibility to simulate the equivalent average
 310 effect of dynamic pump waves in NCWI on a single crack by use of a simplified static model
 311 containing only three parameters: h/L , dL^{pre}/L and ω/h . Thanks to the applied hypothesis,
 312 this simplified model enables imaging quantitatively cracks with NCWI.

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Appendices

A. Nomenclature

	Variable	Description	
Acoustical parameters	$u_i(t)$	reference coda signal	
	$u_p(t)$	perturbed coda signal	
	$CC(\theta)$	correlation coefficient between $u_i(t)$ and $u_p(t)$ for a given θ	
	θ	relative velocity change of the elastic medium	
	K_d	decorrelation coefficient between reference and perturbed coda signals	
	α_θ	coefficient of linear term of θ vs. pump wave stress amplitude	
	α_{K_d}	coefficient of quadratic term of K_d vs. pump wave stress amplitude	
	σ_D	scattering cross section of the crack	
	$\sigma_{dyn}(t)$	time signal of the dynamic pump waves	
	σ_{dyn}^{max}	maximum dynamic pump stress wave amplitude of $\sigma_{dyn}(t)$	
	V_p	velocity of pressure waves	
	V_s	velocity of shear waves	
	Material parameters	E	Young's modulus
		ν	Poisson's ratio
ρ		density	
Static model parameters	L_{medium}	length of the rectangular medium	
	D	height of the rectangular medium	
	L	length of the crack	
	h	aperture at the crack center	
	y_{geo}	function of the crack surface geometry	
	u_x	horizontal displacement of the crack surface	
	u_y	vertical displacement of the crack surface	
	dL	algebraic length variation of the open part of the crack	
	dL^{pre}	pre-close crack segment	
	dL^\pm	length variation of the open part of the crack (dL^+ : opening, dL^- : closure)	
	$[dL/L]_{dyn}(t)$	dynamic time variation of dL/L to $\sigma_{dyn}(t)$	
	\widetilde{dL}/L	average of $[dL/L]_{dyn}(t)$ over time	
	ω	limit aperture above which the crack is considered as open	
	dx	distance between node O and node S of the crack	
	Ω	2D elastic isotropic rectangular medium	
	$\widetilde{\Omega}$	Upper right quarter of the medium	
	Γ_L	embedded horizontal crack	
	$\Gamma_{(L-dL)/2}$	open part of Γ_L	
	$\Gamma_{dL/2}$	close part of Γ_L	
	\wp	total problem of the FE crack model	
	\wp_1	problem 1 of the FE crack model	
	\wp_2	problem 2 of the FE crack model	
	σ_0	far field stress applied on the crack in \wp	
γ	target value of σ_0		
$\boldsymbol{\sigma}^{ff}$	tensor of far field stress		
\mathbf{T}	traction vector		

B. Detailed description of the pump / crack interaction (static model)

B.1. Relative crack length variation under far field stress loading in 2D plane strain

This part examines the equivalent effect of the dynamic pump wave over a crack during NCWI assuming static regime. As illustrated in Fig.B.1 (a), due to symmetries, a quarter of the elastic isotropic 2D domain Ω of dimension $L_{medium} \times D$ with an embedded horizontal crack Γ_L of length L is considered. The design of the crack geometry is illustrated in Fig.B.1 (b) and consists in two parts: an open segment AV and a pre-close segment VV' . this partially closed crack is used to model both crack closure and opening. The geometry of the crack surface along the open part is given by $y = \pm y_{geo}(x) = \pm \frac{h}{4}(1 + \cos(\pi \frac{2x}{L}))$ with $0 \leq x \leq L/2$, $0 \leq y_{geo} \ll D$ and $y_{geo}(-x) = y_{geo}(x)$. The half length of the pre-close part is $dL^{pre}/2$. Domain Ω is subjected to a far field stress state on its outer boundary $\partial\Omega/\Gamma_L$ resulting in the traction vector \mathbf{T} :

$$\mathbf{T} = \boldsymbol{\sigma}^{ff} \cdot \mathbf{n}, \quad \text{with} \quad \boldsymbol{\sigma}^{ff} = \begin{bmatrix} \sigma_{xx}^{ff} & \sigma_{xy}^{ff} \\ \sigma_{yx}^{ff} & \sigma_{yy}^{ff} \end{bmatrix}, \quad (\text{B.1})$$

The configuration in Fig.B.1 can treat both compressive and tensile stress. This stress at the domain boundaries, especially the σ_{yy}^{ff} component if non zero, is likely to induce a crack length variation of $dL^\pm/2$ due to compression or tension from each of its tips. The purpose of this section is to compute $dL^\pm/2$ as a function of $\boldsymbol{\sigma}^{ff}$, under the assumption of plane strain. $\Gamma_{(L-dL)/2}$ and $\Gamma_{dL/2}$ are denoted as the open and closed parts of the crack respectively.

First, it can be noticed that σ_{xx}^{ff} and σ_{xy}^{ff} do not or only barely affect the relative crack length variation. Indeed, assuming that σ_{xy} is zero on the whole crack length even on $\Gamma_{dL/2}$ (i.e. case of negligible contact friction conditions between the two crack surfaces), the change in the vertical displacement induced by can be calculated incrementally within the framework of linear elasticity. Therefore, owing to problem symmetry, the vertical displacement δu_y due to σ_{xx}^{ff} is zero on the x-axis, thus σ_{xx}^{ff} does not affect the crack closure. Also, due to symmetry, σ_{xy}^{ff} leads to similar vertical displacements on both crack surfaces, consequently not changing the existing aperture between the two surfaces.

Hence, only the effect of the component σ_{yy}^{ff} will be considered in the following discussion. In addition, we assume in the following that the closing length $dL/2$ is given, while the corresponding value of σ_{yy}^{ff} denoted $\sigma_{yy}^{ff}(dL/2)$ is considered as the unknown variable to be determined.

B.2. Adimensional analysis

The adimensional analysis aims to reduce the number of variables in order to simplify the parametric study in the following. In our problem, the relative crack length variation dL/L depends on several variables: E (Young's modulus), σ_{yy}^{ff} , L , dL^{pre} , h , ν (Poisson's ratio), which can be written as :

$$\begin{aligned} dL^- &= f(E, \sigma_{yy}^{ff}, L, h, \nu), \\ dL^+ &= g(E, \sigma_{yy}^{ff}, L, dL^{pre}, h, \omega, \nu). \end{aligned} \quad (\text{B.2})$$

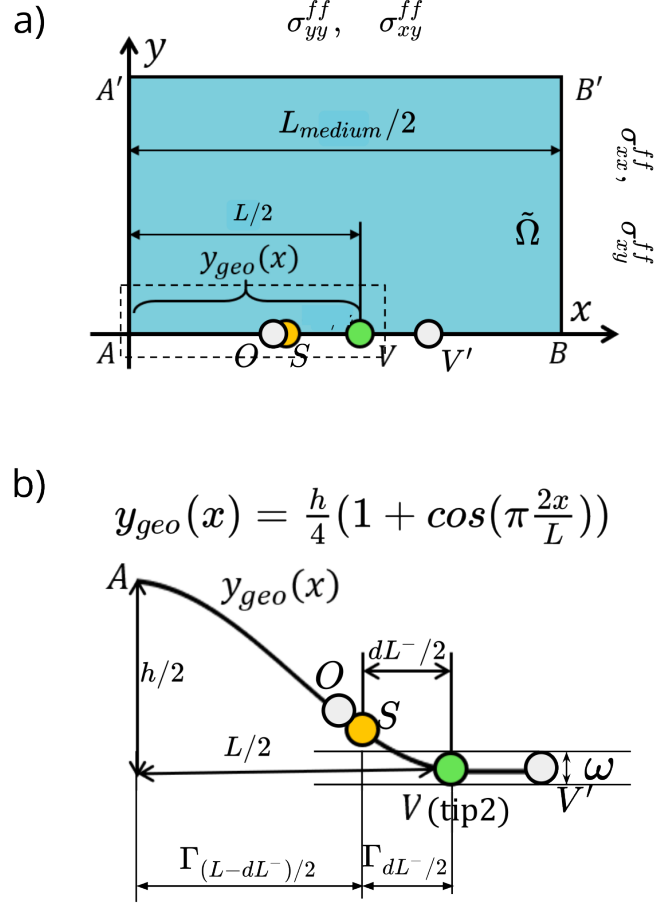


Figure B.1: Schematic representations of the static model :a) Upper right quarter the finite 2D rectangular domain Γ of dimension $L_{medium} \times D$ with a partially closed crack Γ_L centered at the center point A, the 2D domain is subjected to a far field stress σ^{ff} on the boundaries, the segment VA represents a half of total crack length, S is the anticipated new crack tip after the closure of dL^- , $V'V$ of length dL^{pre} represents the extension of AV whose initial aperture is 0, dL^- represents the crack closure length. b) Zoom over the crack segment $V'A$, the geometry of the half crack VA is described by $y_{geo}(x)$. O is an additional node placed at a small distance dx at left side of S . This node is used as a “testing node” in the FEM computation to help approximate the double contact condition to be checked in S , that is geometrical closure of the crack and zero normal stress.

According to Buckingham π theorem [67], Eq.B.2 can be reduced to an adimensional form :

$$\begin{aligned} \frac{dL^-}{L} &= F\left(\frac{\sigma_{yy}^{ff}}{E}, \frac{h}{L}, \nu\right), \\ \frac{dL^+}{L} &= G\left(\frac{\sigma_{yy}^{ff}}{E}, \frac{dL^{pre}}{L}, \frac{h}{L}, \frac{\omega}{h}, \nu\right). \end{aligned} \quad (\text{B.3})$$

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Supposing that ν is fixed, this static model can then be analysed by the 5 adimensional parameters: dL^-/L , dL^{pre}/L , σ_{yy}^{ff}/E , h/L and ω/h . The dynamic pump wave has both positive and negative amplitudes, these correspond to compression and tension respectively. Hence, two cases are considered in the static model: crack closure and crack opening. The first case is solved by using a sub-problem decomposition method.

The second one is solved by applying an external tensile load on the boundaries with the application of crack opening criteria ω .

C. Crack closure and crack opening modeling

C.1. Crack closure: sub-problem decomposition

The process of sub-problem decomposition is presented in Fig.C.1. The total problem \wp (Fig.C.1(a)) describes the crack surface deformation under arbitrary far field tensile stress σ_{yy}^{ff} (or σ_0), which can be solved numerically using the standard Finite Element method. In the sub-problem decomposition, the total problem \wp is decomposed into two sub-problems \wp_1 and \wp_2 . The first sub-problem \wp_1 describes the crack surface deformation under a reference far field tensile stress σ_{yy1}^{ff} . The second sub-problem \wp_2 describes the crack surface deformation due to the application of a normal displacement $u_y(x)$ opposite to $y_{geo}(x)$ which makes it possible to close the crack along SV . Since the crack aperture h is negligible in front of the size of the medium Ω , hence in the FE modeling, the crack segment AV is geometrically represented by a straight line confounded with the x axis. $\tilde{\Omega}$ is the upper right part of the structure $0 \leq x \leq L_{medium}/2, 0 \leq y \leq D/2$, O the point with coordinates $(x = (L - dL^-)/2 - dx, y = 0^+)$, S the point with coordinates $(x = (L - dL^-)/2, y = 0^+)$, where dx has the size of one or several mesh elements in this area. S and O as nodes are included in mesh nodes. As mentioned in Fig.B.1 that O is a "testing node" to approximate the double contact condition, the nodal point S is actually the real left ending point of the crack closure segment. Since only one of the double contact condition (e.g. zero aperture and zero normal stress) can be applied on a nodal point, it can happen, due to numerical error, that the eventual aperture on the chosen nodal point is zero while the normal stress is non zero, or the normal stress is zero but the aperture is non zero. Therefore, The application of the contact condition (zero aperture in our case) on point O , which is at the left side of S by a very small distance dx , is to test if both double contact condition are satisfied. If so, then the point S can be taken as the left ending point of the crack closure segment. $\tilde{\Gamma}_{dL^-/2}$ is the set of the nodes located on the right half part of $\Gamma_{dL^-/2}$ except point O ($x < (L - dL^-)/2, y = 0^+$). All boundary conditions are presented in Table.C.1. For both problems \wp_1 and \wp_2 , the nodal vertical displacement at point O ($u_{y1}(O)$ and $u_{y2}(O)$) is considered as free, while the corresponding nodal vertical force is set equal to 0. For \wp_2 the nodal vertical displacements of $\tilde{\Gamma}_{dL^-/2}$ are taken as equal to the nodal values of $-y_{geo}$, while these are set to zero for \wp_1 . On the upper limit Γ_{sup} of $\tilde{\Omega}$, the stress component σ_{yy}^{ff} is set at 1 MPa for \wp_1 , denoted as σ_{yy1}^{ff} and equal to 0 for \wp_2 , denoted as σ_{yy2}^{ff} . All other boundary conditions of \wp_1 , \wp_2 (zero displacement or stress components) are the same for both problems and assumed to be identical to those of \wp . At the output, \wp_1 and \wp_2 give the nodal vertical displacements at point O $u_{y1}(O)$ and $u_{y2}(O)$ respectively.

It then becomes straightforward to verify, from the boundary conditions of \wp , that the problem can be obtained as the following superposition:

$$\wp = \gamma\wp_1 + \wp_2, \quad (C.1)$$

Table C.1: Boundary conditions applied on all segments in problem 1 and 2.

Segment	Problem 1	Problem 2
AA'	$u_x = 0, \sigma_{xy} = 0$	$u_x = 0, \sigma_{xy} = 0$
A'B'	$\sigma_{yy} = 1\text{MPa (along } -\mathbf{y}), \sigma_{xy} = 0$	$\sigma_{yy} = 0, \sigma_{xy} = 0$
B'B	$\sigma_{xx} = 1\text{MPa (along } -\mathbf{x}), \sigma_{xy} = 0$	$\sigma_{xx} = 0, \sigma_{xy} = 0$
VB	$u_y = 0, \sigma_{xy} = 0$	$u_y = 0, \sigma_{xy} = 0$
SV	$u_y = 0, \sigma_{xy} = 0$	$u_y = -y_{geo}(x), \sigma_{xy} = 0$
AO&OS	$\sigma_{yy} = 0, \sigma_{xy} = 0$	$\sigma_{yy} = 0, \sigma_{xy} = 0$

646 where γ represents the target value of the far field stress σ_0 (in MPa) needed for a crack
647 closure of dL^- . Provided that the total vertical displacement u_y of point O equals zero,
648 and the crack segment AV a straight line. The boundary conditions in \wp_1 and \wp_2 also
649 lead automatically to zero normal stress at O . Therefore, u_y at point O of \wp should
650 equal to $-y_{geo}(O)$. This set-up leads to the target value γ , namely:

$$\begin{aligned} -y_{geo}(O) &= \gamma u_{y1}(O) + u_{y2}(O), \\ \gamma &= -[u_{y2}(O) + y_{geo}(O)]/u_{y1}(O), \end{aligned} \tag{C.2}$$

651 where $u_{y1}(O), u_{y2}(O)$ are the vertical displacements of point O computed for \wp_1 and \wp_2
652 using the Finite Element method.

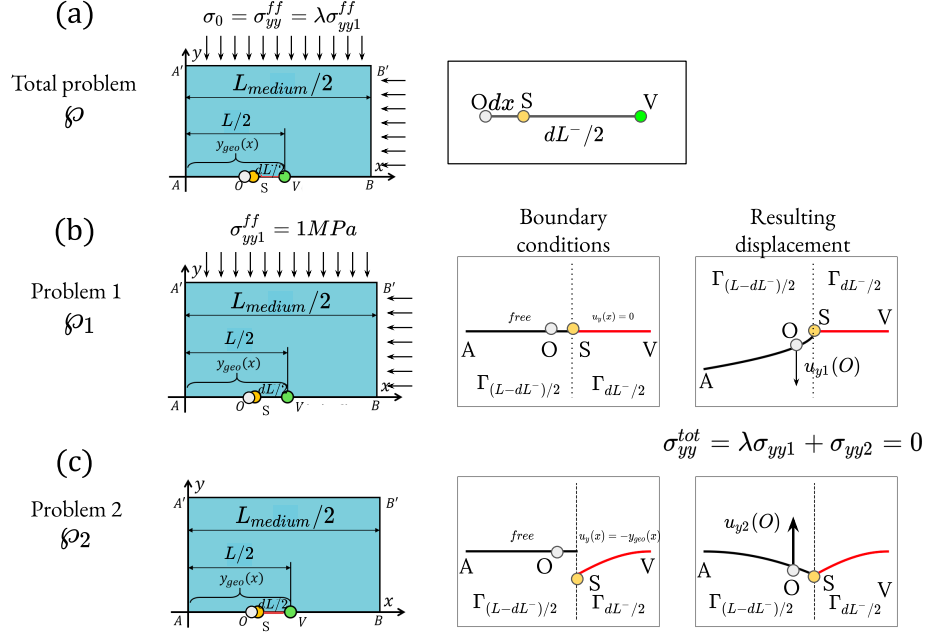


Figure C.1: Schematics of total problem \wp , problem 1 \wp_1 and problem 2 \wp_2 of the crack closure process. The initial half crack is represented by VA and the anticipated half crack closure length $dL^-/2$ is represented by VS . (a). Total problem: glass plate under biaxial static stress load σ_{dyn}^{max} which allows a zero normal stress and crack aperture at point O ($x = (L - dL^-)/2 - dx$). (b). Problem 1: the plate is under biaxial static stress load $\sigma_{yy1}^{ff} = 1\text{MPa}$, the normal displacement u_y along SV is restricted to be zero. (c). Problem 2: the plate is free from external load, the normal displacement u_y along SV is imposed to be the opposite of the initial crack geometry, thus $-y_{geo}(x)$.

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C.2. Crack opening

As illustrated in Fig.3, since $V'V$ is defined as a pre-close segment without prestress (self-stress state), $V'V$ will be completely open as soon as a tensile stress is applied. However the crack length grows gradually with the increase of the tensile load in the real crack opening. Therefore, a limit aperture ω is introduced, only the crack surface whose aperture above ω is considered as open. Denoting the normal displacement along the crack segment AV' due to the external far field tensile stress load σ_{yy}^{ff} as $u_y(x)$, dL^+/L is determined by satisfying the following condition (abscissa for which the crack displacement is equal to ω):

$$u_y(x = (L + dL^+)/2) = \omega. \quad (\text{C.3})$$

The normal crack surface displacement $u_y(x)$ caused by a far field normal tensile stress σ_{yy}^{ff} can be obtained by 2 steps. First, applying a reference far field tensile stress which gives a reference normal crack surface displacement $u_y^{ref}(x)$. Then, since the elastic medium is considered as linear, for any σ_{yy}^{ff} , there is:

$$u_y(x) = \sigma_{yy}^{ff} u_y^{ref}(x). \quad (\text{C.4})$$

Combining Eq.C.3 and Eq.C.4 gives:

$$\sigma_{yy}^{ff} = \omega/u_y^{ref}(x = (L + dL^+)/2). \quad (\text{C.5})$$

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Since $u_y^{ref}(x)$ is obtained in advance from numerical modeling, dL^+ can then be determined implicitly for given values of σ_{yy}^{ff} and ω .