

# Supplementary Material: Statistical performance of local attractor dimension estimators in non-Axiom A dynamical systems

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## SUPPLEMENTARY FIGURES

This section contains all the figures showing the statistical performance of the considered estimators in terms of root mean square error and relative error for the Monte Carlo experiment.

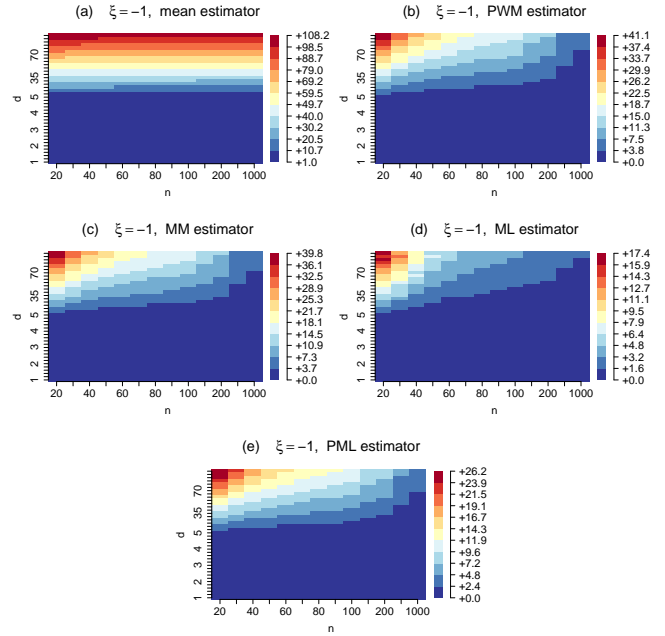


Fig. S1. Root mean square error of the mean estimator (a), PWM (b), MM (c), ML (d) and PML (e) on  $d$  as a function of  $d$  and  $n$ , for  $\xi = -1$ .

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<sup>b)</sup> Also at London Mathematical Laboratory, 14 Buckingham Street, London, WC2N 6DF, UK

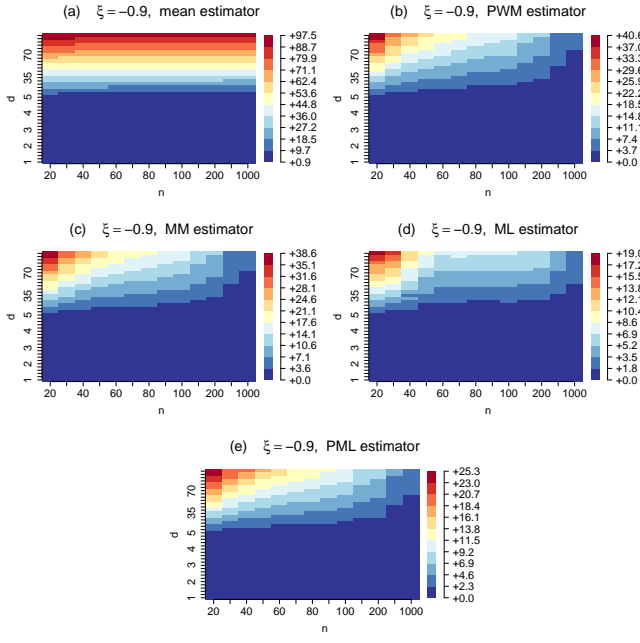


Fig. S2. Root mean square error of the mean estimator (a), PWM (b), MM (c), ML (d) and PML (e) on  $d$  as a function of  $d$  and  $n$ , for  $\xi = -0.9$ .

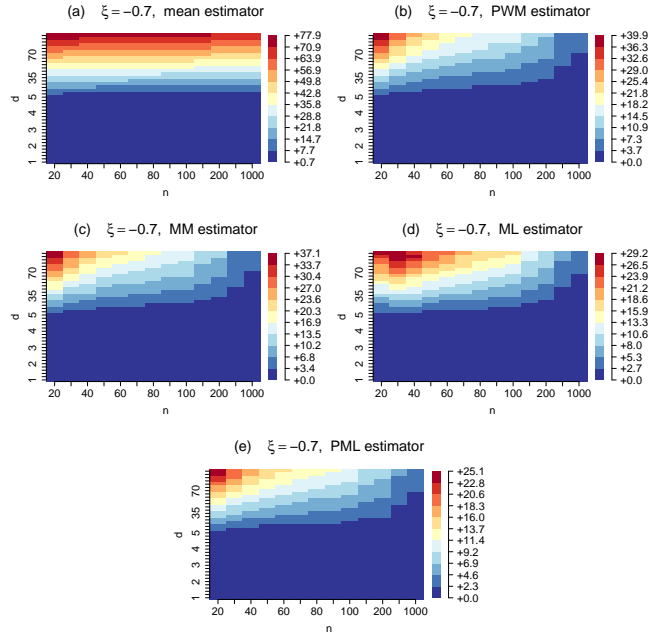


Fig. S4. Root mean square error of the mean estimator (a), PWM (b), MM (c), ML (d) and PML (e) on  $d$  as a function of  $d$  and  $n$ , for  $\xi = -0.7$ .

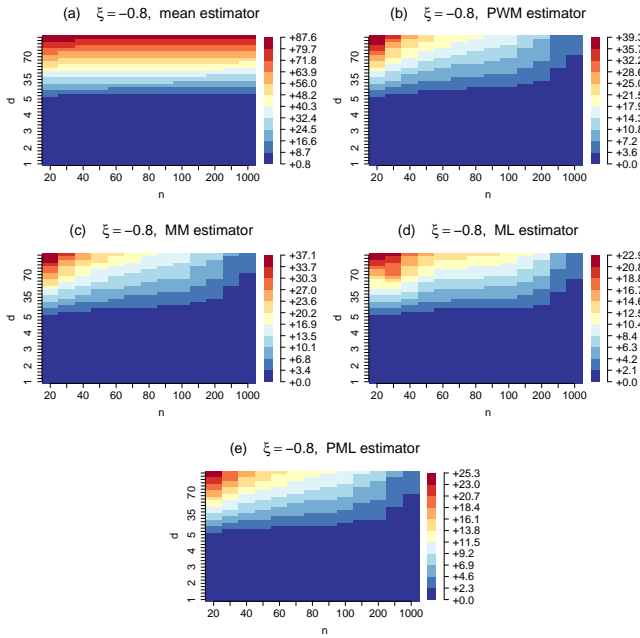


Fig. S3. Root mean square error of the mean estimator (a), PWM (b), MM (c), ML (d) and PML (e) on  $d$  as a function of  $d$  and  $n$ , for  $\xi = -0.8$ .

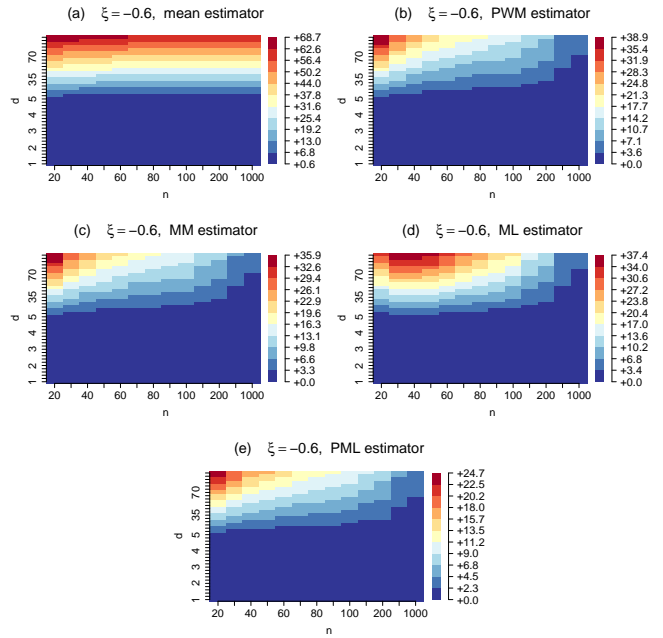


Fig. S5. Root mean square error of the mean estimator (a), PWM (b), MM (c), ML (d) and PML (e) on  $d$  as a function of  $d$  and  $n$ , for  $\xi = -0.6$ .

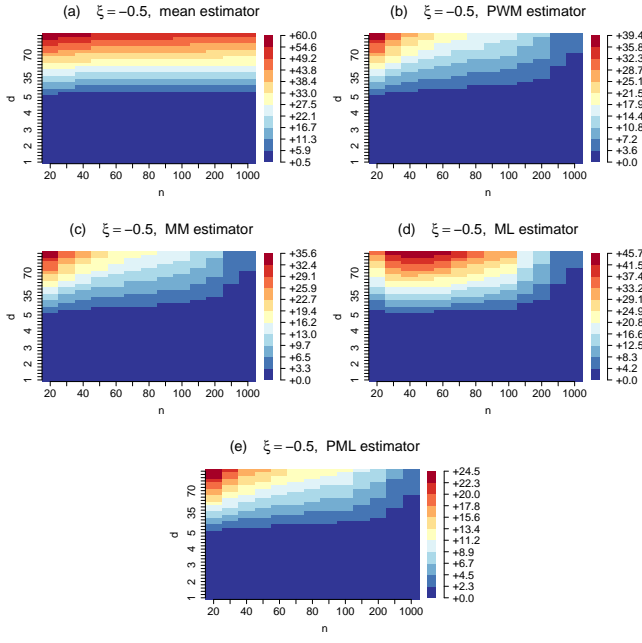


Fig. S6. Root mean square error of the mean estimator (a), PWM (b), MM (c), ML (d) and PML (e) on  $d$  as a function of  $d$  and  $n$ , for  $\xi = -0.5$ .

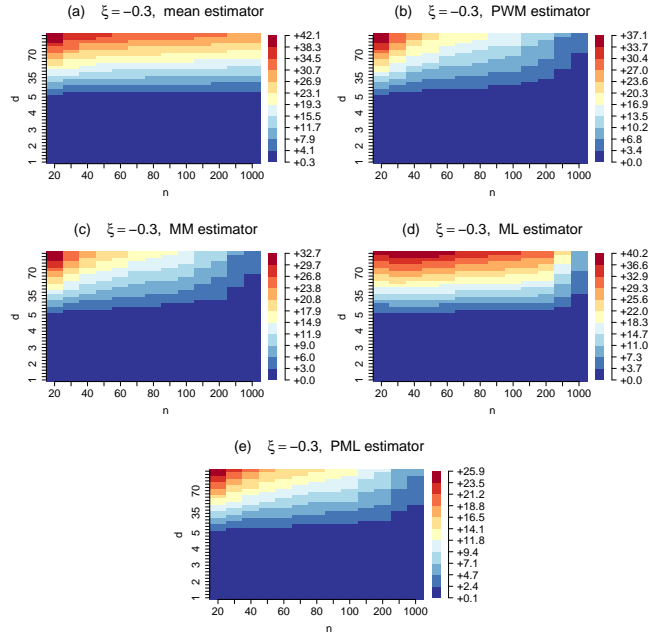


Fig. S8. Root mean square error of the mean estimator (a), PWM (b), MM (c), ML (d) and PML (e) on  $d$  as a function of  $d$  and  $n$ , for  $\xi = -0.3$ .

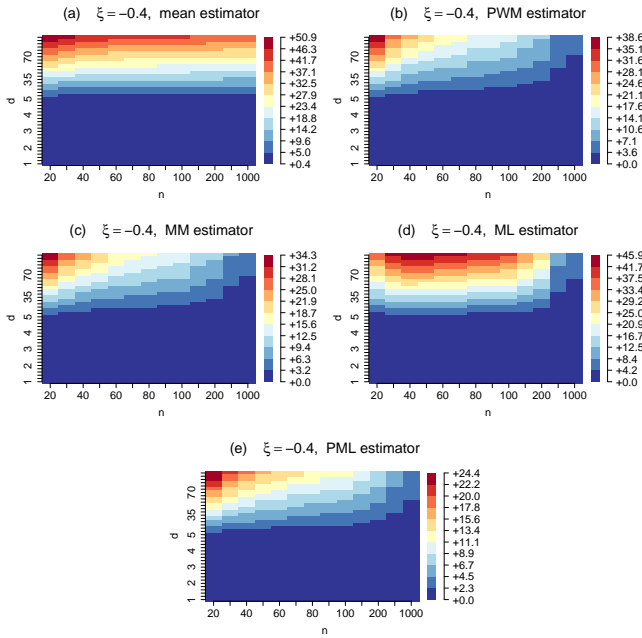


Fig. S7. Root mean square error of the mean estimator (a), PWM (b), MM (c), ML (d) and PML (e) on  $d$  as a function of  $d$  and  $n$ , for  $\xi = -0.4$ .

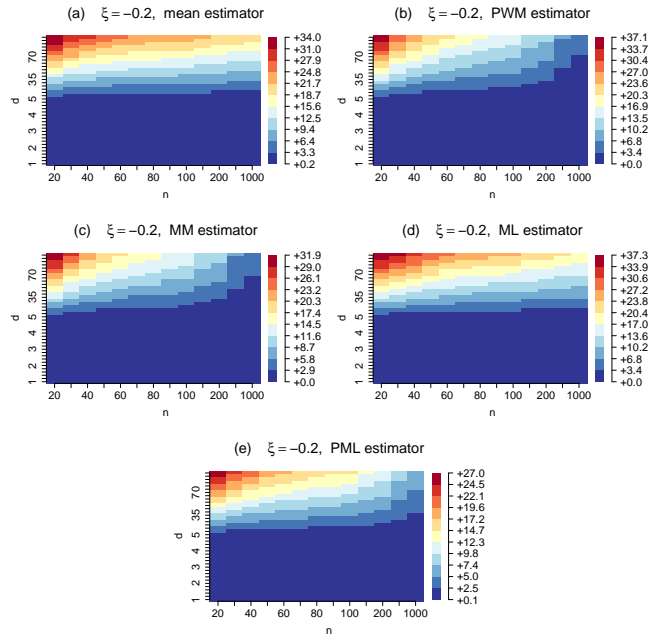


Fig. S9. Root mean square error of the mean estimator (a), PWM (b), MM (c), ML (d) and PML (e) on  $d$  as a function of  $d$  and  $n$ , for  $\xi = -0.2$ .

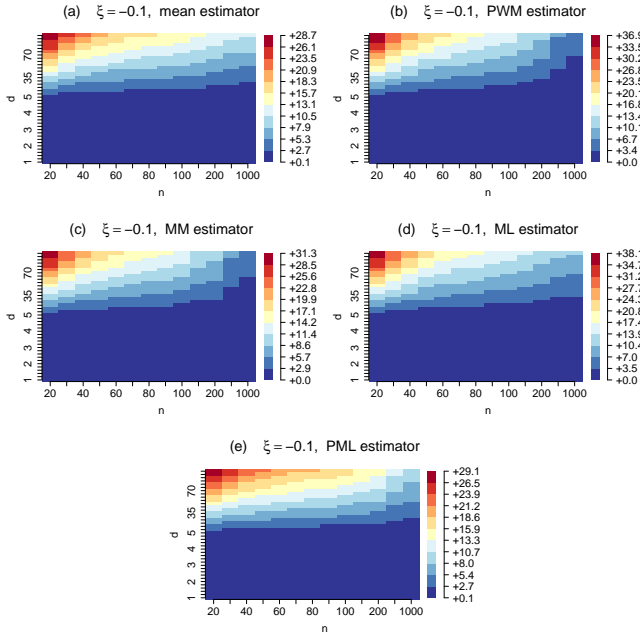


Fig. S10. Root mean square error of the mean estimator (a), PWM (b), MM (c), ML (d) and PML (e) on  $d$  as a function of  $d$  and  $n$ , for  $\xi = -0.1$ .

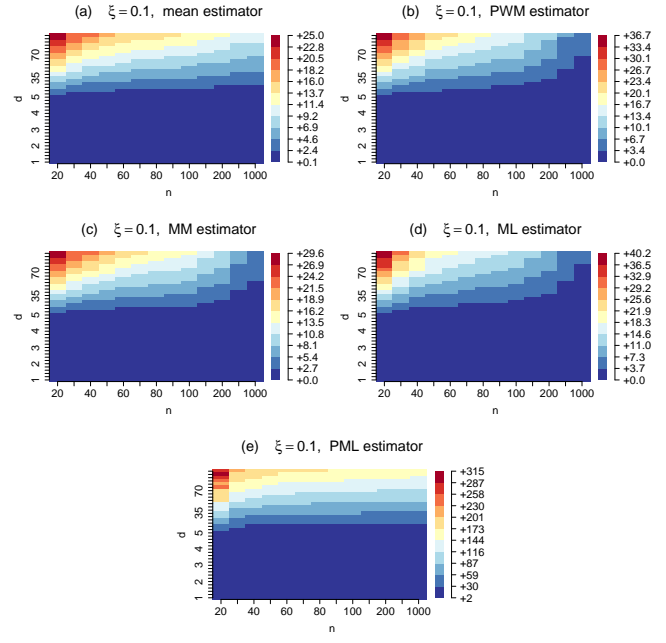


Fig. S12. Root mean square error of the mean estimator (a), PWM (b), MM (c), ML (d) and PML (e) on  $d$  as a function of  $d$  and  $n$ , for  $\xi = 0.1$ .

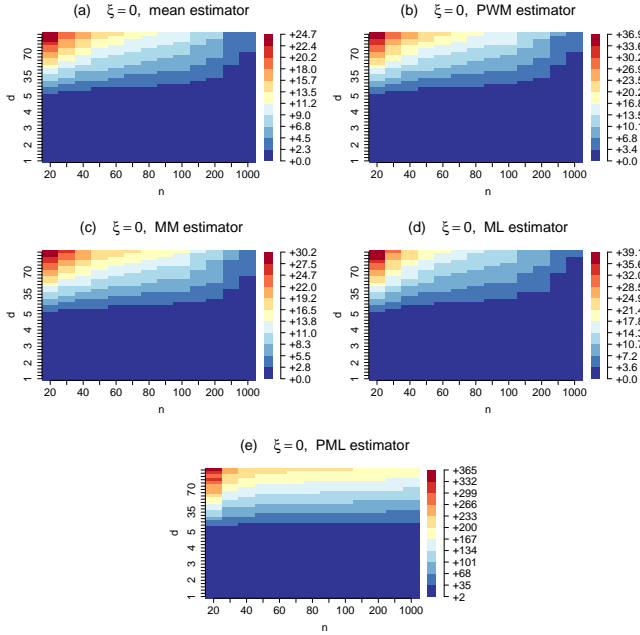


Fig. S11. Root mean square error of the mean estimator (a), PWM (b), MM (c), ML (d) and PML (e) on  $d$  as a function of  $d$  and  $n$ , for  $\xi = 0$ .

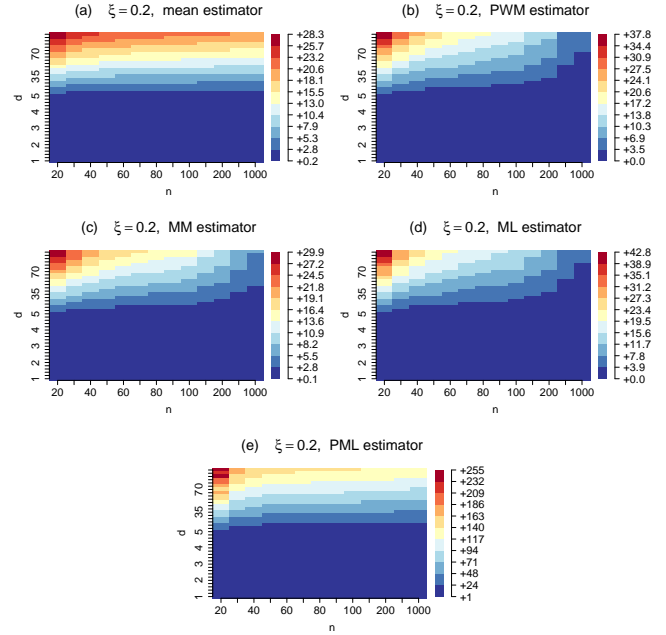


Fig. S13. Root mean square error of the mean estimator (a), PWM (b), MM (c), ML (d) and PML (e) on  $d$  as a function of  $d$  and  $n$ , for  $\xi = 0.2$ .

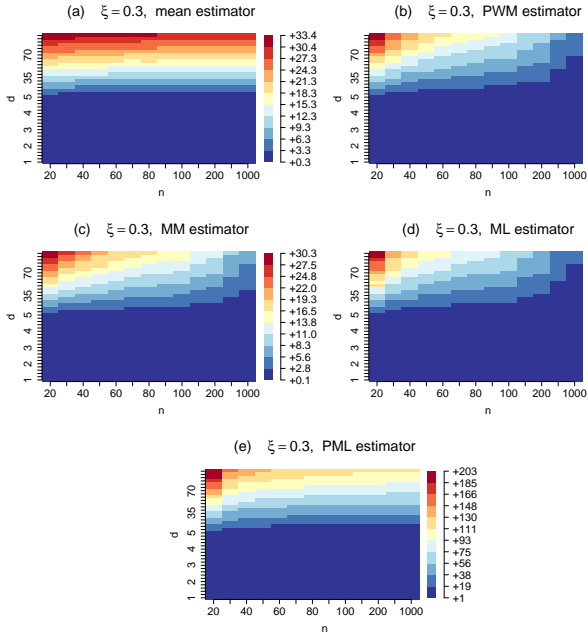


Fig. S14. Root mean square error of the mean estimator (a), PWM (b), MM (c), ML (d) and PML (e) on  $d$  as a function of  $d$  and  $n$ , for  $\xi = 0.3$ .

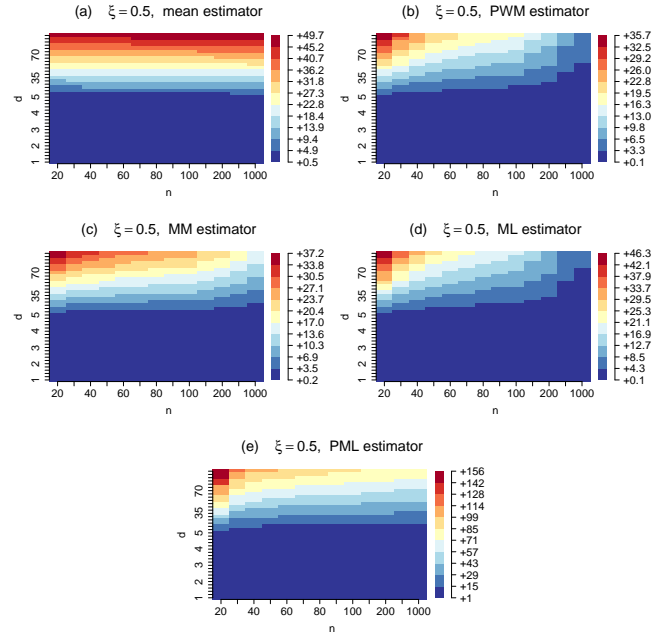


Fig. S16. Root mean square error of the mean estimator (a), PWM (b), MM (c), ML (d) and PML (e) on  $d$  as a function of  $d$  and  $n$ , for  $\xi = 0.5$ .

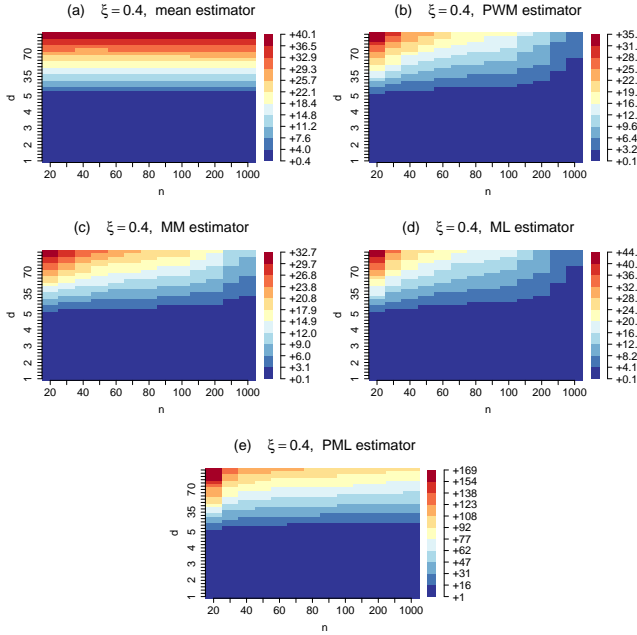


Fig. S15. Root mean square error of the mean estimator (a), PWM (b), MM (c), ML (d) and PML (e) on  $d$  as a function of  $d$  and  $n$ , for  $\xi = 0.4$ .

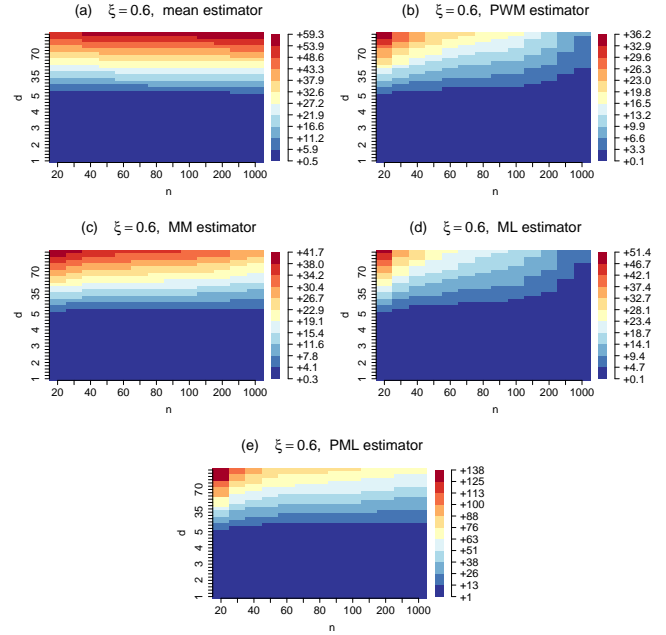


Fig. S17. Root mean square error of the mean estimator (a), PWM (b), MM (c), ML (d) and PML (e) on  $d$  as a function of  $d$  and  $n$ , for  $\xi = 0.6$ .

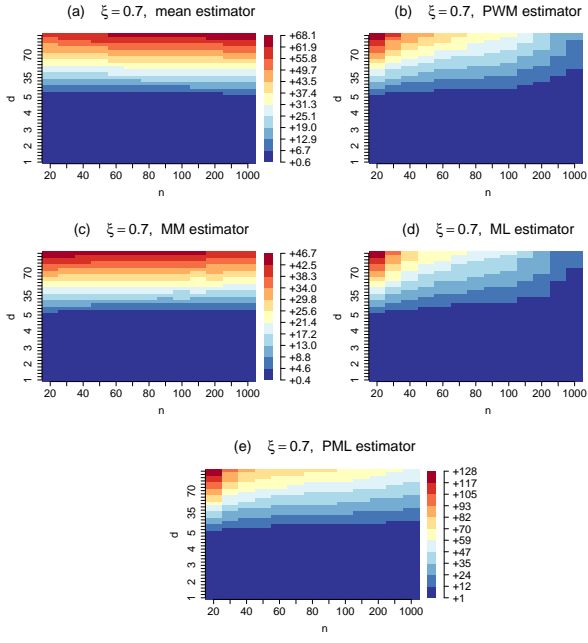


Fig. S18. Root mean square error of the mean estimator (a), PWM (b), MM (c), ML (d) and PML (e) on  $d$  as a function of  $d$  and  $n$ , for  $\xi = 0.7$ .

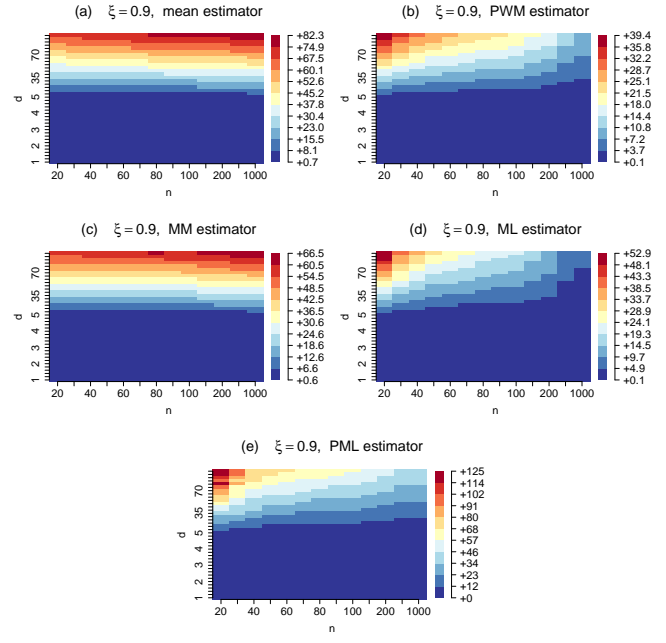


Fig. S20. Root mean square error of the mean estimator (a), PWM (b), MM (c), ML (d) and PML (e) on  $d$  as a function of  $d$  and  $n$ , for  $\xi = 0.9$ .

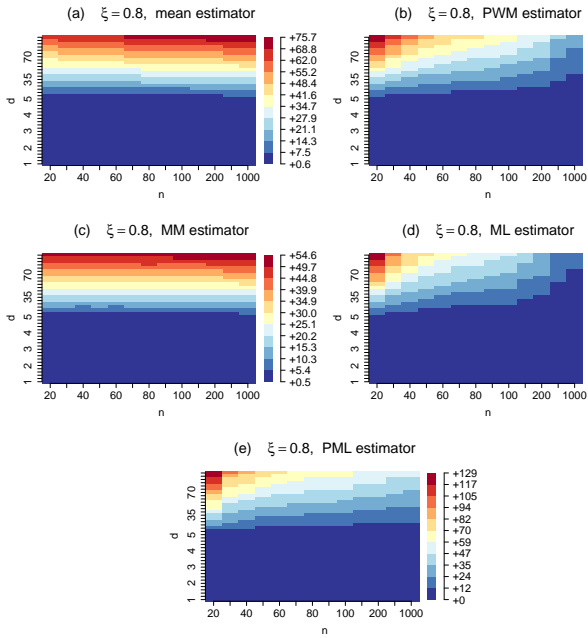


Fig. S19. Root mean square error of the mean estimator (a), PWM (b), MM (c), ML (d) and PML (e) on  $d$  as a function of  $d$  and  $n$ , for  $\xi = 0.8$ .

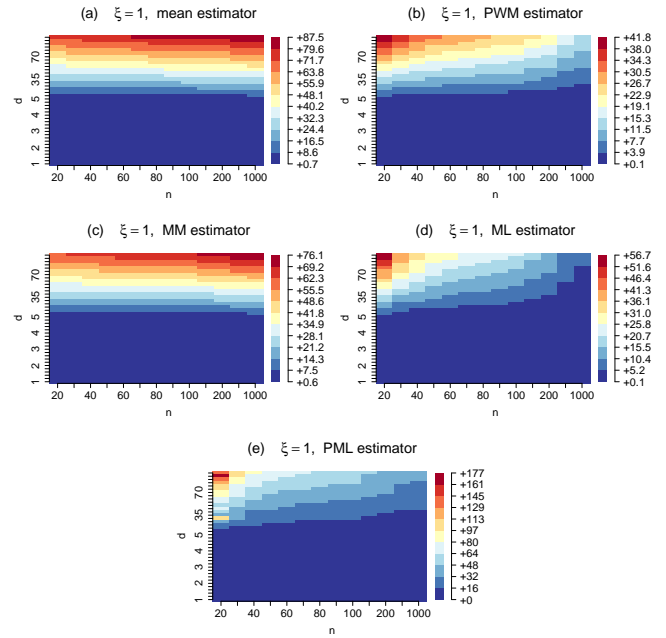


Fig. S21. Root mean square error of the mean estimator (a), PWM (b), MM (c), ML (d) and PML (e) on  $d$  as a function of  $d$  and  $n$ , for  $\xi = 1$ .

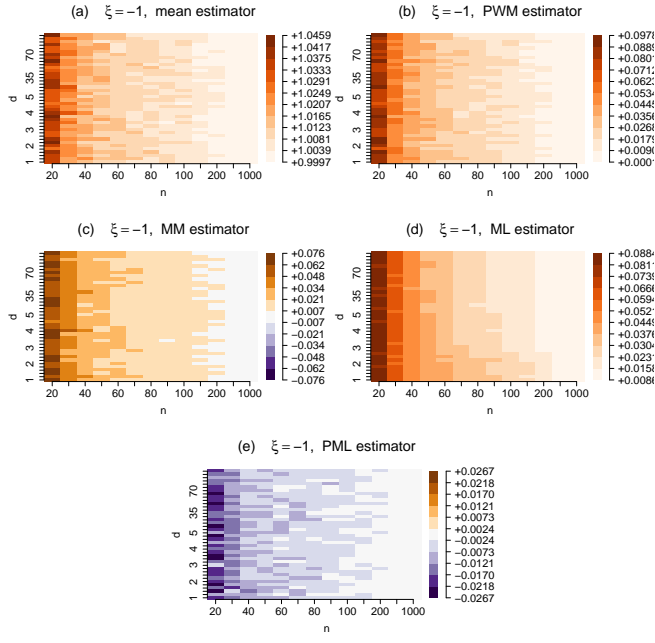


Fig. S22. Relative error of the mean estimator (a), PWM (b), MM (c), ML (d) and PML (e) on  $d$  as a function of  $d$  and  $n$ , for  $\xi = -1$ .

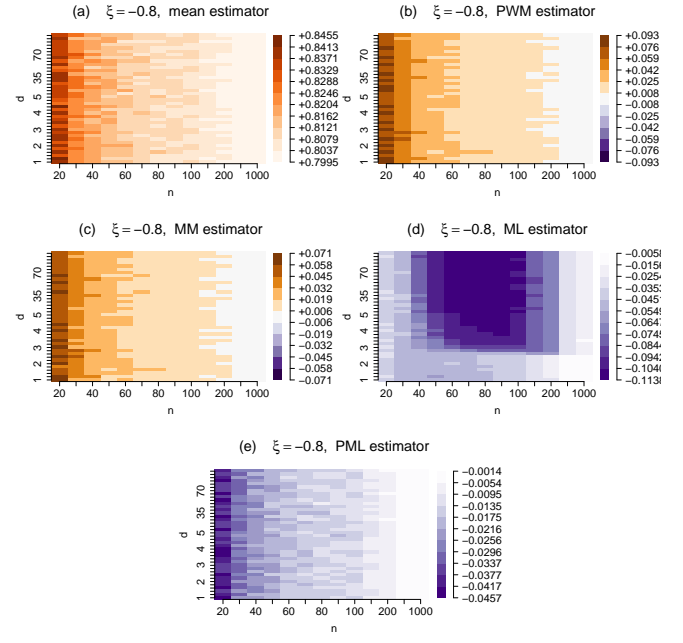


Fig. S24. Relative error of the mean estimator (a), PWM (b), MM (c), ML (d) and PML (e) on  $d$  as a function of  $d$  and  $n$ , for  $\xi = -0.8$ .

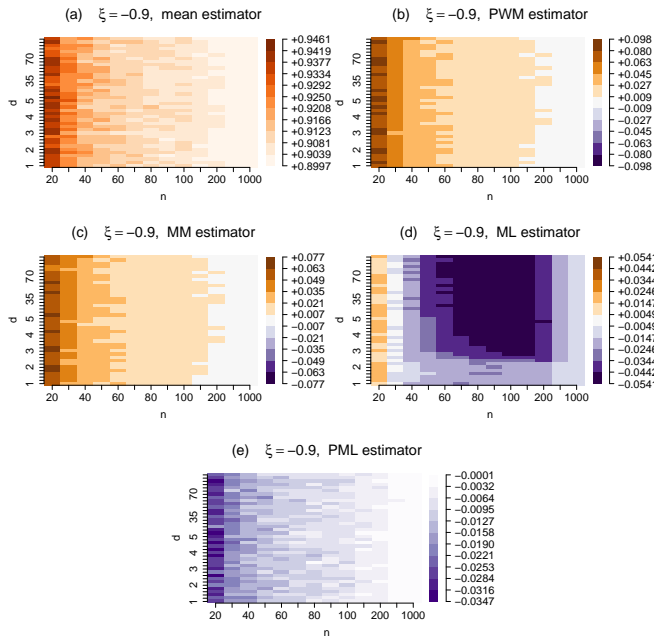


Fig. S23. Relative error of the mean estimator (a), PWM (b), MM (c), ML (d) and PML (e) on  $d$  as a function of  $d$  and  $n$ , for  $\xi = -0.9$ .

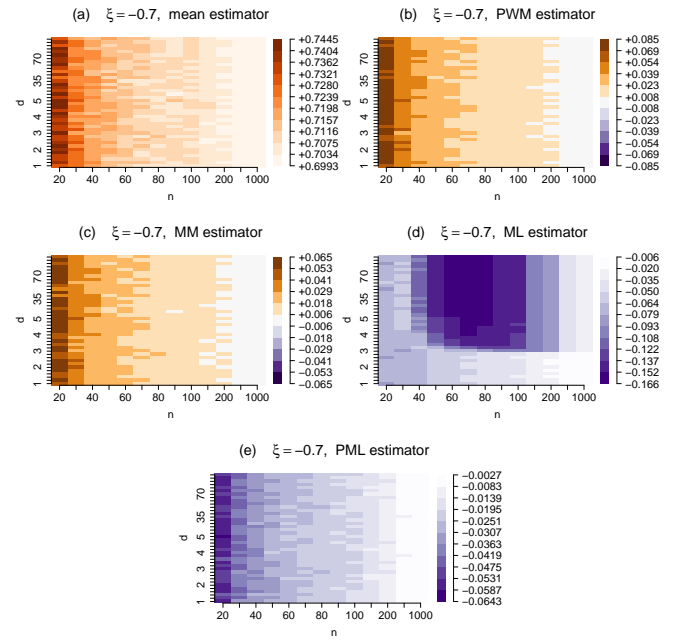


Fig. S25. Relative error of the mean estimator (a), PWM (b), MM (c), ML (d) and PML (e) on  $d$  as a function of  $d$  and  $n$ , for  $\xi = -0.7$ .

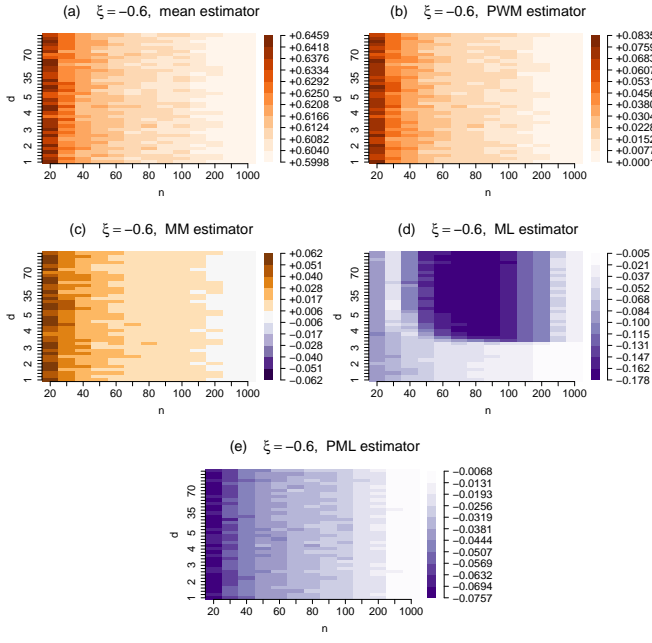


Fig. S26. Relative error of the mean estimator (a), PWM (b), MM (c), ML (d) and PML (e) on  $d$  as a function of  $d$  and  $n$ , for  $\xi = -0.6$ .

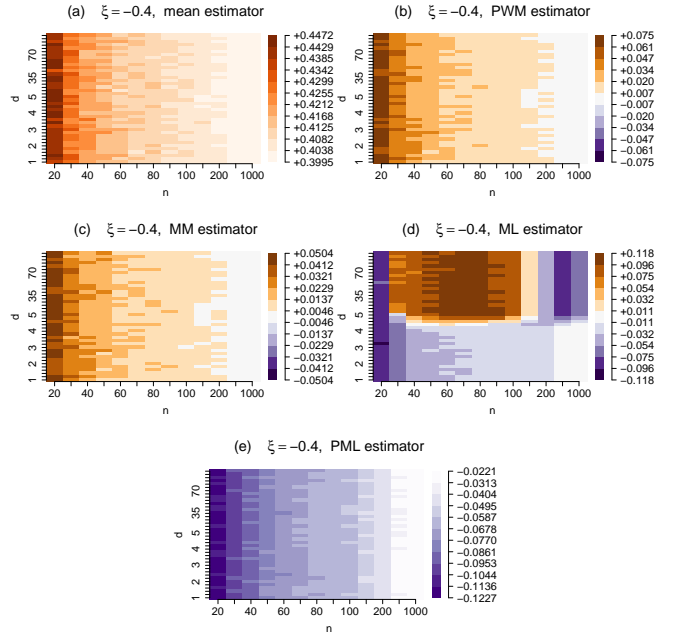


Fig. S28. Relative error of the mean estimator (a), PWM (b), MM (c), ML (d) and PML (e) on  $d$  as a function of  $d$  and  $n$ , for  $\xi = -0.4$ .

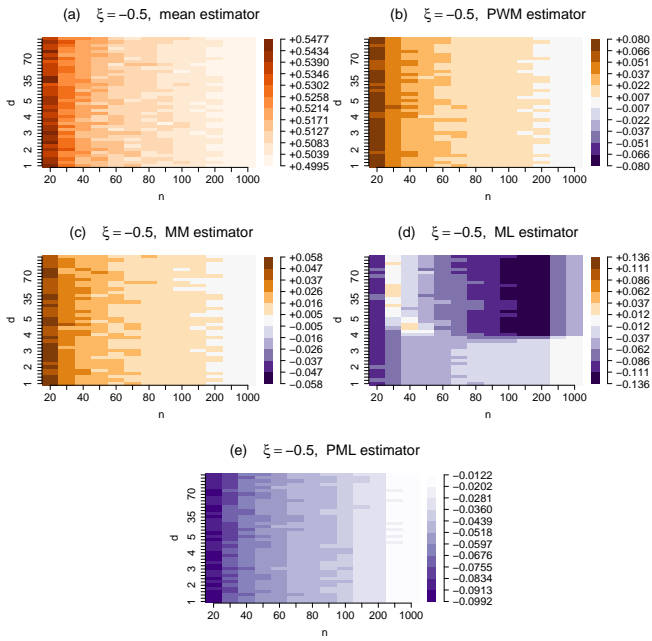


Fig. S27. Relative error of the mean estimator (a), PWM (b), MM (c), ML (d) and PML (e) on  $d$  as a function of  $d$  and  $n$ , for  $\xi = -0.5$ .

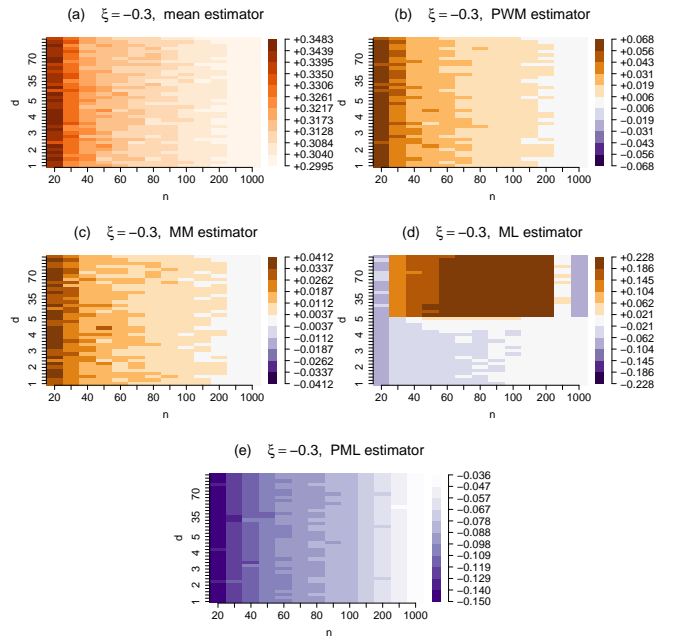


Fig. S29. Relative error of the mean estimator (a), PWM (b), MM (c), ML (d) and PML (e) on  $d$  as a function of  $d$  and  $n$ , for  $\xi = -0.3$ .



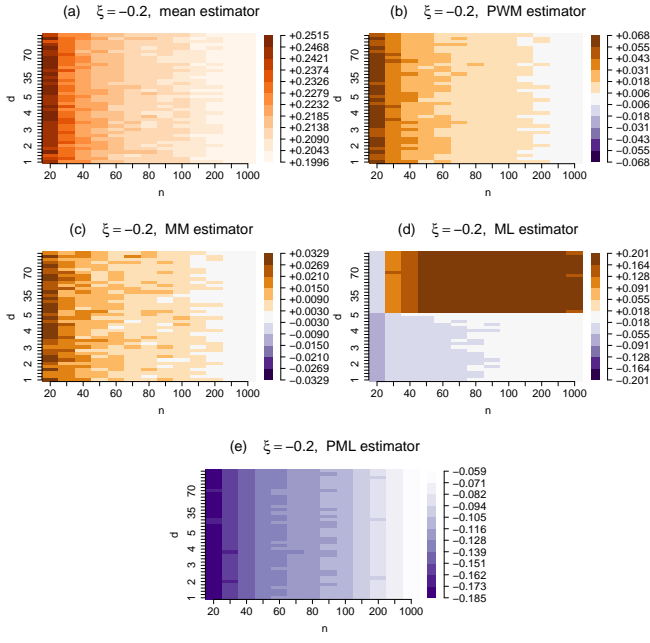


Fig. S30. Relative error of the mean estimator (a), PWM (b), MM (c), ML (d) and PML (e) on  $d$  as a function of  $d$  and  $n$ , for  $\xi = -0.2$ .

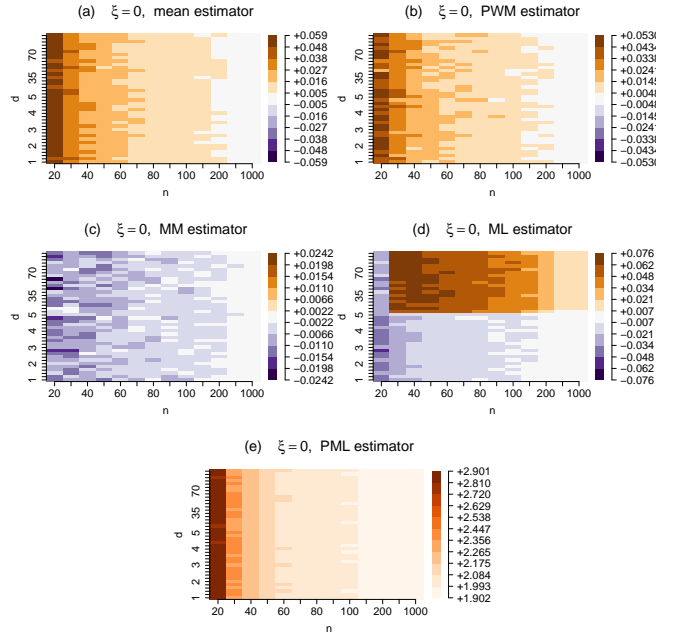


Fig. S32. Relative error of the mean estimator (a), PWM (b), MM (c), ML (d) and PML (e) on  $d$  as a function of  $d$  and  $n$ , for  $\xi = 0$ .

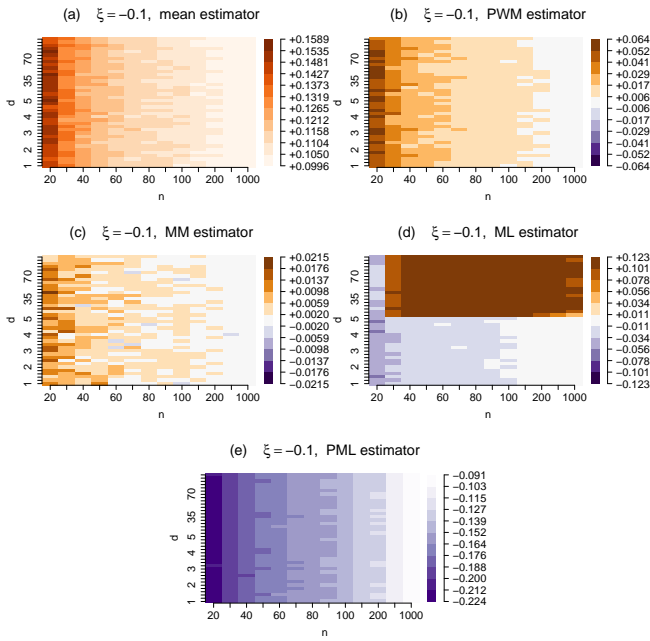


Fig. S31. Relative error of the mean estimator (a), PWM (b), MM (c), ML (d) and PML (e) on  $d$  as a function of  $d$  and  $n$ , for  $\xi = -0.1$ .

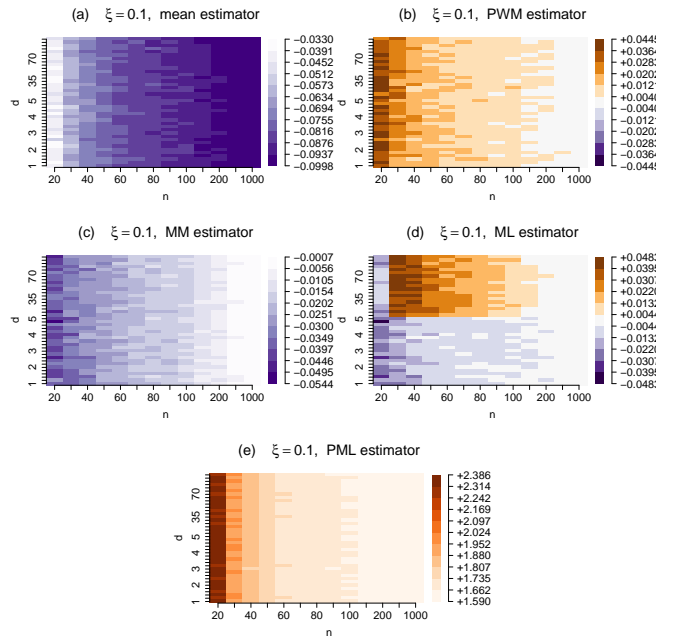


Fig. S33. Relative error of the mean estimator (a), PWM (b), MM (c), ML (d) and PML (e) on  $d$  as a function of  $d$  and  $n$ , for  $\xi = 0.1$ .

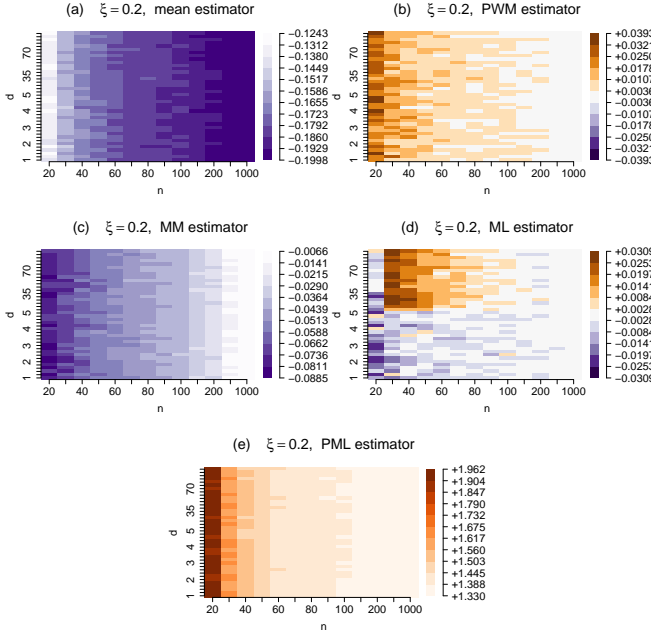


Fig. S34. Relative error of the mean estimator (a), PWM (b), MM (c), ML (d) and PML (e) on  $d$  as a function of  $d$  and  $n$ , for  $\xi = 0.2$ .

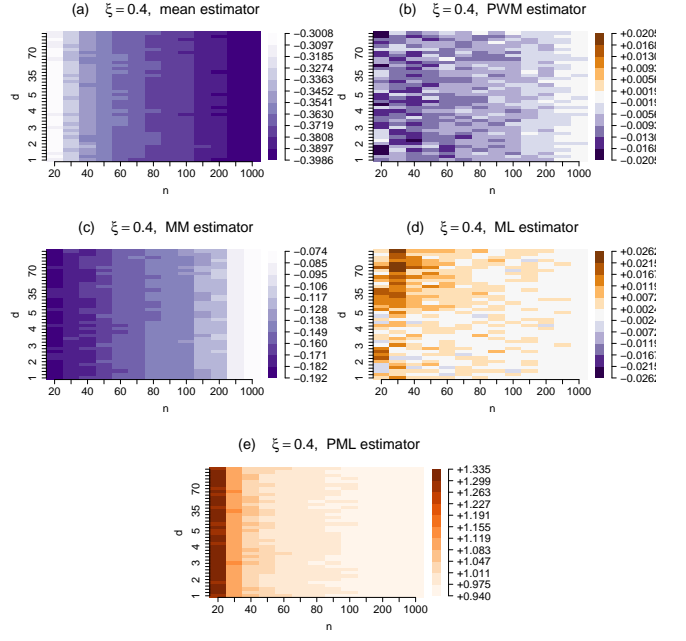


Fig. S36. Relative error of the mean estimator (a), PWM (b), MM (c), ML (d) and PML (e) on  $d$  as a function of  $d$  and  $n$ , for  $\xi = 0.4$ .

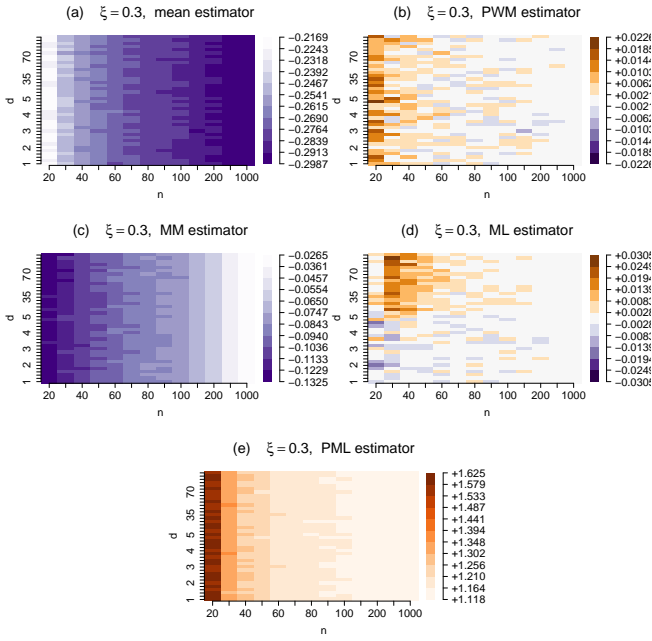


Fig. S35. Relative error of the mean estimator (a), PWM (b), MM (c), ML (d) and PML (e) on  $d$  as a function of  $d$  and  $n$ , for  $\xi = 0.3$ .

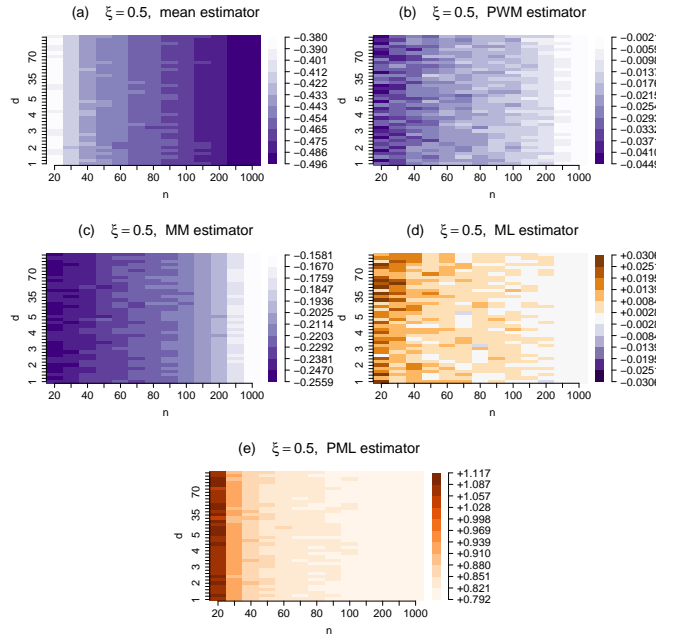


Fig. S37. Relative error of the mean estimator (a), PWM (b), MM (c), ML (d) and PML (e) on  $d$  as a function of  $d$  and  $n$ , for  $\xi = 0.5$ .

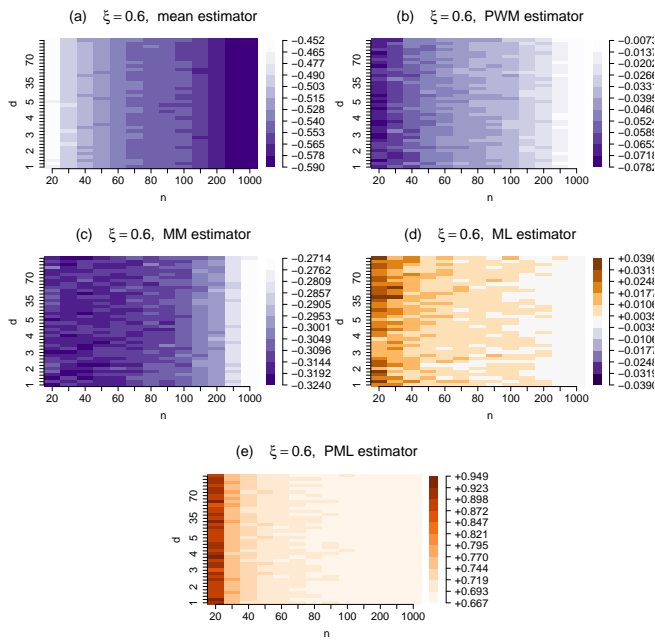


Fig. S38. Relative error of the mean estimator (a), PWM (b), MM (c), ML (d) and PML (e) on  $d$  as a function of  $d$  and  $n$ , for  $\xi = 0.6$ .

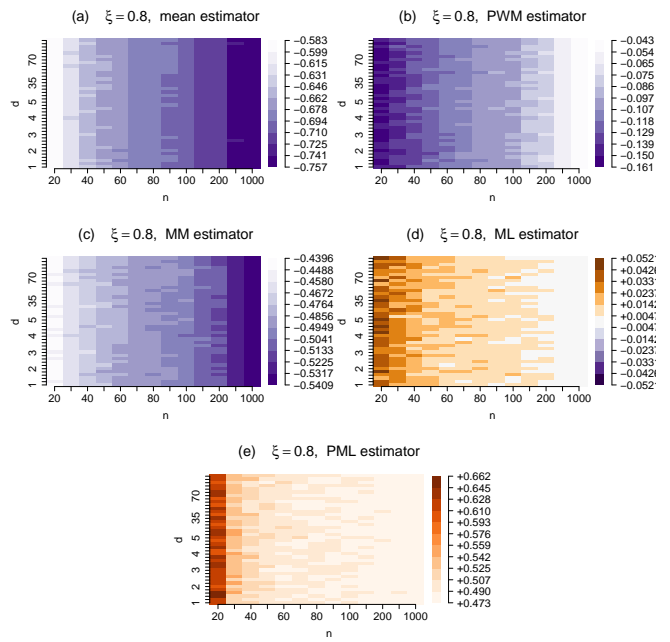


Fig. S40. Relative error of the mean estimator (a), PWM (b), MM (c), ML (d) and PML (e) on  $d$  as a function of  $d$  and  $n$ , for  $\xi = 0.8$ .

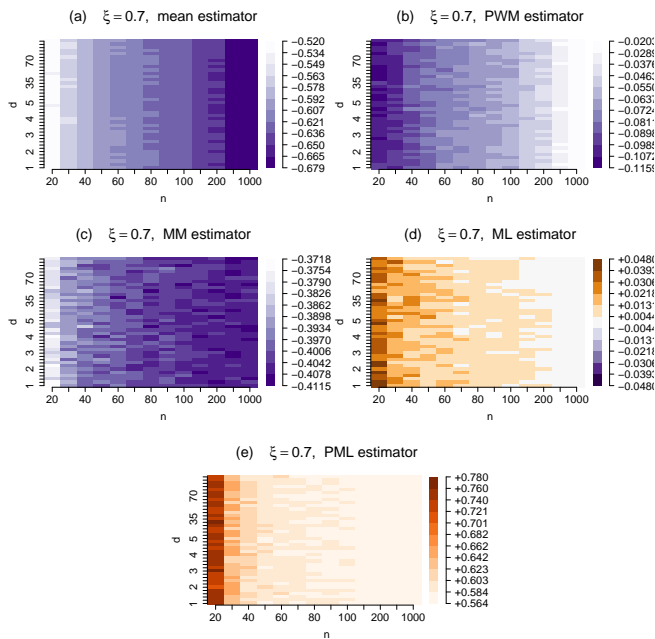


Fig. S39. Relative error of the mean estimator (a), PWM (b), MM (c), ML (d) and PML (e) on  $d$  as a function of  $d$  and  $n$ , for  $\xi = 0.7$ .

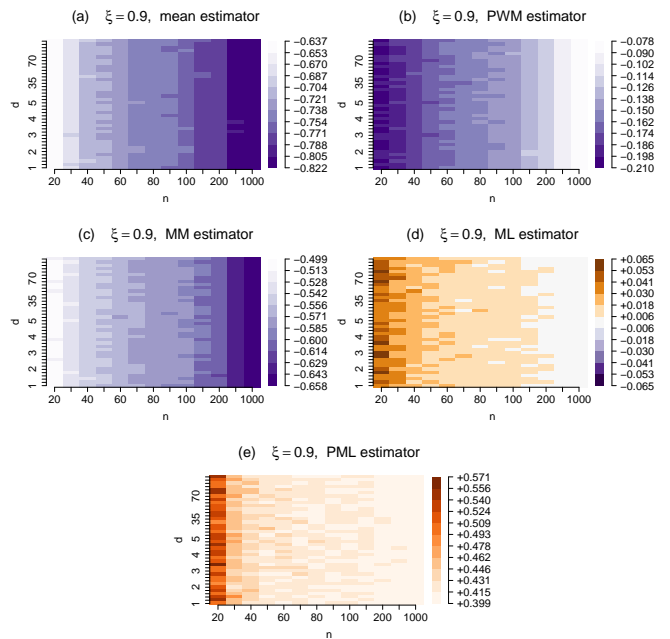


Fig. S41. Relative error of the mean estimator (a), PWM (b), MM (c), ML (d) and PML (e) on  $d$  as a function of  $d$  and  $n$ , for  $\xi = 0.9$ .

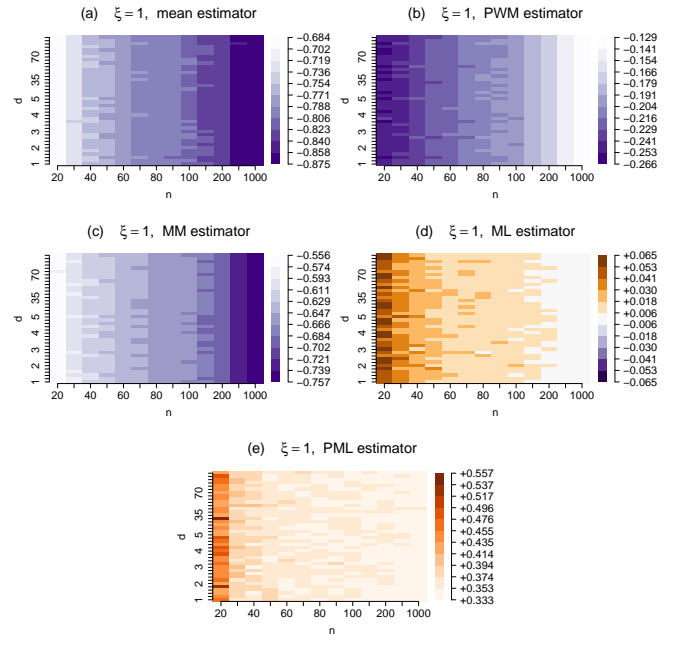


Fig. S42. Relative error of the mean estimator (a), PWM (b), MM (c), ML (d) and PML (e) on  $d$  as a function of  $d$  and  $n$ , for  $\xi = 1$ .