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# On the measurement problems in (relativistic) quantum mechanics

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## Abstract

We explore the main issues that appear in (relativistic) quantum mechanics relative to the implementation of measurements. This approach reveals how fruitful these topics are for theoretical physics. Endeavoring to formulate an exhaustive solution to the measurement problem leads us to a better understanding of the status of the collapse and of the emergence of classicality, thanks to a precise definition of the measurement and some new vocabulary to speak of quantum mechanics and decoherence. The result may be seen as a step beyond the many-worlds interpretation of quantum mechanics. When the relativistic constraints are added, a consistency condition constraining any relativistic quantum theory is formulated, from which can be deduced the locality of physics as well as, in the context of quantum field theory, the microcausality hypothesis. Moreover, it also implies the non-measurability of fermionic fields. On the way, two different arguments are given to show that a deterministic theory superseding quantum mechanics can not be formulated.

**Keywords:** measurement problem, interpretation of quantum mechanics, decoherence, special relativity, locality, determinism.

## Introduction

Measuring a quantum system affects its physics: the quantum Zeno effect or the fact that the presence of a detector in a double slit experiment destroys the fringes confirms this undoubtedly. The measurement must therefore be treated as a physical evolution. Apparent conceptual problems concerning the implementation of measurements in quantum mechanics (QM) fall into three main categories:

1. How comes that the wave function collapse is a non-linear and probabilistic evolution, whereas the Schrödinger equation is linear and deterministic?
2. What is a measurement? What is an observer? When exactly does the projection occur? If it exists, where is the border between the classical and the quantum world? If there is none, how does classicality emerges from the quantum? This is the usual *measurement problem of QM*<sup>1</sup>.

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<sup>1</sup>It is impressive to realize how uncertain the physical community still remain on this fundamental problem. A poll carried out by Schlosshauer et al. (2013) among the participants of a conference on the foundations of QM in 2011 revealed that, when asked about the measurement problem, the participants were approximately equally divided between the five possible answers: a pseudoproblem, a problem solved by decoherence, a problem solved/will be solved in another way, a severe difficulty threatening QM, none of the above. A later survey (Sivasundaram and Nielsen, 2016), gathering more participants in different universities, also led to very divergent answers. Besides, in a recent interview (Peterson, 15 april 2022), Penrose described the measurement problem almost exactly as it was already formulated a century ago, considered it as an inconsistency of quantum theory (mainly because of the absence of clear definition of what constitutes a measurement), and deplored the fact that most physicists ‘shove [this problem] under the carpet’.

3. How to model the effect of a measurement in a way compatible with special relativity? This is what bothered Einstein when considering the entanglement phenomenon and led to the famous EPR paper (Einstein et al., 1935). Indeed, the measurement is generally described as an update happening *instantaneously* in the whole space, as a ‘spooky action at a distance’, but in special relativity the notions of instantaneity or simultaneity are not defined.

The aim of this paper is to formulate properly and propose a solution to all of these three problems. The first two are tackled in Section 1. The reader expecting that the solution to the measurement problem relies on a mathematical feat, or on a stunning simple idea no one ever had, will be disappointed. It rather consists of cautiously changing the way we think and speak of QM. Eventually, the interpretation we develop could perhaps be seen as a many-worlds interpretation with more philosophical rigour, where we are very careful not to give more ontology to the ‘worlds’ than they deserve, so that the problem of the meaning of the probabilities evaporates. To deliver or to receive a new language game, as understood by Wittgenstein (1953), is not an easy task, so the reader must be warned that this section might appear slightly recondite on a first reading.

After some simple preliminaries on quantum theory and decoherence (§1.1), we decompose the problem into two of its main formulations: the status of the collapse (§1.2) and the disappearance of (most) quantum effects at our scales (§1.3). In the first part, we argue that the collapse is not a physical process (§1.2.1, §1.2.2), explain its nature and validity, give a precise definition of the measurement and develop some new vocabulary to interpret QM (§1.2.3). In the second, we introduce some ideas based on Schrödinger’s cat (§1.3.1) further pursued in the sequel, where we tackle the preferred-basis problem and explain the disappearance of interferences (§1.3.2), of quantum correlations due to entanglement (§1.3.3) and of indeterminism (§1.3.4). We finally check that the assumption of the universal validity of QM has no internal inconsistencies (§1.3.5) and discuss the notion of observer (§1.3.6).

The last problem is addressed in Section 2. Two kinds of instantaneities are exhibited in QM but, in the light of the previous section, only one of them is *a priori* problematic. To deal with the latter, a consistency condition constraining the statistics of measurements outcomes in any relativistic quantum theory is formulated, that is necessary to prevent faster-than-light communication and non-covariance (§2.1). This condition can be precisely formulated mathematically, and will turn out to be equivalent to the absence of interactions at a distance in physics (§2.2). In the particular context of quantum field theory (QFT), it also has strong links with the locality and the microcausality hypothesis: it actually allows to justify the former (§2.2), rigorously demonstrate the latter (§2.3) and realize that they are actually two redundant hypotheses of QFT; it will also provide us with a proof that the Dirac field can in no sense be measured.

## 1 The measurement problem in the light of the theory of decoherence

In this section, we investigate methodically the measurement problem of QM in all its ramifications, in view of the ideas of the theory of decoherence. We don’t want to be content with a list of several attempts of solution and their respective drawbacks. Our ambitious aim is rather to understand the relation between the different pieces of the puzzle and propose an exhaustive solution, as well as some linked considerations that may be worth sharing. We will follow the plan announced in the introduction, divided into some brief preliminaries §1.1 and two main parts §1.2 and §1.3, each one addressing a common formulation of the problem. Since the different issues are all very entangled, we will refer a lot, throughout the reasoning, to previous or later paragraphs.

### 1.1 Preliminaries

#### 1.1.1 Preliminary remarks on quantum mechanics

Let’s begin with some obvious but crucial remarks about the nature of quantum theory that are so easily forgotten, thereby often leading to confusion and headaches. Unlike deterministic classical physics,

1. *QM is a probabilistic theory*<sup>2</sup> that predicts the statistics of empirical facts observed following measurements,
2. in QM, the act of measuring a system affects its future statistics because (i) it creates entanglement and therefore produces decoherence, (ii) it requires to interact with the system which perturbs it (see §1.3.4).

It is the combination of these two ingredients that makes almost impossible to built an ontological picture for quantum physics. Indeed, the ‘actual state’ of a particle (*e.g.* through which slit did the electron pass?) can not be based on the ground of its physical state, the density matrix, because the latter is not deterministic and therefore doesn’t provide a unique, well-defined answer. Moreover, it can not be based either (as can be done, for example, in statistical physics) on ‘the outcome that one would have obtained if one had measured’, because then the system would not be the same anymore and the future evolution of the particle would have been different.

The best attempt to assign realistic ontological properties to particles is the fascinating De Broglie - Bohm interpretation, but:

- it is highly non-local (as would any hidden variable theory, due to Bell’s theorem) and therefore its ‘realistic’ trajectories can actually lead to very surrealistic phenomena (Englert et al., 1992). In particular, a ‘detector in an interferometer (...) can become excited even when the electron passes along the other arm of the interferometer’ (Dewdney et al., 1993), while the detector through which the electron actually passes remains unexcited. Consequently, Bohmian trajectories don’t represent either ‘the outcome that one would have obtained if one had measured’. What kind of interesting meaning can they possibly have, then? Besides, does the very notion of physics still makes sense when accepting that we can not trust our measurement devices?
- no one has yet succeeded to make it compatible with special relativity,
- because of item 2. (ii), it is impossible to know perfectly both the initial position and momentum of a particle, therefore it is still a probabilistic theory that adds no predictive power to standard QM.

### 1.1.2 Selective measurement, non-selective measurement

When discussing the implementation of measurements in QM, it is crucial to distinguish clearly between selective and non-selective measurement. The former is what is usually referred to as the ‘wave function collapse’. When measuring an observable  $\hat{A}$ , of spectral decomposition  $\hat{A} = \sum_{x \in \text{spec}\hat{A}} x \Pi_x$ , relative to a system in a pure state (resp. mixed state)  $|\Psi\rangle$  (resp.  $\rho$ ), if the outcome is the eigenvalue  $x_0$ , then the system’s state is standardly postulated to evolve as:

$$|\Psi\rangle \longrightarrow \frac{\Pi_{x_0} |\Psi\rangle}{\|\Pi_{x_0} |\Psi\rangle\|} \quad \text{resp.} \quad \rho \longrightarrow \frac{\Pi_{x_0} \rho \Pi_{x_0}}{\text{tr}(\rho \Pi_{x_0})}. \quad (1)$$

Note that the implementation of such a selective measurement requires to know (or assume) the outcome. Alternatively, a non-selective measurement describes the update of the state without distinguishing the actual outcome: it is only concerned about the statistics that would be obtained if the experiment were repeated. Except for specific cases, the result is always a mixed state:

$$\rho \longrightarrow \sum_{x \in \text{spec}\hat{A}} \Pi_x \rho \Pi_x. \quad (2)$$

Selective measurement is a non-linear probabilistic operation on  $\rho$ , whereas non-selective measurement is a linear deterministic one, since it merely consists of extracting the diagonal part of the density matrix in the eigenbasis of the measured observable. The theory of decoherence explains how non-selective measurements (2) arise in QM, due to the entanglement between the system and its measurement apparatus. Let’s briefly recall why, following our presentation given in more detail in (Soulas, 2023).

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<sup>2</sup>Einstein thought that QM was incomplete because of his famous ‘God does not play dice’. However, note how positivist this remark is: it is not that God necessarily plays dice, but only that the human mind may not be able to access better than statistically the way God plays.

### 1.1.3 Basic notions of decoherence

According to Di Biagio and Rovelli (2021), the deep difference between classical and quantum is the way probabilities behave: all classical phenomena satisfy the total probability formula

$$\mathbb{P}(B = y) = \sum_{x \in \text{Im}(A)} \mathbb{P}(A = x) \mathbb{P}(B = y | A = x)$$

relying on the fact that, *even though the actual value of the variable  $A$  is not known, one can still assume that it has a definite value among the possible ones*. This, however, is not correct for quantum systems, for which the diagonal elements of their density matrix account for the classical behavior (they correspond to the terms of the total probability formula) while the non-diagonal terms are the additional interference terms<sup>3</sup>. The aim of the theory of decoherence is to understand why the latter go to 0 for non-isolated systems.

Here is the typical situation. Consider a system  $\mathcal{S}$ , described by a Hilbert space  $\mathcal{H}_{\mathcal{S}}$  of dimension  $d$ , that interacts with an environment  $\mathcal{E}$  and let  $\mathcal{B} = (|i\rangle)_{1 \leq i \leq d}$  be an orthonormal basis of  $\mathcal{H}_{\mathcal{S}}$ . In the sequel, we will say that each  $|i\rangle$  corresponds to a **possible history** of the system in this basis. In these terms, QM is nothing but a strange geometrical way of calculating probabilities in which all the possible histories interfere, according to the non-diagonal part of the density matrix. For simplicity, let's also assume for now that  $\mathcal{B}$  is a **conserved basis** during the interaction with  $\mathcal{E}$  (in some contexts called a pointer basis) and denote  $|\Psi\rangle = \left(\sum_{i=1}^d c_i |i\rangle\right) \otimes |\mathcal{E}_0\rangle$  the initial state of  $\mathcal{S} + \mathcal{E}$ . After a time  $t$ , the total state evolves to  $|\Psi(t)\rangle = \sum_{i=1}^d c_i |i\rangle \otimes |\mathcal{E}_i(t)\rangle$ . Define  $\eta_{\mathcal{B}}(t) = \max_{i \neq j} |\langle \mathcal{E}_i(t) | \mathcal{E}_j(t) \rangle|$ . Note that this definition actually makes sense in any (potentially non-conserved) basis. We will drop the subscript  $\mathcal{B}$  when the context is clear. By taking the partial trace with respect to the environment, it is straightforward to see that the  $i^{\text{th}}$  diagonal coefficient of the system's state  $\rho_{\mathcal{S}}$  always remains equal to  $|c_i|^2$ , whereas the non-diagonal ones are  $c_i \bar{c}_j \langle \mathcal{E}_j(t) | \mathcal{E}_i(t) \rangle$ , bounded in modulus by  $\eta(t)$ . Therefore  $\eta$  *measures how decohered the system is*, that is how close it is from satisfying the total probability formula. It is now well understood why  $\eta$  quickly tends to 0 for some interesting bases in a wide variety of contexts (Zurek, 2003; Joos, 1996). In particular, when  $\mathcal{E}$  is a measurement apparatus for the observable  $A$ , the eigenbasis of  $\hat{A}$  is clearly a conserved basis<sup>4</sup>, yielding the non-selective measurement evolution (2) in the ideal case of perfect and immediate decoherence.

## 1.2 First formulation: the transition from mixed to pure states

### 1.2.1 Epistemic vs. ontic meaning of the wave function

We are now ready to tackle the measurement problem. The usual puzzle is to make sense of the fact that only one of the possible histories is actually observed, and to understand when and why the state

evolves from  $\rho_{\mathcal{S}} = |c_1|^2 \begin{pmatrix} 1 & & \\ & 0 & \\ & & \ddots \end{pmatrix} + \dots + |c_d|^2 \begin{pmatrix} \ddots & & \\ & 0 & \\ & & 1 \end{pmatrix}$ , which is the state obtained after deco-

herence entailed by the measurement apparatus (non-selective measurement (2)), to the collapsed state  $\begin{pmatrix} 1 & & \\ & 0 & \\ & & \ddots \end{pmatrix}$  (selective measurement (1)).

However, this formulation of the problem makes little sense; troubles appear when one believe more in the mathematics than in the world they are trying to describe. Of course, no observer has ever observed more than one fact at once, and no one has ever felt superposed. Also, one must not demand more to QM than it can give:

<sup>3</sup>As a reminder, this is because the probability to obtain an outcome  $x$  is:

$$\text{tr}(\rho |x\rangle \langle x|) = \sum_{i,j=1}^n \rho_{ij} \langle j|x\rangle \langle x|i\rangle = \sum_{i=1}^n \underbrace{\rho_{ii} |\langle x|i\rangle|^2}_{\mathbb{P}(i)\mathbb{P}(x|i)} + \sum_{1 \leq i < j \leq n} \underbrace{2\text{Re}(\rho_{ij} \langle j|x\rangle \langle x|i\rangle)}_{\text{interferences}}$$

<sup>4</sup>This is how a measurement works: if  $\mathcal{S}$  is prepared in an eigenstate of  $\hat{A}$ , it should remain in this state during the measurement of  $\hat{A}$ . In general, the existence of a conserved basis is not guaranteed unless the interaction Hamiltonian takes the form  $\sum_i \Pi_i^{\mathcal{S}} \otimes H_i^{\mathcal{E}}$ , where  $(\Pi_i^{\mathcal{S}})_{1 \leq i \leq d}$  is a family of orthogonal projectors associated to this basis.

- being a probabilistic theory, it is normal that several possibilities appear, and that only an experimental observation can determine the actual result
- being a probabilistic theory, it is also very natural that an update has to be performed when one obtains new empirical information about the system. This is a common feature of any probabilistic model<sup>5</sup>.

Of course, in QM, things are more confusing because it is a probabilistic theory in which all the possible histories interfere together, so that every potential outcome seem to have had an influence on the result of the experiment, hence a sort of ‘reality’. This is why the collapse looks so physical (at least more than a simple subjective information update as in the example of note 5), so that the

existence of a truly physical process leading from  $\begin{pmatrix} |c_1|^2 & & \\ & \ddots & \\ & & |c_d|^2 \end{pmatrix}$  to the observed  $\begin{pmatrix} 1 & & \\ & 0 & \\ & & \ddots \end{pmatrix}$

has been proposed. This is the starting point for the research on objective collapse models, trying to add a non-linear stochastic modification of the Schrödinger equation which would explain the transition from mixed to deterministic states at macroscopic scales (Bassi et al., 2013). In this case, the question at stake is roughly: when exactly does the collapse occur, and at which speed? However, while those models suffer from issues in their compatibility with special relativity despite numerous attempts (Dove, 1996; Tumulka, 2006; Bedingham et al., 2014), large portions of the set of possible parameters have already been ruled out experimentally. They are also at odds with the phenomenon of coherence revival (see §1.2.2).

On the other hand, if the collapse is not a physical process but a mere epistemic update as in note 5, it remains to understand why it still yields correct predictions. Indeed, the operation of updating the probabilities by suppressing all the possible histories except the one that has actually been observed (or the ones compatible with the observation) is legitimate as long as re-coherence is not permitted, *i.e.* as long as  $\eta$  remains 0 forever. Otherwise, the possible histories interfere again so that suppressing some of them should lead to wrong predictions (as in the experiment of §1.2.2). Said differently, in the case of a non-physical collapse, determining exactly when does the latter occur is no more problematic (you update your mathematical tool whenever you wish), but one has to understand what always justifies its use (see §1.2.3).

### 1.2.2 Is the collapse a physical process?

In Bassi et al. (2013), it is claimed that many-worlds interpretations based on decoherence can not be experimentally distinguished from the Copenhagen interpretation. But this is not correct since no interpretation assuming the reality of the collapse (Copenhagen or collapse models) can explain a phenomenon such as coherence revival, as sketched in the following thought experiment. Note first that when a quantum system like a spin  $\frac{1}{2}$  interacts with a microscopic environment like another spin  $\frac{1}{2}$ , decoherence is possible but re-coherence is not difficult to achieve, for example *via* a C-NOT gate (by default, the spin is considered along the  $z$  axis):

$$\underbrace{\frac{1}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle)}_{\mathcal{S}} \otimes \underbrace{|\uparrow\rangle}_{\mathcal{E}} \xrightarrow{C-NOT} \frac{1}{\sqrt{2}}(|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle) \xrightarrow{C-NOT} \frac{1}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle) \otimes |\uparrow\rangle.$$

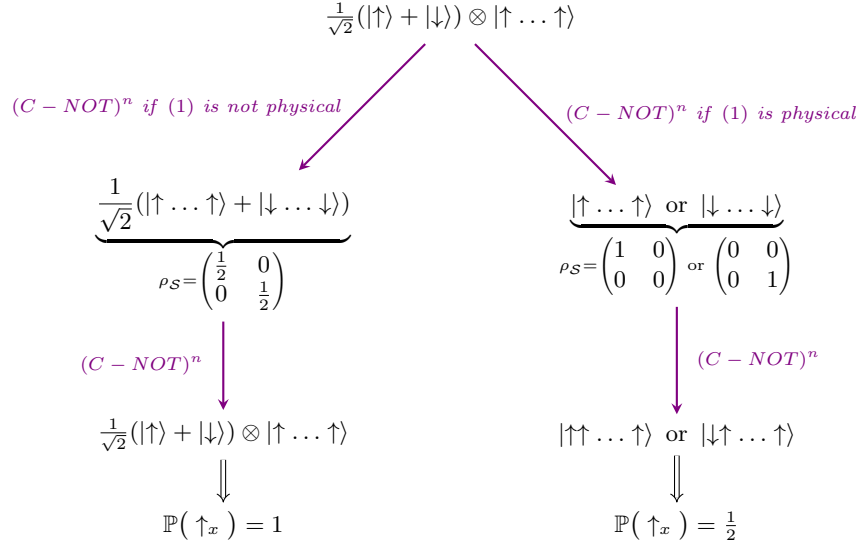
$$\rho_{\mathcal{S}} = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix}$$

Here  $\mathcal{S}$  clearly decoheres and then retrieves its coherence at the end in the superposed state  $\frac{1}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle) = |\uparrow_x\rangle$ , for which a measurement of the spin along  $x$  yields  $\uparrow_x$  with probability 1. The same operation could, in principle, be applied to a system coupled with a much larger environment, modeled for example by a collection of  $n$  spins with huge  $n$ . Note that it doesn’t have to be the genuine spin, but could also stand for any abstract degree of freedom, in particular it could be linked with position which is generally the basis in which the collapse is induced in collapse models. If the collapse had a physical reality, coupling

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<sup>5</sup>As a trivial example, suppose for instance that you ask your computer to choose uniformly a number  $x_1$  in  $\{0; 1\}$ , and then to choose uniformly a second number  $x_2$  in  $\{0; 1\}$  if  $x_1 = 0$ , or to choose uniformly  $x_2$  in  $\{2; 3\}$  if  $x_1 = 1$ . If you don’t look at the result for  $x_1$ , you predict  $x_2$  to follow a uniform law in  $\{0; 1; 2; 3\}$ , but if you know the outcome for  $x_1$ , you obviously update your probabilities.

$S$  with a macroscopic system should make it undergo the collapse (1), that will affect the statistics after re-coherence:



Obviously, this experiment is not feasible in the present technology of quantum computing for a large  $n$ , but more realistic ones have been conducted and coherence revival is already a confirmed phenomenon (Chapman et al., 1995). At the end of the day, we still have no compelling empirical reason to believe that a collapse process exists whereas, as recalled above, an update operation is naturally expected in any probabilistic theory. Moreover, for the first time since the advent of QM, the theory of decoherence allows us to assert consistently that the Schrödinger equation is universally valid at all scales (see §1.3). What can be the status of decoherence in a collapse model, apart from a curious redundant phenomenon that also happen to destroy the quantum interferences? In the following, we will therefore consider that the collapse is not a physical process; it remains to address the problem stated above at the end of §1.2.1.

### 1.2.3 Definition of the measurement

By chance, as explained by the theory of decoherence, it is impossible for an observer to obtain empirical information on a quantum system without (almost) immediately and perfectly destroying the quantum interferences. Said differently, a knowledge update (selective measurement) (1) is inevitably preceded by the physical process (non-selective measurement) (2)<sup>6</sup>. As mentioned above, as long as coherence is not retrieved, the possible histories will never interfere anymore, so that forgetting about those which did not happen will not change any future prediction. It is important to keep in mind, though, that this ideal case is never absolutely fulfilled, because decoherence is not immediate nor perfect, and because re-coherence can happen in particular experiments (as in the example of §1.2.2). It is even *expected* in principle, even though the recurrence time can easily exceed the lifetime of the universe for realistic systems (Zurek, 1982).

However, the collapse follows a gain of information due to a measurement. In this case, the situation is much better because the physicist who reads, *remembers* and stores the data of the outcome is part of the environment too. As long as these data exist materially<sup>7</sup> (be it in her brain, her computer or her notebook), the vectors  $(|\mathcal{E}_i(t)\rangle)_{1 \leq i \leq d}$  are perfectly distinguishable, hence orthogonal, guaranteeing  $\eta$  to be 0. Thus re-coherence of the possible histories can only happen when nothing material remains that allows to know what the outcome was. In other words, when a measurement has been performed, a condition for the collapse to ultimately lead to wrong predictions is that nothing in the universe should remember the outcome... but in this case the person who made the prediction is not here anymore

<sup>6</sup>If it were not the case, the universe would be very different... This is also why it is so hard to untangle the two and clarify their respective status.

<sup>7</sup>By 'matter', we designate all that can be objectively measured and quantified.

willing to check it! (More on this in §1.3.5.)

To build an interpretation of a theory is all about finding a way to present the mathematics, and developing a vocabulary to speak of those mathematics, that raise clear pictures in mind with as few counter-intuitive properties as possible. In §1.1.1, we explained why quantum theory is so reluctant to ontological pictures. Nevertheless, this is not a reason for completely neglecting the search for a (less worse) interpretation. In fact, a good language brings good reasonings and, all things considered, (interpretation of the) theory and experiments co-evolve, as brilliantly pointed by Einstein in this discussion with Heisenberg, related in Heisenberg’s autobiographical notes (Heisenberg, 1973): ‘It may be of heuristic value to recall what one really observes. But from a principal point of view it is quite wrong to insist on founding a theory on observed quantities alone. In reality just the opposite is true. Only the theory decides about what can be observed.’ It is indeed the interpretation of the theory that dictates the language with which we describe our observations, the meaning we give to them, and also influences the instruments we build and the experiments we decide to conduct. In particular, the usual expression ‘the particle is both here and there’ generally used to refer to a superposition *is* an interpretation, and arguably a quite confusing choice of words. Let’s try to introduce some new vocabulary.

**Definition 1.1 (Measurement in quantum mechanics).**

- We say that the observable  $A$  of a system  $\mathcal{S}$  is **recorded** by an environment  $\mathcal{E}$ , when  $\mathcal{S}$  and  $\mathcal{E}$  are entangled with perfect decoherence in the eigenbasis  $\mathcal{B}$  of  $A$  (*i.e.*  $\eta_{\mathcal{B}} = 0$ ). As long as this holds, we say that the possible histories of  $\mathcal{S}$  are **split** by  $\mathcal{E}$  in  $\mathcal{B}$ , or simply that  $A$  is split. If coherence is retrieved, we say that the possible histories **recombine**.
- A **non-selective measurement process** for  $A$  is a unitary evolution that entangles  $\mathcal{S}$  with  $\mathcal{E}$  such that, for any initial state of  $\mathcal{S}$ , the system’s possible histories are split in  $\mathcal{B}$  (*i.e.*  $A$  is recorded by  $\mathcal{E}$ ) after interaction. If moreover  $\mathcal{B}$  is a conserved basis during the process, we call it a **projective** (or non-demolition) measurement.
- A **selective measurement** of  $A$  by an observer  $\mathcal{O}$  occurs when  $A$  is split (in particular) by the act of  $\mathcal{O}$  getting entangled with  $\mathcal{S}$  when storing the data of an outcome. As long as this holds, we say that the observed outcome is a **fact** for  $\mathcal{O}$  and that  $\mathcal{O}$  **remembers** the data of the outcome. The **collapse** is the subsequent update performed by  $\mathcal{O}$  on the probabilities, on the basis of this new knowledge.

The first is only an instantaneous statement: does the environment presently distinguishes the eigenstates of  $A$ ? Mathematically, this means that  $\mathcal{S} + \mathcal{E}$ ’s state takes the form  $\sum_i c_i |A_i\rangle |\mathcal{E}_i\rangle$ , where  $\mathcal{B} = (|A_i\rangle)_i$  are the eigenstates of  $A$  and  $\langle \mathcal{E}_i | \mathcal{E}_j \rangle = 0$  for  $i \neq j$ . In this case, the total probability formula applies, so that one can assume that we are in one — and only one — of the possible histories concerning  $A$ . Note that any entanglement constitutes a recording in the basis where the density matrix of  $\mathcal{S}$  is diagonal after interaction<sup>8</sup> (this basis may depend on the initial state of the system and/or of the environment).

The second, in the projective case, is a condition for an interaction to entail the operation (2). It relies on the existence of a unitary  $U$  and an initial environment state  $|\mathcal{E}_0\rangle$  such that, for all  $i$ ,  $U |A_i\rangle |\mathcal{E}_0\rangle = |A_i\rangle |\mathcal{E}_i\rangle$  with  $\langle \mathcal{E}_i | \mathcal{E}_j \rangle = 0$  for  $i \neq j$ . But the existence of a conserved basis is a strong requirement, and we consider here that it is enough to have  $\mathcal{E}$  split the possible histories in a given basis in order to speak of measurement (only a projective measurement reflects something of the initial state; it also differs from a POVM, in which the environment distinguishes what the system’s state was *before* interaction, no matter if its final state is altered). In this case, we only suppose that for all  $|\Psi\rangle \in \mathcal{H}_{\mathcal{S}}$ ,  $U |\Psi\rangle |\mathcal{E}_0\rangle = \sum_i c_i^\Psi |A_i\rangle |\mathcal{E}_i^\Psi\rangle$  with  $\langle \mathcal{E}_i^\Psi | \mathcal{E}_j^\Psi \rangle = 0$  for  $i \neq j$ . By abuse of language when the context is clear, we may still write ‘measurement’ instead of ‘projective measurement’ as we did above.

The third is a condition for  $\mathcal{O}$  to apply safely the collapse (1) and get correct predictions, at least as long as she remembers the observed outcome. It is based on the fact that obtaining some knowledge about a system necessarily splits its possible histories in the basis corresponding to the question asked, because  $\mathcal{S} + \mathcal{O}$ ’s state takes the form  $\sum_i |A_i\rangle |\mathcal{O}_i(t)\rangle$ , where  $\langle \mathcal{O}_i(t) | \mathcal{O}_j(t) \rangle = 0$  for  $i \neq j$  as long as  $\mathcal{O}$  remembers the outcome. Since we consider that the collapse is not a physical operation, it is natural that

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<sup>8</sup>A slight redefinition of  $\eta_{\mathcal{B}}$  as  $\max_{\substack{i \neq j \\ c_i, c_j \neq 0}} |\langle \mathcal{E}_i(t) | \mathcal{E}_j(t) \rangle|$  with, by convention,  $\eta_{\mathcal{B}} = 0$  when only one  $c_i$  is non zero, may be

necessary to ensure that  $\rho_{\mathcal{S}}$  being diagonal in  $\mathcal{B}$  is equivalent to  $\eta_{\mathcal{B}} = 0$ . Further adaptations could be made to deal with the possible degeneracy of  $\hat{A}$ ’s eigenvalues, which need not necessarily be distinguished by the environment. . .



its definition doesn't entirely rely on physical concepts. By observer, we designate at least any human being that can remember facts; the question of what exactly can be deemed an observer is addressed in §1.3.6. We see that it is not wrong, for example, to say that the primordial universe has collapsed the first time an astrophysicist has looked at the cosmic microwave background, simply because it was the first time that someone acquired knowledge about the latter, although the possible histories of each primordial photon were split long ago by decoherence. We also see that there is one wave function per observer, as any probability distribution depends on the information available to the person making predictions<sup>9</sup>. However, due to the particular way of computing probabilities in QM, it must be checked that no inconsistencies appear when observers who have different knowledge compare their predictions (see §1.3.5).

### 1.3 Second formulation: the transition from quantum to classical physics

The other main aspect of the measurement problem concerns the emergence of classicality. If no physical collapse is assumed, a myriad of interrogations must be solved with regard to the seemingly extremely different aspects of quantum and classical physics. Can the Schrödinger equation really be considered valid at all scales?

#### 1.3.1 Some remarks on Schrödinger's cat

Schrödinger's cat paradox is often cited as a key illustration of the measurement problem. The basic question is: why don't we observe cats in superpositions like  $\alpha |\text{dead}\rangle + \beta |\text{alive}\rangle$ ? Once again, this quick formulation is too simplistic for the reasons listed below, but the thought experiment still contains interesting issues concerning the applicability of QM to our daily environment.

- Even if the room were perfectly isolated, the cat would already be completely decohered by the Geiger counter composing its environment, or even by the unstable atom only, hence in a mixed state with no quantum interferences. In the following, let's rather suppose that the cat is truly isolated.
- The level of interferences is in general basis-dependant, so even if the cat is not superposed in the basis  $\mathcal{B} = (|\text{dead}\rangle, |\text{alive}\rangle)$ , it still presents quantum interferences in the vast majority of other basis. What is so special with  $\mathcal{B}$  from our point of view and, to begin with, of which Hilbert space is it a basis (see §1.3.2)?
- How could we possibly know whether the cat is superposed or not? Clearly not by opening the room and looking at it, which would constitute a measurement in the basis  $\mathcal{B}$ , thus immediately destroying the interferences between the histories  $|\text{dead}\rangle$  and  $|\text{alive}\rangle$ . As mentioned earlier, a cat in superposition should preferably not be thought of as being both dead and alive, but simply as a cat whose statistics are not those of a classical cat either dead or alive. To empirically reveal the presence of a superposition, one has to make the histories interfere, for example by performing a measurement in another orthogonal basis like  $\mathcal{B}' = (\alpha |\text{dead}\rangle + \beta |\text{alive}\rangle, \bar{\beta} |\text{dead}\rangle - \bar{\alpha} |\text{alive}\rangle)$  or, equivalently, to make the cat undergo a unitary evolution sending  $\mathcal{B}'$  to  $\mathcal{B}$  before performing a measurement in  $\mathcal{B}$ , and repeat this process several times to check that we indeed obtain  $\alpha |\text{dead}\rangle + \beta |\text{alive}\rangle$  in the first case, or  $|\text{dead}\rangle$  in the second case, with probability 1. But what kind of apparatus could well perform a measurement with respect to  $\mathcal{B}'$ ? How could we have such absolute quantum control on the cat so that we can rotate its state from  $\mathcal{B}'$  to  $\mathcal{B}$ ? Furthermore, how could we prepare a large number of cats in exactly the same quantum state? The problem is that many aspects of QM are not even testable at our scales, so we can't do better than to check its compatibility, in principle, with what we know of classical physics. As we will see throughout this section, we have no reason to think that quantum theory is not universally valid because, if it is, then the oddities that *could* appear at our scales are actually prevented by the fact that we can not play God. In particular, we can neither perfectly isolate a large system nor apply any unitary evolution we wish (this is why we never observe recombination of macroscopic histories as in the thought experiment 2. of §1.3.5),

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<sup>9</sup>Mathematics have long ago accepted the idea that a theory is a point of view, always embedded in a larger meta-theory, itself subject to reasonings applicable to theories. There is no more absolute wave function of the universe than there is an ultimate mathematical theory.

nor prepare a precise macroscopic state, and we only have access to a very small number of coarse measurements (see §1.3.2).

### 1.3.2 The preferred basis problem

At this point, a comment must be made on the status of what we call the possible histories. In QM, choosing an orthonormal basis  $\mathcal{B}$  of the Hilbert space corresponds to choosing a particular way to tell the possible histories, and the value of  $\eta_{\mathcal{B}}$  in a given basis quantifies to what extent these histories are split. We have  $\eta_{\mathcal{B}} = 0$  only in an eigenbasis of  $\rho_S$ , so decoherence only ensures that the histories are easy to tell in this particular basis where they don't influence each other. The very fact for a system to have its possible histories split or interfere (some might say: for worlds to branch), is not absolute because there are infinitely inequivalent ways to decompose them. The picture of a well-defined graph of histories splitting over time with decoherence, and recombining with coherence revival, is a bit too simple.

Nonetheless, there exists cases in which all the histories are split in any basis. Consider for instance a spin  $\frac{1}{2}$  entangled with a measurement apparatus  $\mathcal{A}$  in the state  $|\Psi\rangle = \frac{1}{\sqrt{2}}(|\uparrow_{\mathcal{A}\uparrow}\rangle + |\downarrow_{\mathcal{A}\downarrow}\rangle)$ , such that the spin along  $z$  is split by  $\mathcal{A}$  *i.e.*  $\langle\mathcal{A}\uparrow|\mathcal{A}\downarrow\rangle = 0$ . It is not difficult to show that one can rewrite, for any other direction  $u$ ,  $|\Psi\rangle = \frac{1}{\sqrt{2}}(|\uparrow_u \mathcal{A}\uparrow_u\rangle + |\downarrow_u \mathcal{A}\downarrow_u\rangle)$ , with  $|\mathcal{A}\uparrow_u\rangle$  and  $|\mathcal{A}\downarrow_u\rangle$  some linear combinations of  $|\mathcal{A}\uparrow\rangle$  and  $|\mathcal{A}\downarrow\rangle$  still satisfying  $\langle\mathcal{A}\uparrow_u|\mathcal{A}\downarrow_u\rangle = 0$ . The reason is that all the possible histories are equiprobable so the density matrix is a scalar matrix which is diagonal in every basis. Therefore, if being split by an environment were a sufficient criterion to define the selective measurement, it would be possible here to apply the collapse in *any* basis, which is manifestly wrong! As stated by Schlosshauer (2005), QM ‘has nothing to say about which observable(s) of the system is (are) being recorded, via the formation of quantum correlations, by the apparatus (...) in obvious contrast to our experience of the workings of measuring devices that seem to be “designed” to measure certain quantities’.

So what can well distinguish the basis in which the spin was really measured? Here, if ever  $|\mathcal{A}\uparrow\rangle$  is the state of a device displaying a definite outcome,  $|\mathcal{A}\uparrow_u\rangle$  is not. Indeed, a measurement apparatus can be seen as a tool associating the eigenvectors of the observable to be measured in  $\mathcal{H}_S$  with (in a good approximation) eigenvectors of the position observable in  $\mathcal{H}_{\mathcal{A}}$ . In this way, reading the outcome is easily accessible to us; in fact, we wouldn't know how to do otherwise anyway: no engineer has ever wondered ‘Into which orthogonal vectors do I want my device to evolve when measuring a spin purely up or down?’. But if  $|\mathcal{A}\uparrow\rangle$  is spatially localized, all the other  $|\mathcal{A}\uparrow_u\rangle$  are spatial superpositions. This characterizes the basis in which the measurement took place.

The position eigenbasis clearly plays a special role in QM (see Bell (1982)), especially when the system goes larger. It is generally a preferred basis for decoherence (Schlosshauer, 2005), certainly because the laws of physics usually involve position variables, hence environments accurately feel variations in position and record them. For the same reason it is also the basis in which we can most easily perform a measurement, be it very imprecise, at our scales (sending a few photons may suffice to roughly localize an object). Said differently, the salient information we have about the world is mostly linked with position, so this is also in this basis that we conceive our interaction with it, as does the engineer cited above<sup>10</sup>. One may speculate that the position basis may be defined as the basis in which measurements require the least amount of energy for an observer (about measurements perturbing the system, see §1.3.4 and §1.3.5).

Even on particles, we are able to measure only a very restricted set of definite observables: position,

<sup>10</sup>This is actually a deep remark, but one should be very careful with its philosophical implications. I reckon that it would not be correct to state that the human body and mind (and presumably all living beings on Earth) evolved so as to develop perceptions strongly based on the notion of space and position because this evolutionary path allowed them to benefit from a stability of facts entailed by decoherence on this observable. Indeed, there was no ‘notion of space and position’ before something developed it, and even less any ‘laws of physics involving position variables’, since such laws only belong to the worlds of those sensitive beings whose minds are already appropriately shaped. I would be surprised to meet an alien, but I would not be surprised if it has absolutely nothing like a ‘notion of space and position’ even though its body, if it is made of matter, will certainly be perceived by us as obeying the laws of physics. This alien will probably not survive long on Earth, but maybe on its planet there is no heredity and therefore no Darwinian evolution; maybe the stability of facts, or the very notion of facts, don't confer there any kind of advantage; maybe what we would deem to be its death would not be a relevant event in its world. If ever it has something like a physics, I would try with all my energy to understand it. When talking about the world from outside the earthly point of view, there is really nothing that can be expected. The earthly minds and bodies on one side, their world of outer perceptions on the other side, co-evolved; it would be misleading but appreciably poetic to say that they emerged out of the amorphous void.

momentum, spin, charge, energy, angular momentum... But on macroscopic systems, we have much less: we are restricted to very coarse measurements that discriminate only between huge subspaces of  $L^2(\mathbb{R}^3)$ , which are typically unions of eigenspaces of the position observable. For instance, measuring the cat in the basis ( $|\text{dead}\rangle, |\text{alive}\rangle$ ) precisely means that, by looking at it (*i.e.* exchanging some photons), we entangle with it and split two large sets of its possible histories, which we term the ‘dead’ and ‘alive’ subspaces. These latter are much more related to the position operator (‘is the cat standing or lying on the floor?’) than any other observable (we don’t care about questions like ‘what is the internal energy of the cat?’).

This is the grand lesson of the theory of decoherence: even though QM is universally valid, our experience of the world is governed by classical probabilities because the structure of the laws of physics make position a preferred basis for splitting histories, and this holds for us as well as for any environment, therefore the observables we are mainly able to measure on a system are rightly the same as those continuously recorded by its environment. We only distinguish between histories of a system that are already split by the rest of the world.

### 1.3.3 The disappearance of quantum correlations

Any physical interaction creates entanglement. If everything is so entangled, why don’t we experience its purely quantum effects at our scales, namely correlations violating Bell’s inequality? In a Bell experiment, one well-chosen measurement is performed on each particle of a Bell state  $|\Psi\rangle = c_1 |\uparrow\downarrow\rangle + c_2 |\downarrow\uparrow\rangle$ . For macroscopic entangled pairs, however, none of the two elements are in practice isolated. They have been separately recorded by their respective environments  $\mathcal{E}$  and  $\mathcal{E}'$  in some bases  $\mathcal{B}$  and  $\mathcal{B}'$  (namely in the eigenbases of their density matrices after interaction). The two interactions taking place in different Hilbert spaces, they are described by commuting Hamiltonians so we can deal with them one after the other. Let’s still take  $c_1 |\uparrow\downarrow\rangle + c_2 |\downarrow\uparrow\rangle$  as the initial state of the pair,  $|\mathcal{E}_0\rangle$  and  $|\mathcal{E}'_0\rangle$  the initial states of the environments. Denote  $\mathcal{B} = (\alpha |\uparrow\rangle + \beta |\downarrow\rangle, \bar{\beta} |\uparrow\rangle - \bar{\alpha} |\downarrow\rangle) \equiv (|\uparrow\rangle_{\mathcal{B}}, |\downarrow\rangle_{\mathcal{B}})$  and  $\mathcal{B}' = (\alpha' |\uparrow\rangle + \beta' |\downarrow\rangle, \bar{\beta}' |\uparrow\rangle - \bar{\alpha}' |\downarrow\rangle) \equiv (|\uparrow\rangle_{\mathcal{B}'}, |\downarrow\rangle_{\mathcal{B}'})$ . For simplicity, we will assume that  $\mathcal{B}$  and  $\mathcal{B}'$  are conserved bases, so that the interactions are simply defined by:  $|\mathcal{E}_0\rangle |\uparrow\rangle_{\mathcal{B}} \rightsquigarrow |\mathcal{E}_{\uparrow_{\mathcal{B}}}\rangle |\uparrow\rangle_{\mathcal{B}}$ ;  $|\mathcal{E}_0\rangle |\downarrow\rangle_{\mathcal{B}} \rightsquigarrow |\mathcal{E}_{\downarrow_{\mathcal{B}}}\rangle |\downarrow\rangle_{\mathcal{B}}$  with  $\langle \mathcal{E}_{\uparrow_{\mathcal{B}}} | \mathcal{E}_{\downarrow_{\mathcal{B}}} \rangle = 0$  by construction, and similarly with the primes. Here is what happens schematically:

$$\begin{aligned}
& |\mathcal{E}_0\rangle \left( c_1 |\uparrow\downarrow\rangle + c_2 |\downarrow\uparrow\rangle \right) |\mathcal{E}'_0\rangle \\
&= |\mathcal{E}_0\rangle \left( c_1 (\bar{\alpha} |\uparrow\rangle_{\mathcal{B}} + \beta |\downarrow\rangle_{\mathcal{B}}) \otimes |\downarrow\rangle + c_2 (\bar{\beta} |\uparrow\rangle_{\mathcal{B}} - \alpha |\downarrow\rangle_{\mathcal{B}}) \otimes |\uparrow\rangle \right) |\mathcal{E}'_0\rangle \\
\mathcal{E} \rightsquigarrow \text{records} & |\mathcal{E}_{\uparrow_{\mathcal{B}}}\rangle |\uparrow\rangle_{\mathcal{B}} \left( c_1 \bar{\alpha} |\downarrow\rangle + c_2 \bar{\beta} |\uparrow\rangle \right) |\mathcal{E}'_0\rangle + |\mathcal{E}_{\downarrow_{\mathcal{B}}}\rangle |\downarrow\rangle_{\mathcal{B}} \left( c_1 \beta |\downarrow\rangle - c_2 \alpha |\uparrow\rangle \right) |\mathcal{E}'_0\rangle \\
&= |\mathcal{E}_{\uparrow_{\mathcal{B}}}\rangle |\uparrow\rangle_{\mathcal{B}} \left( c_1 \bar{\alpha} [\bar{\beta}' |\uparrow\rangle_{\mathcal{B}'} - \alpha' |\downarrow\rangle_{\mathcal{B}'}] + c_2 \bar{\beta} [\bar{\alpha}' |\uparrow\rangle_{\mathcal{B}'} + \beta' |\downarrow\rangle_{\mathcal{B}'}] \right) |\mathcal{E}'_0\rangle \\
&\quad + |\mathcal{E}_{\downarrow_{\mathcal{B}}}\rangle |\downarrow\rangle_{\mathcal{B}} \left( c_1 \beta [\bar{\beta}' |\uparrow\rangle_{\mathcal{B}'} - \alpha' |\downarrow\rangle_{\mathcal{B}'}] - c_2 \alpha [\bar{\alpha}' |\uparrow\rangle_{\mathcal{B}'} + \beta' |\downarrow\rangle_{\mathcal{B}'}] \right) |\mathcal{E}'_0\rangle \\
\mathcal{E}' \rightsquigarrow \text{records} & (c_1 \bar{\alpha} \bar{\beta}' + c_2 \bar{\beta} \bar{\alpha}') |\mathcal{E}_{\uparrow_{\mathcal{B}}}\rangle |\uparrow\rangle_{\mathcal{B}} |\uparrow\rangle_{\mathcal{B}'} |\mathcal{E}'_{\uparrow_{\mathcal{B}'}}\rangle + (-c_1 \bar{\alpha} \alpha' + c_2 \bar{\beta} \beta') |\mathcal{E}_{\uparrow_{\mathcal{B}}}\rangle |\uparrow\rangle_{\mathcal{B}} |\downarrow\rangle_{\mathcal{B}'} |\mathcal{E}'_{\downarrow_{\mathcal{B}'}}\rangle \\
&\quad + (c_1 \beta \bar{\beta}' - c_2 \alpha \bar{\alpha}') |\mathcal{E}_{\downarrow_{\mathcal{B}}}\rangle |\downarrow\rangle_{\mathcal{B}} |\uparrow\rangle_{\mathcal{B}'} |\mathcal{E}'_{\uparrow_{\mathcal{B}'}}\rangle + (-c_1 \beta \alpha' - c_2 \alpha \beta') |\mathcal{E}_{\downarrow_{\mathcal{B}}}\rangle |\downarrow\rangle_{\mathcal{B}} |\downarrow\rangle_{\mathcal{B}'} |\mathcal{E}'_{\downarrow_{\mathcal{B}'}}\rangle
\end{aligned}$$

The pair’s possible histories are now split in  $\mathcal{B} \otimes \mathcal{B}'$  (because  $\langle \mathcal{E}_{\uparrow_{\mathcal{B}}} | \mathcal{E}_{\downarrow_{\mathcal{B}}} \rangle = \langle \mathcal{E}_{\uparrow_{\mathcal{B}'}} | \mathcal{E}_{\downarrow_{\mathcal{B}'}} \rangle = 0$ ). Even if we don’t know what the environments have actually recorded, the total probability formula applies and we can assert that we are in only one of these histories. The important thing is that *they all take the form of pure tensor products*: no violation of Bell’s inequalities can occur with such non-entangled systems. Note that it doesn’t matter if  $\mathcal{B}$  and  $\mathcal{B}'$  are changing over time; the environments don’t have to record the value of any fixed observable.

There are, however, two difficulties with this proof:

- the above proof does not work in general if the bases are not assumed conserved anymore,

- we supposed the two environments initially non-entangled; a justification could be given by the fact that they are themselves already recorded by the rest of the world, but one should be careful to avoid circular arguments. . .

#### 1.3.4 The emergence of determinism

An issue that is not often related to the measurement problem, and yet has much to do with measurements, is the following question: if QM is valid at all scales, hence if classical physics stems from QM, how can the former be deterministic while the latter is probabilistic? Why is our experience of the world so predictable in a quantum universe?

QM, as it stands, can not provide fully deterministic predictions in particular because of Heisenberg inequality, which is a theorem implied by the postulates of the theory. Presented this way, one might think that QM is simply incomplete and that, one day, we could have at our disposal a better theory predicting, for each single experiment, the exact outcome (a hidden variable theory). However, we should recall that Heisenberg initially justified his ‘principle’ with various heuristic arguments before it became a theorem. Most people seem to agree that the theorem formulation is more satisfying and much stronger but, all things considered, it is quite the contrary. The heuristic reasonings are based on the universal fact that *measuring a system requires to exchange energy with it, and therefore perturbs it* (item 2. (ii) in §1.1.1): this fundamental idea might really be called the Heisenberg uncertainty principle, and is no more present in the theorem (the perfect agreement between the principle and the theorem is actually very puzzling, so their status, content and rigidity differ). The consequence of this principle is the impossibility to access exact initial conditions for particles, which is a necessary condition for any theory to provide deterministic predictions. It is valid independently of the theory and this is precisely the reason why, even though we already do have a hidden variable theory compatible with QM, namely Bohmian mechanics, the latter is exactly as uncertain as QM.

Why, then, does this principle doesn’t affect classical physics? There are at least two main reasons for that.

- Until the 19th century, we hadn’t empirical access to systems whose typical action is of order  $\hbar$ , for which a measurement entails a significant uncertainty on the initial condition itself, hence immediate indeterminism. Of course, uncertainties propagate in time and may grow dramatically. But for systems like the solar system for instance, the uncertainty on the initial condition we have (much larger than  $\hbar$ !) remains sufficiently long under the uncertainty of our own measurement apparatus, and so is practically undetectable.
- Classical physics also deals with systems that are chaotic (*i.e.* with large Lyapunov exponents, as does statistical physics) or composed of particles subject to a notable uncertainty (like photons in optics and electromagnetism). In these cases, we restrict ourselves to a well-chosen variables that become practically deterministic thanks to the law of large numbers.

One last word about the largely debated question of whether the brain has quantum properties or should be treated as a classical system. Maybe this is finally of little interest compared to the question of knowing whether the brain can be considered as deterministic *i.e.* fully predictable: given the best initial condition we can hope for a brain, how long will the indeterminacy remain negligible?

#### 1.3.5 Checking the consistency of quantum mechanics at all scales

We now understand why the peculiar quantum features (interferences, quantum correlations, indeterminism) are not observed at our scales. From a theoretical point of view, though, we should also make sure that the assumption of the universal validity of QM has no internal inconsistencies, even for omnipotent observers who could have absolute quantum control on arbitrary systems. Potential contradictions could at first sight occur when different observers having different knowledge, hence using different wave functions, compare their predictions (as evoked in §1.2.3). This situation is captured by the famous Wigner’s friend thought experiment. In what follows, we will grant more and more quantum powers to Wigner and see if he and his friend expect the same statistics for the outcomes of their joint experiment.

1. Let him first be able to perfectly isolate together a spin  $\frac{1}{2}$  and his friend  $\mathcal{F}$  measuring it along  $z$ , and then isolate the particle from the friend to perform a measurement on the spin along a direction  $u$

characterized by  $|\uparrow\rangle = \alpha|\uparrow_u\rangle + \beta|\downarrow_u\rangle$  and  $|\downarrow\rangle = \bar{\beta}|\uparrow_u\rangle - \bar{\alpha}|\downarrow_u\rangle$ . Denoting  $E$  the probabilistic event ‘Wigner obtains  $\uparrow_u$ ’, do we have  $\mathbb{P}_{\mathcal{W}}(E) = \mathbb{P}_{\mathcal{F}}(E)$ ? Things run as follows:

$$|\mathcal{W}_0\rangle \otimes (c_1|\uparrow\rangle + c_2|\downarrow\rangle) \otimes |\mathcal{F}_0\rangle \xrightarrow[\text{Friend measures}]{\text{along } z} |\mathcal{W}_0\rangle \otimes (c_1|\uparrow_{\mathcal{F}_\uparrow}\rangle + c_2|\downarrow_{\mathcal{F}_\downarrow}\rangle) \\ \xrightarrow[\text{Wigner measures}]{\text{along } u} (\alpha c_1|\mathcal{W}_{\uparrow_u \uparrow_u}\rangle + \beta c_1|\mathcal{W}_{\downarrow_u \downarrow_u}\rangle) \otimes |\mathcal{F}_\uparrow\rangle + (\bar{\beta} c_2|\mathcal{W}_{\uparrow_u \uparrow_u}\rangle - \bar{\alpha} c_2|\mathcal{W}_{\downarrow_u \downarrow_u}\rangle) \otimes |\mathcal{F}_\downarrow\rangle.$$

After collapse, the friend’s wave function describing Wigner and the spin is either  $\alpha|\mathcal{W}_{\uparrow_u \uparrow_u}\rangle + \beta|\mathcal{W}_{\downarrow_u \downarrow_u}\rangle$  or  $\bar{\beta}|\mathcal{W}_{\uparrow_u \uparrow_u}\rangle - \bar{\alpha}|\mathcal{W}_{\downarrow_u \downarrow_u}\rangle$  with probability  $|c_1|^2$  or  $|c_2|^2$  respectively, while Wigner’s wave function describing the friend and the spin is (up to a normalization factor) either  $|\uparrow_u\rangle \otimes (\alpha c_1|\mathcal{F}_\uparrow\rangle + \bar{\beta} c_2|\mathcal{F}_\downarrow\rangle)$  or  $|\downarrow_u\rangle \otimes (\beta c_1|\mathcal{F}_\uparrow\rangle - \bar{\alpha} c_2|\mathcal{F}_\downarrow\rangle)$  with probability  $|\alpha c_1|^2 + |\bar{\beta} c_2|^2$  or  $|\beta c_1|^2 + |\bar{\alpha} c_2|^2$  respectively. Consequently,  $\mathbb{P}_{\mathcal{W}}(\uparrow_u) = |\alpha c_1|^2 + |\bar{\beta} c_2|^2 = \mathbb{P}_{\mathcal{F}}(\uparrow_u)$ . As a general argument, we see that each of the histories split by the friend’s measurement is in turn split by Wigner’s measurement, so that their respective collapsed wave functions are in fact two marginals extracted from a single classical probability distribution among four possible outcomes displaying no quantum interferences, which are naturally consistent<sup>11</sup>.

2. What happens if the possible histories split by the friend’s measurement recombine before Wigner’s measurement? For this, suppose now that Wigner can apply any unitary evolution inside the room:

$$\frac{1}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle) \otimes |\mathcal{F}_0\rangle \xrightarrow[\text{Friend measures}]{\text{along } z} \frac{1}{\sqrt{2}}(|\uparrow_{\mathcal{F}_\uparrow}\rangle + |\downarrow_{\mathcal{F}_\downarrow}\rangle) \xrightarrow[\text{Wigner applies}]{U^{-1}} \frac{1}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle) \otimes |\mathcal{F}_0\rangle.$$

If now Wigner performs a measurement of the spin along  $x$ , he will get  $\uparrow_x$  with certainty, evidence of a superposition of the two possible histories  $\uparrow$  and  $\downarrow$  along  $z$ . This means that both histories contribute to the final state... even the one that the friend has not experienced!

Perhaps the most annoying feature of interpretations of QM based on words like ‘worlds’ or ‘histories’ concerns the fate of all the other non-factual histories, which seem very real as long as coherence is preserved, but look so ghostly and unscientific afterwards. But we see here that *when one possible history actually becomes a fact for the friend, all the other non-factual histories are ‘still there’, waiting for re-coherence to interfere again*. If the histories recombine<sup>12</sup>, though, for sure the friend doesn’t remember what she has observed<sup>13</sup> (otherwise this would suffice to keep the histories split), so that it is not anymore a fact for her. Wigner and his friend’s wave functions describing the spin are now identical, so their predictions are obviously consistent.

Considering that QM is universally valid urges us to accept the idea that *facts have a finite lifetime and that they are relative to an observer’s memory* (Brukner, 2020); as Di Biagio and Rovelli (2021) put it: ‘Wigner’s facts are not necessarily his friend’s facts’. It doesn’t even make sense to ask whether Wigner would obtain the same outcome as his friend obtained earlier if he measured the spin along  $z$  too, because the answer doesn’t exist anymore. In Bong et al. (2020), the authors constrained this mathematically with a no-go theorem: if QM applies at all scales, then any non-superdeterministic theory satisfying the independence of spacelike separated events can not consistently treat the friend’s facts (called ‘observed events’ in the paper) as hidden variables,

<sup>11</sup>Classical probabilities for different observers are consistent because the total probability formula is valid, meaning that (as recalled in §1.1.3) even though the actual value of a variable is not known for an observer, she can still assume that it has a definite value. As an illustration, consider again the trivial example of note 5. Suppose that two observers  $\mathcal{O}_1$  and  $\mathcal{O}_2$  predict the statistics of  $x_2$ , but only  $\mathcal{O}_1$  knows the value of  $x_1$ . In average for  $k \in \llbracket 0, 3 \rrbracket$ ,  $\mathbb{P}_{\mathcal{O}_1}(x_2 = k) = \frac{1}{2} \times 0 + \frac{1}{2} \times \frac{1}{2} = \frac{1}{4} = \mathbb{P}_{\mathcal{O}_2}(x_2 = k)$ , but this fundamentally relies on the total probability formula, *i.e.* on the fact that the two possible histories  $x_1 = 0$  and  $x_2 = 1$  did not interfere.

<sup>12</sup>Note that for the unitary evolution imposed by Wigner to entail re-coherence, it must decrease the entanglement entropy between  $\mathcal{S}$  and  $\mathcal{E}$  so, one might say, invert the arrow of time.

<sup>13</sup>To state this, as well as the second paragraph of §1.2.3, we have tacitly assumed that the physicist’s memories are entirely encoded in her brain’s material structure. But do the application of  $U^{-1}$  by Wigner really suppresses such immaterial things as memories? If not, it would in theory be possible, for instance, to both observe interferences fringes and know the which-way information in a double slit experiment. However, this hypothesis is not scientifically testable at present because we can’t apply precise unitary evolutions on human beings, and will probably never. Personally, I am convinced that spirit is not reducible to matter, but that simple cognitive processes, such that the task of remembering whether the detector was turned on or not, are typically operated by the brain and presumably reducible to it.

therefore facts can not exist absolutely. (Note that, for their ‘Local Friendliness (LF) inequalities’ to be violated, they need the superobservers to be able to recombine histories by applying unitary evolutions quite similar to the  $U^{-1}$  of our example.) This is reminiscent of Zurek’s proposal: ‘I strongly suspect that the ultimate message of quantum theory is that the separation between what exists and what is known to exist – between the epistemic and the ontic – must be abolished’ (Zurek, 2022). This also reminds the ideas developed in Auffèves and Grangier (2016) and Auffèves and Grangier (2018), in particular the principle according to which ‘In QM, modalities are attributed jointly to the system and to the context’, and finally the ‘background independence’ guiding principle for quantum gravity (see Markopoulou (2009)), in particular the definition given by Dreyer: ‘a theory is background independent if all observations are made by observers *inside* the system’. See also note 10 and the concept of earthy in the work of Latour (2015).

3. Suppose now that Wigner is able to perform any measurement of the room’s content. Note what a technical feat it must be: for it to be correct, no internal evolution and no external entanglement should occur during the measurement, in particular it must be faster than the already incredibly short (self)decoherence time in position (Joos, 1996, Table 3.1). Thereby, Wigner can make the possible histories of the room interfere without first erasing the friend’s facts. Starting again from the state  $|\Psi_+\rangle = \frac{1}{\sqrt{2}}(|\uparrow \mathcal{F}_\uparrow\rangle + |\downarrow \mathcal{F}_\downarrow\rangle)$ , let Wigner measure in the  $(|\Psi_+\rangle, |\Psi_-\rangle)$  basis where  $|\Psi_-\rangle = \frac{1}{\sqrt{2}}(|\uparrow \mathcal{F}_\uparrow\rangle - |\downarrow \mathcal{F}_\downarrow\rangle)$ , so that he expects to obtain  $|\Psi_+\rangle$  with probability 1. On the other hand, the friend, considering herself in the collapsed state  $|\uparrow \mathcal{F}_\uparrow\rangle$  or  $|\downarrow \mathcal{F}_\downarrow\rangle$ , predicts only a  $\frac{1}{2}$  probability to be measured in  $|\Psi_+\rangle$ , leading to statistical inconsistency. This happens because the friend’s collapse was not justified, since Wigner’s measurement recombines the two possible histories.

At the end of the measurement, the friend is still in the state  $|\Psi_+\rangle$  both for Wigner and herself. How is it like to be superposed? It is probably not a big deal, because (i) everyone is always superposed in most bases, (ii) as seen in 1., Wigner can prepare the friend in a superposed state without even directly interacting with her, (iii) after the measurement, simply by accessing her memories (which may have been rewritten during the measurement, see 4.), the friend splits the histories in the position basis (corresponding for instance to her two possible brain’s shapes) and knows which one she is in, therefore she feels nothing more than being in the state  $|\uparrow \mathcal{F}_\uparrow\rangle$  or  $|\downarrow \mathcal{F}_\downarrow\rangle$ . One is never spatially superposed from one’s point of view.

4. Let’s finish with the following interesting experiment. Let  $(|0\rangle, |1\rangle)$  be two possible orthogonal states for the friend in the position basis (for example ‘friend at the right of the room’ and ‘friend at the left of the room’, or ‘friend remembering the digit 0’ and ‘friend remembering the digit 1’), and let Wigner and his friend initially agree together that the latter is in the state  $|0\rangle$ . Wigner now isolates the friend, measures in the  $(\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle), \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle))$  basis, then opens the door to look at or ask his friend, *i.e.* measures her in the  $(|0\rangle, |1\rangle)$  basis. Their wave functions describing the friend are initially the same, so here again the statistics are consistent, but they both predict a  $\frac{1}{2}$  probability to find the friend in the new state  $|1\rangle$ . How can this be? This is simply an illustration of item 2. (ii) of §1.1.1, namely that measurements require to interact and perturb the system, all the more as they differ from a position measurement. Here, all the friend’s particles have been moved, or the friend’s brain has been completely reshaped by the measurement.

At the end of the day, even for omnipotent observers, QM doesn’t lead to inconsistencies at any scale, provided all collapses are justified. It is not really weirder than what it already is for particles, except that it highlights the relativity of facts.

### 1.3.6 What is an observer?

In §1.2.3, we used the word ‘observer’ in our definition of the selective measurement, but we haven’t specified yet what can be deemed an observer. The problem is raised by Bong et al. (2020): ‘the attribution of a “fact” to the friend’s measurement (...) depends on what counts as an “observer” (and what counts as a “measurement”)', because the experiment they conducted to violate the LF inequalities used photons as friends. So if photons have facts, these are relative, but do we want to consider photons as observers?

This question is not so deep since it is a mere choice of definition; any choice can be acceptable as long as it is consistent with what we have elaborated so far. Does it constrain the definition of an observer? We mainly need observers to have a memory. A few specific degrees of freedom of a particle could suffice,

although in this case very few facts can be stored and they are presumably quite short-lived. Note that the verification carried out in §1.3.5 guarantees the consistency even between arbitrary small observers, because the fewer facts can be stored, the fewer collapses are applied and the more alike the different observers' wave functions are, so the fewer inconsistencies can occur.

That said, what is the point for us to write the wavefunction from a photon's point of view, suppose its facts and apply the collapse for it? Would that even deserve to be called 'doing physics'? In the end, the most interesting characteristic that we may expect from an observer is perhaps the ability to *communicate* facts and subjective experiences. If we agree on that, it is probably better to keep restricting observers to human beings.

## 2 Measurements in relativistic quantum mechanics

### 2.1 The conditions (C) and (MC)

#### 2.1.1 Literal and mathematical formulation of the conditions

Let's now consider the third problem listed in the introduction. Standard QFT textbooks generally focus on constructing the quantum fields and the dynamics in the Fock space, but usually don't tackle the question of the measurements, as though it were a purely non-relativistic topic. More generally, most of them don't check the compatibility of QM with the disappearance of the notion of instantaneity in special relativity. There have been surprisingly few works addressing this question, and we will compare our results to the previous ones in §2.1.4.

There are two different sources of instantaneity in QM that could cause troubles. When two subsystems are entangled, they must be considered as a whole<sup>14</sup>, therefore:

- any physical evolution (for instance a non-selective measurement (2)) on the first instantaneously affects the whole state no matter how far the other part may be,
- any measurement performed on the first allows the observer to apply a collapse (1) on the whole state, which is generally presented as an instantaneous update.

However, according to the understanding of the collapse developed in §1.2, the second item is not problematic at all, because the epistemic update of one particular observer has no reason to be constrained by the speed of light. As ? wrote: 'When the Queen dies in London (may it long be delayed) the Prince of Wales, lecturing on modern architecture in Australia, becomes instantaneously King', and he could have added: from the Queen and her entourage's point of view. On the other hand, what has to be constrained by special relativity are the physically predictable effects, so that no experiment conducted on the King can determine faster than light whether the Queen is alive. For this to be prevented in QM despite the non-locality of the entanglement phenomenon, the following consistency condition has to hold:

**(C)** *For all quantum systems composed of two entangled subsystems, any physical evolution of the first must leave invariant **the statistical results** of any measurement on the second, if the two are spacelike separated.*

In the following, we will also consider the following more specific consistency condition (dealing only with measurements), that will subsequently be referred to as the condition (MC), because it is easier to manipulate mathematically and will turn out to be equivalent to (C):

**(MC)** *For all quantum systems composed of two entangled subsystems, any ideal projective measurement of the first must leave invariant **the statistical results** of any measurement on the second, if the two measurements are spacelike separated.*

Indeed, if (MC) were not satisfied (*a fortiori* (C)), the theory would face two types of inconsistencies:

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<sup>14</sup>It has been experimentally confirmed that non-local correlations are satisfied (almost) immediately: in Zbinden et al. (2001), the authors present an incredibly precise experimental test which has allowed them to 'set a lower bound on the speed on quantum information to  $10^7c$ , *i.e.* seven orders of magnitude larger than the speed of light.'

- *Non covariance* — Consider two entangled quantum systems that violate the condition (MC). Then there exists an experimental protocol concerning the second system that yields different statistical results depending on whether a certain measurement has been performed on the first system or not, such that the two measurements are spacelike separated. Thus one can find a reference frame in which the measurement on the first system happens *before* the other measurement, and another reference frame in which it happens *after*. Consequently, the statistical predictions of the theory depend on the reference frame.
- *Faster-than-light communication* — Consider a reference frame in which Alice and Bob are apart, motionless, and share  $N$  entangled pairs that violate the condition (MC) as well as two synchronized clocks. At  $t = 0$ , Alice performs the suitable measurement on each of the  $N$  subsystems if she wants to communicate the bit 0, or do nothing if she wants to communicate the bit 1. At  $t = 0^+$ , Bob performs the corresponding measurement on each of the subsystems in his possession: the statistical distribution he obtains allows him to distinguish if Alice has sent the bit 0 or 1, with an error margin arbitrarily small when  $N$  goes larger.

The condition (MC) can be given a precise mathematical formulation. Let  $\mathcal{S}_1 + \mathcal{S}_2$  be two entangled systems described by a Hilbert space  $\mathcal{H}_1 \otimes \mathcal{H}_2$ , prepared in a state  $\rho$ . Let also  $\hat{A}$  (resp.  $\hat{B}$ ) be a hermitian operator of  $\mathcal{H}_1$  (resp.  $\mathcal{H}_2$ ) of spectral decomposition  $\hat{A} = \sum_{x \in \text{spec} \hat{A}} x \Pi_x^{(1)}$  (resp.  $\hat{B} = \sum_{y \in \text{spec} \hat{B}} y \Pi_y^{(2)}$ ) where the  $\Pi_x^{(1)}$  (resp.  $\Pi_y^{(2)}$ ) are the spectral projectors of the observable. If an ideal projective measurement of  $A$  is performed on  $\mathcal{S}_1$ , the whole state evolves to  $\sum_{x \in \text{spec} \hat{A}} (\Pi_x^{(1)} \otimes \mathbb{1}_2) \rho (\Pi_x^{(1)} \otimes \mathbb{1}_2)$  (non-selective operation (2)). After this, in a given reference frame, the system evolves according to a unitary operator  $U$  (see Fig. 1). Since the state of  $\mathcal{S}_2$ , obtained by tracing over  $\mathcal{S}_1$ , fully characterizes the probabilities of any measurement on  $\mathcal{S}_2$ , the invariance of the statistics of  $\mathcal{S}_2$  under the measurement of  $\mathcal{S}_1$  is expressed by:

### Mathematical formulation of (MC)

$$(MC) \quad \forall \rho, \forall U, \forall \hat{A}, \quad \text{tr}_1 \left( \sum_{x \in \text{spec} \hat{A}} U (\Pi_x^{(1)} \otimes \mathbb{1}_2) \rho (\Pi_x^{(1)} \otimes \mathbb{1}_2) U^\dagger \right) = \text{tr}_1 (U \rho U^\dagger).$$

Or, equivalently, due to the universal property of the partial trace:

$$\forall \rho, \forall U, \forall \hat{A}, \forall \hat{B}, \forall y_0 \in \text{spec} \hat{B},$$

$$\sum_{x \in \text{spec} \hat{A}} \text{tr} \left( U (\Pi_x^{(1)} \otimes \mathbb{1}_2) \rho (\Pi_x^{(1)} \otimes \mathbb{1}_2) U^\dagger (\mathbb{1}_1 \otimes \Pi_{y_0}^{(2)}) \right) = \text{tr} (U \rho U^\dagger (\mathbb{1}_1 \otimes \Pi_{y_0}^{(2)})). \quad (3)$$

Concerning (C), the most general physical evolution that  $\mathcal{S}_1$  may undergo is a unitary  $V$  in  $\mathcal{H}_1 \otimes \mathcal{H}_\mathcal{E}$  (possibly non-trivial only on  $\mathcal{H}_1$ ), where  $\mathcal{E}$  stands for any external third system. They are initially uncoupled, so that the initial state is  $\rho \otimes \rho_\mathcal{E} \in \mathcal{S}(\mathcal{H}_1 \otimes \mathcal{H}_2) \otimes \mathcal{S}(\mathcal{H}_\mathcal{E})$ . Then, as previously,  $\mathcal{S}_1 + \mathcal{S}_2$  evolves according to a unitary  $U$  in  $\mathcal{H}_1 \otimes \mathcal{H}_2$ . For the theory to be consistent, whether  $V$  has been applied or not must not modify the statistics of  $\mathcal{S}_2$ , hence the following criterion:

### Mathematical formulation of (C)

$$(C) \quad \forall \rho, \forall \rho_\mathcal{E}, \forall U, \forall V, \quad \text{tr}_1 \left( U \text{tr}_\mathcal{E} \left[ (V \otimes \mathbb{1}_2) (\rho_\mathcal{E} \otimes \rho) (V^\dagger \otimes \mathbb{1}_2) \right] U^\dagger \right) = \text{tr}_1 (U \rho U^\dagger).$$

Or, equivalently, due to the universal property of the partial trace:

$$\forall \rho, \forall \rho_\mathcal{E}, \forall U, \forall V, \forall \hat{B}, \forall y_0 \in \text{spec} \hat{B},$$

$$\text{tr} \left( U \text{tr}_\mathcal{E} \left[ (V \otimes \mathbb{1}_2) (\rho_\mathcal{E} \otimes \rho) (V^\dagger \otimes \mathbb{1}_2) \right] U^\dagger (\mathbb{1}_1 \otimes \Pi_{y_0}^{(2)}) \right) = \text{tr} (U \rho U^\dagger (\mathbb{1}_1 \otimes \Pi_{y_0}^{(2)})).$$

Clearly, when  $\mathcal{E}$  is a measurement apparatus that causes perfect and immediate decoherence of  $\mathcal{S}_1$  in the eigenbasis of  $\hat{A}$ , then  $\text{tr}_\mathcal{E} \left[ (V \otimes \mathbb{1}_2) (\rho_\mathcal{E} \otimes \rho) (V^\dagger \otimes \mathbb{1}_2) \right] = \sum_{x \in \text{spec} \hat{A}} (\Pi_x^{(1)} \otimes \mathbb{1}_2) \rho (\Pi_x^{(1)} \otimes \mathbb{1}_2)$  (recall that perfect decoherence in a given basis amounts, by definition, to extracting the diagonal part of the density matrix in this basis) and we recover the previous condition (MC). Therefore **(C) implies (MC)**.



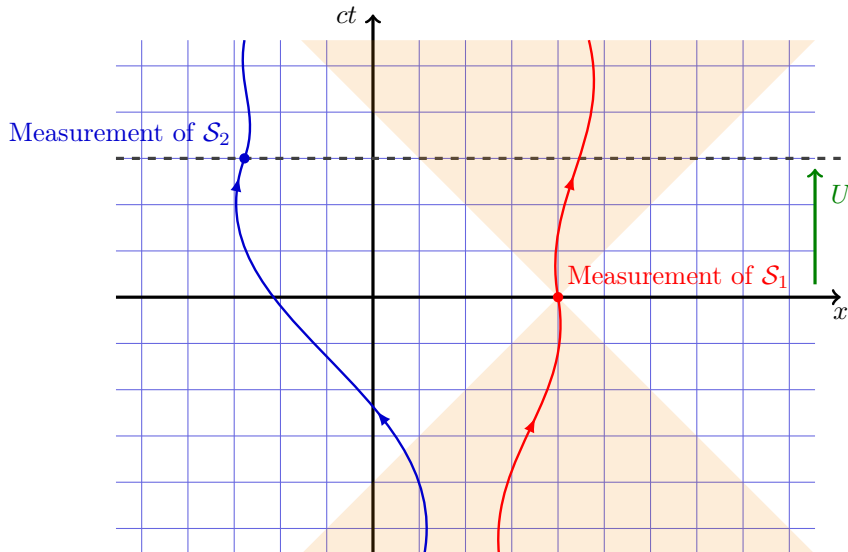


Figure 1: Spacelike measurements

### 2.1.2 Further remarks on the probabilistic nature of QM

In the literal formulations of (C) and (MC) given above, we have highlighted the fact that these conditions only constrain the statistical results of measurements. This is because special relativity only imposes the covariance of what is predictable by a given theory; and QM, being a probabilistic theory, only predicts the statistics. Similarly, the ability to transmit information depends on the best theory available to the communicators. If they can't do better than probabilistic predictions, Alice has to be able to modify the statistical results of a repeated experiment on Bob's side in order to send him a bit.

Alternatively, if they had a deterministic hidden-variable theory superseding QM, only one run could suffice. Such a theory should then be constrained by a stronger version of (C) where the word 'statistical' is removed. But this is precisely forbidden by Bell's theorem (Bell, 1964) which implies that the theory necessarily would display non-local features. More precisely, either the outcome or parameter independence assumption would have to be violated, allowing in any case for superluminal signalling and covariance issues; see Myrvold et al. (2021) for an excellent introduction to everything about Bell's theorem. See also Helling (2019) for a simple but illuminating analytic example of non-locality in Bohmian mechanics compared to QM, where it is also shown that no probabilistic knowledge other than the one given by the wave function is compatible with special relativity. Quantum entanglement seemed spooky to Einstein because the collapse was perceived as physical and the wave function as observer-independent, but it is actually the contrary: QM is the only possible non-spooky theory! This remark provides another proof of the fact that we will never be able to build a deterministic theory supplanting QM, based on an argument completely different from the one already exposed in §1.3.4. This is also why the terms added to the Schrödinger equation in collapse models will always be stochastic, 'because otherwise [they] would allow for faster-than-light communication' (Bassi et al., 2013).

### 2.1.3 (F) implies (C)

Let's now check that the condition (C) (*a fortiori* (MC)) is indeed satisfied in any relativistic quantum theory provided it satisfies the following factorization property (F), generally assumed in QM:

**(F)** For all pairs of isolated systems<sup>15</sup>  $\mathcal{S}_1$  and  $\mathcal{S}_2$ , the unitary evolution operator of  $\mathcal{S}_1 + \mathcal{S}_2$  takes the factorized form  $U = U_1 \otimes U_2$ .

<sup>15</sup>We say that two systems are isolated if no particles of  $\mathcal{S}_1$  and  $\mathcal{S}_2$  can meet each other. The possibility to write  $U$  as  $U_1 \otimes U_2$  is sometimes taken as a definition for being isolated, or we could have replaced 'isolated' by 'spacelike separated' in all this subsection §2.1, but our choice will become clear when discussing locality in §2.2. Of course, the main difficulty with our

Let's keep the notations introduced in §2.1.1. It is clear that the systems are isolated between the two measurements, since they are spacelike separated; therefore, we may write  $U = U_1 \otimes U_2$  and:

$$\begin{aligned}
\mathbb{P}(B = y_0) &= \text{tr} \left( U \text{tr}_\mathcal{E} \left[ (V \otimes \mathbb{1}_2)(\rho_\mathcal{E} \otimes \rho)(V^\dagger \otimes \mathbb{1}_2) \right] U^\dagger (\mathbb{1}_1 \otimes \Pi_{y_0}^{(2)}) \right) \\
&= \text{tr} \left( \text{tr}_\mathcal{E} \left[ (V \otimes \mathbb{1}_2)(\rho_\mathcal{E} \otimes \rho)(V^\dagger \otimes \mathbb{1}_2) \right] (\mathbb{1}_1 \otimes U_2^\dagger \Pi_{y_0}^{(2)} U_2) \right) \\
&= \text{tr} \left( (V \otimes \mathbb{1}_2)(\rho_\mathcal{E} \otimes \rho)(V^\dagger \otimes \mathbb{1}_2) (\mathbb{1}_\mathcal{E} \otimes \mathbb{1}_1 \otimes U_2^\dagger \Pi_{y_0}^{(2)} U_2) \right) \\
&= \text{tr} \left( \rho_\mathcal{E} \otimes (\rho U_2^\dagger \Pi_{y_0}^{(2)} U_2) \right) \\
&= \text{tr} \left( U \rho U^\dagger (\mathbb{1}_1 \otimes \Pi_{y_0}^{(2)}) \right)
\end{aligned}$$

where we have used the fact that  $\text{tr}(\rho_\mathcal{E}) = 1$  in the last step, and the property of the partial trace  $\text{tr}(\text{tr}_1(A_{12})B_2) = \text{tr}(A_{12}(\mathbb{1}_1 \otimes B_2))$  in the third step. [To prove this equality, it is enough by linearity to check it for pure tensors  $A_{12} = A_1 \otimes A_2$ ; it then simply reads:  $\text{tr}(\text{tr}_1(A_1 \otimes A_2)B_2) = \text{tr}(\text{tr}(A_1)A_2 B_2) = \text{tr}(A_1) \text{tr}(A_2 B_2) = \text{tr}(A_1 \otimes A_2 B_2) = \text{tr}((A_1 \otimes A_2)(\mathbb{1}_1 \otimes B_2))$ .] Thus **(F) implies (C)**.

We have therefore established that any relativistic quantum theory which satisfies (F) is consistent with an instantaneous effect of physical interactions on an entangled state, even though the notion of instantaneity is frame dependent. Different observers may write different states for the entangled pair, but they agree on the statistics. In this sense, this consistency check pursues, in the relativistic context, the verifications undertaken in §1.3.5. Once again, understanding that there is one wave function per observer (for a probability distribution always depends on what is known, recall the end of §1.2.3 and Bell's example in §2.1.1) solves a lot of spooky problems, because then the wave function doesn't have to be covariant, only the physics have to. Also, what precedes constitutes the proof that an entangled pair can not convey information in QM, known as the no-communication theorem.

**Remark 2.1.** In §2.1.1, we have motivated the importance of (C) by the fact that two kinds of inconsistencies appear if it were not satisfied. The above proof suffices to exclude the second kind (faster-than-light communication), but lacks an additional argument to get rid of the first kind (non covariance). Indeed, it has assumed the choice of a fixed reference frame, in which the time evolutions of the systems between the instants  $t_{1i}$ ,  $t_{1f}$  and  $t_2$  (corresponding to the beginning and the end of the evolution undergone by  $\mathcal{S}_1$  and the measurement performed on  $\mathcal{S}_2$  respectively) are given by unitary operators  $V_{t_{1i}, t_{1f}}$  and  $U_{t_{1f}, t_2}$  that we have simply denoted  $V$  and  $U$ . In another reference frame, however, neither the temporal axis nor the unitary operators are conserved. Even the possible outcomes of the measurements may undergo a Lorentz transformation, if they correspond to the position observable for example. The invariance of the statistics between two reference frames  $\mathcal{R}$  and  $\mathcal{R}'$  may be written more precisely as follows (one assumes for simplicity  $t_{1i} = t_{1f} = t_1 > t_2$ , with  $t_2 < t_1$  in  $\mathcal{R}$  and denote  $t_0$  an earlier time at which the two entangled systems were separated):

$$\left\{ \begin{array}{ll}
\bullet \text{tr} (U_{t_0, t_2}^{(\mathcal{R})} \rho U_{t_0, t_2}^{(\mathcal{R})\dagger} (\mathbb{1}_1 \otimes \Pi_{y_0}^{(2)})) = \text{tr} (U_{t'_0, t'_2}^{(\mathcal{R}')} \rho U_{t'_0, t'_2}^{(\mathcal{R}')\dagger} (\mathbb{1}_1 \otimes \Pi_{y'_0}^{(2)})) & \text{if } t'_2 < t'_1. \\
\bullet \text{tr} (U_{t_0, t_2}^{(\mathcal{R})} \rho U_{t_0, t_2}^{(\mathcal{R})\dagger} (\mathbb{1}_1 \otimes \Pi_{y_0}^{(2)})) \\
= \text{tr} (U_{t'_1, t'_2}^{(\mathcal{R}')} \text{tr}_\mathcal{E} [(V_{t'_1}^{(\mathcal{R}')} \otimes \mathbb{1}_2)(\rho_\mathcal{E} \otimes U_{t'_0, t'_1}^{(\mathcal{R}')} \rho U_{t'_0, t'_1}^{(\mathcal{R}')\dagger})(V_{t'_1}^{(\mathcal{R}')} \otimes \mathbb{1}_2)] U_{t'_1, t'_2}^{(\mathcal{R}')\dagger} (\mathbb{1}_1 \otimes \Pi_{y'_0}^{(2)})) & \text{if } t'_2 > t'_1
\end{array} \right.$$

The first line is simply the covariance of the theory in the absence of measurements: we suppose it already established. The only case to examine involves the changes of reference frames that reverse the temporal order of  $t_1$  and  $t_2$ . In particular, since the only potential discontinuity occur when the sign of  $t_2 - t_1$  flips, it suffices to write the second line above in the limit  $\varepsilon, \varepsilon' \rightarrow 0$  of an infinitesimal change of frame with  $t_2 = t_1 - \varepsilon$  and  $t'_2 = t'_1 + \varepsilon'$ . In that case,  $U_{t'_a, t'_b}^{(\mathcal{R}')} \rightarrow U_{t_a, t_b}^{(\mathcal{R})}$  for all instants  $t_a$  and  $t_b$ , and  $y'_0 \rightarrow y_0$  so that the condition to verify reads:

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definition is that the concept of particle is ill-defined. In particular, Unruh (1976) showed that the very presence of a particle is frame-dependent. Also, should we consider virtual particles? How to include gravitational interactions in our treatment if the graviton actually doesn't exist?

$$\text{tr} \left( U_{t_0, t_2}^{(\mathcal{R})} \rho U_{t_0, t_2}^{(\mathcal{R})\dagger} (\mathbb{1}_1 \otimes \Pi_{y_0}^{(2)}) \right) = \text{tr} \left( U_{t_1, t_2}^{(\mathcal{R})} \text{tr}_{\mathcal{E}} \left[ (V_{t_1}^{(\mathcal{R})} \otimes \mathbb{1}_2) (\rho_{\mathcal{E}} \otimes U_{t_0, t_1}^{(\mathcal{R})} \rho U_{t_0, t_1}^{(\mathcal{R})\dagger}) (V_{t_1}^{(\mathcal{R})\dagger} \otimes \mathbb{1}_2) \right] U_{t_1, t_2}^{(\mathcal{R})\dagger} (\mathbb{1}_1 \otimes \Pi_{y_0}^{(2)}) \right)$$

but since  $U_{t_0, t_2}^{(\mathcal{R})} = U_{t_1, t_2}^{(\mathcal{R})} U_{t_0, t_1}^{(\mathcal{R})}$ , this is nothing but the condition (C) that has been proved applied to  $U_{t_0, t_1}^{(\mathcal{R})} \rho U_{t_0, t_1}^{(\mathcal{R})\dagger}$  instead of  $\rho$ .

#### 2.1.4 Comparison with previous works

The above verification of (C) is simple but essential to establish the compatibility between QM and special relativity. Of course, some previous works have already investigated these topics, although remarkably few. Bloch (1967) pointed out some apparent inconsistencies and new intuitions, notably: ‘it appears that either causality or Lorentz covariance of wave functions must be sacrificed’. In reply, Hellwig and Kraus (1970) published a paper in which they clarified Bloch’s ideas, checked a simple version (MC) (equation (6)) based on an assumption called locality (commutation of projectors associated to spacelike measurements), and then proposed that the effect of measurements (selective and non-selective) should be implemented along the past light cone so that their description becomes covariant. In a long footnote, Malament (1996) actually proved the logical equivalence between a simple version of (MC) and the fact that spacelike measurements projectors commute. A general proof of (MC), presented as a no-communication theorem, was detailed in Ghirardi et al. (1980). In a very inspiring work, Aharonov and Albert (1981) explained how non-local measurements can actually be implemented based on local interactions only. As a consequence, they claim that ‘the proposal that the reduction be taken to occur covariantly along the backward light cone (...) or along *any* hypersurface other than  $t = 0$  will fail’ (although we must confess that we didn’t understand their argument), and that ‘the covariance of relativistic quantum theories (...) resides *exclusively* in the experimental probabilities, and not in the underlying quantum states. The states themselves make sense only within a given frame’. Surprisingly enough, all these works were concerned with (MC) (even the Wikipedia entry ‘No-communication theorem’ only deals with (MC)), which is after all a very artificial statement compared to (C).

Much more recently, Polo-Gómez et al. (2022) exactly proved the condition (C) (equation (29)) in the context of QFT based on the microcausality hypothesis, used to derive a relation not very different from (F) (equation (28)). They also argued that selective measurements can not be compatible with special relativity unless one accepts to define as many wave functions as there are observers and that the update occurs along the future light cone. Contrary to the latter and in accordance to Aharonov and Albert (1981), we consider that, as soon as the wave function is understood to be observer dependent (hence, in particular, frame dependent), it is pointless and even awkward to ask for frame independent updates. Why would a given observer in her reference frame wait some time before using all the information she has (in the case of a selective measurement), or before taking into account the evolution she knows has happened on  $\mathcal{S}_1$  to write down the whole state of  $\mathcal{S}_1 + \mathcal{S}_2$ ? Moreover, none of these approaches find out how fruitful the precise formulation of (C) and (MC) may be, as we shall now see in the next subsections.

## 2.2 The locality hypothesis

### 2.2.1 (L’) implies (L) and (F)

In §2.1.3, we have seen that (F) ensures a theory to be consistent with special relativity. The property (F) is directly linked to a fundamental principle of physics, namely the locality hypothesis (L) which states the absence of interactions at a distance:

**(L)** *Two localized particles can interact<sup>16</sup> at a given time only if they are located at the same point in space.*

In QFT, this hypothesis has a more specific formulation:

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<sup>16</sup>Again, as in note 15, this term can be confusing. One can think of possible interactions as allowed vertices in Feynman diagrams, but what about the gravitational interaction?

**(L')** The interaction Hamiltonian of any system can be written in the form  $H_{int}(t) = \int_{\mathbb{R}^3} H_{int}(\vec{x}, t) d^3x$  where  $H_{int}$  is a field of operators defined on the whole spacetime.

Why is (L') called a locality hypothesis? Suppose one wants to compute the coefficient  $S_{\alpha\beta}$  of the  $S$ -matrix, where  $|\alpha\rangle$  is the state composed of two particles localized in  $\vec{x}_1$  and  $\vec{x}_2$ , *i.e.*

$$|\alpha\rangle = \int \frac{d^3\vec{p}_1}{(2\pi)^{3/2}} e^{-i\vec{p}_1 \cdot \vec{x}_1} a^\dagger(\vec{p}_1) \int \frac{d^3\vec{p}_2}{(2\pi)^{3/2}} e^{-i\vec{p}_2 \cdot \vec{x}_2} b^\dagger(\vec{p}_2) |0\rangle.$$

Under the hypothesis (L'), the Born approximation reads:  $S_{\alpha\beta} = -i \int d^4x \langle \alpha | H_{int}(x) | \beta \rangle$ . Furthermore, even without specifying  $H_{int}(x)$ , one knows by covariance that the latter can only be built from the different quantum fields of the theory, and that if an interaction is possible it necessarily contains two quantum fields of the type of the particles of the state  $|\alpha\rangle$ , and so includes at least an expression of the form  $\int \frac{d^3p}{(2\pi)^{3/2}} e^{-ipx} u(\vec{p}) a^\dagger(\vec{p}) \int \frac{d^3p'}{(2\pi)^{3/2}} e^{-ip'x} v(\vec{p}') b^\dagger(\vec{p}')$  where  $u$  and  $v$  are objects that depend on the nature of the quantum fields. Using the (anti-)commutation relation for the operators  $a$  et  $b$ , one finds:

$$\langle \alpha | H_{int}(x) | \beta \rangle = \iint d^3\vec{p}_1 d^3\vec{p}_2 [\dots] e^{i\vec{p}_1 \cdot \vec{x}_1} e^{i\vec{p}_2 \cdot \vec{x}_2} e^{-i(p_1+p_2)x} \propto \delta^{(3)}(\vec{x} - \vec{x}_1) \delta^{(3)}(\vec{x} - \vec{x}_2) \propto \delta^{(3)}(\vec{x}_1 - \vec{x}_2).$$

This means that two localized particles can interact at a given time only if they are located at the same point in space, namely (L). The electromagnetic force between two charged particles, for example, is an interaction 'at a distance' only because (virtual) photons are exchanged between them. Furthermore, under the hypothesis (L), two isolated<sup>17</sup> systems  $\mathcal{S}_1$  and  $\mathcal{S}_2$  can not interact, therefore the total Hamiltonian reads  $H = H_1 \otimes \mathbb{1}_2 + \mathbb{1}_1 \otimes H_2$  with  $H_1$  et  $H_2$  the internal Hamiltonians of  $\mathcal{S}_1$  et  $\mathcal{S}_2$ . Therefore, the unitary evolution operator of  $\mathcal{S}_1 + \mathcal{S}_2$  is  $U = e^{itH} = e^{itH_1} \otimes e^{itH_2} = U_1 \otimes U_2$ . We have just shown that **(L') implies (L) which implies (F)**.

### 2.2.2 (F) implies (L) and (almost) (L')

Conversely, suppose that (F) is true. Then, if  $\mathcal{S}_1$  and  $\mathcal{S}_2$  are two isolated systems, the total Hamiltonian of  $\mathcal{S}_1 + \mathcal{S}_2$  reads:

$$H = \frac{1}{i} \frac{dU}{dt} = \frac{1}{i} \frac{dU_1 \otimes U_2}{dt} \Big|_{t=0} = \frac{1}{i} \frac{dU_1}{dt} \Big|_{t=0} \otimes \mathbb{1}_2 + \mathbb{1}_1 \otimes \frac{1}{i} \frac{dU_2}{dt} \Big|_{t=0} = H_1 \otimes \mathbb{1}_2 + \mathbb{1}_1 \otimes H_2 \Rightarrow H_{int} = 0,$$

hence **(F) implies (L)**. Now, it is natural to imagine (this is not a proof!) that in a theory satisfying (L), the interaction Hamiltonian of an arbitrary system should be composed of a combination of local operators of the form  $\int_{\mathcal{D}} H_{int}(\vec{x}, t)$  where  $\mathcal{D}$  is an integration domain *a priori* unknown. However, to preserve the translation invariance of the laws of physics,  $\mathcal{D}$  must be the whole space  $\mathbb{R}^3$ . Thus **(L) justifies the locality hypothesis in QFT (L')**.

### 2.2.3 (MC) implies (F)

We need our theories to satisfy (C). In Section 2.1.3, we have shown that  $(F) \Rightarrow (C)$  and remarked that (F) is generally postulated in QM. But, unlike (F), (C) is a direct consistency condition, so that it seems after all more natural to postulate the latter. One can then try to determine the set of unitaries compatible with (C), yielding a constraint on any unitary evolution operator associated to a system of the form  $\mathcal{S}_1 + \mathcal{S}_2$  with  $\mathcal{S}_1$  and  $\mathcal{S}_2$  spacelike separated. It happens that the more restrictive condition (MC) suffices to deduce (F), as shown in the following theorem.

**Theorem 2.2 ((MC)  $\Rightarrow$  (F)).** *Let  $U(t)$  be the unitary evolution operator, expressed in a fixed reference frame  $\mathcal{R}$ , of a quantum system  $\mathcal{S}_1 + \mathcal{S}_2$  composed of two isolated subsystems  $\mathcal{S}_1$  and  $\mathcal{S}_2$ , described in a relativistic quantum theory. Then there exist two unitary operators  $U_1(t)$  and  $U_2(t)$  such that:*

$$U(t) = U_1(t) \otimes U_2(t)$$

<sup>17</sup>Recall that we defined 'isolated' as the fact that no particles of  $\mathcal{S}_1$  and  $\mathcal{S}_2$  meet each other.

*Proof.* The two subsystems being isolated, it is possible to divide the temporal axis of  $\mathcal{R}$  into small time intervals such that  $\mathcal{S}_1$  and  $\mathcal{S}_2$  are spacelike separated<sup>18</sup> during any of these time intervals. On each interval, the theory satisfies (in particular) the condition (MC) by consistency, and it suffices to show there the factorization result. From now, for the sake of clarity, we will not write the parameter  $t$  which plays no role in the proof anymore. Let's denote  $\mathcal{H}_1$  and  $\mathcal{H}_2$  the Hilbert spaces associated with  $\mathcal{S}_1$  and  $\mathcal{S}_2$ , and assume first that they are finite dimensional with  $n_1 = \dim(\mathcal{H}_1)$  and  $n_2 = \dim(\mathcal{H}_2)$ . One can write  $U$  in the following generic form:

$$U = \sum_{1 \leq k, l \leq n_2} \alpha_{ikl} T_i \otimes |k\rangle \langle l|$$

with  $(|k\rangle \langle l|)_{1 \leq k, l \leq n_2}$  the canonical basis of  $\mathcal{L}(\mathcal{H}_2)$  associated with an orthonormal basis  $(|k\rangle)_{1 \leq k \leq n_2}$ , and  $(T_i)_i$  a collection of operators of  $\mathcal{L}(\mathcal{H}_1)$ . Up to a reorganization of the sum, one can assume that the  $(T_i)_i$  are linearly independent in  $\mathcal{L}(\mathcal{H}_1)$ .

When replacing  $U$  in the expression (3) for the condition (MC), one gets for all hermitian operators  $\hat{A}$  and  $\hat{B}$  of  $\mathcal{H}_1$  and  $\mathcal{H}_2$  and for all  $y_0 \in \text{spec} \hat{B}$ :

$$\begin{aligned} \forall \rho, \quad & \text{tr} \left( \rho \sum_{\substack{i, k, l \\ j, k', l'}} \left[ \sum_{x \in \text{spec} \hat{A}} \Pi_x^{(1)} T_j^\dagger T_i \Pi_x^{(1)} - T_j^\dagger T_i \right] \otimes \alpha_{ikl} \overline{\alpha_{jk'l'}} |l'\rangle \langle k'| \Pi_{y_0}^{(2)} |k\rangle \langle l| \right) = 0 \\ \Rightarrow \quad & \sum_{\substack{i, k, l \\ j, k', l'}} \left[ \sum_{x \in \text{spec} \hat{A}} \Pi_x^{(1)} T_j^\dagger T_i \Pi_x^{(1)} - T_j^\dagger T_i \right] \otimes \alpha_{ikl} \overline{\alpha_{jk'l'}} |l'\rangle \langle k'| \Pi_{y_0}^{(2)} |k\rangle \langle l| = 0. \end{aligned}$$

Note that  $\hat{B}$  may be chosen arbitrarily, as well as  $\Pi_{y_0}^{(2)}$ . In particular, for any pair  $\{k_1, k_2\} \subset \llbracket 1, n_2 \rrbracket$  and  $\mu, \nu \in \mathbb{C}$  such that  $|\mu|^2 + |\nu|^2 = 1$ , one can define  $\Pi_{y_0}^{(2)}$  to be the projector on the vector  $\mu |k_1\rangle + \nu |k_2\rangle$ , that is  $|\mu|^2 |k_1\rangle \langle k_1| + \mu \bar{\nu} |k_1\rangle \langle k_2| + \bar{\mu} \nu |k_2\rangle \langle k_1| + |\nu|^2 |k_2\rangle \langle k_2|$ . Inserting into the previous equation divides it in four sums:

$$\begin{aligned} \forall k_1, k_2, \quad & \sum_{\substack{i, l \\ j, l'}} \left[ \sum_{x \in \text{spec} \hat{A}} \Pi_x^{(1)} T_j^\dagger T_i \Pi_x^{(1)} - T_j^\dagger T_i \right] \\ & \otimes \left[ |\mu|^2 \alpha_{ik_1 l} \overline{\alpha_{jk_1 l'}} + \mu \bar{\nu} \alpha_{ik_2 l} \overline{\alpha_{jk_1 l'}} + \bar{\mu} \nu \alpha_{ik_1 l} \overline{\alpha_{jk_2 l'}} + |\nu|^2 \alpha_{ik_2 l} \overline{\alpha_{jk_2 l'}} \right] |l'\rangle \langle l| = 0. \end{aligned}$$

The particular cases  $\mu = 1, \nu = 0$  or  $\mu = 0, \nu = 1$  imply that the first and fourth terms actually always vanish. Setting  $\mu = \nu = \frac{1}{\sqrt{2}}$  or  $\mu = \frac{1}{\sqrt{2}}, \nu = \frac{i}{\sqrt{2}}$  leads to:

$$\begin{cases} \sum_{\substack{i, l \\ j, l'}} \left[ \sum_{x \in \text{spec} \hat{A}} \Pi_x^{(1)} T_j^\dagger T_i \Pi_x^{(1)} - T_j^\dagger T_i \right] \otimes \left[ \frac{1}{2} \alpha_{ik_2 l} \overline{\alpha_{jk_1 l'}} + \frac{1}{2} \alpha_{ik_1 l} \overline{\alpha_{jk_2 l'}} \right] |l'\rangle \langle l| = 0 \\ \sum_{\substack{i, l \\ j, l'}} \left[ \sum_{x \in \text{spec} \hat{A}} \Pi_x^{(1)} T_j^\dagger T_i \Pi_x^{(1)} - T_j^\dagger T_i \right] \otimes \left[ -\frac{i}{2} \alpha_{ik_2 l} \overline{\alpha_{jk_1 l'}} + \frac{i}{2} \alpha_{ik_1 l} \overline{\alpha_{jk_2 l'}} \right] |l'\rangle \langle l| = 0 \end{cases}$$

and taking appropriate linear combinations of these shows that the second and third terms vanish as well. Therefore:

$$\begin{aligned} \forall k, k', \quad & \sum_{\substack{i, l \\ j, l'}} \alpha_{ikl} \overline{\alpha_{jk'l'}} \left[ \sum_{x \in \text{spec} \hat{A}} \Pi_x^{(1)} T_j^\dagger T_i \Pi_x^{(1)} - T_j^\dagger T_i \right] \otimes |l'\rangle \langle l| = 0 \\ \Rightarrow \quad & \forall k, k', l, l', \quad \sum_{i, j} \alpha_{ikl} \overline{\alpha_{jk'l'}} \left[ \sum_{x \in \text{spec} \hat{A}} \Pi_x^{(1)} T_j^\dagger T_i \Pi_x^{(1)} - T_j^\dagger T_i \right] = 0 \end{aligned}$$

<sup>18</sup>Meaning that every point of the spacetime region spanned by  $\mathcal{S}_1$  is spacelike separated to every point of the spacetime region spanned by  $\mathcal{S}_2$ .

because  $(|l'\rangle \langle l|)_{1 \leq l, l' \leq n_2}$  is a basis of  $\mathcal{L}(\mathcal{H}_2)$ . This being true for all hermitian operators  $\hat{A}$ , the operator  $\sum_{i,j} \alpha_{ikl} \alpha_{jk'l'} T_j^\dagger T_i = (\sum_i \alpha_{ik'l'} T_i)^\dagger (\sum_i \alpha_{ikl} T_i)$  is diagonal in every orthonormal basis of  $\mathcal{H}_1$ , so it is a dilation:

$$\forall k, k', l, l', \exists \lambda_{kk'l'l'} \in \mathbb{C} : \left( \sum_i \alpha_{ik'l'} T_i \right)^\dagger \left( \sum_i \alpha_{ikl} T_i \right) = \lambda_{kk'l'l'} \mathbb{1}.$$

Obviously,  $\lambda_{kk'l'l'} \neq 0$ , otherwise  $\sum_i \alpha_{ikl} T_i = 0$  (for  $\mathcal{L}(\mathcal{H}_2)$  is a  $C^*$ -algebra) which would contradict the linear independence of the  $(T_i)_i$ . Moreover, by unicity of the inverse, we have for all  $k, l$ ,  $\frac{1}{\lambda_{k111}} \sum_i \alpha_{ikl} T_i = \frac{1}{\lambda_{1111}} \sum_i \alpha_{i11} T_i$  and since the  $(T_i)_i$  are linearly independent:

$$\forall k, l, \exists \beta_{kl} \in \mathbb{C} : \forall i, \quad \alpha_{ikl} = \beta_{kl} \alpha_{i11}.$$

It is now possible to factorize:

$$U = \sum_{\substack{i \\ 1 \leq k, l \leq n_2}} \beta_{kl} \alpha_{i11} T_i \otimes |k\rangle \langle l| = \left( \sum_i \alpha_{i11} T_i \right) \otimes \left( \sum_{1 \leq k, l \leq n_2} \beta_{kl} |k\rangle \langle l| \right) = U_1 \otimes U_2$$

where one can identify  $U_1$  and  $U_2$  to the evolution operators of  $\mathcal{S}_1$  and  $\mathcal{S}_2$ , which are necessarily unitary. Finally, this proof is purely algebraic and can be transposed without difficulty in the infinite dimensional case by replacing the sums by integrals.  $\square$

A consequence of this theorem is that (F), (C) and (MC) imply themselves circularly, therefore **(F), (C) and (MC) are logically equivalent**.

#### 2.2.4 Nonlocal measurements

We are now in position to address a thorny problem. Let's reproduce briefly how the aforementioned Aharonov and Albert (1981) proceed to implement what they call 'nonlocal measurements'. Consider a system  $\mathcal{S}$  composed of two spin  $\frac{1}{2}$  particles  $\mathcal{S}_1$  and  $\mathcal{S}_2$  to be measured, and an apparatus  $\mathcal{A}$  composed of two measuring particles  $\mathcal{A}_1$  and  $\mathcal{A}_2$ . Let the interaction be given by  $\hat{H}_{int} = g_1(t) \hat{X}_1 \hat{\sigma}_1^z + g_2(t) \hat{X}_2 \hat{\sigma}_2^z$ , where the  $g_i$  are two couplings which are non zero only during a short interval of time centered around  $t_0$ ,  $\hat{X}_i$  is  $\mathcal{A}_i$ 's position operator and  $\hat{\sigma}_i^z$  is  $\mathcal{S}_i$ 's spin operator along  $z$ . This Hamiltonian is simply the composition of two separate local measurements processes for  $\mathcal{S}_i$ 's spin, recorded in  $\mathcal{A}_i$ 's momentum  $P_i$ , since we have in the Heisenberg picture  $\Delta \hat{P}_i = \hat{P}_i(t > t_0) - \hat{P}_i(t < t_0) \propto \hat{\sigma}_i^z$ . If  $\mathcal{A}$  is initially prepared in an entangled state such that  $(\hat{P}_1 + \hat{P}_2) |\mathcal{A}\rangle = (\hat{X}_1 - \hat{X}_2) |\mathcal{A}\rangle = 0$  (check that this is indeed possible, in particular because  $[\hat{P}_1 + \hat{P}_2, \hat{X}_1 - \hat{X}_2] = 0$ ), then the nonlocal quantity  $J^z$  corresponding to the sum of the two spins is now measured by  $\mathcal{A}$  since  $\hat{J}^z = \hat{\sigma}_1^z + \hat{\sigma}_2^z \propto \hat{P}_1(t > t_0) + \hat{P}_2(t > t_0)$ . Moreover, it is possible to verify whether  $\mathcal{S}$  is in the state  $\frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) = |J^2 = 0, J^z = 0\rangle$  (a nonlocal statement), by measuring  $J^x$ ,  $J^y$  and  $J^z$  simultaneously (using 3 pairs of measuring particles) since this state is uniquely defined by the requirements  $J^x = J^y = J^z = 0$ .

Clearly, no relativistic inconsistencies can occur with such 'nonlocal measurements', because they arise from local Hamiltonians generating a unitary evolution of the form  $U_1 \otimes U_2$  (where  $U_i$  acts on  $\mathcal{H}_{\mathcal{S}_i} \otimes \mathcal{H}_{\mathcal{A}_i}$ ), therefore (C) is satisfied and the statistics of  $\mathcal{S}_1 + \mathcal{A}_1$  (*a fortiori* those of  $\mathcal{S}_1$ ) are unaltered by whatever may happen on  $\mathcal{S}_2 + \mathcal{A}_2$ . Quoting Aharonov and Albert: 'It is a requirement of relativistic causality, then, that although we may measure nonlocal properties of various systems, we must always carry out such measurements by means of local observations on the measuring apparatus; all measurements must ultimately be local ones'. On the other hand, they present a simple experimental setup in which the ability to perform a nonlocal measurement of  $J^2$  would allow for faster-than-light communication. Similarly, Sorkin (1993) points out that being able to measure the observable  $\Pi$  of orthogonal projection onto  $\frac{1}{\sqrt{2}}(|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle)$  would also bypass the speed of light. Certainly, these two measurements can not be realized by means of local Hamiltonians (otherwise they would satisfy (C)). But is there a simple criterion to determine whether a given nonlocal observable can be measured (like  $J^z$ ) or not (like  $J^2$  or  $\Pi$ )? As Sorkin deplors: 'the need to resort to a case-by-case analysis would still leave us without any clear formal criterion for which "observables" can be ideally-measured, and which cannot'.

First note that the answer depends on the definition of the measurement used. In the proposed setups,  $J^2$  and  $\Pi$  had to be *projectively* measured to lead to relativistic inconsistencies. If one rather uses our broader definition 1.1, then  $J^2$  and  $\Pi$  can be measured. Indeed, a few calculations show that a nonlocal

measurement of  $J^x$  as above followed by one of  $J^z$  is an interaction that splits the four possible histories of  $\mathcal{S}$  ( $|\uparrow\uparrow\rangle, |\downarrow\downarrow\rangle, \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle), \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$ ) regardless of its initial state. These are nothing but the eigenbasis of  $J^2$  (a simple flip of spin also yields an eigenbasis of  $\Pi$ ), although the interaction admits no conserved basis. So the question becomes: which observables can be (i) recorded, (ii) measured, (iii) projectively measured, by means of local Hamiltonians? The question (i) could be reformulated this way: what are the possible eigenbases of  $\text{tr}_{\mathcal{A}}(U_1 \otimes U_2(\rho_{\mathcal{S}} \otimes \rho_{\mathcal{A}})U_1^\dagger \otimes U_2^\dagger)$  for arbitrary unitaries  $U_1$  and  $U_2$ ? For (ii), one demands moreover that the eigenbasis be independent of  $\rho_{\mathcal{S}}$ , and for (iii) one adds the hypothesis that there exists a conserved basis in the measurement process. We keep the resolution of these questions for a future work, although we conjecture that the answers are (i) & (ii) any observable (iii) any observable diagonal in a product basis  $\mathcal{B} \otimes \mathcal{B}'$ .

### 2.3 The microcausality hypothesis

In addition of the locality hypothesis, there is another deep postulate of QFT: the microcausality hypothesis (M).

**(M)** *For all quantum fields  $\Phi$  and spacelike intervals  $x - y$ ,  $[\Phi(x), \Phi^\dagger(y)]_\pm = 0$  where  $[\ , ]_\pm$  stands for an anti-commutator or a commutator depending on the fermionic or bosonic nature of  $\Phi$ .*

Usually (Peskin, 2018, p.28) (Itzykson and Zuber, 2012, p.106) (Zee, 2010, p.121), standard QFT textbooks justify this hypothesis by invoking — for once! — the concept of measurement, but they generally make do with the affirmation that two spacelike measurements must be independent... without more explanations. Not only is the argument too vague, but it is hard to see how could the relation differ according to the fermionic or bosonic nature of the field. Weinberg, on the other hand, makes an interesting remark:

‘The condition [(M)] is often described as a causality condition, because if  $x - y$  is spacelike then no signal can reach  $y$  from  $x$ , so that a measurement of  $\Phi$  at point  $x$  should not be able to interfere with a measurement of  $\Phi$  or  $\Phi^\dagger$  at point  $y$ . Such considerations of causality are plausible for the electromagnetic field, any one of whose components may be measured at a given spacetime point, as shown in a classic paper of Bohr and Rosenfeld (1933). However, we will be dealing here with fields like the Dirac field of the electron that do not seem in any sense measurable. The point of view taken here is that [(M)] is needed for the Lorentz invariance of the S-matrix, without any ancillary assumptions about measurability or causality.’ (Weinberg, 1995, p.198)

Nevertheless, as Weinberg notes himself, microcausality is only a sufficient condition for the invariance of the S-matrix:

‘Theories of this class [satisfying (M)] are not the only ones that are Lorentz invariant, but the most general Lorentz invariant theories are not very different. In particular, there is always a commutation condition something like [(M)] that needs to be satisfied. This condition has no counterpart for non-relativistic systems, for which time-ordering is always Galilean-invariant. *It is this condition that makes the combination of Lorentz invariance and QM so restrictive.*’ (Weinberg, 1995, p.145)

However, this argument is valid only when one works with normal-ordered fields, but this writing is only a computation convenience and doesn’t have, as it seems, any physical meaning. The main effect of the normal ordering is to get rid of the infinite constants that appear in certain computations, by making finite all the matrix elements of the operators manipulated. This operation is justified when the divergences have no influence in the considered context. Alternatively, the prediction of the Casimir effect or the Lamb shift by the vacuum energy is only possible *without* normal-ordering (one substantially uses the fact that  $\langle 0|\Phi^2|0\rangle \neq 0$ , see (Itzykson and Zuber, 2012, p.111)). Presumably, a stronger argument is needed to justify the microcausality hypothesis (M). Note that (M) is especially crucial to prove the famous spin-statistics theorem (SS):

**(SS)** *Scalar and vector fields correspond to bosons, while Dirac fields correspond to fermions.*

Here is how most QFT textbooks proceed to establish (SS). Since we don't know the result yet, we have to compute both commutators and anti-commutators. A short calculation first shows that when  $\Phi$  is a Dirac field, we have:

$$[\Phi(x), \Phi^\dagger(y)]_\pm = \Delta_+(x-y) \mp \Delta_+(y-x) \quad (4)$$

depending on  $\Phi$  describing fermions or bosons, *i.e.*  $[a(\vec{p}), a^\dagger(\vec{p}')]_\pm = \delta^{(3)}(\vec{p} - \vec{p}')$ , and where  $\Delta_+(x) = \int \frac{d^3p}{(2\pi)^3 2p_0} e^{ip_\mu x^\mu}$  is shown to be a Lorenz invariant quantity. Likewise, when  $\Phi$  is a scalar or a vector field:

$$[\Phi(x), \Phi^\dagger(y)]_\pm = \Delta_+(x-y) \pm \Delta_+(y-x) \quad (5)$$

depending on  $\Phi$  describing fermions or bosons. It is quite easy to see that when  $x-y$  is spacelike,  $\Delta_+(x-y) - \Delta_+(y-x) = 0$ , while  $\Delta_+(x-y) + \Delta_+(y-x)$  is not identically zero (Weinberg, 1995, p.202). All these remarks, in addition to (M), imply (SS). Not the reverse: these computations generally don't justify (M) but only the fermionic or bosonic nature of a field, whereas one can intuitively feel that they contain much more information.

Let's present now a simultaneous proof of the microcausality hypothesis and the spin-statistics theorem that relies only on the consistency of the theory and on the above (anti-)commutators computations.

**Theorem 2.3** ((MC)  $\Rightarrow$  (M) and (SS)). *Let  $\Phi$  be a quantum field. Then, for all spacelike intervals  $x-y$ ,  $[\Phi(x), \Phi^\dagger(y)]_\pm = 0$  where  $[\ , ]_\pm$  stands for an anti-commutator or a commutator depending on  $\Phi$  being a Dirac field (and in that case it describes a fermion) or a scalar or vector field (and in that case it describes a boson).*

*Proof.* Let's first suppose that  $\Phi$  is effectively a measurable field, in other words  $\Phi(x)$  is an observable for all  $x$ , *i.e.* a hermitian operator. As stated by Weinberg in the previous quote, it is for example the case for the electromagnetic field that describes the photon. One can write its spectral decomposition  $\Phi(x) = \sum_{\lambda \in \text{spec } \Phi(x)} \lambda \Pi_\lambda^{(x)}$  (rigorously speaking, one should work with smeared fields, and the sum could be an integral in the case of a continuous spectrum). Contrary to the previous sections, there is now only one Hilbert space, the Fock space  $\mathcal{H}_{Fock}$ , and the system's state is given by a density matrix  $\rho \in \mathcal{S}(\mathcal{H}_{Fock})$ . For all spacelike intervals  $x-y$ , a measurement of  $\Phi(x)$  doesn't affect the statistics of a measurement of  $\Phi(y)$  if and only if the following condition, variant of (MC), is satisfied:

$$\forall \rho, \forall \mu \in \text{spec } \Phi(y), \quad \text{tr} \left( \left( \sum_{\lambda \in \text{spec } \Phi(x)} \Pi_\lambda^{(x)} \rho \Pi_\lambda^{(x)} \right) \Pi_\mu^{(y)} \right) = \text{tr}(\rho \Pi_\mu^{(y)})$$

where we implicitly moved to a reference frame  $\mathcal{R}$  in which  $x^0 = y^0$ , so as to avoid to introduce the (non covariant) unitary evolution operators<sup>19</sup>. It yields:

$$\begin{aligned} & \forall \rho, \forall \mu \in \text{spec } \Phi(y), \quad \text{tr} \left( \rho \left( \sum_{\lambda \in \text{spec } \Phi(x)} \Pi_\lambda^{(x)} \Pi_\mu^{(y)} \Pi_\lambda^{(x)} - \Pi_\mu^{(y)} \right) \right) = 0 \\ \Rightarrow & \forall \mu \in \text{spec } \Phi(y), \quad \sum_{\lambda \in \text{spec } \Phi(x)} \Pi_\lambda^{(x)} \Pi_\mu^{(y)} \Pi_\lambda^{(x)} = \Pi_\mu^{(y)} \\ \Rightarrow & \sum_{\lambda \in \text{spec } \Phi(x)} \Pi_\lambda^{(x)} \Phi(y) \Pi_\lambda^{(x)} = \Phi(y). \end{aligned}$$

Thus,  $\Phi(y)$  is (block) diagonal in the eigenbasis of  $\Phi(x)$ , so they are codiagonalisable and  $[\Phi(x), \Phi(y)] = 0$ . This relation *a priori* holds in the frame  $\mathcal{R}$ , but when applying the appropriate representation of the Lorenz group under which  $\Phi$  transforms, one sees that  $\Phi(x)$  and  $\Phi(y)$  commute in all reference frames.

If now  $\Phi$  is not supposed hermitian anymore, we still know that  $\Phi\Phi^\dagger$  is. Applying what precedes to  $\Phi\Phi^\dagger$  instead of  $\Phi$ , we obtain that for all  $x-y$  spacelike,  $[\Phi(x)\Phi^\dagger(x), \Phi(y)\Phi^\dagger(y)] = 0$ . Moreover, as  $\Phi$  is

<sup>19</sup>We are aware that there still doesn't exist a proper mathematical framework for the implementation of measurements in QFT (see the introduction of Fewster and Verch (2020)). This is probably why the same computation leads to oddities if one doesn't move to the special reference frame  $\mathcal{R}$  beforehand. Still, it seems to us that this simple reasoning actually captures something of physical interest.



not hermitian, one shows as usual that for all  $x$  and  $y$ ,  $[\Phi(x), \Phi(y)] = [\Phi^\dagger(x), \Phi^\dagger(y)] = 0$ <sup>20</sup>. But on the other hand, it is also possible to compute the commutator  $[\Phi(x)\Phi^\dagger(x), \Phi(y)\Phi^\dagger(y)]$  using the commutation relations (4) and (5). Suppose for instance that  $\Phi$  is a Dirac field. Since we don't know yet if it is a boson or a fermion, let's distinguish the possible cases:

- if the particle described by  $\Phi$  is a fermion, we have (4) with the upper signs. Then:

$$\begin{aligned}
\Phi(x) \underbrace{\Phi^\dagger(x)\Phi(y)}_{\substack{=-\Phi(y)\Phi^\dagger(x) \\ -\Delta_+(x-y) \\ +\Delta_+(y-x)}} \Phi^\dagger(y) &= -\Phi(x)\Phi(y)\Phi^\dagger(x)\Phi^\dagger(y) + \left(\Delta_+(y-x) - \Delta_+(x-y)\right)\Phi(x)\Phi^\dagger(y) \\
&= -\Phi(y) \underbrace{\Phi(x)\Phi^\dagger(y)}_{\substack{=-\Phi^\dagger(y)\Phi(x) \\ +\Delta_+(x-y) \\ -\Delta_+(y-x)}} \Phi^\dagger(x) + \left(\Delta_+(y-x) - \Delta_+(x-y)\right)\Phi(x)\Phi^\dagger(y) \\
&= \Phi(y)\Phi^\dagger(y)\Phi(x)\Phi^\dagger(x) + \underbrace{\left(\Delta_+(y-x) - \Delta_+(x-y)\right)}_{=0 \text{ if } x-y \text{ spacelike}} (\Phi(y)\Phi^\dagger(x) + \Phi(x)\Phi^\dagger(y)),
\end{aligned}$$

- if the particle described by  $\Phi$  is a boson, we have (4) with the lower signs. Then

$$\begin{aligned}
\Phi(x) \underbrace{\Phi^\dagger(x)\Phi(y)}_{\substack{=\Phi(y)\Phi^\dagger(x) \\ -\Delta_+(x-y) \\ -\Delta_+(y-x)}} \Phi^\dagger(y) &= \Phi(x)\Phi(y)\Phi^\dagger(x)\Phi^\dagger(y) - \left(\Delta_+(x-y) + \Delta_+(y-x)\right)\Phi(x)\Phi^\dagger(y) \\
&= \Phi(y) \underbrace{\Phi(x)\Phi^\dagger(y)}_{\substack{=\Phi^\dagger(y)\Phi(x) \\ +\Delta_+(x-y) \\ +\Delta_+(y-x)}} \Phi^\dagger(x) - \left(\Delta_+(x-y) + \Delta_+(y-x)\right)\Phi(x)\Phi^\dagger(y) \\
&= \Phi(y)\Phi^\dagger(y)\Phi(x)\Phi^\dagger(x) + \underbrace{\left(\Delta_+(x-y) + \Delta_+(y-x)\right)}_{\substack{\text{non identically zero} \\ \text{if } x-y \text{ spacelike}}} (\Phi(y)\Phi^\dagger(x) - \Phi(x)\Phi^\dagger(y)).
\end{aligned}$$

As a consequence, one recovers the commutation relation  $[\Phi(x)\Phi^\dagger(x), \Phi(y)\Phi^\dagger(y)] = 0$  imposed by the condition (MC) if, and only if,  $\Phi$  is a fermionic field and in this case, the relation (4) implies that  $\{\Phi(x), \Phi^\dagger(y)\} = 0$  for all spacelike intervals  $x - y$ . These are indeed the statements (M) and (SS). When  $\Phi$  is a scalar or vector field, the proof is similar.  $\square$

**Remark 2.4.** Note that this proof makes use, as usually in physics, of the identification between the notion of observable in the mathematical sense (hermitian operator) and in the physical sense (quantity measurable by a concrete experimental protocol). In §1.3.2, we argued that the second is far more restrictive, since in practice we can only measure a few very specific observables. Rigorously speaking, only the physical notion of observable is constrained by (MC), since the latter must ensure the absence of inconsistencies between *actual* measurements. Therefore, in the above proof, although  $\Phi\Phi^\dagger$  is hermitian, one could question the legitimacy of imposing it (MC). Of course, any mathematical observable could be in principle measured by applying a suitable unitary evolution to the system that would map its eigenbasis to the eigenbasis of a physically measurable observable. But is it satisfactory to rely on the idea that all unitary evolutions are a priori feasible, even though we will never be able to implement them? Nonetheless, it is still possible to adapt the proof by replacing  $\Phi\Phi^\dagger$  by a undoubtedly physical observable, such that a function of the components of  $\hat{T}_{\mu\nu}$  or even the charge  $\hat{Q}$ , that one can express in terms of  $\Phi$ . For example, for a Dirac field,  $\hat{T}_\mu^\mu = m\Phi^\dagger\gamma^0\Phi$ , would allow a quite similar proof.

What precedes has an unexpected consequence, expressed in the corollary below.

**Corollary 2.5.** *A fermionic field is not measurable.*

<sup>20</sup>This is because the only commutators that can appear between a creation and an annihilation operator are  $[a(\vec{p}), a^{c\dagger}(\vec{p}')] = 0$  or  $[a^c(\vec{p}), a^\dagger(\vec{p}')] = 0$  (with the label  $c$  standing for the antiparticle) which are zero if  $a(\vec{p}) \neq a(\vec{p})^c$ .

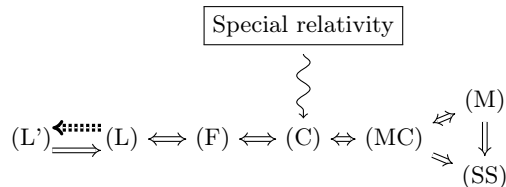
*Proof.* In the previous proof, we have seen that if a field is an observable (*i.e.*  $\Phi = \Phi^\dagger$ ), then the condition (MC) implies that for all  $x - y$  spacelike,  $[\Phi(x), \Phi(y)] = 0$ . But if  $\Phi$  is a fermionic field, it also satisfies  $\{\Phi(x), \Phi(y)\} = 0$ . Adding these two relations yields for all  $x - y$  spacelike,  $\Phi(x)\Phi(y) = 0$ , which is too strong a constraint for a quantum field. In particular,  $\Phi(x)\Phi(y)|0\rangle$  is the state containing two localized particles at  $x$  and  $y$ ; in any case it is a non-zero vector of the Fock space<sup>21</sup>.  $\square$

### 3 Conclusion

The aim of this paper was to investigate fundamental issues concerning the implementation of measurements in (relativistic) QM. As we have seen, such a work is not only useful to found quantum theory on solid bases, but it also delivered fruitful and unexpected results. Various philosophical ideas were formulated on the way, as well as two distinct arguments showing the impossibility to build a deterministic theory superseding QM.

In Section 1, we saw that the measurement problem is much more a problem of words than a problem of maths. More precisely, it is the challenge of integrating the lessons of the theory of decoherence in the physicist’s language, along with accepting the relativity of facts as a fundamental aspect of the universe. We also realized that the usual manichean choice between epistemic and ontic meaning of the wave function is too schematic. To show this, we decomposed the problem into two main questions: the status of the collapse and the emergence of classicality. Examining the first led us to argue that the collapse is not a physical process, but merely a natural consequence of the probabilistic nature of quantum theory. We coined some vocabulary (‘possible histories’, ‘split’, ‘recombine’, ‘fact’...) to speak of the mathematics of QM, in the hope of raising clear pictures in mind with as few counter-intuitive properties as possible. This language was used in the sequel to address the second question: once explained the disappearance of interferences, of quantum correlations and of indeterminism at our scales, we found no reason to believe after all that QM is not universally valid. It also revealed the very special role of the position eigenbasis (both for decoherence and in terms of measurements accessible to humans), shed new lights on the uncertainty principle and highlighted the relativity of facts. The wave function clearly appeared as observer dependent, and we checked that it does not entail any probabilistic inconsistency by revisiting the Wigner’s friend thought experiment.

In Section 2, we considered the apparent incompatibility between special relativity and two kinds of instantaneities that seem to appear in QM. By the time it was historically developed, quantum theory was not built to integrate special relativity, so that it is always surprising to contemplate their ‘peaceful coexistence’ (expression coined by Shimony (1978)). We formulated a mathematical consistency condition that proved to share deep logical links with other fundamental postulates in physics, and it also implied the non-measurability (hence the non-physicality) of the Dirac field. The following diagram summarizes all the logical relationships established in Section 2:



where the dotted arrow stands for ‘justifies’ rather than ‘implies’. The implication  $(M) \Rightarrow (MC)$  is not proved in this paper but in the aforementioned Polo-Gómez et al. (2022). Let’s emphasize again that (C) (*a fortiori* (MC)) is not an arbitrary postulate but a mere consistency criterion that must be valid for any relativistic quantum theory: it ‘costs nothing’ to be assumed because it stems from the constraints of special relativity. Surprisingly, it becomes obvious on this diagram that *the two fundamental postulates (L’) and (M) of QFT are actually redundant*, since locality implies microcausality!

<sup>21</sup>This result is not new, and can also be derived, for instance, by the fact that the equation  $\Phi = \Phi^\dagger$  is neither covariant nor independent of the representation of the gamma matrices (Pal, 2011), so it can’t have a physical meaning for the Dirac field. But it is interesting to see that it can be directly derived from considerations about measurements. Again, our argument relies on the widespread identification between the notions of mathematical and physical observable. Here,  $\Phi$  may indeed be hermitian, but is it a physically measurable quantity subject to the condition (MC)?

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# References

- Yakir Aharonov and David Z Albert. Can we make sense out of the measurement process in relativistic quantum mechanics? *Physical Review D*, 24(2):359, 1981. doi: 10.1103/PhysRevD.24.359.
- Alexia Auffèves and Philippe Grangier. Contexts, systems and modalities: a new ontology for quantum mechanics. *Foundations of Physics*, 46(2):121–137, 2016. doi: 10.1007/s10701-015-9952-z.
- Alexia Auffèves and Philippe Grangier. What is quantum in quantum randomness? *Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences*, 376(2123):20170322, 2018. doi: 10.1098/rsta.2017.0322.
- Angelo Bassi, Kinjalk Lochan, Seema Satin, Tejinder P Singh, and Hendrik Ulbricht. Models of wave-function collapse, underlying theories, and experimental tests. *Reviews of Modern Physics*, 85(2):471, 2013. doi: 10.1103/RevModPhys.85.471.
- Daniel Bedingham, Detlef Dürr, GianCarlo Ghirardi, Sheldon Goldstein, Roderich Tumulka, and Nino Zanghì. Matter density and relativistic models of wave function collapse. *Journal of Statistical Physics*, 154:623–631, 2014. doi: 10.1007/s10955-013-0814-9.
- John S Bell. On the Einstein Podolsky Rosen paradox. *Physics Physique Fizika*, 1(3):195, 1964. doi: 10.1103/PhysicsPhysiqueFizika.1.195.
- John S Bell. On the impossible pilot wave. *Foundations of Physics*, 12(10):989–999, 1982. doi: 10.1007/BF01889272.
- I Bloch. Some relativistic oddities in the quantum theory of observation. *Physical Review*, 156(5):1377, 1967. doi: 10.1103/PhysRev.156.1377.
- Niels Bohr and Leon Rosenfeld. Zur Frage der Messbarkeit der elektromagnetischen Feldgrößen. *Kgl. Danske Vidensk. Selskab. Math.-Fys. Medd*, 12:3, 1933.
- Kok-Wei Bong, Aníbal Utreras-Alarcón, Farzad Ghafari, Yeong-Cherng Liang, Nora Tischler, Eric G Cavalcanti, Geoff J Pryde, and Howard M Wiseman. A strong no-go theorem on the Wigner’s friend paradox. *Nature Physics*, 16(12):1199–1205, 2020. doi: 10.1038/s41567-020-0990-x.
- Časlav Brukner. Facts are relative. *Nature Physics*, 16(12):1172–1174, 2020. doi: 10.1038/s41567-020-0984-8.
- Michael S Chapman, Troy D Hammond, Alan Lenef, Jörg Schmiedmayer, Richard A Rubenstein, Edward Smith, and David E Pritchard. Photon scattering from atoms in an atom interferometer: coherence lost and regained. *Physical Review Letters*, 75(21):3783, 1995. doi: 10.1103/PhysRevLett.75.3783.
- Chris Dewdney, Lucien Hardy, and Euan J Squires. How late measurements of quantum trajectories can fool a detector. *Physics Letters A*, 184(1):6–11, 1993. doi: 10.1016/0375-9601(93)90337-Y.

- Andrea Di Biagio and Carlo Rovelli. Stable facts, relative facts. *Foundations of Physics*, 51(1):1–13, 2021. doi: 10.1007/s10701-021-00429-w.
- C. Dove. Explicit wavefunction collapse and quantum measurement. *Ph.D. thesis, Department of Mathematical Sciences, University of Durham*, 1996.
- Albert Einstein, Boris Podolsky, and Nathan Rosen. Can quantum-mechanical description of physical reality be considered complete? *Physical review*, 47(10):777, 1935. doi: 10.1103/PhysRev.47.777.
- Berthold-Georg Englert, Marian O Scully, Georg Süssmann, and Herbert Walther. Surrealistic bohm trajectories. *Zeitschrift für Naturforschung A*, 47(12):1175–1186, 1992. doi: 10.1515/zna-1992-1201.
- Christopher J Fewster and Rainer Verch. Quantum fields and local measurements. *Communications in Mathematical physics*, 378(2):851–889, 2020. doi: 10.1007/s00220-020-03800-6.
- Gian-Carlo Ghirardi, Alberto Rimini, and Tullio Weber. A general argument against superluminal transmission through the quantum mechanical measurement process. *Lettere al Nuovo Cimento (1971-1985)*, 27(10):293–298, 1980. doi: 10.1007/BF02817189.
- Werner Heisenberg. Development of concepts in the history of quantum theory. In *The physicist's conception of nature*, pages 264–275. Springer, 1973. doi: 10.1007/978-94-010-2602-4.
- Robert C Helling. No signalling and unknowable bohmian particle positions. *arXiv preprint arXiv:1902.03752*, 2019. doi: 10.48550/arXiv.1902.03752.
- K-E Hellwig and K Kraus. Formal description of measurements in local quantum field theory. *Physical Review D*, 1(2):566, 1970. doi: 10.1103/PhysRevD.1.566.
- Claude Itzykson and Jean-Bernard Zuber. *Quantum field theory*. Courier Corporation, 2012. ISBN 9780486134697.
- Erich Joos. Decoherence through interaction with the environment. In *Decoherence and the appearance of a classical world in quantum theory*, pages 35–136. Springer, 1996. doi: 10.1007/978-3-662-03263-3.
- Bruno Latour. *Face à Gaïa: huit conférences sur le nouveau régime climatique*. Empêcheurs de penser rond, 2015. doi: 10.4000/lectures.19763.
- David B Malament. In defense of dogma: Why there cannot be a relativistic quantum mechanics of (localizable) particles. In *Perspectives on quantum reality*, pages 1–10. Springer, 1996. doi: 10.1007/978-94-015-8656-6\_1.
- Fotini Markopoulou. New directions in background independent quantum gravity. *Approaches to quantum gravity*, pages 129–149, 2009. doi: 10.1017/CBO9780511575549.
- Wayne Myrvold, Marco Genovese, and Abner Shimony. Bell's Theorem. In Edward N. Zalta, editor, *The Stanford Encyclopedia of Philosophy*. Metaphysics Research Lab, Stanford University, Fall 2021 edition, 2021.
- Palash B Pal. Dirac, Majorana, and Weyl fermions. *American Journal of Physics*, 79(5):485–498, 2011. doi: 10.1119/1.3549729.
- Michael E Peskin. *An introduction to quantum field theory*. CRC press, 2018. doi: 10.1201/9780429503559.
- Jordan B Peterson. Why quantum mechanics is an inconsistent theory | Roger Penrose & Jordan Peterson, 15 april 2022. <https://www.youtube.com/watch?v=TSBOBJsdEuY>.
- José Polo-Gómez, Luis J Garay, and Eduardo Martín-Martínez. A detector-based measurement theory for quantum field theory. *Physical Review D*, 105(6):065003, 2022. doi: 10.1103/PhysRevD.105.065003.
- Maximilian Schlosshauer. Decoherence, the measurement problem, and interpretations of quantum mechanics. *Reviews of Modern physics*, 76(4):1267, 2005. doi: 10.1103/RevModPhys.76.1267.

- Maximilian Schlosshauer, Johannes Kofler, and Anton Zeilinger. A snapshot of foundational attitudes toward quantum mechanics. *Studies in History and Philosophy of Science Part B: Studies in History and Philosophy of Modern Physics*, 44(3):222–230, 2013. doi: 10.1016/j.shpsb.2013.04.004.
- Abner Shimony. Metaphysical problems in the foundations of quantum mechanics. *International Philosophical Quarterly*, 18(1):3–17, 1978. doi: 10.5840/ipq19781818.
- Sujeevan Sivasundaram and Kristian Hvidtfelt Nielsen. Surveying the attitudes of physicists concerning foundational issues of quantum mechanics. *arXiv preprint arXiv:1612.00676*, 2016. doi: 10.48550/arXiv.1612.00676.
- Rafael D Sorkin. Impossible measurements on quantum fields. In *Directions in general relativity: Proceedings of the 1993 International Symposium, Maryland*, volume 2, pages 293–305, 1993. doi: 10.1017/CBO9780511524653.
- Antoine Soulas. Decoherence as a high-dimensional geometrical phenomenon. *arXiv preprint*, 2023. doi : 10.48550/arXiv.2302.04148.
- Roderich Tumulka. A relativistic version of the ghirardi–rimini–weber model. *Journal of Statistical Physics*, 125:821–840, 2006. doi: 10.1007/s10955-006-9227-3.
- William G Unruh. Notes on black-hole evaporation. *Physical Review D*, 14(4):870, 1976. doi: 10.1103/PhysRevD.14.870.
- Steven Weinberg. *The quantum theory of fields*, volume 2. Cambridge university press, 1995. doi: 10.1017/CBO9781139644167.
- Ludwig Wittgenstein. *Philosophical investigations*. Basil Blackwell, Oxford, 1953. ISBN 9780631119005.
- Hugo Zbinden, Jürgen Brendel, Nicolas Gisin, and Wolfgang Tittel. Experimental test of nonlocal quantum correlation in relativistic configurations. *Physical Review A*, 63(2):022111, 2001. doi: 10.1103/PhysRevA.63.022111.
- Anthony Zee. *Quantum field theory in a nutshell*, volume 7. Princeton university press, 2010. doi: 10.1088/0264-9381/28/8/089003.
- Wojciech Hubert Zurek. Environment-induced superselection rules. *Physical review D*, 26(8):1862, 1982. doi: 10.1103/PhysRevD.26.1862.
- Wojciech Hubert Zurek. Decoherence, einselection, and the quantum origins of the classical. *Reviews of modern physics*, 75(3):715, 2003. doi: 10.1103/RevModPhys.75.715.
- Wojciech Hubert Zurek. Emergence of the classical world from within our quantum universe. In *From Quantum to Classical*, pages 23–44. Springer, 2022. doi: 10.1007/978-3-030-88781-0\_2.