Evidential data association based on Dezert–Smarandache Theory

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Mohammed Boumediene[1,2] **· Hossni Zebiri**[1] **· Jean Dezert**[3]

Abstract

Data association has become pertinent task to interpret the perceived environment for mobile robots such as autonomous vehicles. It consists in assigning the sensor detections to the known objects in order to update the obstacles map surround-ing the vehicle. Dezert–Smarandache Theory (DSmT) provides a mathematical framework for reasoning with imperfect data like sensor's detections. In DSmT, data are quantified by belief functions and combined by the Proportional Conflict Redistribution rule in order to obtain the fusion of evidences to make a decision. However, this combination rule has an exponential complexity and that is why DSmT is rarely used for real-time applications. This paper proposes a new evidential data association based on DSmT techniques. The proposed approach focuses on the significant pieces of information when combining and removes unreliable and useless information. Consequently, the complexity is reduced without degrading substantially the decision-making. The paper proposes also a new simple decision-making algorithm based on a global optimization procedure. Experimental results obtained on a well-known KITTI dataset show that this new approach reduces significantly the computation time while preserving the association accuracy. Consequently, the new proposed approach makes DSmT framework applicable for real-time applications for autonomous vehicle perception.

Keywords Data association · Belief functions · Dezert–Smarandache theory · Proportional conflict redistribution 6 · Dezert–Smarandache probability

1 Introduction

Multi-Target Tracking (MTT) is a fundamental system to interpret the perceived environment of mobile robots such as autonomous vehicles (Armingol et al. 2018; Brummelen et al. 2018). These cars require precise knowledge of their surrounding environment in order to ensure safe and comfortable driving (Boumediene et al. 2014, 2014; Steyer et al. 2018). The MTT system estimates the status of detected objects surrounding the vehicle at different times by single or multiple sensors. Data Association is a central problem in MTT which assigns *targets* to the predicted *tracks* in order to update their status. Targets refer to the *detected objects* at the current time and tracks refer to the *known objects* in the scene. A dynamic environment, like the road environment, makes the object association more difficult because of the appearance/disappearance of objects in the perceived scene.

Usually, the assignment problem is resolved by the probability theory. Several methods have been proposed as the well-known Global Nearest Neighbour (GNN) method and the Joint Probability Data Association Filter (JPDAF) (Blackman 1986; Fortmann et al. 1983; Bar-Shalom et al. 2011). GNN provides the optimal pairing by minimizing the global distance between detections and known objects. JPDAF is based on a weighted linear combination of all detections to estimate status of known objects. More details about these methods can be found in (Bar-Shalom and Li 1995; Blackman and Popoli 1999; Bar-Shalom et al. 2011).

Recently, the belief function theory has also been used to cope with the association problem (Boumediene et al. 2014; Mercier et al. 2011). This theory, also called Dempster-Shafer Theory (DST) (Dempster 1968; Shafer 1976) allows to reason about uncertainty thanks to the belief functions

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that are often interpreted as lower and upper bound of unknown probability measures. In fact, sensor's detections can be inaccurate and incomplete. However, the DST models these imperfect information through a distribution of belief masses which quantify the confidence granted. Thereafter, these masses are combined by Dempster's rule to make decisions. Because Dempster's rule has been used and promoted by Shafer in his mathematical theory of evidence, it is also often denoted as DS rule in the literature.

Rombaut Rombaut (1998) formalizes the association problem by DST to reconstruct the environment of intelligent vehicles. This approach measures the confidence of the association hypotheses between perceived and known obstacles by combining belief masses using DS rule. This approach is extended in Gruyer et al. (2016); Mercier et al. (2011) to track vehicles where the association process is based on the Transferable Belief Model (TBM) (Smets and Kennes 1994). This latter is a subjective and non-probabilistic interpretation of the Belief theory. In TBM, the decision-making is based on the pignistic probabilities derived from the belief quantities. Several alternative probabilistic transformations have been proposed in the literature. Our previous work (Boumediene and Dezert 2020) evaluates some of them on real-data in the context of the DST framework. In Mercier et al. (2011), the decision is performed by maximizing the joint pignistic probability. However, this probability is computed for all possible associations which grows the computation time exponentially with the objects number. To tackle this problem, the decision is made by selecting associations corresponding to local maxima of pignistic probabilities (Boumediene et al. 2014; Daniel and Lauffenburger 2012). More recently, *Denœux et al.* Denœux et al. (2014) express DS rule in terms of contour functions and plausibility functions which reduces the complexity and makes this approach applicable for real-time applications.

All those aforementioned approaches use Dempster's rule which provides a counter-intuitive behavior specially in high and low conflicting situations (Zadeh 1979; Smarandache and Dezert 2015). In fact, DS rule redistributes the conflicting mass on all elements which can cause the lost of the information specificity and then generates unacceptable results. In addition, serious mistakes have been shown in logical fundamentals of the DST framework (Dezert et al. 2012; Tchamova and Dezert 2012; Smarandache et al. 2013). To overcome those drawbacks, a more sophisticate rule has been proposed and defined in the framework of Dezert–Smarandache Theory (DSmT) (Smarandache and Dezert 2015). Based on the Proportional Conflict Redistribution (PCR) process, PCR6 rule preserves the information specificity by transferring the conflicting mass only to the elements involved in the conflict and proportionally to their individual masses. However, PCR6 has an exponential complexity and that is why it is rarely used for real-time applications.

In this paper, we propose a new evidential data association based on the DSmT framework. The first contribution is to reduce the complexity of the combination step based on PCR6 rule developed originally in the framework of Dezert–Smarandache Theory. The proposed approach focuses on the significant pieces of information when combining and removes unreliable and useless information. Consequently, the complexity is reduced without degrading substantially the decision-making. The second contribution is to propose a new simple decision-making algorithm based on a global optimization. Experimental results obtained on a well-known intelligent transportation systems dataset show the benefits of this new approach in terms of computation time reduction and association accuracy.

The rest of this paper is organized as follows. In Sect. 2, few basics of the DSmT are presented. Section 3 details the new proposed evidential data association approach and its experimental validation is presented in Sect. 4. Finally, Sect. 5 concludes this paper.

2 Fundamentals of DSmT

In the Belief theory context, a problem is modelled by a finite set of hypotheses H_i likely to be the solutions, called *Frame of Discernment* (FoD). In the general DSmT framework, the elements of the FoD do not need to be mutually exhaustive as in the DST framework, but in the particular context of our application presented in this paper, we work with Shafer's model of the FoD where all elements of the FoD are mutually exclusive and exhaustive, that is:

$$\Theta = \bigcup_{i=1}^{k} \{H_i\} \text{ with } H_i \cap H_j = \emptyset \tag{1}$$

where H_i are denoted as *singletons*, the lowest piece of discernible knowledge in the FoD.

2.1 Basic belief assignment

A *basic belief assignment* (*bba*) or mass function associated to a given source is defined as a function $m : 2^\Theta \rightarrow [0, 1]$ satisfying:

$$\sum_{A\in 2^\Theta} m(A) = 1 \tag{2}$$

where $m(A)$ is the mass of belief that supports A. The source is totally ignorant if $m(\Theta) = 1$ and so the *bba* is considered as vacuous function. Whether $m(A) > 0$, A is called a focal element of the *bba* $m(.)$. Thus $\mathcal{F}(m) = \{A \in 2^\Theta / m(A) > 0\}$ defines the set of focal elements.

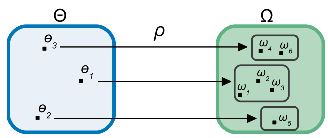

Fig. 1 Illustration of the refinement function ρ (Mercier et al. 2011)

2.2 Vacuous extension

Some sources of information can express on different FoDs but related. However, in order to combine them, it is necessary to work with the same common frame. For that, it can be defined a finer FoD (Shafer 1976). Let Ω a finer frame of Θ where every element of Θ is mapped into one or more elements of Ω (Cf. Fig. 1). Therefore, the refinement function ρ matches proposition A from 2^Θ to 2^Ω according to:

$$\begin{cases} \{\rho(\{\theta\}), \theta \in \Theta\} \text{ is a partition of } \Omega \\ \forall A \subseteq \Theta, \rho(A) = \bigcup_{\theta \in A} \rho(\{\theta\}). \end{cases} \tag{3}$$

The vacuous extension $m^{\Theta\uparrow\Omega}$ defines the *bba* on Ω from the *bba* m^Θ defined on Θ and the refinement ρ:

$$m^{\Theta\uparrow\Omega}(\rho(A)) = \begin{cases} m^\Theta(A), & \forall A \subseteq \Theta \\ 0, & \text{otherwise.} \end{cases} \tag{4}$$

2.3 Belief combination

The belief combination consists in merging the measures of evidence m_i^Θ of M distinct sources S_i, defined on the same frame Θ, to a new distribution of evidence. For that, the Proportional Conflict Redistribution rule 6 (PCR6) have been proposed in Martin and Osswald (2006) and theoretically justified in Smarandache and Dezert (2015). In fact, PCR6 rule overcomes the drawbacks of the Dempster rule (Shafer 1976) by redistributing proportionally the partial conflict only on elements involved in this conflict. The formula of PCR6 is defined by $m_{\text{PCR6}}(\emptyset) = 0$ and $\forall A \in 2^\Theta \backslash \{\emptyset\}$ by Dezert and Dezert (2021); Dezert et al. (2021):

$$m_{\text{PCR6}}(A) = m_{\text{Conj}}(A) + \sum_{j \in \{1,\ldots,\mathcal{F}\} | A \in \mathbf{A}_j \wedge \pi_j(\emptyset)} \left[\left(\sum_{i \in \{1,\ldots,M\} | A_{j_i} = A} m_i^\Theta(A_{j_i}) \right) \cdot \frac{\pi_j(\emptyset)}{\sum_{A \in \mathbf{A}_j} \left(\sum_{i \in \{1,\ldots,M\} | A_{j_i} = A} m_i^\Theta(A_{j_i}) \right)} \right], \tag{5}$$

where $\wedge$ is the logical conjunction[1] and $\mathbf{A}_j$ is a possible M-uple of focal elements with $A_{j_i} \in \mathcal{F}(m_i^\Theta)$, that is $\mathbf{A}_j \triangleq (A_{j_1}, A_{j_2}, \ldots, A_{j_M})$. $\mathcal{F}$ is the cardinality of $\mathcal{F}(m_1^\Theta, m_2^\Theta, \ldots, m_M^\Theta)$ which is the set of all possible M-uple. And where $\pi_j(A_{j_1} \cap A_{j_2} \cap \cdots \cap A_{j_M}) \triangleq \prod_{i=1}^M m_i^\Theta(A_{j_i})$, and $\pi_j(\emptyset) = \pi_j(A_{j_1} \cap A_{j_2} \cap \cdots \cap A_{j_M})$ defines the *conflicting mass product* of $\mathbf{A}_j$ if $A_{j_1} \cap A_{j_2} \cap \cdots \cap A_{j_M} = \emptyset$ and the conjunctive rule m_{Conj} is given by:

$$m_{\text{Conj}}(A) = \sum_{A_{j_1} \cap \ldots \cap A_{j_M} = A} \prod_{i=1}^{M} m_i^\Theta(A_{j_i}). \tag{6}$$

2.4 Probabilistic transformation

Decision-making consists of selecting a solution among all possible hypotheses. Usually, the decision must be made among elements of the frame. However, the belief combination also generates masses for disjunctive propositions. Therefore, it is necessary to redistribute the masses of these unions on elements of Θ in order to make a decision. For that, Dezert–Smarandache Probability (DSmP) transformation is defined (Dezert et al 2012) where $DSmP(\emptyset) = 0$ and $\forall A \in 2^\Theta \backslash \{\emptyset\}$:

$$DSmP_\varepsilon(A) = \sum_{Y \in 2^\Theta} \frac{\sum_{\substack{Z \subseteq A \cap Y \\ \mathcal{C}(Z) = 1}} m(Z) + \varepsilon \cdot \mathcal{C}(A \cap Y)}{\sum_{\substack{Z \subseteq Y \\ \mathcal{C}(Z) = 1}} m(Z) + \varepsilon \cdot \mathcal{C}(Y)} \tag{7}$$

Where $\epsilon \geq 0$ is used to adjust the effect of element's cardinality ($\mathcal{C}(.)$) in the proportional redistribution. In addition, ϵ permits to compute *DSmP* when encountering zero masses. Typically, $\epsilon = 0.001$ because with a smaller ϵ the Probabilistic Information Content (PIC) (Sudano 2002) is higher. The PIC indicates the level of the available knowledge to make a correct decision. $PIC = 0$ indicates that no knowledge exists to make a correct decision.

3 Data association using DSmT

Four steps are needed to solve the data association problem: modeling, estimation, combining, and decision-making. However, PCR6 rule combination has an exponential complexity which makes it not appealing for real-time applications. This is why in this paper, only k-significant sources are combined (with k lesser than the original number or sources available). Thereafter, a simple global optimization is used to make association decisions.

[1] i.e. $x \wedge y$ means that conditions x and y are both true.

3.1 Data modelling

Let us consider n detected objects at time t and m known objects at previous time $t-1$. In this context, data association aims at matching the n detected objects X_i to the m known ones Y_j under certain conditions:

- multiple associations are not accepted, a detected object is associated with only one known object at most and *vice versa*,
- multiple new objects can appear,
- multiple known objects can disappear.

The distances between the attributes of objects (position, velocity, etc.) are considered as pieces of evidence. For a given distance, its belief will be expressed on the elementary FoD $\theta_{i,j} = \{\text{yes}_{(i,j)}, \text{no}_{(i,j)}\}$ which models the relevance of the association between X_i and Y_j. Therefore, three *bba* masses are constructed for each pairwise objects (X_i, Y_j):

- $m^{\theta_{i,j}}(\text{yes}_{(i,j)})$: degree of belief that X_i is associated with Y_j,
- $m^{\theta_{i,j}}(\text{no}_{(i,j)})$: degree of belief that X_i is not associated with Y_j,
- $m^{\theta_{i,j}}(\theta_{i,j})$: represents the ignorance.

3.2 Belief estimation

The estimation of belief masses is related to the considered application. The most suitable model for data association applications (Boumediene 2019) is the non-antagonist model (Gruyer et al. 2016; Rombaut 1998) defined by:

$$m_j^{\Theta_{i,.}}(Y_{(i,j)}) = \begin{cases} 0 & , I_{i,j} \in [0,\tau] \\ \Phi_1(I_{i,j}) & , I_{i,j} \in [\tau,1] \end{cases} \tag{8}$$

$$m_j^{\Theta_{i,.}}(\bar{Y}_{(i,j)}) = \begin{cases} \Phi_2(I_{i,j}) & , I_{i,j} \in [0,\tau] \\ 0 & , I_{i,j} \in [\tau,1] \end{cases} \tag{9}$$

$$m_j^{\Theta_{i,.}}(\Theta_{i,.}) = \begin{cases} 1 - m_j^{\Theta_{i,.}}(\bar{Y}_{(i,j)}) & , I_{i,j} \in [0,\tau] \\ 1 - m_j^{\Theta_{i,.}}(Y_{(i,j)}) & , I_{i,j} \in [\tau,1], \end{cases} \tag{10}$$

where $I_{i,j} \in [0,1]$ is an index of similarity between X_i and Y_j. $\Phi_1(.)$ and $\Phi_2(.)$ are two cosine functions defined as follows:

$$\begin{cases} \Phi_1(I_{i,j}) = \frac{\alpha}{2}\left[1 - \cos(\pi \frac{I_{i,j}-\tau}{\tau})\right] \\ \Phi_2(I_{i,j}) = \frac{\alpha}{2}\left[1 + \cos(\pi \frac{I_{i,j}}{\tau})\right] \end{cases} \tag{11}$$

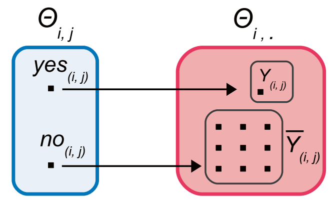

Fig. 2 The refinement frames of $\theta_{i,j}$: $\Theta_{i,.}$

where $0 < \alpha < 1$ is the reliability factor of the data source and $0 < \tau < 1$ represents the impartiality of the association process.

3.3 k-Significant sources combination

Before decision-making, sources should be combined which is possible only if they express on the same FoD. Hence, to determine who is associated to the detected object X_i, a new FoD is defined $\Theta_{i,.}$ (12). This new frame is composed of the m possible X_i-to- Y_j associations denoted $Y_{(i,j)}$ and the appearance hypothesis of object X_i denoted by $Y_{(i,*)}$:

$$\Theta_{i,.} = \left\{Y_{(i,1)}, Y_{(i,2)}, \ldots, Y_{(i,m)}, Y_{(i,*)}\right\}. \tag{12}$$

Therefore, $\Theta_{i,.}$ is a refinement frame of the previous FoDs $\theta_{i,j}$ in which the belief is initially expressed (Cf. Fig. 2). Based on a vacuous extension (3), initial belief functions $m^{\theta_{i,j}}$ are expressed on $\Theta_{i,.}$ as follows:

$$\begin{cases} m_j^{\Theta_{i,.}}(Y_{(i,j)}) = m^{\theta_{i,j}}(\text{yes}_{(i,j)}) \\ m_j^{\Theta_{i,.}}(\bar{Y}_{(i,j)}) = m^{\theta_{i,j}}(\text{no}_{(i,j)}) \\ m_j^{\Theta_{i,.}}(\Theta_{i,.}) = m^{\theta_{i,j}}(\theta_{i,j}) \end{cases} \tag{13}$$

where $\bar{Y}_{(i,j)}$ represents the hypothesis "X_i is not associated to Y_j" which corresponds to the union of all association hypotheses expect the $Y_{(i,j)}$, i.e. $\bar{Y}_{(i,j)} = \{Y_{(i,1)}, \ldots, Y_{(i,j-1)}, Y_{(i,j+1)}, \ldots, Y_{(i,m)}, , Y_{(i,*)}\}$. It should be noted that no information is initially considered on $Y_{(i,*)}$. This information appears during combination step.

Once the sources are expressed on the same frame, the *bbas* are combined with the PCR6 rule. However, combining all sources increases the time-consuming and can be reach an exponential complexity when the number of sources is important. To overcome this drawback, this paper proposes a new method to reduce the combination complexity without sacrificing too much the decision quality.

The proposed approach selects only information having belief in top k highest masses. Formally, for each X_i object, initial masses on association hypotheses are sorted:

Table 1 *DSmP* probabilities of detected-to-known object associations

$\Theta_{i,.}$	$Y_{(i,1)}$	$\cdots$	$Y_{(i,m)}$	$Y_{(i,*)}$
$DSmP_{1,.}(.)$	$DSmP_{1,.}(Y_{(1,1)})$	$\cdots$	$DSmP_{1,.}(Y_{(1,m)})$	$DSmP_{1,.}(Y_{(1,*)})$
$DSmP_{2,.}(.)$	$DSmP_{2,.}(Y_{(2,1)})$	$\cdots$	$DSmP_{2,.}(Y_{(2,m)})$	$DSmP_{2,.}(Y_{(2,*)})$
$\vdots$	$\vdots$		$\vdots$	$\vdots$
$DSmP_{n,.}(.)$	$DSmP_{n,.}(Y_{(n,1)})$	$\cdots$	$DSmP_{n,.}(Y_{(n,m)})$	$DSmP_{n,.}(Y_{(n,*)})$

$$\begin{cases} b_1 \geq b_2 \geq \cdots \geq b_z \geq \cdots \geq b_m \\ b_z = m^{\theta_{i,j}}(yes_{(i,j)}), \text{ and } z, j \in \{1, \ldots, m\} \end{cases} \tag{14}$$

where b_1 is highest mass of belief, so the source that generated it is the most significant for matching X_i. On other hand, the least important source is that which generates the lowest belief b_m.

Now, only k most significant sources are selected for their combination. Therefore, for each X_i assignment, $\Theta_{i,.}$ is defined as follows:

$$\Theta_{i,.} = \left\{ Y_{(i,z)} / b_z \geq b_k, Y_{(i,*)} \right\} \tag{15}$$

with $z \in \{1, \ldots, m\}$ and $k < m$. Consequently, $\Theta_{i,.}$ contains only the most relevant hypotheses and ignores others ($b_z < b_k$). By this simple selection procedure one reduces the computation complexity of the combination process.

If $b_k = 0$, b_{k-1} is used to select significant sources. In the case where no $b_k > 0$, the object X_i is considered as an appearance and is associated directly to $Y_{(i,*)}$. Thereafter, initial mass functions $m^{\theta_{i,j}}(.)$ is hence transferred to $\Theta_{i,.}$ by the refinement defined in (13) and the PCR6 rule of combination (5) is applied.

3.4 Decision-making

The assignment decision is based on the $DSmP_{i,.}$ matrix which is the probabilistic approximation of the combined masses. Table 1 presents the $DSmP_{i,.}$ of the detected-to-known objects association. Each line defines the association probabilities of the detected object X_i with all known ones Y_j. $DSmP_{i,.}(Y_{(i,*)})$ defines the appearance probability of X_i. It is useful to note that multiple objects can appear/disappear.

Different decision-making strategies have been proposed according to the desired objectives (Daniel and Lauffenburger 2012; Mercier et al. 2011). There are two approaches depending on the type of optimization: global or local. The first approach selects the "best" associations optimizing a global cost function (Gruyer and Berge-Cherfaoui 1999; Royère et al 2000). The Joint Pignistic Probability (JPP) $BetP_{\prod_{i=1}^{n}}$ is defined as the cost function in Mercier et al. (2011):

$$BetP_{\prod_{i=1}^{n}} = BetP_{1,.}(Y_{(1,j_1)}) \times \ldots \times BetP_{n,.}(Y_{(n,j_n)}) \tag{16}$$

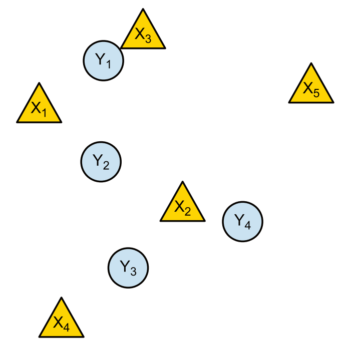

Fig. 3 Scenario showing 5 detected objects (triangle) and 4 known objects (circle)

with $j_i \in \{1, 2, \ldots, m, *\}$. Among all possible solutions for the detected-to-known association, the *best* is that maximizing $BetP_{\prod_{i=1}^{n}}$. However, when the number of possible associations is important, this optimization generates a high computational complexity. To cope with this inconvenience, another approach consists of resolving the assignment problem by a local optimization. The Local Pignistic Probability (LPP) (Daniel and Lauffenburger 2012) makes the association decisions according to local maxima of the pignistic matrix ($BetP_{i,.}$). The LPP method performs a successive selection of n local maxima while respecting the association constrains (Cf. Sect. 3.1). However, local optimization is considered as a sub-optimal solution.

In this paper, a new simple global optimization is applied/proposed. Firstly, the last column ($Y_{(i,*)}$) of the *DSmP* matrix is removed in order to select "best" associations by using the well-known Munkres algorithm (Munkres 1957). The complexity of this algorithm is only $O(n^3)$ (Munkres 1957). Secondly, for each selected association $Y_{(i,j)}$, if $DSmP_{i,.}(Y_{(i,j)}) < DSmP_{i,.}(Y_{(i,*)})$ the association $Y_{(i,j)}$ is removed and the object X_i is considered to be a new object ($Y_{(i,*)}$).

4 Illustrative example

Let us consider the simulated example presented in Fig. 3. The scenario shows 5 detected objects and 4 known objects. By observing the corresponding initial *bba* presented in Table 2, one can already assume some associations. For instance, with $m^{\Theta_{3,.}}(yes_{(3,1)}) = 0.85$ and $m^{\Theta_{2,.}}(yes_{(2,4)}) = 0.75$, X_3 and X_2 are most likely to be associated respectively to Y_1 and Y_4. As for the detected object X_5, no source supports its association with a known objects, so it can be an appearance.

Therefore, it is possible to make decisions by combining only some information? To answer, the proposed

Table 2 Initial mass functions for the scenario in Fig. 3

$S_{1,1}\begin{cases} m^{\theta_{1,1}}(yes_{(1,1))}) = 0.45 \\ m^{\theta_{1,1}}(no_{(1,1)}) = 0.35 \\ m^{\theta_{1,1}}(\theta_{1,1}) = 0.20 \end{cases}$	$S_{1,2}\begin{cases} m^{\theta_{1,2}}(yes_{(1,2)}) = 0.48 \\ m^{\theta_{1,2}}(no_{(1,2)}) = 0.32 \\ m^{\theta_{1,2}}(\theta_{1,2}) = 0.20 \end{cases}$	$S_{1,3}\begin{cases} m^{\theta_{1,3}}(yes_{(1,3)}) = 0.00 \\ m^{\theta_{1,3}}(no_{(1,3)}) = 0.95 \\ m^{\theta_{1,3}}(\theta_{1,3}) = 0.05 \end{cases}$	$S_{1,4}\begin{cases} m^{\theta_{1,4}}(yes_{(1,4)}) = 0.00 \\ m^{\theta_{1,4}}(no_{(1,4)}) = 0.99 \\ m^{\theta_{1,4}}(\theta_{1,4}) = 0.01 \end{cases}$
$S_{2,1}\begin{cases} m^{\theta_{2,1}}(yes_{(2,1)}) = 0.00 \\ m^{\theta_{2,1}}(no_{(2,1)}) = 0.99 \\ m^{\theta_{2,1}}(\theta_{2,1}) = 0.01 \end{cases}$	$S_{2,2}\begin{cases} m^{\theta_{2,2}}(yes_{(2,2)}) = 0.32 \\ m^{\theta_{2,2}}(no_{(2,2)}) = 0.58 \\ m^{\theta_{2,2}}(\theta_{2,2}) = 0.10 \end{cases}$	$S_{2,3}\begin{cases} m^{\theta_{2,3}}(yes_{(2,3)}) = 0.47 \\ m^{\theta_{2,3}}(no_{(2,3)}) = 0.43 \\ m^{\theta_{2,3}}(\theta_{2,3}) = 0.10 \end{cases}$	$S_{2,4}\begin{cases} m^{\theta_{2,4}}(yes_{(2,4)}) = 0.75 \\ m^{\theta_{2,4}}(no_{(2,4)}) = 0.15 \\ m^{\theta_{2,4}}(\theta_{2,4}) = 0.10 \end{cases}$
$S_{3,1}\begin{cases} m^{\theta_{3,1}}(yes_{(3,1)}) = 0.85 \\ m^{\theta_{3,1}}(no_{(3,1)}) = 0.05 \\ m^{\theta_{3,1}}(\theta_{3,1}) = 0.10 \end{cases}$	$S_{3,2}\begin{cases} m^{\theta_{3,2}}(yes_{(3,2)}) = 0.00 \\ m^{\theta_{3,2}}(no_{(3,2)}) = 0.90 \\ m^{\theta_{3,2}}(\theta_{3,2}) = 0.10 \end{cases}$	$S_{3,3}\begin{cases} m^{\theta_{3,3}}(yes_{(3,3)}) = 0.00 \\ m^{\theta_{3,3}}(no_{(3,3)}) = 0.90 \\ m^{\theta_{3,3}}(\theta_{3,3}) = 0.10 \end{cases}$	$S_{3,4}\begin{cases} m^{\theta_{3,4}}(yes_{(3,4)}) = 0.00 \\ m^{\theta_{3,4}}(no_{(3,4)}) = 0.99 \\ m^{\theta_{3,4}}(\theta_{3,4}) = 0.01 \end{cases}$
$S_{4,1}\begin{cases} m^{\theta_{4,1}}(yes_{(4,1)}) = 0.00 \\ m^{\theta_{4,1}}(no_{(4,1)}) = 0.99 \\ m^{\theta_{4,1}}(\theta_{4,1}) = 0.01 \end{cases}$	$S_{4,2}\begin{cases} m^{\theta_{4,2}}(yes_{(4,2)}) = 0.00 \\ m^{\theta_{4,2}}(no_{(4,2)}) = 0.90 \\ m^{\theta_{4,2}}(\theta_{4,2}) = 0.10 \end{cases}$	$S_{4,3}\begin{cases} m^{\theta_{4,3}}(yes_{(4,3)}) = 0.50 \\ m^{\theta_{4,3}}(no_{(4,3)}) = 0.40 \\ m^{\theta_{4,3}}(\theta_{4,3}) = 0.10 \end{cases}$	$S_{4,4}\begin{cases} m^{\theta_{4,4}}(yes_{(4,4)}) = 0.00 \\ m^{\theta_{4,4}}(no_{(4,4)}) = 0.99 \\ m^{\theta_{4,4}}(\theta_{4,4}) = 0.01 \end{cases}$
$S_{5,1}\begin{cases} m^{\theta_{5,1}}(yes_{(5,1)}) = 0.00 \\ m^{\theta_{5,1}}(no_{(5,1)}) = 0.90 \\ m^{\theta_{5,1}}(\theta_{5,1}) = 0.10 \end{cases}$	$S_{5,2}\begin{cases} m^{\theta_{5,2}}(yes_{(5,2)}) = 0.00 \\ m^{\theta_{5,2}}(no_{(5,2)}) = 0.85 \\ m^{\theta_{5,2}}(\theta_{5,2}) = 0.15 \end{cases}$	$S_{5,3}\begin{cases} m^{\theta_{5,3}}(yes_{(5,3)}) = 0.00 \\ m^{\theta_{5,3}}(no_{(5,3)}) = 0.90 \\ m^{\theta_{5,3}}(\theta_{5,3}) = 0.10 \end{cases}$	$S_{5,4}\begin{cases} m^{\theta_{5,4}}(yes_{(5,4)}) = 0.00 \\ m^{\theta_{5,4}}(no_{(5,4)}) = 0.90 \\ m^{\theta_{5,4}}(\theta_{5,4}) = 0.10 \end{cases}$

method is applied with $k = 2$. The selected information for the detected-to-known association are represented by (17):

$$\begin{cases} \Theta_{1,.} = \{Y_{(1,1)}, Y_{(1,2)}, Y_{(1,*)}\} \\ \Theta_{2,.} = \{Y_{(2,3)}, Y_{(2,4)}, Y_{(2,*)}\} \\ \Theta_{3,.} = \{Y_{(3,1)}, Y_{(3,*)}\} \\ \Theta_{4,.} = \{Y_{(4,3)}, Y_{(4,*)}\} \\ \text{direct decision: } X_5 \text{ appears.} \end{cases} \tag{17}$$

Regarding the association of X_1, the two highest belief masses (0.48 and 0.45) are respectively related to the $Y_{(1,2)}$ and $Y_{(1,1)}$ hypotheses which makes them relevant for decision-making. Thus, we work with the frame $\Theta_{1,.} = \{Y_{(1,1)}, Y_{(1,2)}, Y_{(1,*)}\}$ instead the set of all hypotheses $\{Y_{(1,1)}, Y_{(1,2)}, Y_{(1,3)}, Y_{(1,4)}, Y_{(1,*)}\}$ which decreases the complexity of combination. In the same way, $\Theta_{2,.} = \{Y_{(2,3)}, Y_{(2,4)}, Y_{(2,*)}\}$ because the highest beliefs (0.75 and 0.47) are related to the $Y_{(2,4)}$ and $Y_{(2,3)}$ hypotheses. In this case, $Y_{(2,1)}$ and $Y_{(2,2)}$ are ignored because their beliefs are less significant than those of $Y_{(2,3)}$ and $Y_{(2,4)}$ $(0.75 > 0.47 > 0.32 > 0.00)$. For X_3 and X_4, there is only one piece of information with a non-null belief for their association. Therefore, $\Theta_{3,.} = \{Y_{(3,1)}, Y_{(3,*)}\}$ and $\Theta_{4,.} = \{Y_{(4,3)}, Y_{(4,*)}\}$. Concerning X_5, no source believes on its association, so X_5 is a new detected object which means an appearance $Y_{(5,*)}$. In this case, the decision is directly made without combination. Consequently, the cardinality of each $\Theta_{i,.}$ (17) is reduced which means less computation time when combining.

To make decision, the selected information are combined by (5) and transformed to *DSmP* probabilities by (7). Table 3 represents $DSmP_{i,.}(.)$ based on the two most significant mass functions. The dimension of each $DSmP_{i,.}$ vector is smaller than usual (Cf. Table 4) and corresponds to the number of relevant associations. In this context, the complexity of decision-making can be reduced too. In addition, it can be observed that the proposed approach preserves the relevant association probabilities. Therefore, the same decisions (18) are made through Tables 3 and 4.

Table 3 $BetP_{i,.}$ based on 2-significant mass functions.

$\Theta_{i,.}$	$Y_{(i,1)}$	$Y_{(i,2)}$	$Y_{(i,3)}$	$Y_{(i,4)}$	$Y_{(i,*)}$
$DSmP_{1,.}$	0.40	0.45	–	–	0.15
$DSmP_{2,.}$	–	–	0.27	0.66	0.07
$DSmP_{3,.}$	0.94	–	–	–	0.06
$DSmP_{4,.}$	–	–	0.56	–	0.44

Table 4 $BetP_{i,.}$ based on all mass functions

$\Theta_{i,.}$	$Y_{(i,1)}$	$Y_{(i,2)}$	$Y_{(i,3)}$	$Y_{(i,4)}$	$Y_{(i,*)}$
$DSmP_{1,.}$	**0.39**	**0.43**	0.00	0.00	**0.18**
$DSmP_{2,.}$	0.00	0.11	**0.22**	**0.61**	**0.06**
$DSmP_{3,.}$	**0.95**	0.00	0.00	0.00	**0.05**
$DSmP_{4,.}$	0.00	0.00	**0.56**	0.00	**0.44**
$DSmP_{5,.}$	0.00	0.00	0.00	0.00	1.00

$$\begin{cases} X_1 \rightarrow Y_2 \\ X_2 \rightarrow Y_4 \\ X_3 \rightarrow Y_1 \\ X_4 \rightarrow Y_3 \\ X_5 \text{ appears.} \end{cases} \tag{18}$$

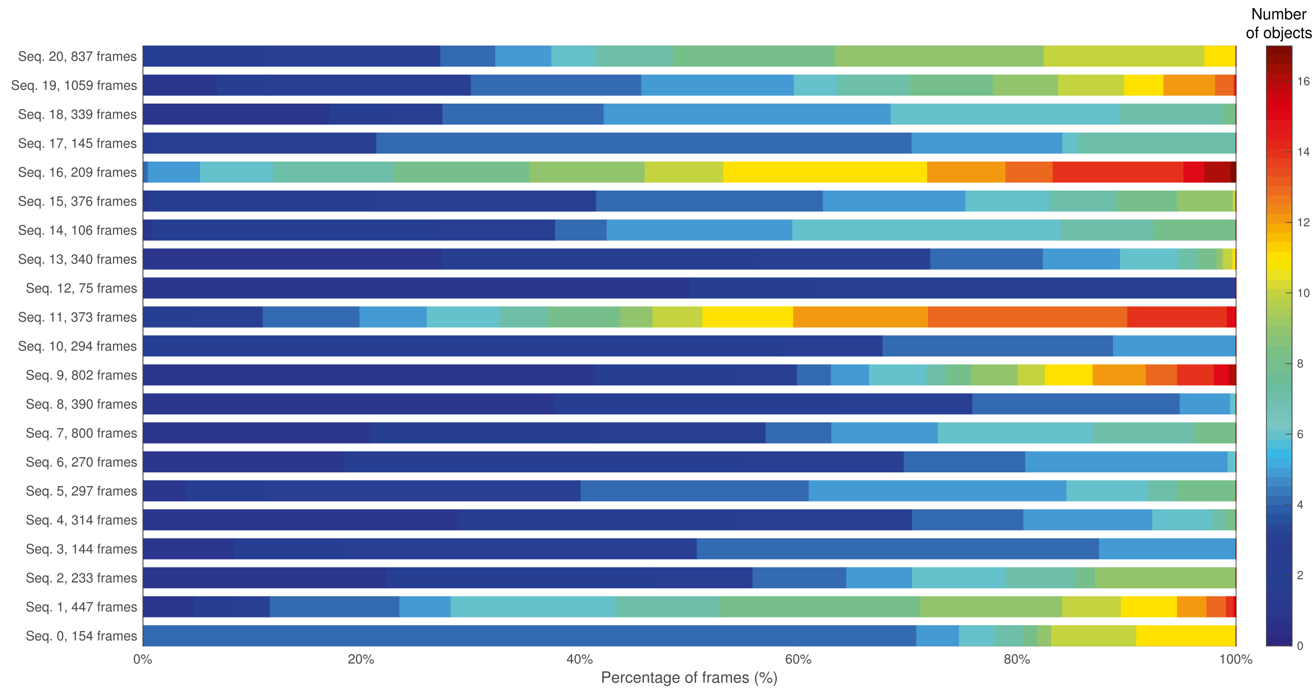

Fig. 4 The number of objects per frame vs. percentage of frames

Table 5 KITTI image sequence characteristics

	Seq. 2	Seq. 4	Seq. 6	Seq. 7	Seq. 8	Seq. 13	Seq. 14	Seq. 16	Seq. 18	Seq. 19	Seq. 20
Number of frames	233	314	270	800	390	340	106	209	339	1059	837
Number of associations	668	545	474	2083	492	617	744	1872	1130	4968	4673
Max vehicle speed (km/h)	43	56	33	34	62	26	35	0	55	21	54
Min vehicle speed (km/h)	0	20	0	1	38	8	1	0	0	0	0
Speed < 30 km/h (%)	66	15	93	75	0	100	87	100	66	100	51
Speed > 30 km/h (%)	34	85	7	25	100	0	13	0	34	0	49

5 Experimental results

This section evaluates the proposed approach on real data coming from the well-known KITTI dataset (Geiger et al. 2012). First, the dataset description is presented, followed by the experimental setting. Secondly, the obtained results are analyzed and commented. It is noted that this evaluation focuses only on data association, so no tracking is done.

5.1 Datasets

The KITTI vision dataset provides data recorded from different sensors mounted on a moving vehicle on urban roads (Geiger et al. 2012). It contains camera images, laser scans, and GPS/IMU data. The dataset also includes object labels classified in 8 categories. For this evaluation, only image data have been used where detections are defined by 2D bounding box tracklets. Four object classes have been considered: pedestrian, cyclist, car, and van. Table 5 presents a part of these sequences according to their different road context and the number of detections. On some sequences, the vehicle mainly moving at a speed less than 30 *km*/*h* which is common in urban areas, e.g. sequences 6, 13, 14, and 19. Sequence 16 was recorded when the vehicle stopped at a crosswalk, i.e. $speed = 0\ \mathrm{km/h}$. On other sequences, the vehicle was moving at a speed sometimes exceeding 50 *km*/*h*, e.g. sequences 4 and 8. Figure 4 illustrates the number of objects per image and their proportion on each of the sequences where more than 30000 associations have been evaluated. To the best of our knowledge, no study has been evaluated on so many real data. These latter cover different road scenarii containing various objects as shown in Fig. 5.

5.2 Experimental setting

The matching process is based on the distance between objects attributes. In this work, only 2D position in the image plane

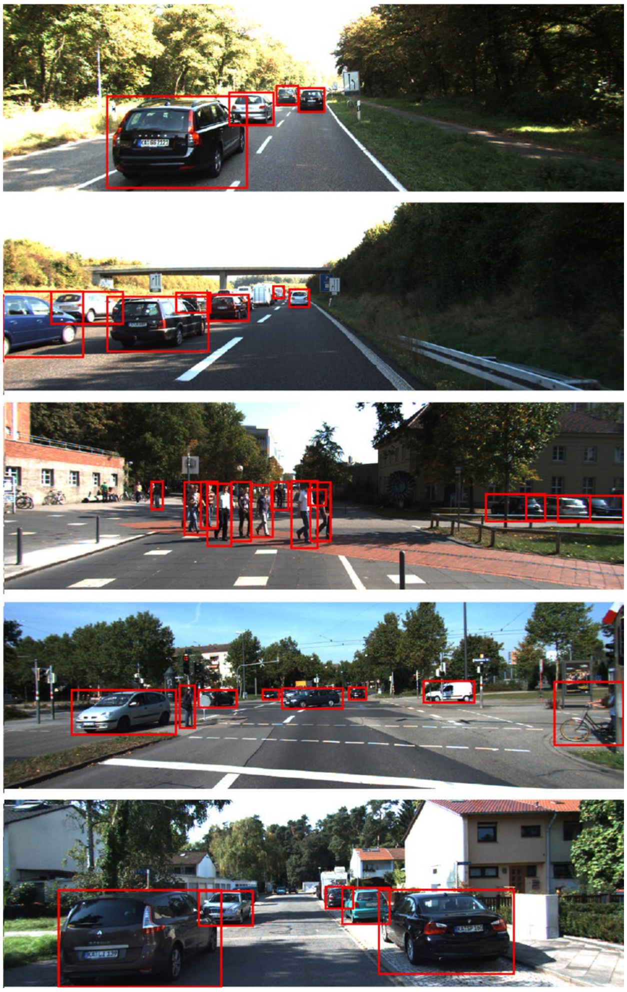

Fig. 5 Examples of images provided by KITTI (Boumediene 2019)

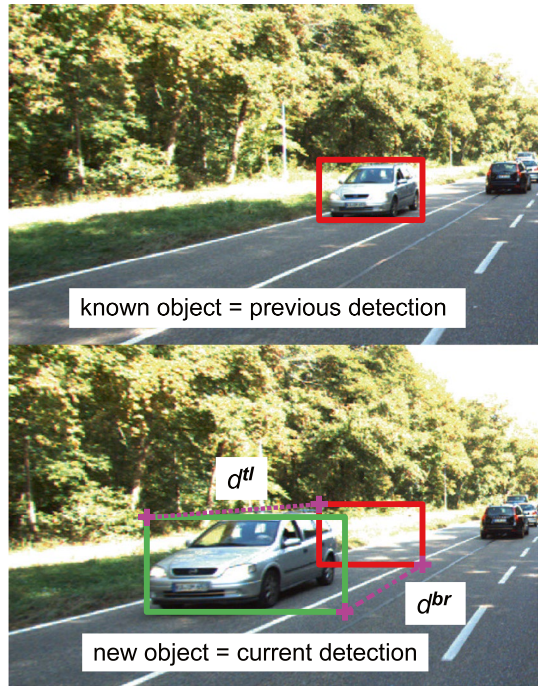

Fig. 6 The illustration of the distance between a detected and a known object (Boumediene 2019)

is considered as pieces of evidence. Thus, the distance $d_{i,j}$ is defined as follows:

$$d_{i,j} = 0.5 \times (d_{i,j}^{left} + d_{i,j}^{right}) \tag{19}$$

where $d_{i,j}^{left}$ ($d_{i,j}^{right}$) is the Euclidean distance between top-left (bottom-right) points of the bounding boxes of objects X_i and Y_j as illustrated in Fig. 6.

The critical parameters to estimate belief masses are: $\alpha = 0.9$, $\tau = 0.5$ and $\epsilon = 0.001$ for *DSmP* transformation. The proposed approach is written in C++ and runs on Intel core i7 2.20 GHz with 8 GB RAM.

5.3 Results and analysis

The performance of the k-significant sources combination refers to its capacity to reduce complexity while maintaining a high decision quality. Therefore, the evaluation focuses on the Computation Time (*CT*) and the *recall* which are defined as follows:

$$\begin{cases} CT = \sum_t ET_t \\ recall = \frac{\sum_t TA_t}{\sum_t GT_t} \end{cases} \tag{20}$$

where ET_t is the execution time of the frame t, TA_t and GT_t are the numbers of true associations and ground truth associations respectively.

Table 6 compares the running time of the combination step using two approaches according to the number of objects. The first is to combine all the sources and the second combines the k-significant sources where $k \in [2, 4]$. To show the real-time aspect of the proposed approach, the association process is applied for 24 frames. The results confirm that the proposed approach needs low computation time than combining all sources. The smaller the number of combined sources, the shorter the computation time. With $n = m = 13$, the proposed approach ($k = 2$) needs $1.33ms$ on 24 frames while combining all sources takes $\simeq 4$ minutes which is not acceptable for real-time applications. In addition, combining all sources grows exponentially the computation cost with (n, m) while the time complexity of the proposed approach is polynomial which makes it well-suited for real-time applications (Cf. Fig. 7).

Table 7 compares the complexity of the proposed decision-making algorithm with the JPP method according to

Table 6 Computation time (ms) of the combination step for 24 frames containing (n, m) objects

(n, m)	All sources	4-Sig. Src.	3-Sig. Src.	2-Sig. Src.
(4, 4)	1.33	1.49	0.60	0.39
(7, 7)	$> 0.1s$	2.27	0.92	0.59
(10, 10)	$> 5s$	3.54	1.35	0.89
(13, 13)	$\simeq 4$ min	5.28	2.20	1.33

Table 7 Computation time (ms) of the decision-making step for 24 frames containing (n, m) objects

(n, m)	JPP	Our method	Comp. time gain (%)
(2, 2)	0.21	0.16	23.91
(3, 3)	1.2	0.16	86.66
(4, 4)	9	0.21	97.66
(5, 5)	104	0.27	99.74
(6, 6)	$> 9s$	0.33	99.99
(7, 7)	$> 46min$	0.90	99.99

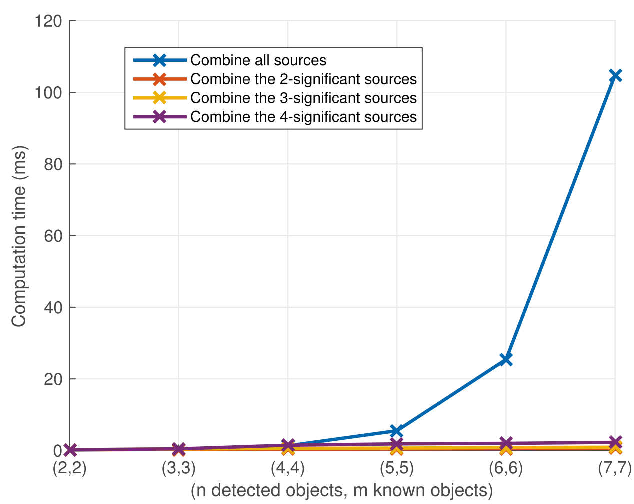

Fig. 7 Computation time of the combination step as a function of the number of objects

the number of objects. Both of these methods are based on a global optimization. The results show that the proposed algorithm needs low computation time than JPP to make association decisions. With more than 4 perceived/detected objects, the complexity is reduced by more than 97%. For instance, with $n = m = 7$, our proposed algorithm needs less than $1ms$ to assign perceived objects on 24 frames while JPP takes too large time, more than $\simeq 46$ minutes. Figure 8 confirms that our algorithm is characterized by a polynomial complexity while JPP has a high exponential complexity which makes impossible its application on the KITTI

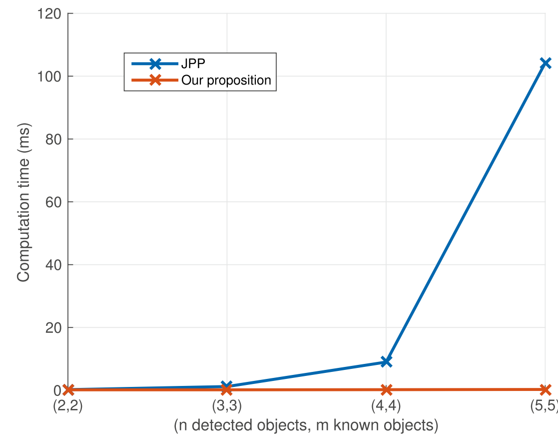

Fig. 8 Computation time of the decision-making step as a function of the number of objects

sequences. For this reason, the rest of the results presented in this section are obtained by our simple decision-making algorithm.

To measure the gain on complexity, the variation in the computation time of a system without $(CT^i_{\ w})$ and with the k-significant sources combination (CT^i_k) is computed for each sequence (i) (21). The higher gain, the better complexity reduction we get. In the same manner, the recall gain is computed (22) . The higher $Gain^i_{\ recall}$, the better decision-quality we get. A higher $Gain^i_{\ recall}$ preserves well the decision-quality.

$$Gain^i_{\ CT} = \frac{(CT^i_{\ w} - CT^i_{\ k})}{CT^i_{\ w}} 100. \tag{21}$$

$$Gain^i_{\ recall} = \frac{(recall^i_{\ k} - recall^i_{\ w})}{recall^i_{\ w}} 100. \tag{22}$$

The weighted average of gain based on all sequences is given by:

$$\begin{cases} Gain^{avg}_{CT} & = \sum_{i=0}^{20} w_i Gain^i_{\ CT} \\ Gain^{avg}_{recall} & = \sum_{i=0}^{20} w_i Gain^i_{\ recall} \end{cases} \tag{23}$$

where the weight w_i is $w_i = n_i / \sum_{i=0}^{20} n_i$ and n_i being the number of associations of the i-th sequence.

Figure 9 presents the weighted average of the computation time gain versus k. These results are obtained by varying the number of significant sources selected, i.e. k. For all dataset, more than 30000 associations, the gain exceeds 99.90% which is well-suited for real-time applications. This gain is explained by the fact that our approach has a polynomial complexity while combining all sources is characterized by an exponential

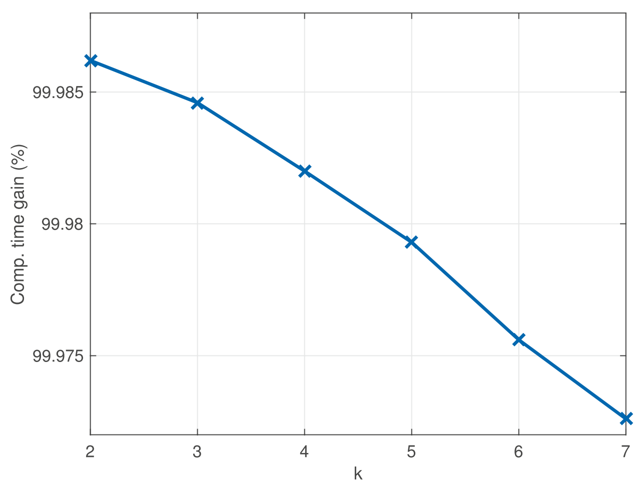

Fig. 9 Computation time gain as a function of the parameter k

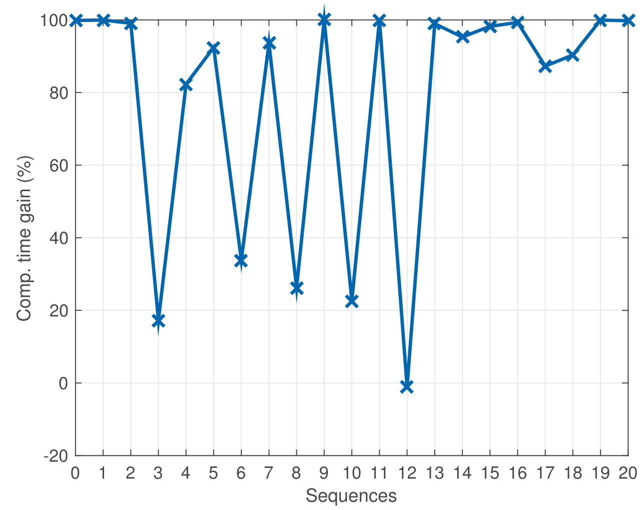

Fig. 10 Computation time gain of 3-sig. Sources approach on each sequence

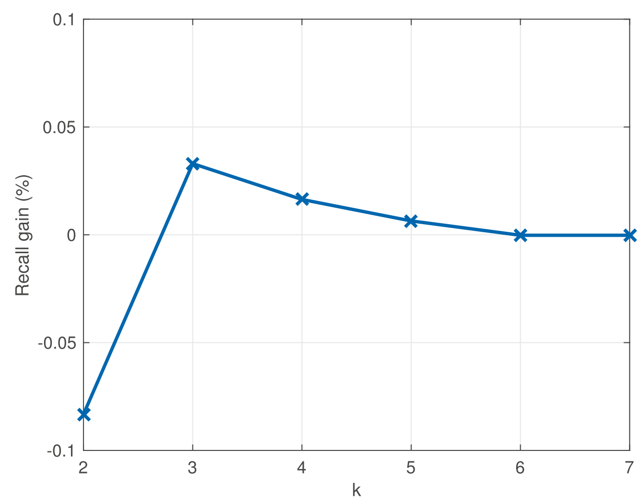

Fig. 11 Recall gain as a function of the parameter k new

complexity (Cf. Fig. 7). In addition, the obtained results show that the computation time reduction is inversely proportional to the k parameter as shown in Table 6. Indeed, by reducing the number of significant sources, the combination complexity decreases which allows a more important gain. Although if the gain, which is expressed as a percentage, seems small between the different values of $k \in [2, 7]$, it remains important for real-time constrain.

The gain depends also on the number of perceived objects. In fact, contrary to our approach, combining all sources increases exponentially the computation time with perceived/detected objects (n, m). Therefore, the more objects in the scene, the greater the gain will be (Cf. Fig. 10). That is why for sequences 3, 6, 8, 10, and 12 where the number of detections is mostly less than 4, the gain is less than 40% while for other sequences is more than 80%. Therefore, the obtained results lead to conclude that the more complex is the sequence, the larger is the computation time reduction.

Now, how about the decision quality? Combine just the significant sources, affects the decisions or not? Fig. 11 presents the weighted average of the recall gain versus k. it is clear that the gain is insignificant, $-0.1\% < Gain_{\ recall} < 0.05\%$. This result proves that focusing only on significant information does not necessary affect the decision quality. Furthermore, the obtained results also show that ignoring the useless information can improve slightly the quality of decisions. For instance, on sequences 11, 17, and 18 the association decisions are improved by more than 4% (Cf. Fig. 12). Therefore, the solution proposed provides good performances by reducing significantly the computation time while preserving the association decisions.

The choice of parameter k depends on the application context and on the desired performances. For the object association in road environment and based on our tests, $k = 3$ appears to be a good setting threshold parameter.

6 Conclusion

This paper presented a new evidential data association based on significant sources combination and a simple decision-making algorithm. The main objective of the proposed approach is to reduce the complexity and time consumption of data fusion based on DSmT techniques (PCR6 and DSmP). This approach focuses only on information having belief in top k highest masses and removes useless information. Therefore, only k-significant sources are combined to deal with the association problem.

Applied to intelligent vehicles perception, the experimental results show the effectiveness of the proposed approach in the reduction of the complexity by more than 99% in dense scenes. Besides, experimental results show that the proposed solution preserves well the decision-quality. It

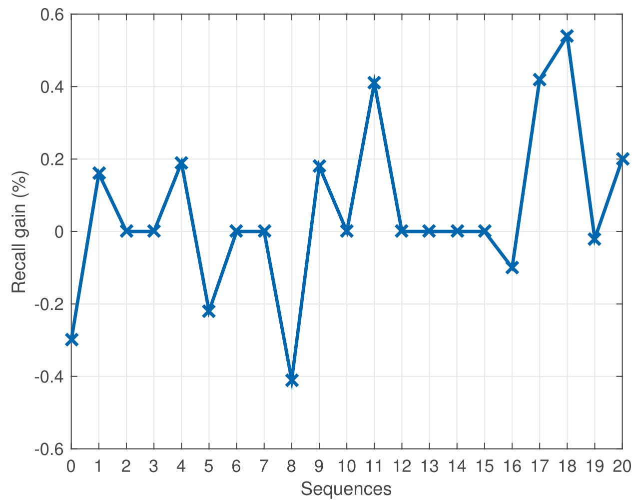

Fig. 12 Recall gain of 3-sig. sources approach on each sequence

can be noted that the k-significant sources combination is not intended only for road environment perception. It can be applied to any data association process based on these DSmT techniques.

Future work should combine heterogeneous sensor data to enhance the object association. Also, we plan to evaluate if an improvement of PCR6 rule of combination would be helpful for the data association problems.

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