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Mathematical Modeling of Cluster Dynamics

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Abstract

This paper deals with the modeling and the numerical simulation of computational cluster networks (grid) dynamics and is more precisely focused on the management of the computational load offered by transferring jobs from one cluster to another. In order to tackle the complexity arising from dealing with the sheer number of processes existing in the system, we develop a comprehensive model of the global population of jobs using the kinetic theory. It takes into account the main characteristics of the clusters, their interactions and allows for a thorough description of the jobs. We then study the resulting system of macroscopic conservation laws and illustrate numerically its ability to capture some interesting behaviors of the grid. This model allows for real-time insight of the performance of a given policy for load management in the grid.

Keywords: Fluid models, Kinetic models, Computational Grid, Numerical simulation, Cluster network

2000 MSC: 35Q68, 35Q94, 68M10, 68M20, 35F50

1. Introduction

1 According to [1], the electric consumption of the information technology raised to 270 TWh
2 in 2012 which is roughly equivalent to 1.4% of the worldwide electrical consumption while
3 the complete Information and Communication Technology sector (excluding manufacturing) ac-
4 counts for 4.7% of it. Moreover, the data center power needs increased annually by 5% between
5 2006 and 2012 (see also [2]). For the year 2030, the data centers alone may use between 3%
6 (best case) and 13% (worst scenario) of the global electricity production [3]. We are interested
7 here in the management of clusters running in several data centers and in the improvement of
8 their energetic consumption while maintaining a certain quality of service. There are currently
9 many ways to limit their energetic impact [4]. One way consists in enhancing the use of the clus-
10 ters particularly thanks to a cooperation between different clusters. There exist different policies:
11

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12 some are based on an *a priori* study of the state of the cluster network before launching any
13 job on it [5]; others are based on a dynamic management of the clusters *via* the displacement of
14 services or jobs between computers or clusters. These movements are facilitated by the use of
15 virtual machines in which the jobs are encapsulated [6]. However, the evaluation of the quality
16 of these policies is particularly difficult through the use of direct methods [7]. Indeed, the com-
17 plexity increases greatly as we are reaching exa-scale systems and other methods are required in
18 order to not treat individually each of the millions or billions of processes [8]. We propose here
19 an evaluation method based on a macroscopic point of view where the value of interest is the
20 local density of jobs rather than the individual jobs themselves.

21 These macroscopic representations are already widely used in other application domains
22 where the sheer number of individuals makes it difficult to run a complete simulation (e.g.
23 dynamics of animal swarms, crowd dynamics, etc.). Dealing with these representations in-
24 volve a rather large panel of mathematical methodology which are the subject of several pa-
25 pers [9, 10, 11]. A comprehensive survey of these methods can for example be found in [12].
26 In the field of computer networks, this kind of approach was first referenced as fluid methods
27 and is based on a set of coupled Ordinary Differential Equations (ODE) [13, 14, 15], each one
28 describing the load on a given network node based on only two parameters (the input and the
29 output rate). An extension of this kind of models, using Partial Differential Equations (PDE) has
30 also been introduced for a group of processes with heterogeneous parameters, such as Mice and
31 Elephant processes in TCP/IP protocol [16]. However, these models are provided thanks to con-
32 servation considerations and are phenomenological. Thus, their generalization or the inclusion
33 of more parameters require a complete rethinking of the model.

34 In this article, we propose a set of conservation laws describing the evolution of local density
35 of jobs on a network derived from a proper kinetic model of the cluster dynamics. The idea is
36 to use the microscopic / mesoscopic / macroscopic formalism rather than an *ad hoc* model of the
37 cluster load. Taking as a basis the individual based model of the jobs and of their movements
38 between the clusters, we define a density distribution function for the jobs, the main part of
39 the work being devoted to the derivation of the corresponding kinetic equation (or system of
40 equations) solved by this distribution function. We are then finally able to obtain a macroscopic
41 model that describes the evolution of the cluster network. The foundations of this model have
42 been laid in [17] and developed in the thesis of De Cecco [18]. One main difference with the fluid
43 models derived in the context of crowds or swarms of animals where the amount of interactions
44 with the other participants (be it collisions or repulsion forces) lead to a global dynamics of the
45 swarms, is that in the context of computer networks, the dynamics of one element is uniquely
46 decided by the environment. The interactions between these elements occur through their own
47 impact on the environment so that no collision operator has to be considered.

48 We will detail in Section 2 a microscopic description of the clusters and the jobs running on
49 them and highlight the asymptotics allowing us to derive the fluid model in Section 3. In Section
50 4 we mathematically justify the models we have obtained and finally, we present in Section 5
51 some numerical illustrations on simple cases to highlight the potential benefits of different job
52 management policies.

53 **2. A comprehensive cluster description**

54 *2.1. Cluster*

55 We consider in this study a network of several data centers which will be modeled as a
56 network of C clusters (denoted by C_j , $j \in \{1, \dots, C\}$). This network is assumed to be managed

57 for each cluster by a centralized middle-ware which decides whether a given job running on
58 a cluster will still be executed on site or will be sent on another cluster in order to improve
59 a cost (computational time, energetic, financial, ... or any combination of them). To evaluate
60 the possible gain arising from moving a job from one cluster to another, each cluster C_j will
61 be characterized by a small number of parameters such as its performance index $v_j \in \mathbb{R}^+$ (its
62 computational speed), its energetic power $Z_j(t, q) \in \mathbb{R}^+$, depending on the time t and the load
63 of the cluster q (it could depend on more parameters such as the number of processors used),
64 its working cost $C_j(t) \in \mathbb{R}^+$, (rental cost for example, possibly depending on the number of
65 processors/cores used by a job), its total number of processors $\pi_j \in \mathbb{N}^*$, its maximal number of
66 simultaneous jobs $T_j \in \mathbb{N}^*$.

67 In our case, we assume each clusters to be homogeneous and to be defined only by its number
68 of processors. Moreover, the transfer time between each interconnected cluster of the network
69 will be defined thanks to a matrix of transfer time $\tau = (\tau_{jk})_{j,k \in \{1, \dots, C\}} \in M_C(\mathbb{R}^+)$ where τ_{jk}
70 is the time needed to transfer a job from the cluster C_j onto the cluster C_k . Given that there is no
71 transfer between a cluster and itself and that all connections are bidirectional, we assume that for
72 each j , $\tau_{jj} = 0$ and that τ is symmetric.

73 2.2. Jobs

74 We consider that N jobs, denoted by J_i , $i \in \{1, \dots, N\}$ are executed on the network and that
75 there is no deadline constraint. In the current model, interactive applications or services are not
76 included. Each one is described by its size $s_i \in \mathbb{R}_+^*$ (the memory size occupied by the job on a
77 cluster), the remaining load of computations required to complete the job $q_i(t) \in \mathbb{R}_+^*$, its position
78 $P_i(t) \in \{C_1, \dots, C_C\}$ in the network, *i.e.* the cluster on which the job is executed at time t , the
79 minimum number of processors required to perform the job $p_i \in \mathbb{N}^*$, the age of the job $a_i(t) \in \mathbb{R}^+$
80 and its waiting time $\theta_i(t) \in \mathbb{R}$. We introduce this time to take into account the transfer time of
81 a job. When the job is moved from a cluster to another, the resources on the arrival cluster are
82 reserved for the incoming job and will thus not be available for new jobs. We assume that the job
83 is transferred immediately but that it arrives on the new cluster with the waiting time θ_i equal to
84 the transfer time needed to transfer its data through the network.

85 In this study we do not use the parameters s_i and p_i and a_i . The transfer time between clusters
86 are given and do not depend on the memory size of the job.

87 The decision to move a job from a cluster to another at a time t depends on a cost function.
88 It can for example be the remaining execution time for a job actually located on the cluster C_j if
89 moved to the cluster C_k ,

$$t^{exe}(q_i(t), P_i(t) = C_j, C_k) = \frac{q_i(t)}{v_k} + \tau_{jk}, \quad \forall (j, k) \in \{1, \dots, C\}^2,$$

90 sum of the transfer time from C_j to C_k and the required time to execute the job on the cluster C_k ,
91 the associated energetic consumption,

$$K^{exe}(q_i(t), C_k) = \int_0^{\frac{q_i(t)}{v_k}} Z_k(t, q) dt, \quad \forall k \in \{1, \dots, C\},$$

92 the working cost of the material. . .

93 The functional to minimize will be a weighted combination of the remaining execution time
94 and of the energy consumption:

$$K(q_i(t), C_k) = c_t t^{exe}(q_i(t), C_j, C_k) + c_e K^{exe}(q_i(t), C_k),$$

95 with the weights c_t and $c_e \in \mathbb{R}^+$ such that $c_t + c_e = 1$.

96 A job is transferred to a cluster C_{k^*} (arbitrarily chosen if there is not uniqueness) if it min-
 97 imizes its cost function. This cluster may eventually be the one on which the job is currently
 98 executed ($P_i(t) = C_{k^*}$). A decision function Dec is associated to the cost function such that

$$\text{Dec}(t, q_i(t), P_i(t), P) = \begin{cases} 1 & \text{if } P_i(t) \neq C_{k^*} \text{ and } P = C_{k^*}, \\ 0 & \text{else.} \end{cases}$$

99 The middle-ware acts like a black box that sends the result 1 if the current job has to be moved
 100 from a cluster to another one and 0 otherwise. We introduce τ_{pr} the time taken to compute the
 101 decision function for a job. It is supposed to be identical for all jobs. We assume that the middle-
 102 ware tests a job after another continuously in an infinite loop. We give a number $\alpha_i \in \{1, \dots, N\}$
 103 to each job that determines the order in which each job will be tested so that the time needed to
 104 test the job J_i is $\alpha_i \tau_{pr}$. We moreover assume that if the remaining load of a job is small enough,
 105 it will never be moved so the function Dec will be equal to zero for such a job.

106 We then obtain the following equations for each job:

$$q_i'(t) = -v_{P_i(t)} \mathbf{1}_{q_i(t)>0} \mathbf{1}_{\theta_i(t)\leq 0}, \quad (1)$$

107 *i.e.* the remaining workload q_i of the job decreases linearly until it finishes ($q_i = 0$) if it is not
 108 being transferred from a cluster to another ($\theta_i = 0$);

$$\theta_i'(t) = -\mathbf{1}_{\theta_i(t)>0} + \sum_{\substack{k=1 \\ k \neq j}}^C \tau_{jk} \text{Dec}(t, q_i(t), C_j, C_k) \delta_{\theta_i(t)=0}, \quad (2)$$

109 *i.e.* the transfer time θ_i decreases linearly during the transfer until the execution begins ($\theta_i = 0$)
 110 or it jumps from 0 to τ_{jk} if the job is transferred from C_j to C_k .

111 **Remark 1.** We remark that θ_i (related to the transfer) and q_i (related to the execution) do not
 112 evolve simultaneously. We also allow $\theta_i(t)$ to be negative. It will never happen in practice but this
 113 technical assumption will facilitate the derivation of the kinetic model in the following. Finally,
 114 let us note that a job waiting on a cluster is not tested until its waiting time is 0 which is relevant
 115 since the job is currently transferred.

116 3. Toward a fluid model

117 3.1. Defining a distribution function

118 To obtain the global behavior of the jobs on the network without studying them individually,
 119 we introduce the distribution function of jobs f , valued in \mathbb{R}^C , whose each component f^j is the
 120 local density of jobs at load q and waiting time θ on the cluster C_j :

$$f(t, P, q, \theta) = \left(f^j(t, q, \theta) \delta_{P=C_j} \right)_{j \in \{1, \dots, C\}}$$

121 with

$$f^j(t, q, \theta) = \frac{1}{T_j} \sum_{\substack{i=1 \\ P_i(t)=C_j}}^{N_j} \delta_{q=q_i(t)} \delta_{\theta=\theta_i(t)}$$

4

122 and N_j the number of jobs on the cluster C_j . Then $\int_{q,\theta} f^j dq d\theta = N_j/T_j$ is the filling rate of the
 123 cluster C_j .

124 We now study the evolution of f^j between the times t and $t + \delta t$ (with $\delta t > 0$) in the distribu-
 125 tional sense. Let $\Phi = (\phi_j)_{1 \leq j \leq C} \in (\mathcal{D}(\mathbb{R}_+^*))_q \otimes \mathcal{D}(\mathbb{R})_\theta$ be a test function associated to the whole
 126 network:

$$\Phi(P, q, \theta) = \sum_{j=1}^C \phi^j(q, \theta) \mathbf{1}_{P=C_j} \quad \text{then} \quad \langle f^j(t), \phi^j \rangle = \frac{1}{T_j} \sum_{\substack{i=1 \\ P_i(t)=C_j}}^{N_j} \phi^j(q_i(t), \theta_i(t))$$

Along the evolution between t and $t + \delta t$, either a job is moved from one cluster to another (case
 B) or it stays in place (case A).

$$\begin{aligned} \langle f^j(t + \delta t) - f^j(t), \phi^j \rangle &= A + B \\ &= \frac{1}{T_j} \left\{ \sum_{\substack{i=1 \\ P_i(t+\delta t)=P_i(t)=C_j}}^{N_j} + \sum_{\substack{i=1 \\ P_i(t+\delta t) \neq P_i(t)}}^{N_j} \right\} \left(\phi^j(q_i(t + \delta t), \theta_i(t + \delta t)) - \phi^j(q_i(t), \theta_i(t)) \right) \end{aligned} \quad (3)$$

127 By distinguishing whether the job is waiting or executing between t and $t + \delta t$ (or partially waiting
 128 then executing, executing and finished or waiting, executing and finished) we obtain

$$\begin{aligned} A = & -v_j \delta t \langle f^j(t), \partial_q \phi^j \mathbf{1}_{q > v_j \delta t} \mathbf{1}_{\theta \leq 0} \rangle && \text{[jobs in execution state]} \\ & -\delta t \langle f^j(t), \partial_\theta \phi^j \mathbf{1}_{q > 0} \mathbf{1}_{\theta > \delta t} \rangle && \text{[jobs in waiting state]} \\ & -\langle \theta f^j(t), \partial_\theta \phi^j \mathbf{1}_{0 < \theta \leq \delta t} \mathbf{1}_{q_i(t) > v_j(\delta t + \theta)} \rangle && \text{[jobs which waited during } \theta \dots \\ & -v_j \langle (\delta t - \theta) f^j(t), \partial_q \phi^j(q, 0) \mathbf{1}_{0 \leq \theta \leq \delta t} \mathbf{1}_{q > v_j(\delta t + \theta)} \rangle && \text{and were executed during } \delta t - \theta] \\ & -\langle f^j(t), \phi^j \mathbf{1}_{q \leq v_j \delta t} \mathbf{1}_{\theta \leq 0} \rangle && \text{[jobs ended during } \delta t \dots \\ & -\langle f^j(t), \phi^j \mathbf{1}_{q \leq v_j(\delta t - \theta)} \mathbf{1}_{0 < \theta \leq \delta t} \rangle && \text{or waited during } \theta \text{ before ending]} \\ & + \mathcal{O}(\delta t^2). \end{aligned} \quad (4)$$

129 To describe simply the term B and avoid any redundancy with the terms presented in A we
 130 assume that δt is small enough so that a job can not be displaced and executed during the same
 131 time interval. This leads to

$$\delta t < \min_{j,k} \tau_{jk}.$$

132 This implies that we only have to look either at the jobs arriving at the cluster C_j (case B_1) or
 133 leaving it (case B_2) during the time interval δt . We also impose that

$$\alpha_i \tau_{pr} \leq \delta t, \forall i \in \{1, \dots, N\},$$

134 so that the job has been tested between t and $t + \delta t$ by the middle-ware. We obtain

$$B = B_1 + B_2 = \frac{1}{T_j} \left[\sum_{\substack{i=1 \\ P_i(t+\delta t)=C_j \neq P_i(t)=C_k}}^{N_j} \phi^j(q_i(t + \delta t), \theta_i(t + \delta t)) - \sum_{\substack{i=1 \\ P_i(t)=C_j \neq P_i(t+\delta t)=C_k}}^{N_j} \phi^j(q_i(t), \theta_i(t)) \right],$$

where C_k is the cluster from which the job is either arriving or leaving. The term B_1 is the density of jobs arriving on the cluster C_j at time $t + \alpha_i \tau_{pr}$ from the clusters C_k with $k \neq j$. They arrive with a waiting time equals to τ_{kj} :

$$\begin{aligned}
B_1 &= \sum_{\substack{k=1 \\ k \neq j \\ P_i(t+\delta t)=C_j}}^C \frac{1}{T_j} \sum_{\substack{i=1 \\ P_i(t)=C_k \\ \alpha_i \tau_{pr} \leq \delta t}}^{N_j} \phi^j(q_i(t) - v_k \alpha_i \tau_{pr}, \tau_{kj} - (\delta t - \alpha_i \tau_{pr})) \mathbf{1}_{\theta_i(t) \leq 0} \\
&\quad \mathbf{1}_{q_i(t) > v_k \delta t} \mathbf{1}_{\text{Dec}(t+\alpha_i \tau_{pr}, q_i(t) - v_k \alpha_i \tau_{pr}, k, j)=1} \\
&= \sum_{\substack{k=1 \\ k \neq j}}^C \frac{T_k}{T_j} \frac{1}{T_k} \sum_{\substack{i=1 \\ \alpha_i \tau_{pr} \leq \delta t}}^{N_k} \left[\phi^j(q_i(t), \tau_{kj}) - v_k \alpha_i \tau_{pr} \partial_q \phi^j(q_i(t), \tau_{kj}) \right. \\
&\quad \left. - (\delta t - \alpha_i \tau_{pr}) \partial_\theta \phi^j(q_i(t), \tau_{kj}) \right] \mathbf{1}_{\theta_i(t) \leq 0} \\
&\quad \mathbf{1}_{q_i(t) > v_k \delta t} \mathbf{1}_{\text{Dec}(t+\alpha_i \tau_{pr}, q_i(t) - v_k \alpha_i \tau_{pr}, k, j)=1} + O(\delta t^2).
\end{aligned}$$

135 The term B_2 describes the jobs leaving the cluster C_j :

$$\begin{aligned}
B_2 &= - \sum_{\substack{k=1 \\ k \neq j}}^C \frac{1}{T_j} \sum_{\substack{i=1 \\ \alpha_i \tau_{pr} \leq \delta t}}^{N_j} \phi^j(q_i(t), \theta_i(t)) \mathbf{1}_{\theta_i(t) \leq 0} \mathbf{1}_{q_i(t) > v_j \delta t} \\
&\quad \mathbf{1}_{\text{Dec}(t+\alpha_i \tau_{pr}, q_i(t) - v_j \alpha_i \tau_{pr}, j, k)=1}.
\end{aligned} \tag{5}$$

136 To take the limit as $\delta t \rightarrow 0$ we first need to scale the characteristic times of the problem with
137 δt so that the behavior of the jobs is correctly taken into account. Since we are looking at the
138 transfer of jobs between clusters, many jobs have to be tested during the time interval δt to obtain
139 a pertinent model. This implies that the time to test a job τ_{pr} is much smaller than δt , $\tau_{pr} \ll \delta t$.
140 However, if all the jobs are tested during the time interval δt , it corresponds to the case of a
141 middle-ware taking instantaneous decisions. In our model the time to test all the jobs $\mathcal{T} = N \tau_{pr}$
142 should remain constant in the limit $\delta t \rightarrow 0$. We then assume in the following that

$$\tau_{pr} = \delta t^2 \ll \delta t \ll \mathcal{T} = O(1). \tag{6}$$

143 **Remark 2.** This scaling implies that the number of jobs on the network fulfills $N = O(1/\delta t^2)$.
144 Note however that the number C of clusters is constant. The number of jobs tested on the network
145 during δt is defined by

$$N_{\delta t} = \frac{\delta t}{\tau_{pr}} = O\left(\frac{1}{\delta t}\right) \gg 1.$$

146 Moreover, under the assumption that the distribution of the tested jobs is uniform on $[1, N]$ (it
147 depends neither on the cluster number nor on its state), the number of jobs tested on the cluster
148 C_j during the time interval δt is

$$\frac{N_j}{N} N_{\delta t} = N_j \frac{\delta t}{\mathcal{T}} = O\left(\frac{1}{\delta t}\right) \gg 1.$$

149 Finally, aside from this choice of asymptotic, we impose that the filling rate of each cluster
150 remains bounded (and consequently not null):

$$\int_{q, \theta} f^j dq d\theta = \frac{N_j}{T_j} = O(1).$$

Lemma 1. Formal limit as $\delta t \rightarrow 0$

The kinetic equation governing the evolution of the distribution function in each cluster C_j , $j = \{1, \dots, C\}$, for all time $t > 0$ is given in $(\mathcal{D}'(\mathbb{R}_+^*))_q \otimes (\mathcal{D}'(\mathbb{R}))_\theta$ by:

$$\begin{aligned} \partial_t f^j(t) - \mathbf{1}_{\theta \leq 0} v_j \partial_q f^j(t) - \mathbf{1}_{\theta > 0} \partial_\theta f^j(t) &= -\frac{1}{\mathcal{T}} \sum_{\substack{k=1 \\ k \neq j}}^C f^j(t) \mathbf{1}_{\theta \leq 0} \text{Dec}(j, k) \\ &+ \frac{1}{\mathcal{T}} \sum_{\substack{k=1 \\ k \neq j}}^C \frac{T_k}{T_j} \langle f^k(t) \text{Dec}(k, j), \mathbf{1}_{\theta \leq 0} \rangle_\theta \delta_{\theta = \tau_{kj}}. \end{aligned} \quad (7)$$

151 *Proof.* We assume that $f_N^j \xrightarrow[N \rightarrow +\infty]{\delta t \rightarrow 0} f^j$ weak-*. Taking the limit of $\frac{A}{\delta t}$ we obtain, using a first order
152 development :

$$\frac{1}{\delta t} A \xrightarrow[N \rightarrow +\infty]{\delta t \rightarrow 0} -v_j \langle f^j(t), \partial_q \phi^j \mathbf{1}_{q \geq 0} \mathbf{1}_{\theta \leq 0} \rangle - \langle f^j, \partial_\theta \phi^j \mathbf{1}_{q \geq 0} \mathbf{1}_{\theta > 0} \rangle.$$

Since $\alpha_i \tau_{pr} \leq \delta t$ then $\alpha_i \tau_{pr} \xrightarrow{\delta t \rightarrow 0} 0$ and the term B_1 reduces to

$$\begin{aligned} B_1 &= \sum_{\substack{k=1 \\ k \neq j}}^C \frac{T_k}{T_j} \frac{1}{T_k} \sum_{\substack{i=1 \\ \alpha_i \tau_{pr} \leq \delta t}}^{N_k} \phi^j(q_i(t), \tau_{kj}) \mathbf{1}_{\theta_i(t) \leq 0} \mathbf{1}_{q_i(t) > v_k \delta t} \\ &\quad \mathbf{1}_{\text{Dec}(t + \alpha_i \tau_{pr}, q_i(t) - v_k \alpha_i \tau_{pr}, k, j) = 1} + \mathcal{O}(\delta t^2). \end{aligned}$$

153 The main difficulty in the formal limit of B lies in determining on a given cluster the number of
154 jobs moved since the Dec function depends on the characteristics of each job on the cluster. We
155 then get around this problem thanks to the following assumption:

156 **Assumption 1.** Let us assume that $\text{Dec}^{-1}\{1\}$ is an open set.

157 This assumption is valid as soon as the cost function is continuous and represents the gain
158 obtained through the minimization process. Then $q \mapsto \text{Dec}(t, q, k, j)$ is piece-wisely constant.
159 Let $I_{t,jk} = \text{Dec}^{-1}(t, \cdot, j, k)(\{1\})$. It also implies that Dec only depends on the jobs characteristics
160 and not the jobs themselves. Since the number of tested jobs during δt is equal to $\frac{\delta t}{\mathcal{T}}$ we obtain
161 the following weak- \star limit:

$$\frac{1}{\delta t} \frac{1}{T_j} \sum_{\substack{i=1 \\ \alpha_i \tau_{pr} \leq \delta t \\ P_i(0) = C_j}}^{N_j} \delta_{q=q_i(t)} \delta_{\theta=\theta_i(t)} \xrightarrow[N \rightarrow +\infty]{\delta t \rightarrow 0} \frac{1}{\mathcal{T}} f^j. \quad (8)$$

162 If $\text{supp } \phi^j \subset I_{t,jk} \otimes \mathbb{R}$, then:

$$\frac{1}{\delta t} B_2 \xrightarrow[N \rightarrow +\infty]{\delta t \rightarrow 0} - \sum_{\substack{k=1 \\ k \neq j}}^C \frac{1}{\mathcal{T}} \langle f^j(t), \phi^j \mathbf{1}_{\theta \leq 0} \mathbf{1}_{q > 0} \rangle \mathbf{1}_{\text{Dec}(t, q, j, k) = 1}. \quad (9)$$

163 Since the variable q and θ do not vary simultaneously we let

$$\phi^j(q, \theta) = (\phi_q^j \otimes \phi_\theta^j)(q, \theta) \in (\mathcal{D}(\mathbb{R}_+^*))_q \otimes (\mathcal{D}(\mathbb{R}))_\theta, \forall j = \{1, \dots, C\}. \quad (10)$$

and denote by $\langle \cdot, \cdot \rangle_\theta$ the dual product on \mathbb{R}_θ . Then if $\text{supp } \phi_q \subset I_{tjk}$,

$$\begin{aligned} \frac{1}{\delta t} B_1 &= \frac{1}{\delta t} \sum_{\substack{k=1 \\ k \neq j}}^C \frac{T_k}{T_j} \left(\frac{1}{T_k} \sum_{\substack{i=1 \\ \alpha_i \tau_{pr} \leq \delta t}}^{N_k} \phi_q^j(q_i(t)) \mathbf{1}_{\theta_i(t) \leq 0} \mathbf{1}_{q_i(t) > v_k \delta t} \right. \\ &\quad \left. \mathbf{1}_{Dec(t+\alpha_i \tau_{pr}, q_i(t) - v_k \alpha_i \tau_{pr}, k, j)=1} \right) \phi_\theta^j(\tau_{kj}) + O(\delta t) \\ &\xrightarrow[\delta t \rightarrow 0]{N \rightarrow +\infty} \sum_{\substack{k=1 \\ k \neq j}}^C \frac{T_k}{T_j} \frac{1}{\mathcal{T}} \langle f^k(t) \mathbf{1}_{q>0} \mathbf{1}_{Dec(t,q,k,j)=1}, \phi_q^j \otimes \mathbf{1}_{\theta \leq 0} \rangle \phi_\theta^j(\tau_{kj}) \\ &\xrightarrow[\delta t \rightarrow 0]{N \rightarrow +\infty} \sum_{\substack{k=1 \\ k \neq j}}^C \frac{T_k}{T_j} \frac{1}{\mathcal{T}} \langle \langle f^k(t) \mathbf{1}_{q>0} \mathbf{1}_{Dec(t,q,k,j)=1}, \mathbf{1}_{\theta \leq 0} \rangle_\theta, \phi_q^j \otimes \phi_\theta^j(\tau_{kj}) \rangle. \end{aligned}$$

164 We now let $Dec(j, k) = \mathbf{1}_{Dec(t,q,j,k)=1}$, and we get:

$$\frac{1}{\delta t} B_1 \xrightarrow[\delta t \rightarrow 0]{N \rightarrow +\infty} \frac{1}{\mathcal{T}} \sum_{\substack{k=1 \\ k \neq j}}^C \frac{T_k}{T_j} \langle \langle f^k(t) Dec(k, j), \mathbf{1}_{\theta \leq 0} \mathbf{1}_{q>0} \rangle_\theta \delta_{\theta=\tau_{kj}}, \phi^j \rangle, \quad (11)$$

165 and

$$\frac{1}{\delta t} B_2 \xrightarrow[\delta t \rightarrow 0]{N \rightarrow +\infty} -\frac{1}{\mathcal{T}} \sum_{\substack{k=1 \\ k \neq j}}^C \langle f^j(t), \phi^j \mathbf{1}_{\theta \leq 0} \mathbf{1}_{q>0} \rangle Dec(j, k), \quad (12)$$

166 which gives the kinetic equation (7). \square

167 Since the evolution on q and θ are not simultaneous we split the distribution function f^j as

$$f^j(t, q, \theta) = \rho_j^{exe}(t, q) \delta_{\theta=0} + \rho_j^{wait}(t, q, \theta) \mathbf{1}_{\theta>0} + \rho_j^{garb}(t, q, \theta) \mathbf{1}_{\theta<0}, \quad (13)$$

168 where the functions ρ_j^{exe} (for the working jobs), ρ_j^{wait} (for the waiting jobs) and ρ_j^{garb} are defined
169 by:

$$\begin{aligned} \rho_j^{exe}(t, q) &= f^j(t, q, \theta) \mathbf{1}_{\theta=0}, & \rho_j^{wait}(t, q, \theta) &= f^j(t, q, \theta) \mathbf{1}_{\theta>0}, \\ \rho_j^{garb}(t, q, \theta) &= f^j(t, q, \theta) \mathbf{1}_{\theta<0}. \end{aligned} \quad (14)$$

170 **Remark 3.** As the evolution in θ stops as soon as $\theta \leq 0$, ρ^{garb} is taken into account for the
171 sake of generality and more importantly so that the definition domain in θ is an open set. This
172 formulation is coherent with the solutions obtained for transport equations with discontinuous
173 coefficients [19].

174 **Lemma 2. Characterization of the solutions of the kinetic equation**

175 We assume that $f^j(t = 0, q, \theta)$ fulfills the decomposition (13) with $\rho_j^{exe}(t = 0, q) \in \mathbb{W}^{1,\infty}(\mathbb{R}_+^*)$,
176 $\rho_j^{wait}(t = 0, q, \theta) \in \mathbb{W}^{1,\infty}(\mathbb{R}_+^* \times \mathbb{R}_+^*)$ and $\rho_j^{garb}(t = 0, q, \theta) \in \mathbb{W}^{1,\infty}(\mathbb{R}_+^* \times \mathbb{R}_-^*)$. Then the solutions
177 in $(\mathcal{D}'(\mathbb{R}_+^*))_q \otimes (\mathcal{D}'(\mathbb{R}))_\theta$ of (7) for $t > 0$ fulfill the decomposition (13), with $\rho_j^{wait}(t, q, \theta) \in$
178 $\mathbb{L}^\infty(\mathbb{R}^+, \mathbb{W}^{1,\infty}(\mathbb{R}_+^* \times \mathbb{R}_+^*))$ and $\rho_j^{garb}(t, q, \theta) \in \mathbb{L}^\infty(\mathbb{R}^+, \mathbb{W}^{1,\infty}(\mathbb{R}_+^* \times \mathbb{R}_-^*))$.

179 *Proof.* We show that no singularity in θ arises along the time evolution of ρ_j^{wait} and ρ_j^{garb} . We test
 180 the kinetic equation (7) with a test function ϕ^j such that $\text{supp } \phi^j \subset (\mathbb{R}_+^*)_q \otimes (\mathbb{R}_-^*)_\theta$ and get

$$\langle \partial_t f^j(t), \phi^j \rangle = \langle \partial_t \rho_j^{garb}(t), \phi^j \rangle \quad (15)$$

181 with

$$\rho_j^{garb}(t, q, \theta) = \rho_j^{garb}(0, q + v_j t, \theta) - \int_0^t \frac{1}{\mathcal{T}} \sum_{\substack{k=1 \\ k \neq j}}^C \rho_j^{garb}(s, q + v_j(t-s), \theta) \text{Dec}(s, q + v_j(t-s), j, k) ds. \quad (16)$$

182 By testing (7) with a test function ϕ^j such that $\text{supp } \phi^j \subset (\mathbb{R}_+^*)_q \otimes (\mathbb{R}_+^*)_\theta$, we get

$$\langle \partial_t f^j(t), \phi^j \rangle = \langle \partial_t \rho_j^{wait}(t), \phi^j \rangle \quad (17)$$

183 with

$$\rho_j^{wait}(t, q, \theta) = \rho_j^{wait}(0, q, \theta + t) + \int_0^t \frac{1}{\mathcal{T}} \sum_{\substack{k=1 \\ k \neq j}}^C \frac{T_k}{T_j} \int_{\theta \leq 0} f^k(s, q, \theta + t - s) \text{Dec}(s, q, k, j) d\theta \delta_{\theta+t-s=\tau_{kj}} ds. \quad (18)$$

184 Since the initial data are regular enough, ρ_j^{exe} , ρ_j^{wait} and ρ_j^{garb} are regular for $t > 0$. \square

185 Let us note that if $\rho_j^{garb}(0, \cdot, \cdot) = 0$ in (16) (which is the case in our model), then $\rho_j^{garb}(t, \cdot, \cdot) = 0$
 186 for all $t > 0$ and if the solution exists, it writes

$$f^j(t, q, \theta) = \rho_j^{exe}(t, q) \delta_{\theta=0} + \rho_j^{wait}(t, q, \theta) \mathbf{1}_{\theta>0}. \quad (19)$$

187 with the functions ρ_j^{exe} and ρ_j^{wait} defined by

$$\rho_j^{exe}(t, q) = \langle f^j(t, q, \theta), \mathbf{1}_{\theta \leq 0} \rangle \quad \text{and} \quad \rho_j^{wait}(t, q, \theta) = f^j(t, q, \theta) \mathbf{1}_{\theta > 0}.$$

188 3.2. Asymptotics for conservation laws

189 We study here the fluid system fulfilled by ρ_j^{exe} and ρ_j^{wait} for all $j \in \{1, \dots, C\}$. We use the
 190 tensor test functions $\phi^j \in (\mathcal{D}'(\mathbb{R}_+^*))_q \otimes (\mathcal{D}'(\mathbb{R}))_\theta$ such that

$$\phi^j(q, \theta) = \phi^{exe}(q, \theta) \mathbf{1}_{\theta \leq 0} + \phi^{wait}(q, \theta) \mathbf{1}_{\theta > 0} \quad (20)$$

191 Testing (7) with ϕ^j , we get in the distribution sense $\forall j \in \{1, \dots, C\}$

$$\begin{cases} \partial_t \rho_j^{exe}(t, q) - v_j \partial_q \rho_j^{exe}(t, q) = S_j^{exe}(t, q), \\ \partial_t \rho_j^{wait}(t, q, \theta) - \partial_\theta \rho_j^{wait}(t, q, \theta) = S_j^{wait}(t, q, \theta), \end{cases} \quad (21)$$

192 with

$$S_j^{exe}(t, q) = \rho_j^{wait}(\theta = 0^+) - \frac{1}{\mathcal{T}} \sum_{k \neq j} \rho_j^{exe}(t, q) \text{Dec}(j, k), \quad (22)$$

193 and

$$S_j^{wait}(t, q, \theta) = \frac{1}{\mathcal{T}} \sum_{k \neq j} \frac{T_k}{T_j} \rho_k^{exe}(t, q) \text{Dec}(k, j) \delta_{\theta=\tau_{kj}}. \quad (23)$$

194 **3.3. Model extension**

195 Let us note that we could use more parameters to characterize the jobs but they were not
 196 used in this model. These parameters could be $s_i \in \mathbb{R}^+$ the size of a job, $p_i \in \mathbb{N}^*$ the number of
 197 processors required for the job J_i , $a_i \in \mathbb{R}_+^*$ the age of the job.

198 These parameters evolve following

$$a_i'(t) = 1, \quad s_i'(t) = 0, \quad p_i'(t) = 0. \quad (24)$$

199 By denoting $x = (a, p, s)$ we then get the following kinetic equation in the distribution sense

$$\begin{cases} (\partial_t \rho_j^{exe} - v_j \partial_q \rho_j^{exe} + \partial_a \rho_j^{exe})(t, q, x) = S_j^{exe}(t, q, x), \\ (\partial_t \rho_j^{wait} - \partial_\theta \rho_j^{wait} + \partial_a \rho_j^{wait})(t, q, \theta, x) = S_j^{wait}(t, q, \theta, x). \end{cases} \quad (25)$$

200 where S_j^{exe} and S_j^{wait} are defined as in (22)-(23).

201 To get the fluid equations on the momentum of the distribution functions ρ_j^{exe} and ρ_j^{wait} , we
 202 let:

$$n_j^{exe/wait} = \int_x \rho_j^{exe/wait} dx, \quad (26)$$

203

$$\bar{a}_j n_j^{exe/wait} = \int_x \alpha \rho_j^{exe/wait}, \quad \text{for } \alpha = \{a, p, s\}, \quad (27)$$

204 with $n_j^{exe/wait}$ the job density, $(n_j \bar{a}_j)^{exe/wait}$ the mean age of the jobs, $(n_j \bar{s}_j)^{exe/wait}$ the mean size
 205 of the jobs, $(n_j \bar{p}_j)^{exe/wait}$ the mean number of needed processors for the jobs, whether they are
 206 executed or waiting.

207 **4. Mathematical study of the model (21)-(22)-(23)**

208 **4.1. Links between mesoscopic and macroscopic models**

209 To prove the existence of the fluid system we use a theorem from Jabin [20] that we adapt to
 210 our case.

211 We let the initial data:

$$f(0, \cdot, \cdot) = f_0, \quad \rho_j^{exe}(0, \cdot) = \rho_{j,0}^{exe} \quad \text{and} \quad \rho_j^{wait}(0, \cdot, \cdot) = \rho_{j,0}^{wait}. \quad (28)$$

212 **Lemma 3.** ρ_j^{exe} and ρ_j^{wait} , with the initial data given in (28), are solutions of the fluid system (21)
 213 in $\mathcal{D}'(\mathbb{R}_+^*)_q$ and $(\mathcal{D}'(\mathbb{R}_+^*))_q \otimes (\mathcal{D}'(\mathbb{R}))_\theta$ respectively if and only if f^j written in the form (19) is
 214 solution of the kinetic equation (7) with an initial data fulfilling the closure assumption (19).

215 *Proof.* Let us first assume that ρ_j^{exe} and ρ_j^{wait} , with their initial data, are solutions if the fluid
 216 system (21) and that

$$f^j(0, q, \theta) = \rho_j^{exe}(0, q) \delta_{\theta=0} + \rho_j^{wait}(0, q, \theta) \mathbf{1}_{\theta>0}. \quad (29)$$

217 Since we have $\rho_j^{garb}(0, \cdot, \cdot) = 0$ and (29), we apply Lemma 2 to characterize the solutions of the
 218 kinetic equation and get for all $t > 0$ that the distribution function writes

$$f^j(t, q, \theta) = \rho_j^{exe}(t, q) \delta_{\theta=0} + \rho_j^{wait}(t, q, \theta) \mathbf{1}_{\theta>0}. \quad (30)$$

219 By writing the kinetic equation in the distribution sense with the test functions defined in (20),
 220 replacing f^j by the closure assumption and testing this equation with ϕ^j we get that ρ_j^{exe} and ρ_j^{wait}
 221 are solutions of the fluid system (21). \square

222 **4.2. Local theory**

223 The fluid system (21) is nonlinear due to the decision function Dec. We introduce the natural
 224 distribution spaces such that $\rho_j^{exe} \in B^{exe}$ and $\rho_j^{wait} \in B^{wait}$, $\forall j \in \{1, \dots, C\}$, where the spaces are
 225 defined by

$$B^{exe} = \mathbb{L}^\infty([0, T], \mathbb{W}^{1,\infty}(\mathbb{R}_+^*)_q) \cap F_q \quad (31)$$

226 and

$$B^{wait} = \mathbb{L}^\infty([0, T], \mathbb{W}^{1,\infty}(\mathbb{R}_+^*)_q \times \mathbb{W}^{1,\infty}(\mathbb{R})_\theta) \cap F_\theta \quad (32)$$

227 where F_X is the set of function which admit a right-handed limit in the X -variable.

228 We introduce the vectors ρ^{exe} and ρ^{wait} such that $\rho^{exe} = (\rho_j^{exe})_{j \in \{1, \dots, C\}} \in (B^{exe})^C$ and $\rho^{wait} =$
 229 $(\rho_j^{wait})_{j \in \{1, \dots, C\}} \in (B^{wait})^C$. We moreover define the norm of ρ^{exe} such that

$$\|\rho^{exe}\|_{exe} = \sum_{j=1}^C T_j \|\rho_j^{exe}\|_{B^{exe}}. \quad (33)$$

230 **Proposition 1. Study of a model problem**

231 Let $a \in \mathbb{R}$ and $\rho \in B = \mathbb{L}^\infty([0, T], \mathbb{W}^{1,\infty}(\mathbb{R}))$. We consider the problem

$$\begin{cases} \partial_t \rho(t, x) - \partial_x \rho(t, x) = S(t) \delta_{x=a}, & \forall (t, x) \in [0, T] \times \mathbb{R}, \\ \rho(0, x) = \rho_0(x), & \forall x \in \mathbb{R}. \end{cases} \quad (34)$$

232 There exists then a solution $\rho(t, x) \in B \cap F_x$ which is given by

$$\rho(t, x) = \begin{cases} \rho_0(x+t) & , \text{ if } x < a-t \text{ or if } x \geq a, \\ \rho_0(x+t) + S(x+t-a) & , \text{ if } a-t \leq x < a, \end{cases} \quad (35)$$

233 if $S(x+t-a)$ is right limited.

234 *Proof.* We are dealing with a transport equation with negative speed. We find that $\rho(t, x) =$
 235 $\rho_0(\xi) + \int_0^t S(s) \delta_{s=(x+t)-a} ds$, which gives the result as soon as $S(x+t-a)$ admits a limit on the
 236 right-hand side. \square

237 **4.3. Global existence and uniqueness theorem**

238 **Theorem 1. Existence and uniqueness of fluid solutions**

239 Let $\rho_{j,0}^{exe} \in B^{exe}$ and $\rho_{j,0}^{wait} \in B^{wait}$ for all $j \in \{1, \dots, C\}$. There exists then an unique solution ρ^{exe} and
 240 ρ^{wait} to (21) for all $t \in \mathbb{R}^+$ with the initial data $\rho_j^{exe}(0, q) = \rho_{j,0}^{exe}(q)$ and $\rho_j^{wait}(0, q, \theta) = \rho_{j,0}^{wait}(q, \theta)$.

241 *Proof.* We use a fixed point theorem. Let $\Psi : (B^{exe})^C \rightarrow (B^{exe})^C$ the application that maps ρ^{exe}
 242 to $\rho^{exe(1)}$ where $(\rho^{exe(1)}, \rho^{wait})$ is solution of

$$\begin{cases} \partial_t \rho_j^{exe(1)}(t, q) - v_j \partial_q \rho_j^{exe(1)}(t, q) = S_j^{exe(1)}(t, q), \\ \partial_t \rho_j^{wait}(t, q, \theta) - \partial_\theta \rho_j^{wait}(t, q, \theta) = S_j^{wait}(t, q, \theta), \end{cases} \quad (36)$$

243 with

$$\begin{aligned} S_j^{exe(1)}(t, q) &= \rho_j^{wait}(t, q, \theta = 0^+) - \frac{1}{\mathcal{T}} \sum_{\substack{k=1 \\ k \neq j}}^C \rho_k^{exe(1)}(t, q) \text{Dec}(j, k), \\ S_j^{wait}(t, q, \theta) &= \frac{1}{\mathcal{T}} \sum_{\substack{k=1 \\ k \neq j}}^C \frac{T_k}{T_j} \rho_k^{exe}(t, q) \text{Dec}(k, j) \delta_{\theta=\tau_{kj}}, \end{aligned} \quad (37)$$

and the initial condition

$$\begin{aligned}\rho^{exe(1)}(0, q) &= \rho_0^{exe}(q) = \left(\rho_{j,0}^{exe}(q)\right)_{j \in \{1, \dots, C\}}, \forall q \in \mathbb{R}_+^*, \\ \rho^{wait}(0, q, \theta) &= \rho_0^{wait}(q, \theta) = \left(\rho_{j,0}^{wait}(q, \theta)\right)_{j \in \{1, \dots, C\}}, \forall (q, \theta) \in \mathbb{R}_+^* \times \mathbb{R}_+^*.\end{aligned}$$

244 We define the sequence $\rho^{exe(l+1)} = \Psi(\rho^{exe(l)})$ starting from $\rho^{exe(0)} = \rho_0^{exe}$. In the first part of the
245 proof we compute explicitly $\Psi(\rho^{exe})$.

Thanks to Prop. 1 we obtain that

$$\rho_j^{wait(l)}(t, q, \theta) = \begin{cases} \rho_{j,0}^{wait}(q, \theta + t) \text{ if } \theta < \tau_{kj} - t \text{ or } \theta \geq \tau_{kj}, \\ \rho_{j,0}^{wait}(q, \theta + t) + \frac{1}{\mathcal{T}} \sum_{\substack{k=1 \\ k \neq j}}^C \frac{T_k}{T_j} \rho_k^{exe}(\theta + t - \tau_{kj}, q) \mathbf{1}_{Dec(\theta+t-\tau_{kj}, q, k, j)=1}, \\ \text{if } \tau_{kj} - t \leq \theta < \tau_{kj}, \end{cases}$$

if the second member of the second equation of (36) admits a limit on the right hand side in $\theta + t - \tau_{kj}$, i.e. if ρ_k^{exe} has a limit on the right hand side in time. This is satisfied since $\rho^{exe} \in \mathcal{B}^{exe}$. Then we have

$$\rho_j^{wait}(t, q, 0^+) = \begin{cases} \rho_{j,0}^{wait}(q, t), \text{ if } t < \tau_{kj}, \\ \rho_{j,0}^{wait}(q, t) + \frac{1}{\mathcal{T}} \sum_{\substack{k=1 \\ k \neq j}}^C \frac{T_k}{T_j} \rho_k^{exe}(t - \tau_{kj}, q) \mathbf{1}_{Dec(t-\tau_{kj}, q, k, j)=1}, \text{ if } t \geq \tau_{kj}. \end{cases}$$

Back to the first equation in (36) we get

$$\begin{aligned}\rho_j^{exe(1)}(t, q) &= \rho_{j,0}^{exe}(q) \\ &+ \int_0^t \frac{1}{\mathcal{T}} \sum_{k \neq j} \frac{T_k}{T_j} \rho_k^{exe}(s - \tau_{kj}, q + v_j(t-s)) \mathbf{1}_{Dec(s-\tau_{kj}, q+v_j(t-s), k, j)=1} ds \\ &+ \int_0^t \rho_{j,0}^{wait}(q + v_j(t-s), s) ds \\ &- \int_0^t \frac{1}{\mathcal{T}} \sum_{k \neq j} \rho_j^{exe(1)}(s, q + v_j(t-s)) \mathbf{1}_{Dec(s, q+v_j(t-s), j, k)=1} ds.\end{aligned}\tag{38}$$

We now look for conditions so that Ψ is a contraction. Let $\mu_j^{exe} \in \mathcal{B}^{exe}$, and $\mu_j^{exe(1)} \in \mathcal{B}^{exe}$, $\mu_j^{wait} \in \mathcal{B}^{wait}$ solutions of the fluid system (36) so that $\mu^{exe(1)} = \Psi(\mu^{exe})$. Then

$$\begin{aligned}|\rho_j^{exe(1)}(t, q) - \mu_j^{exe(1)}(t, q)| &= \chi(t, q) - \int_0^t \psi(s, q) \left(\rho_j^{exe(1)}(s, q) - \mu_j^{exe(1)}(s, q)\right) ds \\ &\leq |\chi(t, q)| + \int_0^t |\psi(s, q)| |\rho_j^{exe(1)}(s, q) - \mu_j^{exe(1)}(s, q)| ds\end{aligned}$$

with

$$\chi(t, q) = \int_0^t \frac{1}{\mathcal{T}} \sum_{k \neq j} \frac{T_k}{T_j} \left(\rho_k^{exe} - \mu_k^{exe}\right)(s - \tau_{kj}, q + v_j(t-s)) \mathbf{1}_{Dec(s-\tau_{kj}, q+v_j(t-s), k, j)=1} ds,$$

and

$$\psi(s, q) = \frac{1}{\mathcal{T}} \sum_{k \neq j} \mathbf{1}_{Dec(s, q + v_j(t-s), j, k) = 1}.$$

Thanks to Gronwall inequality we get

$$|\rho_j^{exe(1)}(t, q) - \mu_j^{exe(1)}(t, q)| \leq |\chi(t, q)| + \int_0^t |\chi(s, q)| |\psi(s, q)| \exp\left(\int_s^t |\psi(u)| du\right) ds.$$

and then

$$\|\rho_j^{exe(1)} - \mu_j^{exe(1)}\|_{\mathbb{L}^\infty([0, T], \mathbb{L}^\infty(\mathbb{R}_+^*))} \leq \frac{1}{C-1} \sum_{k \neq j} \frac{T_k}{T_j} \|\rho_k^{exe} - \mu_k^{exe}\|_{\mathbb{L}^\infty([0, T], \mathbb{L}^\infty(\mathbb{R}_+^*))} \left(-1 + \exp\left(\frac{T(C-1)}{\mathcal{T}}\right)\right).$$

By taking the derivative in q of (38) we get in the same way

$$T_j \|\rho_j^{exe(1)} - \mu_j^{exe(1)}\|_{B^{exe}} \leq \frac{1}{C-1} \sum_{k \neq j} T_k \|\rho_k^{exe} - \mu_k^{exe}\|_{B^{exe}} \left(-1 + \exp\left(\frac{T(C-1)}{\mathcal{T}}\right)\right)$$

and finally

$$\|\Psi(\rho^{exe}) - \Psi(\mu^{exe})\|_{exe} \leq \frac{C}{C-1} \|\rho^{exe} - \mu^{exe}\|_{exe} \left(-1 + \exp\left(\frac{T(C-1)}{\mathcal{T}}\right)\right).$$

If

$$T \leq \frac{\mathcal{T}}{C-1} \ln\left(\frac{3C-1}{2C}\right) = T_{max},$$

246 the application Ψ is 1/2-Lipschitz. Since the time T_{max} only depends on the model we also have
 247 existence of solutions on the time interval $[T, 2T], \dots$ as long as $T \leq T_{max}$ and then the existence
 248 of an unique solution on \mathbb{R}^+ . \square

249 **Theorem 2. Existence and uniqueness of the kinetic solution under closure assumption**

250 Let ρ^{exe} and ρ^{wait} such that $\rho_j^{exe} \in B^{exe}$ and $\rho_j^{wait} \in B^{wait}$ satisfy (21) $\forall j \in \{1, \dots, C\}$, with the initial
 251 data $\rho_j^{exe}(0, q) = \rho_{j,0}(q)$ and $\rho_j^{wait}(0, q, \theta) = 0$. Let us assume that $f^j(t = 0, q, \theta)$ splits according
 252 to the closure assumption (13). There exists then an unique solution to the kinetic equation that
 253 splits according to the closure assumption for all $t \in \mathbb{R}_+^*$.

254 *Proof.* The assumptions on the distribution function allow us to apply Lemma 2 to characterize
 255 the kinetic equations. For all time the solution f then splits according to the closure assumption.
 256 Thanks to Theorem 1, we moreover have uniqueness of the solutions ρ_j^{exe} and ρ_j^{wait} . Since the
 257 distribution function admits a unique decomposition according to the closure assumption, defined
 258 through its moments, we obtain the uniqueness of the kinetic equation and of its decomposition.
 259 \square

260 **5. Numerical results**

261 *5.1. Numerical method*

262 We now numerically solve the system through the fluid model. We have on each cluster C_j :

$$\begin{cases} \partial_t \rho_j^{exe}(t, q) - v_j \partial_q \rho_j^{exe}(t, q) = S_j^{exe}(t, q), \\ \partial_t \rho_j^{wait}(t, q, \theta) - \partial_\theta \rho_j^{wait}(t, q, \theta) = S_j^{wait}(t, q, \theta), \end{cases} \quad (39)$$

263 with the source terms S_j^{exe} and S_j^{wait} defined by (22)-(23)

264 In order to simulate this system of conservation equations, a Finite Volume approach has
 265 been used, along with a first order Explicit Euler method for time integration. We discretized the
 266 respective phase spaces for these two equations. We then impose $q \in [0, q_{max}]$ and $\theta \in [0, \theta_{max}]$,
 267 where q_{max} is obtained by taking the worst case scenario (maximal execution time and flow rate
 268 on the grid) and $\theta_{max} = \max_{j,k \in \{1, \dots, C\}} \tau_{jk}$.

269 The numerical domain for the workload is divided into N_q intervals of uniform length $\Delta q =$
 270 q_{max}/N_q . The $N_q + 1$ points of discretization are noted $q_{i-1/2} = (i-1)\Delta q$ where $i \in \{1, \dots, N_q + 1\}$.
 271 By definition, $0 = q_{1/2} < q_{3/2} < \dots < q_{N_q-1/2} < q_{N_q+1/2} = q_{max}$. Q_i denotes the interval
 272 $[q_{i-1/2}, q_{i+1/2}[$, so that $\bigcup_{i=1}^{N_q} Q_i = [0, q_{max}]$.

273 We use the same approach for the θ variable, introducing N_θ cells of uniform length, noted
 274 $\Theta_m = [\theta_{m-1/2}, \theta_{m+1/2}]$ with $\theta_{m-1/2} = (m-1)\Delta\theta$, for $m \in \{1, \dots, N_\theta + 1\}$.

275 T will denote the time horizon and the interval $[0, T]$ is split within N_t subintervals of length
 276 $\Delta t = T/N_t$. We will note $t^n = n \times \Delta t$, for $n \in \{0, \dots, N_t\}$.

Finally, with these notations, we define :

$$\rho_{j,i}^{exe}(t) \simeq \frac{1}{\Delta q} \int_{q_{i-1/2}}^{q_{i+1/2}} \rho_j^{exe}(t, q) dq$$

and

$$\rho_{j,i,m}^{wait}(t) \simeq \frac{1}{\Delta q \Delta \theta} \int_{q_{i-1/2}}^{q_{i+1/2}} \int_{\theta_{m-1/2}}^{\theta_{m+1/2}} \rho_j^{wait}(t, q, \theta) dq d\theta.$$

We are then able to define the fluxes of jobs with the "running" status on a cluster C_j . The
 equation (39) being a 1D-transport equation, a first-order upwind scheme is used. In this parti-
 cular case, the velocity v_j is known to be positive so that we have :

$$\mathcal{F}_{j,i+1/2}^{exe}(t) = -v_j \rho_{j,i+1}^{exe}(t)$$

Similarly, we use a first-order upwind scheme for the fluxes of "waiting" jobs :

$$\mathcal{F}_{j,i,m+1/2}^{wait}(t) = -\rho_{j,i,m+1}^{wait}(t).$$

277 which yields the following semi-discretized scheme :

$$\begin{aligned} \frac{d}{dt} \rho_{j,i}^{exe}(t) + \frac{1}{\Delta q} \left[\mathcal{F}_{j,i+1/2}^{exe}(t) - \mathcal{F}_{j,i-1/2}^{exe}(t) \right] &= S_{j,i}^{exe}(t), \\ \frac{d}{dt} \rho_{j,i,m}^{wait}(t) + \frac{1}{\Delta \theta} \left[\mathcal{F}_{j,i,m+1/2}^{wait}(t) - \mathcal{F}_{j,i,m-1/2}^{wait}(t) \right] &= S_{j,i,m}^{wait}(t), \end{aligned}$$

where $S_{j,i}^{exe}(t)$ and $S_{j,i,m}^{wait}(t)$ are the source term values, assuming that Dec is constant on any $]q_{i-1/2}, q_{i+1/2}[$, given by :

$$S_{j,i}^{exe}(t) \simeq -\mathcal{F}_{j,i,1/2}^{wait}(t) - \frac{1}{\mathcal{T}} \sum_{\substack{k=1 \\ k \neq j}}^C \rho_{j,i}^{exe}(t) \mathbf{1}_{Dec(t,q_i,j,k)=1},$$

and

$$S_{j,i,m}^{wait}(t) \simeq \frac{1}{\mathcal{T} \Delta\theta} \sum_{\substack{k=1 \\ k \neq j}}^C \frac{T_k}{T_j} \rho_{k,i}^{exe}(t) \mathbf{1}_{Dec(t,q_i,k,j)=1} \delta_{\theta_m=\tau_{kj}}.$$

278 To take into account the transport term and the source term we use a first order Lie splitting. We
279 first solve on a time step the sourceless transport equation:

$$\begin{cases} \frac{d}{dt} \rho_{j,i}^{exe}(t) + \frac{1}{\Delta q} [\mathcal{F}_{j,i+1/2}^{exe}(t) - \mathcal{F}_{j,i-1/2}^{exe}(t)] = 0, \\ \frac{d}{dt} \rho_{j,i,m}^{wait}(t) + \frac{1}{\Delta\theta} [\mathcal{F}_{j,i,m+1/2}^{wait}(t) - \mathcal{F}_{j,i,m-1/2}^{wait}(t)] = 0, \end{cases} \quad (40)$$

280 We then inject the solution of the previous system in the ordinary differential equation with only
281 the source term that we solve on a time step

$$\frac{d}{dt} \rho_{j,i}^{exe}(t) = \tilde{S}_{j,i}^{exe}(t) \frac{d}{dt} \rho_{j,i,m}^{wait}(t) = \tilde{S}_{j,i,m}^{wait}(t), \quad (41)$$

282 with $\tilde{S}_{j,i}^{exe}(t)$ and $\tilde{S}_{j,i,m}^{wait}(t)$ defined by

$$\tilde{S}_{j,i}^{exe}(t) = \tilde{\rho}_{j,i,1/2}^{wait}(t) - \frac{1}{\mathcal{T}} \sum_{\substack{k=1 \\ k \neq j}}^C \tilde{\rho}_{j,i}^{exe}(t) \mathbf{1}_{Dec(t,q_i,j,k)=1}, \quad (42)$$

283 and

$$\tilde{S}_{j,i,m}^{wait}(t) = \frac{1}{\mathcal{T}} \sum_{\substack{k=1 \\ k \neq j}}^C \frac{T_k}{T_j} \tilde{\rho}_{k,i}^{exe}(t) \mathbf{1}_{Dec(t,q_i,k,j)=1} \delta_{\theta_m=\tau_{kj}}. \quad (43)$$

284 In both cases we use a first order explicit Euler scheme whose CFL condition for the \mathbb{L}^∞ -stability
285 is

$$\Delta t \leq \min \left\{ \frac{\Delta q}{\max_{j \in \{1, \dots, N\}} v_j}, \Delta\theta, \mathcal{T} \right\}. \quad (44)$$

286 The simulations in the following sections were performed using the numerical parameters
287 described in table 1, on a laptop equipped with an Intel® Core™ i5-2540M CPU @ 2.60GHz and
288 4 GB of RAM. The mesh sizes have been chosen so that their influence on the final results are
289 quite unnoticeable.

Parameter	q_{max}	θ_{max}	N_q	N_θ	\mathcal{T}	T
Value	50	20 s	100	100	1 s	100 s

Table 1: Numerical simulation parameters

290 In order to accurately capture the dynamics of all clusters, a complete save of the simulation
 291 state is performed each 0.2 second of physical time simulated, yielding to a total of 500 save
 292 points for each simulation. In these conditions, each simulation required roughly 89.9 s to go to
 293 completion, the saves accounting for the vast majority of it (67.4 s). It has to be noted that such a
 294 high amount of save is unnecessary if the only purpose is to evaluate a cluster policy efficiency,
 295 allowing for faster than real-time simulations on a laptop.

296 5.2. A 3-cluster dynamics example

It is quite easy to find a 2-cluster configuration with an analytical solution in order to validate
 the numerical solver [18]. We will here present a 3-cluster configuration, exhibiting a more
 complex behavior. To this extent, we consider a cost function depending on both the remaining
 execution time for a job t^{exe} and the electrical consumption K_e of the target cluster C_k . These
 two items read :

$$K(q_i, C_k) = \alpha_t t^{exe}(q_i, C_j, C_k) + \alpha_e K_e(q_i, C_k), \text{ with } \forall j, k \in \{1, \dots, C\} :$$

$$K_e(q_i, C_k) = \int_0^{\frac{q_i}{v_k}} Z_k(t) dt \text{ and } t^{exe}(q_i(t), C_j, C_k) = \frac{q_i(t)}{v_k} + \tau_{jk},$$

297 Using a dimensionless parameter $\lambda \in [0, 1]$, the coefficients α_e and α_t are respectively taken
 298 as λJ^{-1} and $(1 - \lambda)s^{-1}$ so that K is dimensionless and that we can define a cluster policy by
 299 changing the value of λ . For example, taking $\lambda = 0$ means that we are only interested in opti-
 300 mizing the time of execution regardless of the energy needed. Conversely, $\lambda = 1$ means that the
 301 only objective is to execute the jobs using the less energy possible with no constraint on the time
 302 needed to complete them.

We choose to model the part of the electrical consumption $Z_k(t)$ due to the computers as a
 linear function of the total load $\tau(t)$ on the cluster :

$$Z_k^1(t) = c_k \left(\int_q \rho_k^{exe}(t, q) dq + \int_{q, \theta} \rho_k^{wait}(t, q, \theta) dq d\theta \right) = c_k \cdot \tau(t),$$

303 where $c_k \in \mathcal{R}_+^*$ is a coefficient accounting for the energetic efficiency of the cluster. This kind of
 304 formula is in agreement with the estimates for energy consumption used in real computational
 305 clusters [21].

306 In order to obtain a non-trivial dynamic, we are interested in a configuration for which a
 307 cluster seems at first attractive performance-wise and becomes less and less interesting energetic-
 308 wise as its workload rises, until the point where some of the jobs are moved towards another
 309 cluster. To take this effect of over-consumption into account (due for example to the use of
 310 cooling systems), we define a cost function with a significant rise of the slope for an occupation
 311 rate $\tau(t)$ above 50%.

$$Z_k(t) = c_k * \tau(t) + c_k * \max(0, \tau(t) - 0.5) \quad (45)$$

312 The parameters at disposal to describe such a set-up are the performance indexes and ener-
 313 getic consumption of the clusters but also the transfer rates between them. Let us begin with the
 314 hierarchy of performances for the clusters. We fix the cluster C_0 as the less interesting one setting
 315 reference values $v_0 = 1.0$ and $c_0 = 1.0$. The two other clusters will both be more attractive than
 316 C_0 , the second cluster C_1 being the best choice energetic-wise and the third C_2 being the best
 317 choice for the performance but also the most energy consuming (see table 2).

Cluster	Capacity	Performance	Consumption
0	1700	1.0	1.0
1	3000	2.0	1.0
2	600	10.0	11.0

Table 2: Grid configuration

Cluster	Rate (Nb.s ⁻¹)	Waiting time (s)	Duration (s)
0	100	2.0	20.0
1	65	4.0	20.0
2	20	10.0	10.0

Table 3: Jobs submission scenario for each cluster

318 *5.2.1. A reference case with no transfer*

319 In order to describe the test-case used in terms of workload scenario, we first perform a
320 reference simulation where all transfers are forbidden. In our model, this is strictly equivalent
321 to choose a cost function only based on performance ($\lambda = 0$) and insanely high transfer times
322 between clusters.

323 As for the initial condition, all clusters are considered completely empty with no job waiting.
324 At $t = 0$, jobs are submitted to each cluster. Submission rate (total number of jobs submitted per
325 second), duration of submission and associated waiting time varies with the cluster but remains
326 constant over time. These parameters are listed in the table 3. All jobs submitted are assumed to
327 have a constant distribution along the computational load variable (q).

328 The dynamics of ρ^{exe} and ρ^{wait} for the the cluster 0 are respectively represented on figures 1
329 and 2. We can easily distinguish four phases. At first, all jobs submitted are waiting and no one
330 is in execution ($t = 1s$). Then in a second phase, the jobs submitted have partly entered execution
331 state and as the job submission rate is big enough, ρ^{exe} is growing over time. In a third step, the
332 job submission ends and all jobs are leaving the waiting state, ρ^{exe} reaching its maximum value.
333 In the last phase, all remaining jobs are being executed. We can observe that the behavior of ρ^{exe}
334 and ρ^{wait} are consistent with the exact solutions of equations 21.

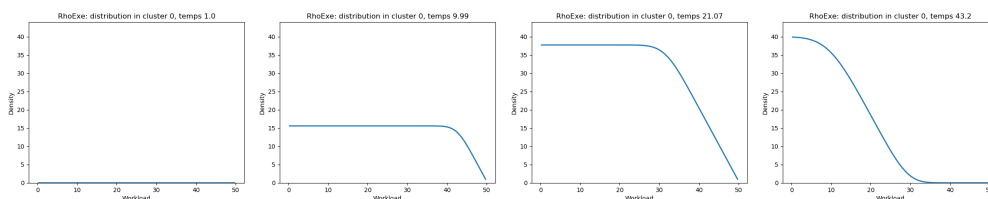


Figure 1: ρ^{exe} distribution for cluster 0 at times $t = 1s, 9.99s, 21.07s$ and $43.2s$

335 As for the occupation rates, as expected, the results first show a rise of the occupation rate
336 (as new jobs are submitted) and then a decrease due to the execution of the jobs on the cluster
337 (see figure 3). All values for this test-case have been fixed in order to keep the occupation rate
338 under 100%.

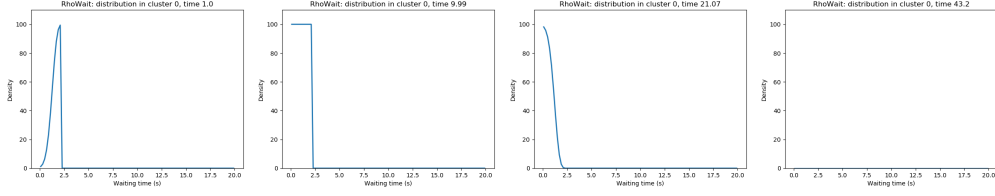


Figure 2: ρ^{wait} distribution for cluster 0 at times $t = 1s, 9.99s, 21.07s$ and $43.2s$

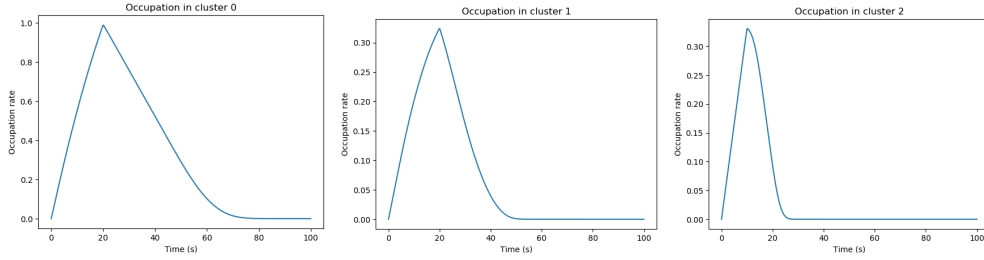


Figure 3: Occupation rate over time for clusters 0 to 2

339 5.2.2. Application to protocol monitoring and performance assessment

340 In this section, we propose to illustrate the impact of the cluster policy on the performance of
 341 the grid with respect to its energy consumption and time needed to execute all the jobs submit-
 342 ted. To this extent, we have performed some numerical simulations with various values for the
 343 parameter λ . These cases will be labeled with the relative importance λ of the energy in the cost
 344 function K . For example, a simulation labelled "80% energy" corresponds to $\lambda = 0.8$.

345 We propose a total of 5 test-cases, with $\lambda \in \{0, 0.01, 0.2, 0.8, 1\}$. The reference case with no
 346 transfer will also be displayed. It has to be noted that the reference case differs from the test-case
 347 with $\lambda = 0$ since for the reference case, the transfer times had been changed in order to prevent
 348 any transfer.

349 As for the transfer times used for these simulations, these are summarized in table 4. For
 350 simplicity sake, these times are considered to be symmetric but this aspect is not mandatory.
 351 These values have been chosen so that they have a non-negligible impact on the time needed to
 352 execute a job with a high workload but also do not render the transfer cost prohibitive.

353 As represented on the figure 4, enabling the transfers between clusters does have an impact
 354 on both the energy consumption and the time needed to complete all the jobs submitted. On
 355 the left side is represented the energy consumption with respect to time for all the test-cases
 356 performed. On the middle figure is indicated the number of jobs remaining in the system while

	Cluster 0	Cluster 1	Cluster 2
Cluster 0	0.0	3.0	8.0
Cluster 1	3.0	0.0	7.0
Cluster 2	8.0	7.0	0.0

Table 4: Transfer times (s) between clusters

357 on the right side is represented the remaining computational load (since all the jobs do not have
 358 the same computational load at submission). All these values are calculated for the whole grid.

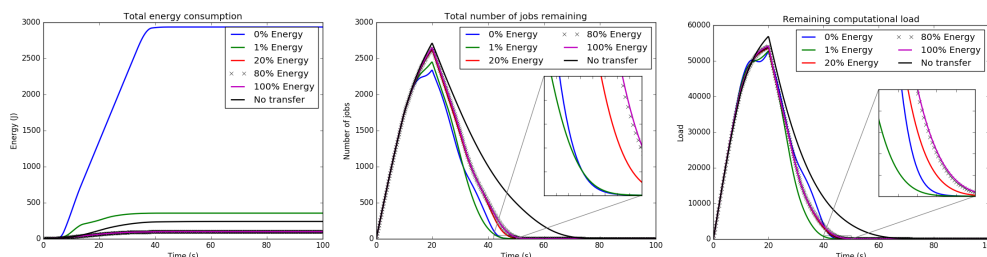


Figure 4: policies efficiency with respect with the energy consumption (left), number of jobs remaining (middle) and the total workload remaining (right)

359 Without any surprise, enabling transfers allows for smaller computational times in any scenario,
 360 even for the one focused only on energy consumption, since a lot of jobs are submitted
 361 in Cluster 0 whereas the grid contains better clusters in every aspect. Conversely, focusing only
 362 on the time aspect leads to a much higher energy consumption than the reference case, which is
 363 easily explained by the fact that the fastest cluster has a very high energy consumption compared
 364 to the two others.

365 On the middle and right figures, all curves seem to be well-ordered with respect to the im-
 366 portance of execution time, except for two of them which correspond to the scenarios with 0%
 367 and 1% energy. However, when zooming near the completion time for the two of them, it can be
 368 seen that the case with 0% energy does manage to finish before the other, even if barely. This
 369 can be explained by a transfer of jobs with high computational load to the fastest cluster delaying
 370 the completion of some of them (due to the transfer time) but in the end allowing to gain some
 371 seconds (or even milliseconds) to complete all of them. These curves also show that taking into
 372 account the consumption, even for a very small amount, can save a lot of energy while having a
 373 quite negligible impact on the computational time.

374 It has to be noted that the transfer decision is based only on the state of the grid at the time
 375 of the transfer, without having any knowledge of what will be submitted in the future, and that in
 376 our model no job can be transferred back so that it makes room for another one. Hence there is
 377 no guarantee that, for a given scenario, lowering the focus on energy will lead to a better global
 378 computational time. That makes the existence of a quick-time simulator even more attractive.

379 *Dynamics split for every cluster.* The results of energy consumption, occupation rate (normal-
 380 ized) and number of jobs ended for every cluster are displayed on figure 5. Each line corresponds
 381 to a different cluster while each column focus on one aspect (energy, occupation, jobs).

382 The results shown on figure 5 are in accordance with the global ones (figure 4) and what can
 383 be expected from the grid dynamics. For example, the occupation rate shows that the Cluster 0
 384 is not the preferred one regardless of the policy used (as soon as transfers are enabled), which
 385 is in accordance with its characteristics. Both the occupation rate and the number of jobs ended
 386 grow on the cluster 2 as the focus on the energy decreases. The effect of the occupation rate on
 387 the consumption of the cluster (cooling system (45)) can be seen with energy-focused policies as
 388 some waves appear on the occupation rate as well as on the number of jobs ended.

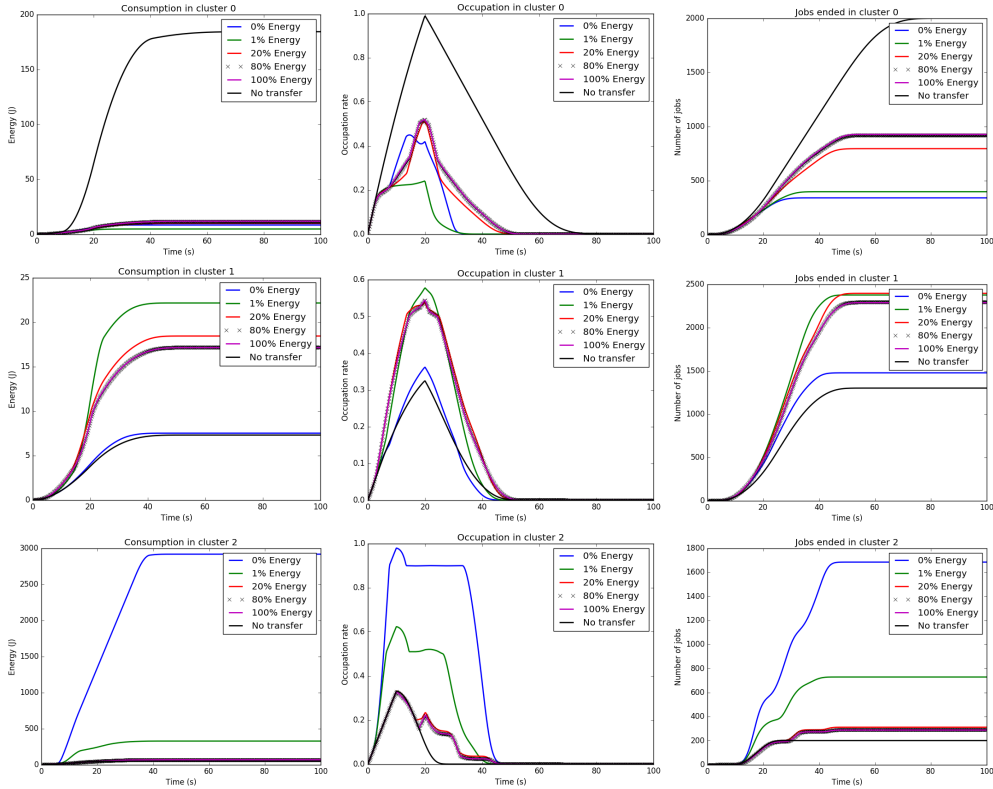


Figure 5: Impact of the grid policies on the energy consumption (left column), occupation rate (middle column) and number of jobs ended (right column) for cluster 0 (top line), cluster 1 (middle line) and cluster 2 (bottom line)

389 *Transfers.* The amounts of transfers performed between the clusters are all represented on figure
 390 6.

391 As prescribed in our model, no transfer is allowed from one cluster to itself (flat lines on
 392 the diagonal of figure 6). As expected, the number of transfers towards Cluster 2 rises when the
 393 focus on energy decreases. It can be noted that some transfers exist from cluster 2 to cluster 1
 394 when energy is the main focus. In these cases, even transfers from cluster 2 to cluster 0 exist,
 395 suggesting that the occupation rate of the cluster 1 yields a rise of its consumption. Yet, this
 396 amount remains negligible.

397 It can also be noted that the timing for the transfers from cluster 1 to cluster 2 are very differ-
 398 ent for the scenarios 0%- and 1%- energy, the 0% case having a bigger transfer occurring later in
 399 the simulation. This explains the crossing lines for both the number of jobs and computational
 400 load remaining observed on figure 4.

401 6. Conclusion and prospects

402 Starting from a microscopic model based on the realistic behavior of jobs submitted in clus-
 403 ters, and from a generic description of the job transfer policy between different clusters of the

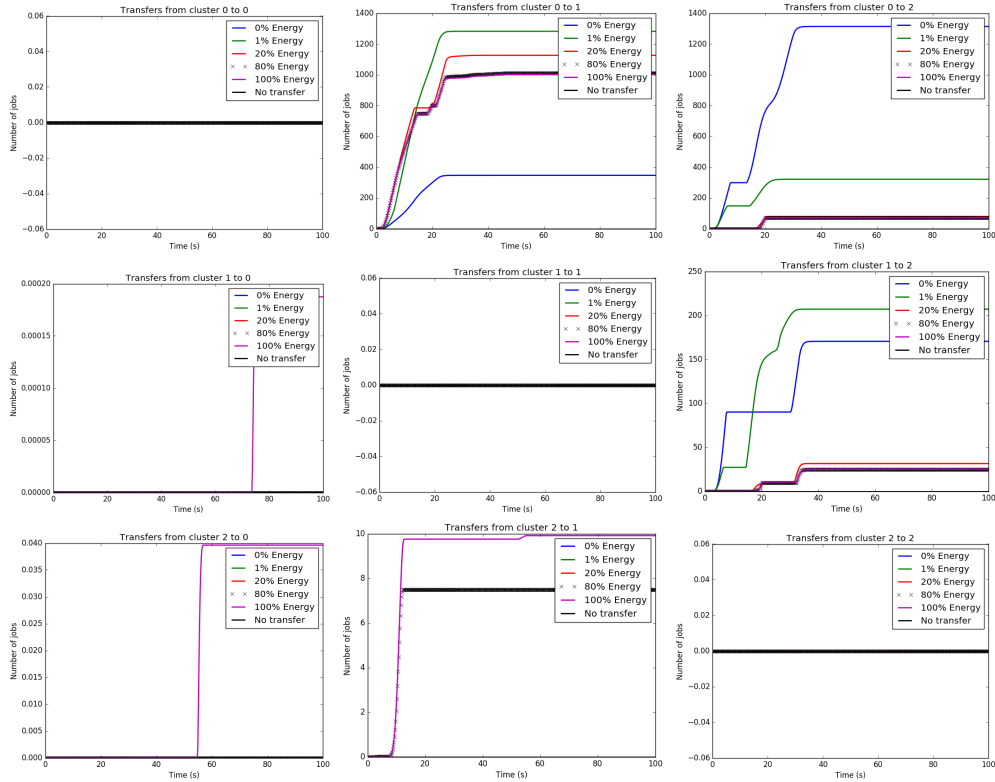


Figure 6: Impact of the grid policies on the number of transfers between clusters, following the structure of table 4

404 same computing grid, we proposed a derivation of a macroscopic model able to represent the
 405 grids dynamics.

406 Existence and uniqueness of a global solution to the resulting fluid system can then be proven
 407 under minimal assumptions of regularity of the decision function used for the transfer of jobs.
 408 This guarantees that the model covers a wide range of decision functions used for the job trans-
 409 fers, where the associated cost-functions can be seen as a "black-box". Hence, this kind of model
 410 enables the evaluation of the impact of a transfer policy on the performance of the cluster.

411 Using a finite volume discretization of the fluid equations and a simple cost function mixing
 412 both the aspects of computing time and energy consumption, we have illustrated a simple test
 413 case exhibiting a non trivial dynamics. It shows that our model has a consistent behavior with
 414 the one which can be expected from a computing grid. The ability of the numerical model to
 415 provide real-time estimates of the performance of a given set of policies for one job submission
 416 scenario, has also been highlighted.

417 This kind of model can easily be extended to a more complex set of parameters. For ex-
 418 ample, on the cost function side, it would be quite straightforward to include the mean time of
 419 execution for each job or the different kinds of fees paid for the use of the cluster. An interesting
 420 generalization of the model would be to take into account, at the microscopic level, the memory
 421 size of each job (as a new variable) which would impact its transfer time. This would add one

422 more conservation equation (on the mean memory size of the jobs) in the system (21). Imple-
423 menting this aspect and using a known submission history would allow to simulate a real cluster
424 dynamics, like the Grid'5000 Cluster in France [22].

425 One important assumption in our model is the fact that N remains constant through time,
426 implying that all jobs are always tested, even the finished ones. If we allow the middle-ware
427 to only focus on the jobs in execution state, it would introduce a variable characteristic time
428 $\mathcal{T}(t)$, yielding to a more complex analysis. This aspect will be tackled in future studies. Finally,
429 another extension of our model may focus on the effect of *exceptional* (oversized or prioritized
430 for example) jobs on the cluster dynamics. Such jobs should be numerically treated with a
431 specific particular solver which would need to be coupled with our fluid model through source
432 terms and/or dynamic values for the clusters characteristics. Such an approach may allow to
433 tackle some practical problems such as cluster shutdowns.

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