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A Numerical Approach for Buildings Reduced Thermal Model Parameters Evaluation

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Abstract. This paper introduces an efficient PSO-based optimization method to identify parametric values of buildings reduced thermal RC network models. A high order reference model representing energy flow through building envelope is developed based on Crank-Nicolson finite difference method. Thermal network models performance with random resistance and capacitance values are compared with a reference model then PSO is used for parameters identification to match the actual reference model thermal dynamics. The accuracy and computational cost of these optimized models are validated against reference model for different construction classes. The proposed methodology presents an effective way to develop a reliable thermal network model representing realistic thermal dynamics in building that can be used to improve efficiency of controlling techniques.

1. Introduction

Buildings are responsible for around 40% of total primary energy consumption in Europe and are the fastest growing energy usage sectors than the transportation and industrial sectors. Inefficiency in the conventional building controlling techniques, specifically in controlling HVAC (heating, ventilation, and air conditioning) systems have resulted in huge energy consumption and poor comfort management. To improve buildings performance, there is a growing interest in the development of multi-objective intelligent controllers to reduce energy consumption. These techniques often require an accurate and computationally efficient thermal dynamics model that is related to the buildings actual thermal energy flow [1].

A transient thermal response model can be developed from heat and mass balance equations. An attempt to develop physical model will lead to a set of partial differential equations (PDE), which can be solved numerically by finite difference method, finite element method, and finite volume method [2]. Models based on such physical equations are always high computational cost and complex in modeling. However, intelligent controllers require a quick response model that process multi-variable data for real time control of buildings. Hence, an accurate low computational cost reduced models are needed for such controlling techniques. Thermal RC (resistor-capacitor) network models are normally used as building reduced model for representing thermal behaviour of building envelope [1]. The parametric values of resistor and capacitor are to be carefully chosen for modeling purposes.



There are mainly two approaches to identify parametric values of a thermal network model: analytical [3] and numerical. In analytical approach, the parametric values will be obtained by using set of algebraic equations and in numerical methods, an approximation of RC values are achieved by using optimization techniques. Fraisse et al. [4], developed an analytical methodology based on electric analogy to evaluate RC values and compared the performance of variety of RC combinations (2R1C, 1R2C, 3R2C, and 3R4C). Numerical approaches have shown greater accuracy than the analytical approach because of their optimization techniques and limitation of analytical approaches for few combinations [5]. Xu and Wang [6], proposed a numerical method for parametric identification based on frequency characteristics analysis and genetic algorithm. These studies ensured that the thermal network models can be used as an alternative for white box models for improved building performance. This paper presents then a numerical approach for parameters evaluation of second-order thermal network model (3R2C) by using a particle swarm optimization (PSO) technique. For parametric identification optimization purposes, a high-order reference model is developed by a set of partial differential equations representing thermal flow through building envelope and solved numerically using Crank-Nicolson finite difference method [2]. The step response of both reference model and 3R2C models are compared, and the root mean square error of a both models are minimized by using a PSO algorithm [7]. The optimized models accuracy is also analyzed and validated for different thermal capacity materials.

2. Reference model

The presented methodology involves the development of a reference model for validation and parameters identification by optimization technique for 3R2C thermal model. Heat transfer through building envelope includes several heat and mass transfer processes. Conduction, convection, and radiation heat transfer through buildings are some major factors to be considered in model development. One of most important types of heat transfer in buildings is heat conduction through building envelope mainly through walls, floors, and roofs.

Heat transfer through conduction under steady state can be expressed using fundamental Fourier heat conduction law [2].

$$\dot{Q}_{conduction} = -\lambda A \frac{dT}{dx} (W) \quad (1)$$

where $\dot{Q}_{conduction}$ = conduction heat transfer through wall (W), T = temperature (K), λ = thermal conductivity (W/m.K), A = surface area (m^2), and x = spatial coordinate (m).

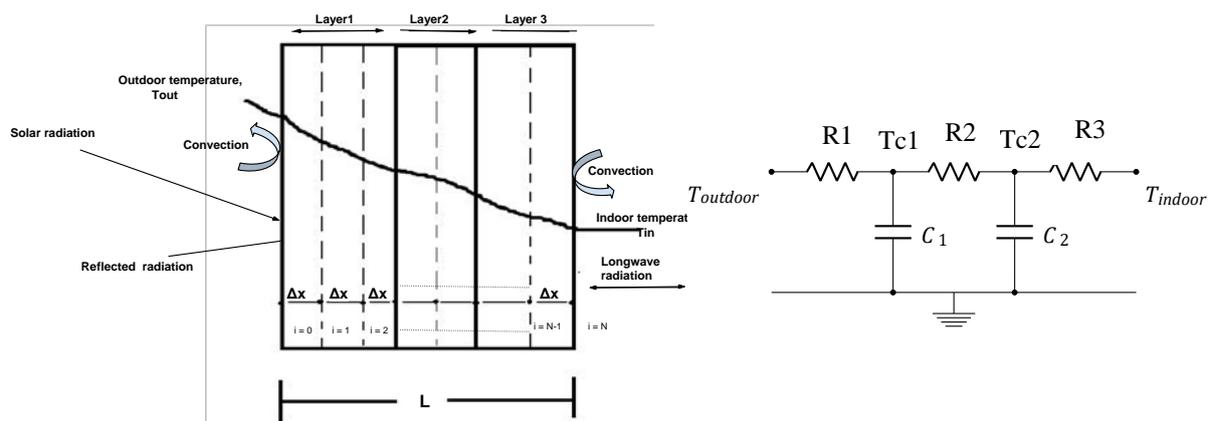


Figure 1. Thermal dynamics of buildings (a) and equivalent 3R2C model (b).

Figure 1 a represents the thermal energy flow through multilayer wall, the temperature at a given point is the function of space x and time t , i.e., $T = T(x, t)$. For walls which have higher ratio difference between its vertical height to thickness, the heat flows in a direction perpendicular to the wall surface, hence conduction is treated as one dimensional and for systems without thermal sources and sinks the equation (1) becomes:

$$\frac{\partial T(x, t)}{\partial t} = \alpha \frac{\partial^2 T(x, t)}{\partial x^2} \quad \text{for } 0 < x < L, t > 0, \quad (2)$$

where $\alpha = \lambda / \rho C_p$ = thermal diffusivity (m^2/s), L = thickness of the wall (m), λ = thermal conductivity ($W/m.K$), ρ = density (kg/m^3), C_p = heat capacity of the wall material ($J/kg.K$). To solve this problem, one initial condition, and two boundary conditions are required. The surface temperatures of both sides of wall are affected by convective heat transfer between air in contact and surface. Hence, convective boundary conditions are formulated as:

$$\lambda \left(\frac{\partial T}{\partial x} \right)_{x=0} = h_c [T_s(t) - T_{x=0}(t)], \quad \lambda \left(\frac{\partial T}{\partial x} \right)_{x=L} = h_c [T_{x=L}(t) - T_i(t)] \quad (3)$$

The thermal heat transfer modeling using above equations is carried out with some hypothesis:

- Thermal energy flow is considered in only spatial x coordinate, because the ratio between height and thickness is very large that results in negligible amount of thermal flow along other directions, i.e., y and z directions. The text should be set to single line spacing.
- Thermal energy distribution in the material is isotropic in nature and thermal properties are temperature independent.
- There is no sink/source present within the material and thermal bridge effects are neglected.

The PDE (2), with boundary conditions (3), can be solved analytically, by using numerous readily available methods: Separation of variables, Laplace transforms, or others. Nevertheless, heat transfers in buildings are time dependent and are generally difficult to model by analytical methods. The finite difference numerical approach is therefore employed to approximate solutions at finite space and time difference, because of its complexity handling capabilities.

2.1. Development of numerical model

To develop heat transfer model, the composite wall (Figure 1a) is subdivided into spatial sub-layer sections of uniform finite difference and thermo-physical properties. A system of simultaneous algebraic equations was developed by transforming (2) and (3) into a set of simultaneous algebraic equations corresponding to each inner and boundary nodes, using unconditionally stable Crank-Nicolson (CN) finite difference method. Using Crank-Nicolson method, we develop finite difference interrelated equations to represent the discrete nodal network. As shown in Figure 1, the composite wall is divided into sub-layers with spatial width (Δx), each sub-layer represents a node of a wall. The general CN finite difference equations is expressed as follows:

$$T_i^{(t+\Delta t)} - T_i^{(t)} = \Psi [(T_{i-1}^{(t+\Delta t)} - 2T_i^{(t+\Delta t)} + T_{i+1}^{(t+\Delta t)}) + (T_{i-1}^{(t)} - 2T_i^{(t)} + T_{i+1}^{(t)})] \quad (4)$$

where $\Psi = k\Delta t / 2\rho C\Delta x^2$, and for the interior nodes (4) becomes:

$$(2 + 2\Psi_l)T_i^{(t+\Delta t)} = (2 - 2\Psi_l)T_i^{(t)} + \Psi_l(T_{i-1}^{(t)} + T_{i+1}^{(t)} + T_{i-1}^{(t+\Delta t)} + T_{i+1}^{(t+\Delta t)}) \quad (5)$$

where $\Psi_l = k_l\Delta t / 2\rho_l C_l\Delta x^2$ for layers $l=1,2,3,4..n$, the material layers of composite wall. These equations shows transient heat conduction in each material layer. For the nodes at the boundary surface of two intermediate material layers (node between $i = 2$ and $i = 3$ (Figure 1)), the heat transfer is affected by the material thermo-physical properties, the resulting equations based on CN method yields:

$$(2 + \Psi_{l1} + 2\Psi_{l2})T_i^{(t+\Delta t)} = (2 - \Psi_{l1} - 2\Psi_{l2})T_i^{(t)} + \Psi_{l1}T_{i-1}^{(t)} + \Psi_{l2}T_{i+1}^{(t)} + \Psi_{l1}T_{i-1}^{(t+\Delta t)} + \Psi_{l2}T_{i+1}^{(t+\Delta t)} \quad (6)$$

$\Psi_{l1} = \frac{\lambda_{l1}\Delta t}{(\rho_{l1}C_{l1} + \rho_{l2}C_{l2})\Delta x^2}$ and $\Psi_{l2} = \frac{\lambda_{l2}\Delta t}{(\rho_{l1}C_{l1} + \rho_{l2}C_{l2})\Delta x^2}$ for discretized layers between material layer 1 and 2, and boundary nodes at $x = 0$ and $x = L$ are represented as follows:

$$(2 + 2\Psi + 2H)T_1^{(t+\Delta t)} = (2 - 2\Psi - 2H)T_1^{(t)} + 2\Psi(T_2^{(t)} + T_2^{(t+\Delta t)}) + H(T_e^{(t)} + T_e^{(t+\Delta t)}) \quad (7)$$

$$(2 + 2\Psi + 2H)T_N^{(t+\Delta t)} = (2 - 2\Psi - 2H)T_N^{(t)} + 2\Psi(T_{N-1}^{(t)} + T_{N-1}^{(t+\Delta t)}) + H(T_e^{(t)} + T_e^{(t+\Delta t)}) \quad (8)$$

where $H = h_c \Delta t / \rho C \Delta x$, T_e = Sol-air temperature for outer boundary and indoor temperature for inner boundary surfaces. To solve these set of energy conservation equations, the system of equations are expressed by Thomas algorithm matrix [2] notation,

$$AT_{1..n}^{(t+\Delta t)} = BT_{1..n}^{(t)} \quad (9)$$

where A and B are matrices with future and present values with respect to time coefficients. T is a temperature vector, t and $(t + \Delta t)$ represents present and future temperature values. By using initial conditions, boundary conditions, and Thomas algorithm the future temperature values are determined.

3. Thermal network model

The RC thermal network methods help to obtain simplified/reduced state space models of the buildings thermal dynamics model. Building reduced models are often developed on the basis of linear networks with lumped parameters [3]. The principle idea is to have an analogy between two different domains, which can be described by the equivalent mathematical equations. Applying this analogy, building models with the help of lumped parameters are expressed as electrical circuits and state-space equations are deduced from these circuits.

Lorenz and Masy [8] described one of the early applications of a lumped capacity model using two resistors and one capacitor (2R1C) for buildings. A similar approach with a 3R2C network was applied for modeling thermal dynamics of a whole room including building walls, ceiling, and floor [9]. Fraisse et al. [4] conducted extensive study on performance of various thermal model configurations of increasing complexity for the same wall and concluded that the 3R2C network is the most suited model. Furthermore, Gouda et al. [5] presented a methodology to obtain higher accuracy models by using non-linear optimisation. This study concluded that 3R2C models are suitable enough for practical applications with proper parameter values.

The implementation of lumped parameter methods can be divided into two types: 1) models for building envelope and 2) models for building space. In this paper, building envelope model is considered as thermal network one. Building envelopes are generally modeled as 2R1C or 3R2C networks [3,4]. However, application of 3R2C networks can be seen more often in the literature because of its accuracy and computational efficiency. The 3R2C network model shown in Figure 1, where R_1 , R_2 , and R_3 are resistors representing thermal resistivity, and C_1 and C_2 are capacitors representing thermal capacitance of the wall, respectively. The number capacitors determine the differential equations degree. In this context, a 3R2C model will lead to the second-order differential equations (10) and (11).

$$\frac{dT_{c1}}{dt} = \frac{T_{out}}{R_1 C_1} - \frac{T_{c1}}{R_1 C_1} - \frac{T_{c1}}{R_2 C_1} + \frac{T_{c2}}{R_2 C_1} \quad (10)$$

$$\frac{dT_{c2}}{dt} = \frac{T_{c1}}{R_2 C_2} - \frac{T_{c2}}{R_2 C_2} + \frac{T_{c2}}{R_3 C_2} + \frac{T_{in}}{R_3 C_2} \quad (11)$$

The above differential equations can be re-written in a state-space representation as:

$$\begin{aligned} \dot{T} &= AT + BU \\ y &= CT + DU \end{aligned} \quad (12)$$

where T = temperature vector consists of wall nodal temperatures, U = input vector with temperatures and internal gains, Y = output vector (indoor temperature), A = square matrix with all values related to

state vectors, B = coefficients related to input matrix, C = coefficients related to state vectors, and D = direct transition matrix (D is zero matrix for cases, where the system model does not have a direct connection from input to output).

4. Parameter identification through PSO optimization technique

Parameters R and C identification through optimization will enhance the performance of the building 3R2C thermal network model-based controllers. The heat conduction through composite wall with multilayer materials, which have different thermo-physical properties is expected to behave as the reference model (building envelope real behavior). Hence, it is important to obtain proper distributed values for each resistor and capacitor of the 3R2C model matching the building envelope real dynamics. In this study, a constrained particle swarm optimization technique was developed to find the best model parameters values by optimizing the root mean squared error between the simplified and the reference models. The objective function for the constrained PSO technique is described as follows:

$$\min f(x_1, x_2, x_4) = \sqrt{\frac{\sum_{k=1}^n (Tfdm_k - T_{reduced_k})^2}{n}} \quad (13)$$

$$\text{subject to constraints: } \begin{aligned} x_1 + x_2 + x_3 &= 1; \quad x_1, x_2, x_3 > 0 \\ x_4 + x_5 &= 1; \quad x_4, x_5 > 0; \quad 1 - (x_1 + x_2) > 0; \quad 1 - x_4 > 0 \end{aligned}$$

R 's and C 's values of 3R2C model are obtained from below expressions

$$R_1 = x_1 R_{total}, \quad R_2 = x_2 R_{total}, \quad R_3 = x_3 R_{total}, \quad C_1 = x_4 C_{total}, \quad C_2 = x_5 C_{total}$$

$$R_{total} = \sum_{j=1}^M R_{layer,m} \quad \text{and} \quad C_{total} = \sum_{j=1}^M C_{layer,m}$$

where R_{total} and C_{total} = are the total resistance and capacitance values of composite wall, respectively, $f(x_1, x_2, x_4)$ = fitness function to be minimized, $Tfdm_k$ and $T_{reduced_k}$ = step response temperature at interval of (k).

4.1. Particle swarm optimization

In the current study, the PSO technique is used to identify the parametric values of the 3R2C model. PSO algorithm is a population-based stochastic optimization technique [7]. PSO-based optimization techniques are well adapted for discontinuous nonlinear systems with a convergence behavior and robustness as key features. Hence, among all evolutionary algorithms, it can be considered as better suited for parametric optimization processes.

The PSO method is initialized with a number of random swarm particles, which then searches for an optimal fitness by updating the generations. At each iteration, each particle is updated by its personal best ' $pbest$ ' and global best ' $gbest$ ' values. After finding $pbest$ and $gbest$ values, the position and the velocity of a particle may be modified. This modification is performed on the following equations basis.

$$\begin{aligned} V_{i,j}^{k+1} &= wV_{i,j}^k + c_1r_1(Pbest_{i,j}^k - X_{i,j}^k) + c_2r_2(Gbest_j^k - X_{i,j}^k) \\ X_{i,j}^{k+1} &= X_{i,j}^k + V_{i,j}^{k+1} \end{aligned} \quad (14)$$

where $Pbest_{i,j}^k$ = personal best j^{th} generation of i^{th} particle, $Gbest_j^k$ = global best of j^{th} generation, $V_{i,j}^{k+1}$ = updated velocity of the particle, $X_{i,j}^{k+1}$ = updated position of particle.

5. Simulation results and discussion

The main objective of the proposed methodology based on a constrained PSO optimization is demonstrated on 2 types of datasets, lighter and medium construction composite walls. The thermo-physical properties and composite wall configurations given in Table 1. Those values are taken from the ASHRAE Handbook of Fundamentals - 2017 [10]. To validate the optimized model, a reference model

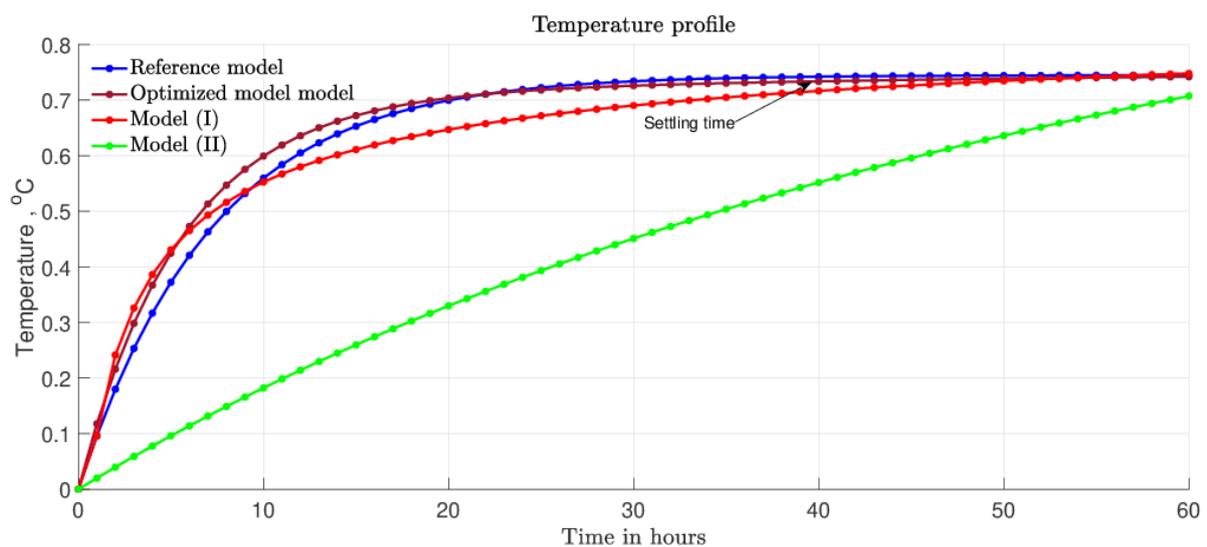
is developed (section 2.1) and two other 3R2C models are considered for comparison purposes. In the first model (model (I)), the R_1 , R_2 , and R_3 values are considered identical as the composite wall layer thermal resistances (i.e. the first (outside) material layer thermal resistance is assigned to the resistor representing outside conductive resistance). Similarly, capacitance C_1 and C_2 values are assigned exactly half of the total capacitance. In the second model (model (II)), the total resistance and capacitance values are distributed evenly for R_1 , R_2 , R_3 , C_1 , and C_2 parameters.

5.1. Case studies

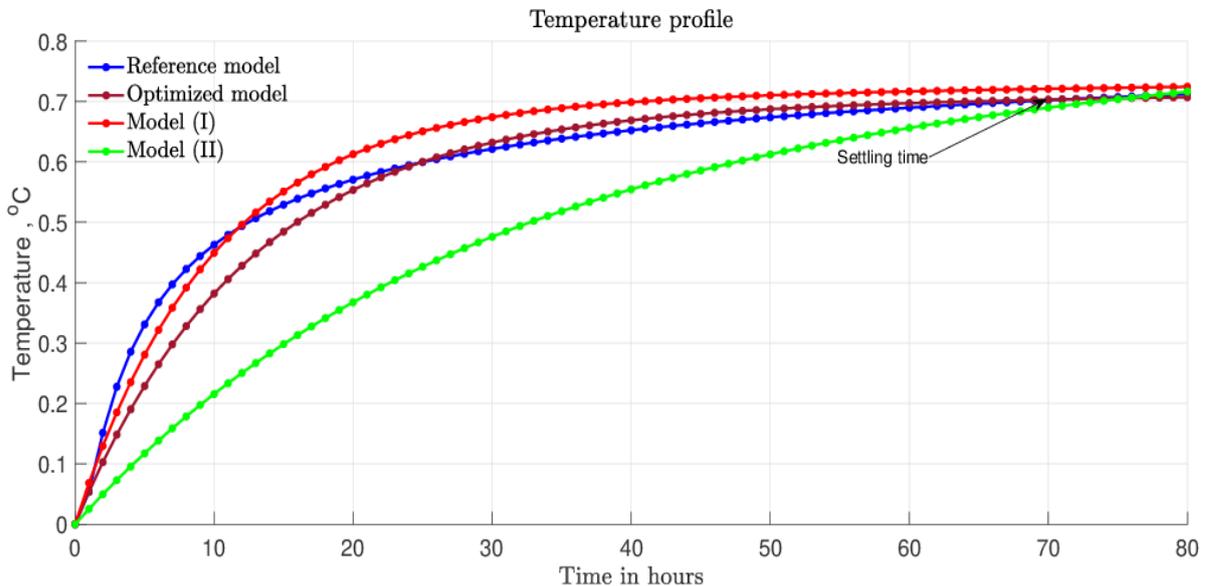
In order to simulate and validate the 3R2C model, the developed reference model spatial domain is divided into 40 layers to have more accurate transient conduction dynamics with initial conditions $T(x, t) = 0$ at $t = 0$, $\forall x \in [0, L]$, boundary conditions $T(x, t) = u(t)$ (unit step) = ($U \forall t > 0$), at $x = 0$, $T(x, t) = T_e(t)$ at $x = L$ and $\forall t > 0$. The developed 3R2C network is modeled for a single zone building. In this study, inter-zone heat transfer and radiative heat transfer between walls in single zone are not considered for modeling simplicity. Internal gains from the occupants and electrical appliances are also neglected. The 3R2C model is simulated with inputs of outdoor temperature and solar heat gains. Outputs are measured indoor temperature with step input in outdoor temperature. The same input and outputs are given to model (I) and (II).

Table 1. Classification and thermal properties of composite wall, and optimised R and C

Construction class	Thermal properties				Parametric values R ($m^2 \cdot K/W$) and C ($kJ/m^2 \cdot K$)						
	Thickness mm	Thermal conductivity $W/m \cdot K$	Density kg/m^3	Specific heat, $kJ/kg \cdot K$	R_{total}	C_{total}	R_1	R_2	R_3	C_1	C_2
Light-weight (LW)											
Stucco	25.00	0.692	1858	0.84							
Insulation (batt)	125.00	0.043	91	0.96	3.1498	76.852	0.0705	2.9468	0.1325	43.738	33.114
Plaster/gypsum	20.00	0.727	1602	0.84							
Medium-weight (MW)											
Brick	101.60	0.89	1920	0.79							
Insulation board	50.80	0.03	43	1.21	2.1450	183.66	0.2290	1.5725	0.3435	51.426	132.24
Air space	50.00	-	-	-							
Gypsum	20.00	0.727	1602	0.84							



(a) Light thermal mass composite wall.



(a) Medium thermal mass composite wall.

Figure 2. Temperature dynamics of reference, optimized and comparison models for light (a) and medium weight constructions (b).

Temperature behavior comparison of reference model, 3R2C optimized model, and models (I) and (II) for a step response is shown in Figure 2(a) and (b) for both light construction around (100 kg/m^3) and medium construction (around 500 kg/m^3). The temperature profile of the optimized model as shown in Figure 2, closely follows the temperature profile of the reference one with optimally identified parametric values (Table 1). However, for the lighter construction composite wall, the optimized model reaches steady-state little before the reference model as finite difference model with high number spatial domains accounts for more accurate heat conduction. Model (I) also closely followed the reference dynamics but with a longer time to reach steady-state. However, model (II) never reaches steady state. It should be considered as unstable. Except model (II), the steady-state is reached before 50 hours, which highlight that lighter thermal mass constructions have lesser time-lag of heat transfer from the outer surface to the inner one.

For the medium thermal mass construction, as shown in Figure 2b, the models (I) and (II), and the optimized one follow the reference dynamics. The model with optimized values reaches steady-state almost at the same time as the reference model quite accurately. However, model (I) leads to over-prediction and model (II) exhibits a longer time to reach steady-state. The medium thermal mass construction time lag is higher than that of a lighter one. The proposed method can further be used to determine suitable construction material for particular climatic conditions based on the time lag values with decrement factor values [11] (determination of decrement factor values is out of the scope of this study).

In term of implementation, the PSO model has been simulated using Matlab using the following computational facilities: Intel Core i5-7300U, CPU 2.60 GHz and 8 GB (RAM) under operating. In this context, the PSO algorithm took 8.92 seconds execution time with 1000 maximum iterations. 3R2C models were almost 3 times computationally efficient when compared to the reference numerical model. Model (I) with same R and C values as layers values has shown good accuracy in this study. However, in the case of medium thermal mass construction, the error values are high with temperature over-prediction. In this context, an optimized model is highly recommended as it yields much better accuracy with a higher computational efficiency.

6. Conclusion

Buildings reduced energy thermal models can represent realistic thermal behavior and predict the same thermal dynamics of a building system. They are significant tool for building thermal performance evaluation and control. Previous studies have shown that reduced 3R2C models lead to significant performance accuracy and are practically feasible. However, these models parameter values identification is a challenging task. In this paper, we have therefore developed an optimization approach to identify parameters of a building reduced model. A numerical approach-based reference model was developed for validation of the 3R2C model using parameters identified by a particle swarm optimization algorithm.

This optimized model was compared with the reference model and 3R2C models with random values. Its accuracy was evaluated for different thermal capacity materials. The achieved results have clearly shown the optimized thermal model accurate performance with significant reduction in computational cost. The proposed modeling identification and optimization approach can be further used to analyze buildings performance and to develop base models for control applications. Future investigations should be carried out on the applicability of the proposed approach on multi-zone buildings considering inter-zonal heat transfers.

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