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Deepening a conceptual framework in travelling between languages and cultures: the case of the documentational approach to didactics

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Abstract

Translation is a critical issue in international mathematics education studies but has seldom been carefully investigated. How can a theoretical approach be translated and subsequently used in different languages, and to what extent could the translation processes deepen the approach itself? This paper addresses such issues in the case of the Documentational Approach to Didactics (DAD). We explore possibilities of translating selected critical notions expressed in English terms into French, Arabic, and Chinese, and of developing insights from these. We further deal with the translation issues 'in situ', i.e., in an English mathematics teacher's lesson where we can identify the embodiments of the DAD concepts. The most challenging part in the translation is the embedding of the English terms in the English cultural sphere, and 're-bedding' them into another cultural sphere. It is claimed that examining the nuances between the different possibilities of translation for the same concept in connection with their use 'in situ', and drawing inspiration from the educational, cultural, and theoretical traditions in other cultural spheres, can help to deepen the theoretical constructs.

Keywords Conceptual equivalence; Cultural embedding; Documentational approach to didactics; Translation in 'situ'

1. Introduction

The Documentational Approach to Didactics (DAD) is a theoretical framework analysing mathematics teachers' professional development through the lens of their interactions with resources (Trouche, Gueudet & Pepin 2020). Even though studies using the frame of DAD have been conducted in a variety of educational cultures and languages, the theoretical model has been originally conceptualised in French (Gueudet & Trouche 2008), and further developed mainly in English (Gueudet & Trouche 2009). But how can a theoretical framework be adapted, when travelling between educational spheres and languages? These issues are discussed by Trouche (2019, p. 453), proposing 10 programs of research/development for enriching this approach, the first one being “designing a DAD living multi-language glossary”.

The present paper aims to deepen this reflection by focusing on the translation of this framework into three languages: Arabic, Chinese and French. The research question is: To what extent can the translation processes be enhanced by considering a concrete situation ('in situ'), and in turn add to the understanding of the DAD approach itself?

The following section addresses the theoretical and methodological issues at stake. Section three presents one English case; section four presents the issues arising from the translation of this case in French, Arabic, and Chinese; and section five discusses the limitations and scope of this study for DAD.

2. Theoretical and methodological issues

In this section, we propose: a general reflection on translation issues; some core concepts of DAD whose translations we want to study here; the DAD-Multilingual project as a theoretical attempt to achieve these translations; and a practical attempt that we coined 'translation in situ'.

2.1 The issue of translation and 'equivalence'

In the literature, the issue of translation has been taken seriously (Ruthven 2022), in particular for cross-national comparative achievement studies (e.g. TIMSS 2019; Pepin 2000). The literature claims that one of the core issues is that of 'equivalence' – 'how to study the same problem/issue in different societies and cultures' (Warwick & Osherson 1973). The main issue is to compare like with like, in order to develop a deeper understanding of the phenomena under study. This is particularly difficult, as Kawanaka et al. (1999) concluded that “Every single aspect of mathematics education, from a particular teacher's behaviour to a national policy, must be considered and evaluated within a socio-cultural context.” (p. 103)

If we regard the choice of research method as an accessory lens for a camera, it depends on the type of picture the researcher wants: broad panoramic; or intense concentration on detail (Warwick & Osherson 1973). In this sense, the comparative perspective can serve as a systematic setting in which to consider a theory formulated in terms of general factors, and the comparative focus becomes central when an attempt is made to consider social reality in a specific setting (Hantrais et al. 1985)- 'in situ' as we call it in this paper.

Warwick and Osherson (1973) have investigated the different facets of equivalence: (1) conceptual equivalence, referring to the question of whether the concepts under study have equivalent, or any, meaning in the cultures which are considered; (2) equivalence of measurement, involving the challenge of developing equivalent indicators for the concept under study; and (3) linguistic equivalence, referring to the problem of translation itself. Warwick and Osherson (1973) contend that the issue of linguistic equivalence is inseparable from the theory and concepts guiding the study, amongst others, and they suggest that the primary emphasis should be on conceptual equivalence ('comparability of ideas').

2.2 The documental approach to didactics (DAD)

DAD (Trouche, Gueudet & Pepin 2020) proposes a frame for analysing a teacher's activity when interacting with resources in a given teaching *situation* (e.g., preparing a lesson on a given mathematical topic for a particular grade). It emerged at a time when the Internet became a prominent feature for teachers' professional activity. This extended the number of resources available for achieving a given teaching goal, and the ways they could be adapted and appropriated by teachers.

The DAD frame is based on two essential dualities (Fig. 1):

- *instrumentation - instrumentalisation*: the *instrumentation* process means the way teachers' activity is oriented by the resources they use, due to their affordances and constraints; whilst the *instrumentalisation* process means the way teachers permanently adapt these resources to adjust them to their didactical goals;
- *resources - document*: a *resource* is defined as everything (e.g., a mathematical problem or task, a software, a textbook) that can *re-source* the teacher's activity (Adler 2000). When adopting a set of resources for facing a teaching situation, a teacher develops a *document*, that is a hybrid entity, composed of this set of resources, and a scheme of usage of them. A *scheme*, according to Vergnaud (2009), is the way somebody organises his/her activity for facing not only a situation (e.g., "teaching how to compute the area of a trapezium in a given grade 9 class"), but a *class of situations* as a generalised case of a situation (e.g., "teaching how to compute the area of surfaces in the Euclidean plane"). A scheme is structured around four components: 1-goals and subgoals; 2-rules of action; 3-operational invariants (implicit knowledge both guiding teacher activity, and developed through this activity); and 4-possibilities of inferences that make it possible to manage the singularities of certain situations that belong to the same class. A scheme can evolve in different ways, when new resources are used and appropriated, or new situations appear. Kayali and Biza (2021) coined the term *re-scheming* for such evolutions.

The process of development from a set of resources to a document is coined a documental genesis (Fig.1).

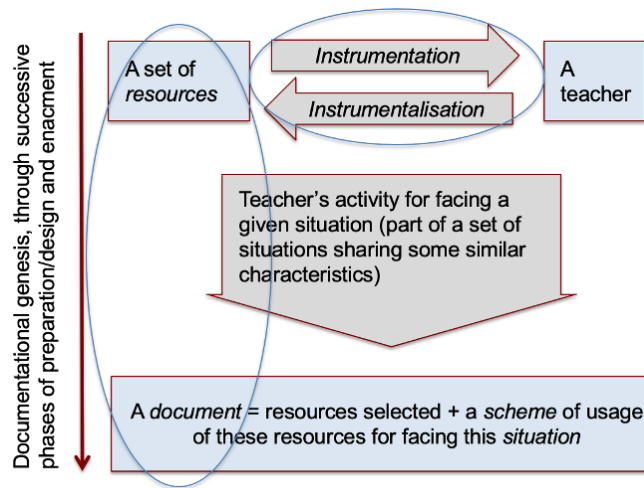


Fig. 1 The DAD model and its two dualities

In this paper, we focus on the translation issues related to these two dualities.

2.3 The DAD-Multilingual project as a theoretical attempt to address the translation issues

The DAD-Multilingual project (Trouche, 2020) aimed at gathering and confronting the translations of a DAD synthetic presentation (Trouche, Gueudet & Pepin 2020) in 14 languages, including Arabic, Chinese and French. It had two major goals:

- making available the presentation of DAD in various languages;
- deepening the DAD concepts by considering their possible expressions in different languages.

For the translation in each target language, the project involved (at least) one translator and one reviewer who were native speakers of the target language, and had participated in the development of the approach. Each pair of translator-reviewer produced a translation of the DAD presentation at stake and a “translating issues report”¹:

- In the French case, it demonstrates the relative ease of translating from English to French, possibly because the framework was originally developed in French. However, even in this case, it is not easy to keep an original metaphor, as for the translation of *resource system*, which is *ystème de ressources*. In the English expression, *resource* is written as a singular (a system resourcing teacher activity), whereas in French we had to use a plural, pointing to the diversity of this system;
- In the Chinese case, it balances between two options: creating a new word, aligned with an explanation in the targeted language; or borrowing a word already existing in the scientific literature;
- In the Arabic case, it retains the need for sometimes leaving the original lexical field, and finding translations reflecting the content of the given concept in Arabic.

¹ See the set of reports here:

<https://hal.archives-ouvertes.fr/DAD-MULTILINGUAL/page/translation-issues>

The French, Chinese and Arabic cases all reflect the translation issue of linguistic-conceptual equivalence. The extent to which the equivalence is realised varies from term to term.

The DAD-Multilingual project did not fully achieve its first goal: even if the website of the project² provides a preliminary presentation of the DAD framework in 14 different languages, some translated terms are still problematic and cannot be used or easily understood in the target languages/contexts. For making the presentations in 14 languages fully available, i.e. usable in the corresponding contexts, it appears necessary to consider the structural, cultural and social aspects of each language. This necessity led us to design another methodology, the ‘in situ’ translation, aiming also to achieve the second goal of this project - deepening the DAD concepts - which connects to the research question at the centre of this article.

2.4 Considering DAD concepts ‘in situ’

We address the translation issues ‘in situ’. That is, we determine the embodiment of selected DAD theoretical concepts in a particular case, in a given language, through its analysis *in this* language. Then, we investigate the translation of (1) the terms naming the DAD concepts, and (2) their use within the text of the case analysis. The adopted method has been:

- selecting and analysing a particular case and teaching situation;
- selecting, from the case analysis, occurrences of the DAD terms, embedded in their surrounding textual sentences (possibly leading to using other grammatical forms of a DAD concept, for instance the verb form, e.g. a given resource *instrumented* the teacher; the teacher *instrumentalises* a given resource);
- translating those analysis sentences into the targeted languages, identifying translation issues, in terms of equivalence;
- conducting a synthetic reflection/discussion cutting across the different translations, elaborating the constraints and affordances encountered in this process;
- reflecting on the constructs in the DAD framework, exploring the extent to which the translation processes can deepen the framework itself.

Thus, instead of addressing the translation issues only from a theoretical perspective, we address them from a practical perspective, by putting the theory’s concepts within the analysis of a case and presenting the analysis in a target language.

For implementing this translation in the study presented here, we made the following choices:

- We focused on the two core DAD dualities (2.2);
- We chose a case of an experienced English teacher, who expressed confidence and willingness to use computer-based resources and other classical resources in his teaching, offering rich data for the analysis in the DAD perspective;
- We chose three languages: French, Chinese and Arabic: French is chosen for its special interplay with English; Chinese and Arabic are chosen for being far from English, with rich cultural traditions related to mathematics teaching resources;

² <https://hal.archives-ouvertes.fr/DAD-MULTILINGUAL>

The second author, hereafter referred to as Lina, has proposed a case study which was a part of her Ph.D. thesis. The authors are respective native speakers of the target languages and have experience in using DAD. For the Arabic and Chinese languages, the authors have not been involved in the DAD-Multilingual project, which allows them to keep a critical distance from this experience of translation.

3. Presentation of the case and its analysis

In this section, we present the case; the situation at stake; and its analysis in English.

3.1 The presentation of the case

Lina investigated a secondary mathematics teacher's (George's) documentation work (Kayali & Biza 2021). The data collected were video-recorded lesson observations, followed by one audio-recorded interview providing information about the resources that the teacher used in general.

George held a Bachelor of Science in mathematics and a Postgraduate Certificate in Education. At the time of data collection, he had fifteen years of teaching experience in upper secondary mathematics education. He was the head of the mathematics department at his school. In his classroom, he had access to an interactive whiteboard, Autograph software (www.autograph-math.com), and iPads for students' use.

We focus here on three fifty-minute sessions of teaching Year 13 students (aged 17-18 years) the topic of 'volume of revolution'. Sessions 1 and 2, dedicated to lesson L1, aimed at introducing the topic to two groups of students (G1, thirteen students, and G2, nine students). Session 3, dedicated to lesson L2, was taught to G1. George also aimed to prepare students for the national examination scheduled at the end of Year 13 (before progressing into university).

3.2 Presentation of the situation of George teaching 'volume of revolution'

In Session 1, George started by asking G1 students about the area of a circle, confirming it is πr^2 . He used the interactive whiteboard to show the two-dimensional graph of $y = x(3 - x)$ in 3D mode on Autograph (Fig. 2) and rotated it to show it was in 3D. He added that he aimed to find the area under the curve and that he would use the trapezium rule (i.e. estimate the area under the curve by dividing it into a number of trapeziums) and Simpson's rule (i.e. estimate the area under the curve by dividing it into smaller parts where the slanted-top sides in trapezium rule are replaced with parabolas)³. He illustrated both rules on Autograph using the graph of $y = x(3 - x)$ (Fig. 2 and Fig. 3, respectively), compared the shaded area in both (Simpson's rule would give, in this case, the exact value) and rotated the shaded area in Fig. 3 around the x-axis (Fig. 4).

³ For detailed explanation of Trapezium and Simpson's rules, see Wiseman and Searle (2005).

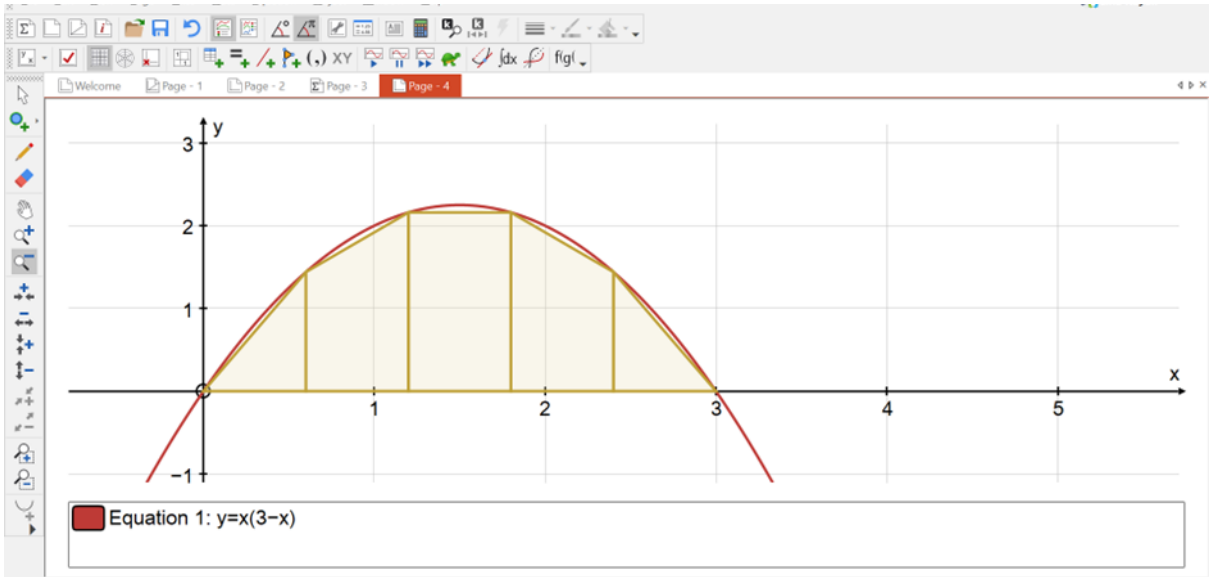


Fig. 2 Trapezium rule for $y = x(3 - x)$ (reproduction)

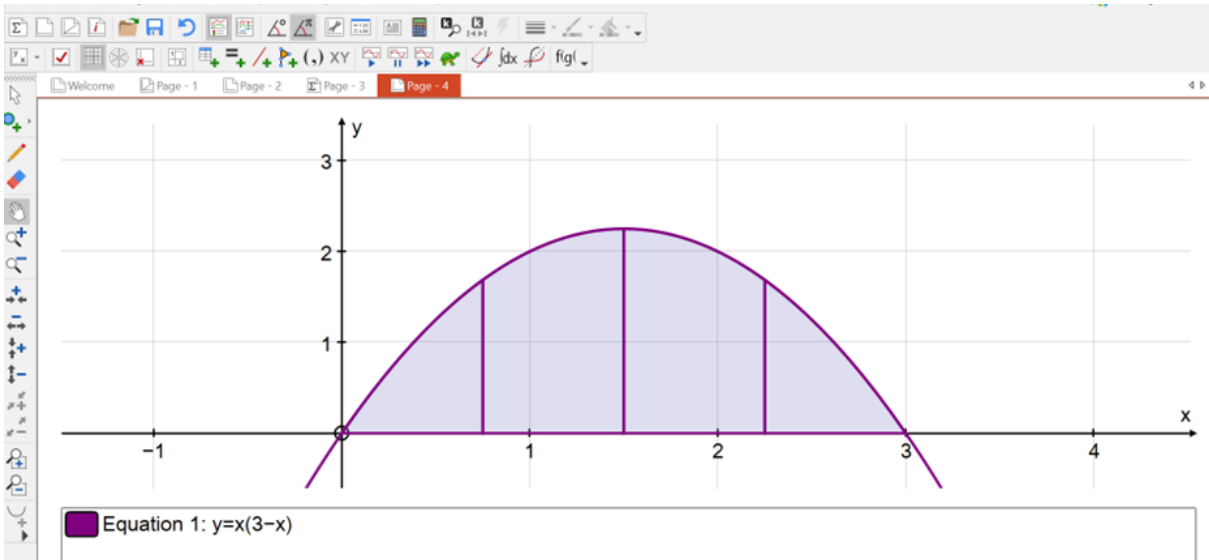


Fig. 3 Simpson's rule for $y = x(3 - x)$ (reproduction)

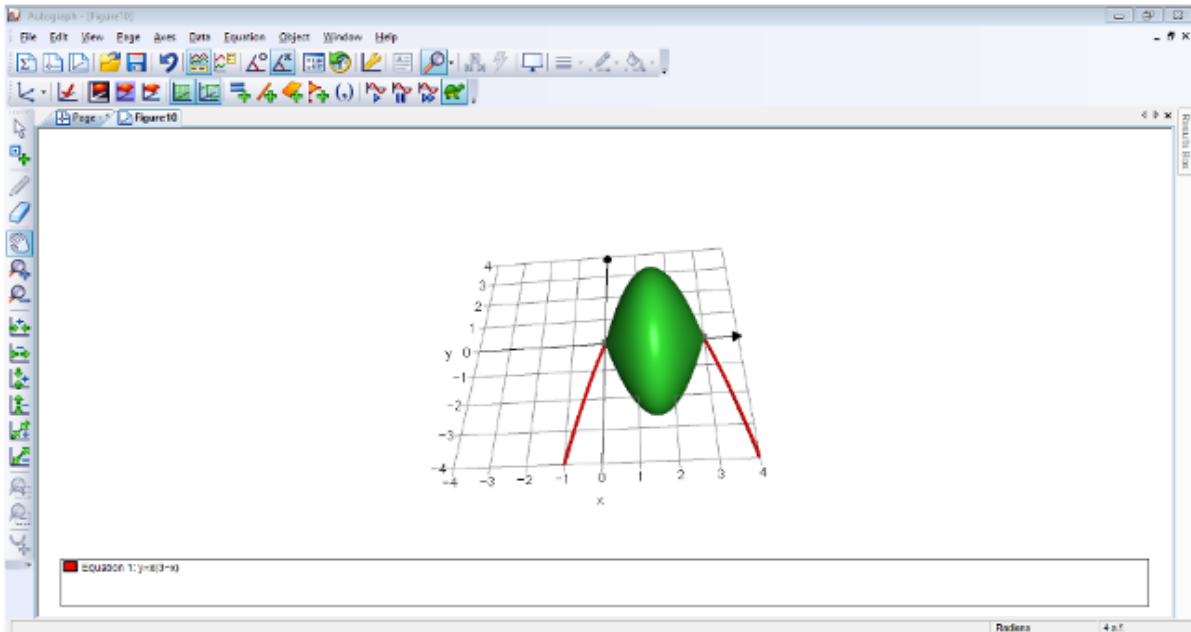


Fig. 4 Rotating the shaded area in Fig. 3 around the x-axis

Also on Autograph, George showed the rotation around the x-axis of the area between the pre-prepared graph of $y = \sqrt{x}$ and the x-axis over a bounded interval (Fig. 5). He stated that the idea was to work out the volume of the shapes resulting from the rotations and for that the area of a circle was needed. He tried to use Autograph to demonstrate how to work out the formula for the volume of revolution. He rotated the graph on Autograph with a small angle in order to show a “thin slice” of the shape resulting from the rotation (Fig. 6). It seemed that George was not satisfied with the demonstration he offered, as the software did not show what he was planning for. He tried to explain how the formula could be concluded, highlighting that the formula would include integrating πr^2 over a given interval.

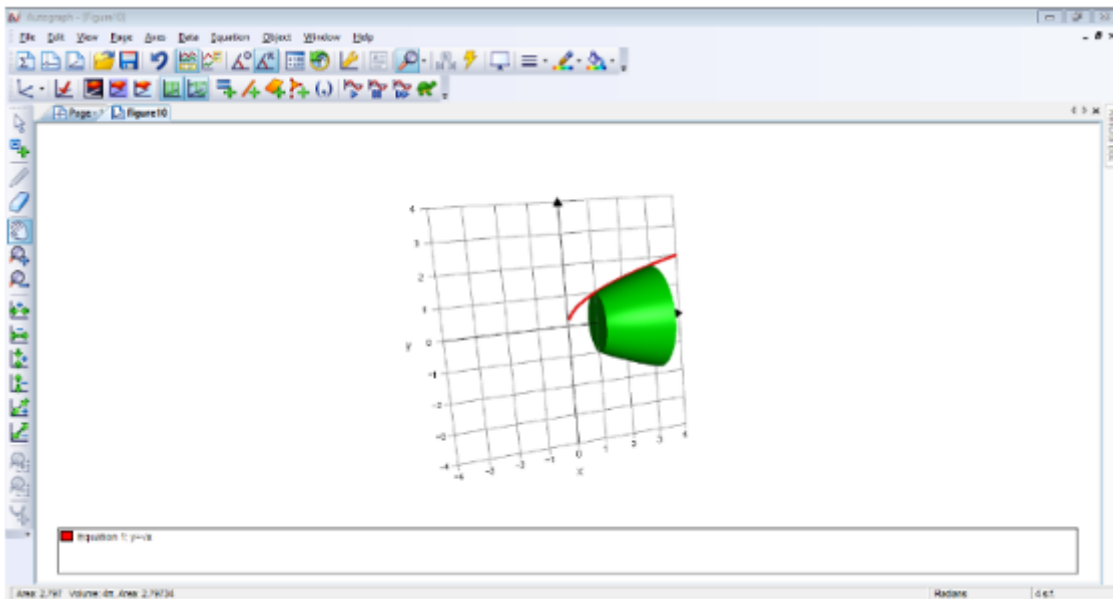


Fig. 5 The rotation of the area between $y = \sqrt{x}$ and the x-axis, around the x-axis (reproduction)



Fig. 6 George's attempt to show a thin slice of the shaded area on Autograph

Next, George invited his students to practise some textbook questions and identified the questions that exceeded the examination standard and thus could be skipped. At that point, he noticed a figure (Fig. 7) in the textbook (Wiseman & Searle 2005) illustrating the process of finding the formula for the volume of revolution, and used it to re-demonstrate the formula.

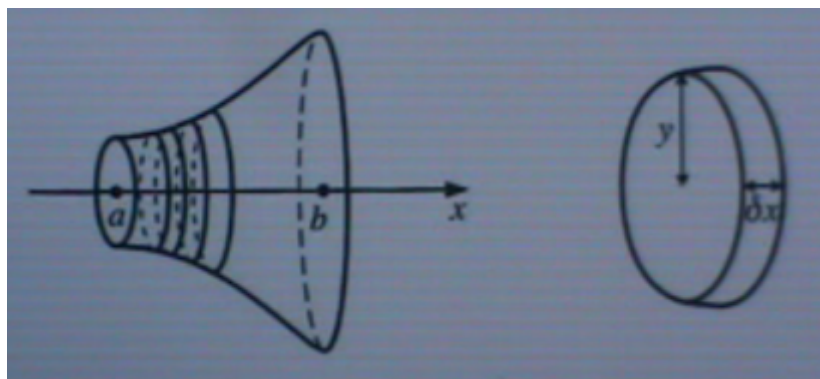


Fig. 7 Diagram illustrating the formula of volume of revolution (Wiseman and Searle 2005, p. 108)

In Session 2, dedicated to the same lesson with G2, George firstly displayed on the interactive whiteboard a pre-scanned copy of the textbook illustration (Fig. 7). To explain what the diagram displayed, he showed the graph of $y = x(3 - x)$ on Autograph in 3D, and continued working on Autograph like he did in the lesson before: using the trapezium rule; then using Simpson's rule. He explained that the volume of the shape resulting from the rotation of the shaded area around the x-axis was called the volume of revolution, and presented different rotations of the shaded area. Instead of using Autograph to explain how to calculate the volume of revolution, George used the textbook diagram he displayed previously. He explained that the volume could be viewed as "lots of slices" to be added up. He asked the students to consider one "slice" and think of its area which was the area of a circle (πr^2). Then, he used the equation he entered on Autograph to find the volume of revolution over a given interval.

In Session 3, which was devoted to doing more work on the volume of revolution with G1, George started by offering a quick review of the volume of revolution. He displayed on Autograph the same example as in the previous two lessons. He used the textbook image to explain how the formula was deduced (Fig. 7) and used a pre-prepared formula card to remind the students of the formula of volume of revolution. George also presented and explained past-examination questions.

From the three sessions, we notice that George's choices in terms of how to introduce the concept of the volume of revolution and its formula became clearer from one lesson to another, with Autograph becoming the resource to introduce the concept and the textbook becoming the resource to introduce the formula. In the next section, we present the analysis of George's sessions.

3.3. Analysis of George's documentation work

We focus on the following analysis on the two dualities: resources – document, and instrumentation – instrumentalisation.

3.3.1. Duality of resources-document

In the three sessions, George worked with a set of resources including: text curriculum resources, digital curriculum resources, social resources, and cognitive resources (Pepin & Guedet 2018). His curriculum resources were the national curriculum of year 13, the school's scheme of work for mathematics, the year 13 textbook, past-examination questions and mark schemes, his personal website, the school website, and formulae cards. The digital curriculum resources were the educational technology utilised within the lessons: calculators, Autograph and pre-prepared graphs of $y = x(3 - x)$ and $y = \sqrt{x}$, and the interactive whiteboard. The social resources were the students' contributions in answering or asking questions or commenting. George's cognitive resources were his knowledge of the Trapezium rule, Simpson's rule, circles, the formula for volume of revolution, and integration as well as his appreciation of the role of visualisation (using Autograph) in teaching mathematics. In his use of Autograph, he re-sourced the volume of revolution activity by using the task and graphs he previously created and used.

In Session 1, the resources George used were associated with a scheme he developed for achieving his two goals: to introduce the formula for the volume of revolution and to prepare students for the examination. The scheme is inferred from his actions within the session and his reflections during the interview. We summarise it in Table 1 and refer to it in this analysis. George's rules of action R1-R12 reflect his actions during Session 1. His operational invariants O1-O4 were derived from the knowledge he expressed or claimed during the session and the interview. His experience in Session 1 led him to two inferences (I1-I2), he expressed during the interview. George's inference that the textbook image is better (than Autograph) to explain the formula of the volume of revolution encouraged him to include the textbook image in Sessions 2 and 3 earlier than he did in Session 1. We notice in Session 2, that he tried to include the textbook image from the start, then moved to the use of Autograph and then returned to the textbook image. In Session 3, he decided to start with Autograph and then moved straight away to the textbook image. In the interview, George confirmed that this

use of Autograph and the textbook image is what he would like to rely on in future sessions on the volume of revolution. We see these adaptations from one session to another as ‘re-scheming’ (2.2).

Table 1 George’s scheme of using the resources in Session 1

<i>Scheme</i>			
<i>Goals</i>	<i>Rules of action</i>	<i>Operational invariants</i>	<i>Inferences</i>
G1. To prepare students for the exam G2. To introduce the volume of revolution	R1. Offer an introduction to the volume of revolution R2. Remind the students of the formula of the area of a circle R3. Use Autograph to show the students a graph in 3D R4. Show graphs of a function they previously learnt R5. Use trapezium rule and Simpson’s rule on Autograph to show the area between a graph and the x-axis, and compare the results R6. Rotate the shaded area around the x-axis R7. Show the students different rotations of the shaded area R8. Introduce the formula for volume of revolution using Autograph R9. Explain why integration is used to find the volume of revolution R10. Use a textbook image to explain the formula of volume of revolution R11. Refer to questions from the textbook that are relevant to the exam questions, starting with an easy question R12. Use exam questions to familiarise students with questions on volume of revolution	O1. Autograph offers students visual representation O2. Using a familiar graph helps reinforcing students’ previous knowledge O3. Using graphs drawn before helps in focusing on the new topic during the lesson O4. The use of exam-style questions is in response to students’ needs	I1. The textbook image is better (than Autograph) to explain the formulae of the volume of revolution I2. It is useful to use Autograph and textbook for the introduction of the volume of revolution and its formula

3.2.2. Duality of instrumentation-instrumentalisation

The duality instrumentation-instrumentalisation is reflected in the interplay between George and his resources. Autograph instrumented George with its affordances for 3D visual display, for creating 2D or 3D. George instrumentalised Autograph by creating 2D graphs in its 3D interface and generating the solids of revolution, because he thought it was helpful for students to visualise these. He also instrumentalised functions familiar to the students by using them in Autograph because he wanted to link the volume of revolution with students’ previous knowledge on integration and functions. George attempted to instrumentalise Autograph by creating a thin slice of a 3D shape on it, to explain how to derive the formula of volume of revolution. However, it was difficult for him to do so; instrumented by that he abandoned the idea of using Autograph to introduce the formula of volume of revolution, and

used it mainly to provide 3D visual display for the solids of revolution and the rotation process. As well, the textbook instrumented George by stating that the students must be able to use definite integration to find the area under a curve. So, although the area under a curve is not used to compute the volume of revolution, George tried to link it to the volume of revolution. Later, George was instrumented by the textbook which provided him with an illustrative image (Fig. 7) and a source of questions. He instrumentalised the textbook by deciding to use its illustrative image, and selecting some of its questions for student practice and contrasting them with the examination standards.

4. Reflection on the translation of DAD dualities in three languages.

In this section, we undertake the ‘in situ’ translation and discuss the emerging translation issues. As mentioned previously, we are not undertaking full translations of the case analysis in 3.3. We only translate, from English to the three chosen languages, the terms of the selected DAD concepts (the two dualities) and identify the main key issues that emerge from their use within the text of the case analysis in the target language. We identify five types of translation issues finally, respectively named ‘conceptual-cultural equivalence’, ‘conceptual-theoretical equivalence’, ‘embodied equivalence’, ‘artificial linguistic-conceptual equivalence’, and ‘false linguistic-conceptual equivalence’. We will evidence them in 4.1-4.3 and summarise them with a table in 5.1.

4.1 Reflection on the translation in French

4.1.1 Duality resources-document

George developed a document, encompassing a set of resources and a scheme of usage of these resources. Translating the analysis (3.3.1) into French, we noted issues linked with the resources used, and with the concept of scheme.

When we translate the names of George’s resources (3.3.1) into French, we note an issue about *curriculum resources*. The direct translation would be *ressources curriculaires*, but this expression, whilst correct, is not used in French; *ressources pédagogiques* would probably better convey the intended meaning. The English expression *curriculum* would be translated into French as *programme*, meaning an official list of contents and competencies to be taught. But *programme* and *curriculum* are not equivalent, and an expression like *ressources de programme* does not exist in French.

A French teacher for the same lesson would probably use an additional resource: his *progression annuelle*, a sequencing of the topics over the whole year. Writing the *progression annuelle* is a professional skill that should be mastered by teachers (Ministry of Education 2013). At the same time, some of the resources used by George are not available in France. Institutions like “mathematics departments” gathering all the mathematics teachers in a school likely to produce a *scheme of work for mathematics* do not exist (at an official level). While past examination questions (coming from the Baccaalaureate - examination at the end of secondary school) could certainly be used by a French teacher, the associated *marking*

schemes (*barème* in French) are confidential and thus would not be present. This is linked with the differences in teachers' resources (and thus in their documents) that could be identified from teachers' activities in different educational contexts.

Concerning the document developed by George, an interesting translation issue concerns the word *scheme*. The corresponding French term is *schème*, which is a direct and transparent translation. Nevertheless, *schème* is a specialised term in French, belonging to philosophy (coming from the work of Kant) or psychology. In psychology it was defined as a "general form having not only an internal organisation but an organising and structuring action" (Mucchielli & Mucchielli 1969). In English the term *scheme* can also be used outside of a scientific context, indicating a frame or a plan (as in *marking scheme*, one of the resources used by George). The expression *re-scheming* (proposed in 3.3.1) also raises a translation issue: would it be in French *adaptation du schème* (adaptation to the particular situation) or *évolution du schème* (new stable organisation)? This issue is theoretical and methodological: the scheme is a stable organisation that can be adapted to different situations. Distinguishing between a real evolution towards a new stable organisation and a local adaptation is complex.

4.1.2 Duality instrumentation-instrumentalisation

Concerning the duality of instrumentation-instrumentalisation, the interpretations by the French members in our team of authors would be similar to those in section 3.2.2. This is probably because of the uncommon character of these words both in French and in English: their use has been developed mostly within the international mathematics education research community.

We note nevertheless some processes in George's case that seem not likely to appear in France, which may be linked to the educational differences between the two countries. In Gueudet and Lebaud (2016), we analysed the answers of 126 teachers to a survey proposed to upper secondary teachers about their choice and use of the class textbook. Teachers particularly used the textbook on paper, mostly to give homework (84%). Only 57% of the teachers surveyed used the digital version of the textbook in class, and they projected exercises to be solved by the students (92%). Thus, the document developed for introducing a new formula seems unlikely to include the digital textbook in the French context.

Concerning the instrumentalisation of the software, a national survey organised in 2019 about the uses of dynamic geometry software by French mathematics teachers (Soury-Lavergne, 2020) evidenced that 65% of the 1147 respondents prepared their dynamic geometry files out of class during their lesson preparation. The on-the-spot instrumentalisation of dynamic figures as observed in George's case is not a common practice in France.

Drawing on this brief analysis, we note several points:

- The problems of translation from English to French are less significant than in the case of Chinese or Arabic. Nevertheless, we note the translation issue of 'false linguistic-conceptual equivalence': *scheme* and *curriculum* can be directly translated as *schème* and *curriculum* in French at first sight, but at a closer look, the two French terms are 'false friends' of the two English terms. The term *curriculum* even has no actual equivalent in French.

- The introduction in the English case of a new concept, ‘re-scheming’ and the need to translate it, raises an issue of measurement equivalence: finding indicators of a re-scheming process and finding the relevant French translation are strongly associated;
- The educational and cultural context influences not only the available resources but also the *instrumentalisation-instrumentation* processes that teachers engage with. The usage schemes and their components are not only individual constructs; they are also influenced by the cultural context. This calls for a deepening of DAD, by introducing, for example, categories of operational invariants shared in a given cultural context or by a given group of teachers.

4.2 Reflection on the translation in Arabic

4.2.1 Duality resources-documents

The Arabic term for *resources* is مَوَارِد (mawaarid). This term means the set of available potential sources of information or ideas. From a cultural point of view, we note a historic connection between مَوَارِد and the vital need for water in the desert. The singular form of the term is مَوْرِد (mawrid), originally meaning both a *water spring* and a *place to come to*. In the Arabic language, we find a frequent use of a metaphor that strongly associates knowledge with the act of drinking water. One example, among many, is the expression: “ينهلون من منابع المعرفة”, which literally translates into: “they drink from the springs of knowledge”. We see this as an instance of ‘cultural-conceptual equivalence’. Seeking knowledge from its resources is seen as being of the same value as the vital act of drinking water from its springs, in a culture that originated in the desert.

In the analysis of the case, it was found that “George worked with a set of resources including: text curriculum resources, digital curriculum resources, social resources, and cognitive resources”.

We will focus on the fourth type, cognitive resources, which includes, for example, George’s knowledge of the subject being taught, beliefs about efficient techniques and tools for teaching it.

George’s cognitive resources included his knowledge of the Trapezium rule, ...

We translate the above sentence as follows:

شملت مَوَارِد جورج الإدراكية معرفته لقاعدة شبه المنحرف، ...

There are two commonly used Arabic terms for *cognitive*: إدراكية (idraakiiah) and مَعْرِفِيَّة (ma’arifiiah). Both terms are found in the Arabic literature related to cognitive sciences. Their meanings are close, but we adopt الإدراكية not المَعْرِفِيَّة, because الإدراكية is derived from الإدراك, underlying the *dynamic* process of realising or cognizing, while المَعْرِفِيَّة is derived from المَعْرِفَة, meaning *knowledge*, in a *static* sense; which makes الإدراكية more *conceptually equivalent* to the dynamic meaning of *cognitive* as used in English.

As to the terms *document* and *scheme*, they appeared in the analysis of the case as follows:

The resources George used were associated with a scheme he developed to create a document...

In the Arabic literature, there are two commonly used Arabic terms for *scheme*: *صيغة* (sighah) and *مُخَطَّط* (moukhattat). Both can be found in the body of publications in Arabic translations of Piaget's work involving the concept of scheme. Here we use *صيغة* to translate the sentence above as:

ارتبطت الموارد التي استخدمها جورج بـ*صيغة* استخدام طَوَّرَها لإنشاء مُسْتَنَد... .

The reason for adopting *صيغة* rather than *مُخَطَّط* is that *مُخَطَّط* in Arabic commonly means a *plan* or a *roadmap* in a linear sense, while *صيغة*, the adopted term, reflects an intertwining and flexible structure, open to redesign, additions and modifications, which is more *conceptually equivalent* to the meaning of *scheme* in DAD or in cognitive theories.

4.2.2 Duality Instrumentation – Instrumentalisation

The two terms in this duality are the trickiest to translate, as there are no terms in Arabic, derived from the same root *instrument*, that render the same meaning.

The analysis of the case gives the example of the instrumentation by the textbook that inspired George's actions. This process can be expressed in Arabic as *إمداد* (imdaad) meaning *supply*.

The textbook instrumented George, as it stated that the students...

The sentence would be translated as:

أَمَدَّ الكتاب جورج، إذ ذكر أن الطلاب...

Examples of instrumentalisation are given regarding George's use of Autograph while trying to adapt it to his needs. This process is expressed in Arabic as *تَسْخِير* (taskhiir), which means using and adapting according to one's needs and practices.

George instrumentalised Autograph to create graphs in 3D environment [...]

سَخَّرَ جورج اوتوغراف لبناء رسوم بيانية ثلاثية الأبعاد [...]

A most significant example which reflects the difference between instrumentation *إمداد* and instrumentalisation *تَسْخِير* is about the figure that George spotted in the textbook by coincidence

George did not a priori look for the figure. The book instrumented his work with the figure and the idea of using it. Then he instrumentalised that figure to show an illustration of the formula.

In Arabic:

لم يبحث جورج في البداية عن الصورة التوضيحية. إنما أَمَدَّ الكتاب عَمَلَه بالصورة وبفكرة استخدامها. ثم سَخَّرَ تلك الصورة لتقديم إيضاح تمثيلي للقاعدة.

The translation process raised many critical issues deeply rooted in the subtleties of the targeted language, in its structure and morphology, and its relation to the culture. Below are some of these issues:

- With the lens of Warwick and Osherson (1973), we can see that the translations of different DAD concepts in Arabic rely on different facets of equivalences: while a heavy ‘conceptual-cultural equivalence’ characterises the translation of *resource*, the simple ‘linguistic-conceptual equivalence’ characterises the translations of other concepts, such as *scheme*.
- As regards the translation of *instrumentation-instrumentalisation*, there are no equivalents in Arabic. Actually, when we searched for the occurrences of the two, or related, terms in the DAD-based literature, we found that the *instrumentation-instrumentalisation* are rarely, almost never, used except as nouns to label the process. They are only found, either as headings for paragraphs or within one standard form of sentence: “this is an instrumentation process”. So, we decided to include, in the analysis of the case, sentences that use other forms of the terms, for instance the verb form, to express their embodiments, such as: “Autograph *instrumented* George with its affordances...” and “George *instrumentalised* Autograph by creating...”, and we translated the whole embodiments, seeking for the Arabic terms equivalent to *instrumented* and *instrumentalised*. In this way, we achieved an ‘embodied equivalence’. The translation process created the need for a reflection and revision of the ways the DAD concepts are used as analysis tools in their original language of development. It made us aware of the importance of integrating different forms of the terms expressing the concepts into analysis sentences, i.e., in ‘situ’.
- While focusing the translation on some concepts in a theory, one cannot avoid reflecting on other related terms that are not included as core concepts in the theory. For example, the term *sources* is related to *resources*. While *resources* is a core concept in DAD, *sources* is not considered as such. The two terms are sometimes, however, wrongly used interchangeably, partly because of their morphological closeness. In Arabic, however, the two concepts are expressed in two totally different terms: مصادر (*sources*) and موارد (*resources*), which reduces the chances of using them interchangeably and highlights their different meanings within the theory. The example of the textbook shows a dynamic relation by which one *resource* (the textbook), which can re-source some situation (e.g., to prepare students for the exam) and a *source* for examination questions. The translation process and the differences between the corresponding two terms in Arabic motivated the reflection on the dialectic connections of the two concepts: can we think of a duality *Resources–Sources*?

4.3 Reflection on the translation in Chinese

4.3.1 Duality resources-document

In the analysis in section 3.3, the term *resource(s)* is used together with examples such as the national curriculum, Autograph, and students' contributions, to refer to anything that can re-source the teacher's activity or support his engagement in teaching. The Chinese term for *resource(s)* is 资源 (zī yuán), which means the sources of materials that support human livelihood and production. In Chinese education literature, national curriculum, educational software, and students' contributions will also be identified as 资源 for teachers (Jia et al. 2016). Hence there are no specific equivalence issues regarding the translation of *resource(s)*.

While *resource(s)* is well equivalent to the 资源, we observe non-equivalence between particular examples of resources and their Chinese names, like *textbook* and 课本 (kè běn). 课本 in Chinese means a book published according to the curriculum prescriptions and used by both teachers and students for teaching or learning a subject. Although *textbook* is often translated as 课本, their usages in English and Chinese secondary schools are quite different. The 课 and 本 in 课本 respectively mean 'lesson' and 'book', suggesting it is a book setting foundation for lessons. Chinese secondary teachers tend to comply with textbooks qualified by the national ministry of education and they are always familiar with the textbook content before giving lessons (Li et al. 2009). That is not the case for George, who did not strictly comply with the textbook and noticed the "illustrative image" in the textbook only near the end of the first session. We thus notice non-equivalence between *textbook* and 课本 with respect to their usages and status in the corresponding educational contexts. To make up for the non-equivalence, it is necessary to add a sentence explaining how textbooks are generally used in the English educational context while translating *textbook* to 课本.

4.3.2 Duality instrumentation-instrumentalisation

Concerning *instrumentation*, the analysis sentences in section 3.3 mainly integrate the verb form of this term. For example,

It was difficult for George to do so (create thin slices of a 3D shape on Autograph); instrumented by that he abandoned the idea of using Autograph to introduce the formula of volume of revolution, and used it mainly to provide 3D visual display for the solids of revolution...

In Chinese, it is difficult to find a word that precisely conveys the idea of *instrumenting* or being *instrumented*. The Chinese report (Wang & Xu 2020, see 2.3) directly translated *instrumentation* as 工具性 (gōng jù xìng), a word meaning that some object is functional in promoting a process, plan, or system. Sticking to the original meaning and syntax rule of 工具性, we translate the above analysis sentence as

Autograph 能够让乔治展示旋转体的三维视觉效果, 但却很难让他构造一个立体图形的薄切片, 因而具有工具性;

which will be understood by Chinese readers as

Autograph is functional because it allows George to provide a 3D visual display for the solids of revolution, but causes difficulties for him to create thin slices of a 3D shape.

Here, we identify non-equivalence between 工具性 and *instrumentation* in DAD. They are not linguistically equivalent in the sense that 工具性 refers to a property of an object whilst *instrumentation* refers to a process. They are not conceptually equivalent either because 工具性 focuses on functionalities or affordances of an object whereas *instrumentation* involves both affordances and constraints of the object. To address the non-equivalence in the conceptual aspect, the Chinese report later attached the original definition of *instrumentation* to 工具性, letting 工具性 specifically refer to the process where an object influences a teacher's practice with both its affordances and constraints. In this paper, we suggest adding 过程 (guò chéng, meaning 'process') after 工具性, so that the new term – 工具性过程 (gōng jù xìng guò chéng) – means a process by nature and becomes more linguistically equivalent to *instrumentation*. With the new term, the translation of the analysis sentence becomes

乔治很难在Autograph中构造一个立体图形的薄切片，于是他放弃了用Autograph引入旋转体的体积公式，而只用它展示旋转体的三维视觉效果；这体现了Autograph对教师的工具性过程。

For Chinese readers, that means

It was difficult for George to create a thin slice of a 3D shape on Autograph, so he abandoned the idea of using Autograph to introduce the formulae of volume of revolution, and used it mainly to provide 3D visual display for the solids of revolution; this reflects Autograph's instrumentation on the teacher.

Concerning the term *instrumentalisation*, the analysis sentences in 3.3 mainly integrate its verb form, *instrumentalise*. For example,

George instrumentalised Autograph by creating 2D graphs in its 3D interface and generating the solids of revolution; because he thought it was helpful for students to visualise these.

In Chinese, the idea of *instrumentalisation* or *instrumentalising* can be expressed by a word 工具化 (gōng jù huà), which appears in both daily life and philosophical literature in China. In daily Chinese, 工具化 refers to the process of turning one object/person into an instrument for achieving certain (usually economic) purpose. That is almost equivalent to *instrumentalisation* both conceptually and linguistically, as the *instrumentalisation* also refers to a process, in which it is the teacher who adapts some object turning it into an instrument for achieving a certain didactical goal. Based on the meaning in daily Chinese, 工具化 also involves 'impoverishment and enrichment' when used as a concept in the philosophical theory of 'instrumental rationality' in China (Kong et al., 2018). So, when Chinese readers see the translation of the above sentence with 工具化 –

通过在Autograph的3D视图中构造二维图形并生成相应的旋转体，帮助学生直观地认识它们，乔治实现了他对Autograph的工具化：

they may reflect on the impoverishment and enrichment in George’s instrumentalisation of Autograph and understand the sentence in the following way:

George realised his instrumentalisation (enrichment and meanwhile impoverishment) of Autograph by creating 2D graphs in its 3D interface and by generating the solids of revolution to help students visualise them.

That is, during the *instrumentalisation* of Autograph, George could ‘enrich’ Autograph by exploiting its advantages for creating 2D graphs in the 3D interface and generating solids of revolution, and meanwhile ‘impoverish’ Autograph by weakening its other attributes. The idea of impoverishment and enrichment in the philosophical concept 工具化 is not included in the original definition of *instrumentalisation* in DAD, but when the Chinese term is used, it may influence Chinese readers’ interpretation of the concept. Here, the non-equivalence between 工具化 and *instrumentalisation* offers new opportunities for rethinking the terrain of DAD and the concept therein: can we deepen *instrumentalisation* by integrating the idea of impoverishment and enrichment into its definition?

To summarise, we draw the following points:

- Similar to the Arabic case, the translations of different DAD concepts in Chinese also rely on different facets of equivalences. For the term *instrumentalisation*, we have an issue of ‘conceptual-theoretical equivalence’, as the meaning of its Chinese translation is also influenced by an existing theory in China and not equivalent to the theoretical meaning of *instrumentalisation* in DAD. The term *textbook* has no actual equivalent in Chinese, so we translated the embodiment of *textbook* in the English educational context into Chinese, to achieve an ‘embodied equivalence’.
- The term *instrumentation* also has no equivalent in Chinese, but instead of ‘translating the embodiment of a concept/term’, we adopted another approach. That is, we artificially created a new Chinese term and artificially added an explanation aligned with the original definition of the DAD concept, to achieve a linguistic-conceptual equivalence between *instrumentation* and the created Chinese term. We label the translation issue as ‘artificial conceptual-linguistic equivalence’.
- The translating process raises opportunities for deepening the DAD framework itself. For example, a multilingual living glossary that connects resources with their names in different languages can potentially be a window for examining the different usages of the resources specific to the corresponding educational contexts. DAD concepts can be deepened by borrowing the ideas from an existing theory in a target language, with the framework itself becoming better available for local teaching/research activities.

5. Discussion of results in light of the translation processes

In this section we discuss the constraints and affordances encountered in translating a theoretical approach ‘in situ’, and ask what we can learn from these processes in terms of deepening the DAD approach itself.

5.1 Constraints and affordances

As explained earlier, translation issues in cross-national comparative studies are closely connected to the concern for ‘equivalence’. For our study of translating a theoretical approach (DAD), we concentrate on linguistic and conceptual equivalence. It became clear throughout the translations, but in particular with the Arabic and Chinese translations, that:

- 1) a word-for-word linguistic translation did not help for the understanding of the DAD concepts; hence, the translators needed a deeper understanding of the theory of DAD in relation to their linguistic context;
- 2) conceptual equivalence was preferred, and necessary, where the translation of concepts was established and the translation occurred at the concept level. It also meant that the *meaning* always took priority over the *form* in translation (Wallin & Ahlström, 2006). In that sense, it was helpful, and necessary to translate terms ‘in situ’;
- 3) by focusing on particular phenomena and using the theoretical concepts to interpret/analyse them, the translations were related to the full analysis sentences in which the terms/concepts were embedded (which imposed new constraints on the analysis itself).

At the same time, the emerging set of translation issues in 4.1-4.3 led us to assume that these issues should be re-examined from an analytical perspective, in order to set the scene for developing a tentative typology: we propose here (Table 2), in the global frame of linguistic-conceptual equivalence, five types of translation issues that emerged from our discussion in 4.1-4.3 and hypothesise that they could emerge over the translation of a general theoretical framework.

Table 2 Five types of translation issues

<i>Type of translation issues</i>	<i>Where the issue emerges</i>
Conceptual-cultural equivalence: certain terms could be better translated using cultural metaphorical equivalents in the target language.	Arabic translation of <i>resources</i>
Conceptual-theoretical equivalence: the existing translations of certain terms were influenced by and based on existing theories in the target language, but were not actually equivalent to the original theoretical meaning of the translated terms.	Chinese translation of <i>instrumentalisation</i>

Embodied equivalence: certain terms did not have equivalents in the target language, but could be translated through the embodiment of the terms (e.g., situating these in George’s classroom).	Arabic translation of <i>instrumentation-instrument alisation</i> ; Chinese translation of <i>textbook</i>
Artificial linguistic-conceptual equivalence: certain terms did not have equivalents in the target language, but could be translated by creating new terms in the target language and artificially adding an explanation aligned with the theoretical definition of the term.	Chinese translation of <i>instrumentation</i>
False linguistic-conceptual equivalence: for certain terms, a direct translation appeared to be possible at first sight, but at a closer look was identified as a ‘false friend’, as it did not have the same meaning as the translated term.	French translation of <i>scheme</i> and <i>curriculum</i>

As can be seen, the names of the translation issues sometimes also suggest the approaches for addressing the issues. It is the translation ‘in situ’ that allowed us to identify these types of translation issues and explore the corresponding approaches. The typology is not thorough of course; it requires further study and research.

The translation issues emerging in 4.1-4.3 made us reflect on the factors imposing constraints on the translation practice. It became clear that the translation of a mathematics education theoretical framework cannot stay on pre-defined simple conceptual and lexical equivalences, but involves the ‘interpretation’ with reference to:

- the educational context involved ‘in situ’;
- the educational, cultural and theoretical traditions in the cultural spheres where a language is used;
- the language use and nuances in the language.

First, the educational context imposed a constraint as it determined what embodiments of the theoretical concepts could exist and be interpreted in that context; some embodiments in one educational context had no linguistic equivalences, or even did not exist, in the educational context corresponding to the target language. Regarding the embodiments of *instrumentalisation*, French mathematics teachers would avoid the on-the-spot instrumentalisation process that George engaged with, and refer back to their lesson preparation notes. With respect to *resources*, teachers in France appeared to prepare their individual *annual progression*, while the schools would not provide a *scheme of work* as the English school would. A 课本 in Chinese educational context means something very different than a *textbook* in English educational context.

Second, the translator needed to ‘interpret’ the terms of concepts with reference to educational cultural traditions and theories that are/were perceived to be prevalent in the corresponding cultural spheres (Pepin 2005). Halai (2007) also describes this process as

“cultural decoding” of the processes, which can never be a neutral practice. To attempt cultural decoding requires the translator to have a deep understanding of both languages and both cultural spheres. The cultural embedding of the text to be translated, and its ‘re-bedding’ into the ‘new’ language, seemed to propose most of the constraints (e.g., Mesiti et al. 2021): a) The researchers from different cultural spheres would produce different interpretations in terms of one concept with the same data (e.g., the French researchers in our team of authors would not identify “prepare students for the exam” as a teacher’s *goal* in George’s case); b) For some terms, like *instrumentation*, when translating them into Chinese, an entire sentence or expression was required to explain the concept; c) Even when words had similar equivalent terms in different languages, they might not always carry the same connotations (*curriculum* is differently interpreted in English and French (‘false friends’) due to the different traditions of ‘curriculum theory’ in the Anglo-Saxon/American sphere versus ‘didactical theories’ in the European sphere; d) Drawing from the educational, cultural or theoretical traditions of other spheres was perceived to help enrich the original definitions of the theoretical concepts, and also to lead to an accentuation of educational and cultural specificities of different countries. For example, the connection between the Arabic term for *resources* and the source of water has ‘refreshed’ our view on the relationships between *source*, *resource*, and *re-source*.

Third, the language use and nuances in the language put another constraint on the translation; sometimes it was difficult to find a completely equivalent term in a target language. For example, in Chinese *instrumentation* was translated in such a way that reflects Autograph’s influence on George’s practice, a process from the object dedicated towards the user. Another translation of instrumentation would only refer to the tool – Autograph. In our study, these nuances were only evident to Chinese translators and Chinese readers. Another nuance links to the grammatical use of nouns like ‘document’: in the English language, the term ‘document’ is a noun. However, there are uncountable and countable nouns (e.g. fruit – fruits). Hence, it is important to distinguish between (several countable) documents, and the ‘document’ as referring to the concept of document in DAD, which can encompass several documents that the teacher has developed.

From the discussion above, we can identify three levels which shall be worth considering when translating theoretical frameworks:

- 1- The first level concerns the translation issues themselves;
- 2- The second level concerns the factors imposing constraints on the translation practice (e.g., educational context; educational, cultural and theoretical traditions);
- 3- The third level concerns the approaches for addressing translation issues.

5.2 Moving between multiple languages - a chance for deepening a theoretical frame?

In terms of deepening the concepts related to the DAD, we claim that to truly understand the concepts and their use, we need to understand the original cultural sphere in which they were

developed, and those in which they are/were used, and further how the concepts can be translated to and used in other cultural spheres. If these conditions are not understood, a double discontinuity might develop: the discontinuity between the meaning of a term that was translated into different languages; and the discontinuity between the different uses or practices in different cultural spheres (e.g. of teaching volume of revolution) – this double discontinuity, we argue, might provide grounds for research and the deepening of theoretical terms. Besides, contrasting the different possibilities of translation for the same concept, in connection with their use ‘in situ’, can help clarify some critical aspects of the concept at stake.

The Chinese translations of *instrumentation* – *instrumentalization* can serve as an example: for each term, the dual aspect of process and result, represented by a verb and a noun, is reflected in the translation of the term. For *instrumentation*, the nuances between the several Chinese terms highlight that both the affordances and constraints of the resources could shape the teacher’s activity. In terms of *instrumentalisation*, the connection between the Chinese term and the philosophical theory makes us realise that the *instrumentalisation* may involve an enrichment and meanwhile a weakening of the resources at stake.

In Arabic, the translation of *instrumentation* provided a very powerful image showing that there are active processes at stake, whilst the English term seems to suggest ‘affordances’ or ‘opportunities’ which does not suggest an active process. As another example, the term *scheme* also needs to be understood in connection with the theory/theoretical frame on which it has been built. In the Arabic language, two potential translations were compared and contrasted: "a sequential, linear structure" versus an "intertwining, interconnected and flexible structure, open to redesign, additions, and modifications”.

However, we can/should also regard these differences as (missed) opportunities how the DAD framework might be expanded (to account for the context-specific differences). To provide different examples of schemes (in different contexts) makes the notion of scheme clearer, which helps to understand what a scheme consists of (e.g., concept-in-action; theorem-in-action). In turn, different examples of the notion of teachers’ schemes might help to theorise the work of mathematics teaching and teachers’ use of resources. Whilst theories are expected to be ‘context-free’, we claim that they are actually not, as only the connection with the situations makes them conceptually understandable. This is in line with Vergnaud (2009), who emphasises that different concepts and situations are tied together to form a conceptual field; hence the importance of seeing concepts and situations tied together.

At the same time, we network theories here (e.g., Vergnaud’s theory; DAD), a work which always necessitates a ‘translation’ effort: not translating from real-life context to another, but from one theoretical/conceptual context to another. Reflecting on and understanding the theories and their constituents is likely to advance the field, in particular if a theory is regarded as ‘dynamic’: it is a tool ‘in use’ rooted in particular kinds of theoretical/philosophical background (to be developed in a particular way to be of use for a particular question) and embedded in the practical work of researchers (Prediger et al. 2008).

In our study, we have opened up the issues surrounding translations and interpretations again, in particular concerning the translation of theory. Delving deeper into translation issues may help researchers to understand the potential nuances of abstract concepts, and how they are connected to different practices in different cultural spheres. Considering theoretical concepts from different perspectives, and juxtaposing meanings in different contexts, is likely to deepen our understanding of such concepts, which in turn is likely to enrich the theory at stake.

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