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A δ-persistently-exciting formation controller for non-holonomic systems over directed graphs

Maitreyee Dutta Antonio Loría Elena Panteley Srikant Sukumar Emmanuel Nuño

***Abstract*— We present a simple smooth time-varying formation controller for autonomous vehicles, both in leaderless and leader-follower scenario. The controller is efficacious for scouting missions in which a group of robots is required to meet at a specified rendezvous location and acquire a non-specified consensual orientation, with the purpose of advancing in formation at the pace of a virtual leader. Hence, the controller that we propose achieves stabilization in Cartesian coordinates and consensus in orientation, for robots interconnected over generic directed graphs. To overcome the effect of the nonholonomic constraints it relies on persistency of excitation.**

***Index Terms*— Formation consensus, non-holonomic constraints, Lyapunov stability**

I. Introduction

THE control of multiple vehicles with non-holonomic constraints brings about two problems related to consensus. One is partial consensus, in which case the vehicles are controlled to acquire either a common position [1], [2] or a common orientation [3]. The other is full consensus, *i.e.*, consensus both in position and orientation is staked for all the vehicles [4]–[7].

On the consensus manifold, all the robots may stabilize at a common equilibrium point [4]–[9] or follow a common time-varying trajectory [10]–[18]. The first scenario is a static formation control problem in which the vehicles are required to form a geometric shape called formation with a non-pre-specified center and a common non-predefined orientation. These are determined by the graph topology, the systems' initial conditions and their dynamics—see [4], [6], [7], [8], and [9]. In a leader-follower scenario, a specified posture is imposed [2], [19]. In the case of formation-tracking control a desired trajectory for the common centroid of the formation is predominantly dictated by a single leader or the root node of the network. The well-known difficulty to control nonholonomic systems is that they are not stabilizable by smooth time-invariant feedback. Therefore, numerous works have been proposed, relying either on time-varying controllers—see [4], [7], [9], and [8] or on non-smooth controllers, as seen, *e.g.*, in [5], [19]. As a matter of fact, smooth controllers must rely on persistency of excitation [20] both in stabilization and tracking control tasks [21]. Such controllers are increasingly popular since their first appearance in the literature, for tracking [22], as for stabilization [23].

Besides the difficulties imposed by the nonholonomic constraints, controlling a group of such autonomous vehicles is stymied by the additional encumbrances introduced by the nature of the connectivity among the vehicles. In this regard, the literature on consensus is very rich, specifically for linear systems. Many works also address problems of formation and consensus of multiple robots interacting over a communications network, but the vast majority pertain to the case in which the network's topology is undirected, whether in tracking or stabilization problems— see, *e.g.*, [2], [5], [7], [8], [10], [12], [13], [16], [17], [19], [24], and [25]. Comparatively, there are relatively few works on consensus-based stabilization of multitude of vehicles interacting via a generic directed network, if not for particular directed topologies [4] popular leader-follower scenario which, in terms of graph theory, pertains to the case in which the graph consists in a spanning-tree graph [6]. See also [1] and [9].

In this letter we address a problem of stabilization and formation consensus. The problem may be recast as part of a complex multi-vehicle maneuver in which a group of robots is required to stabilize around a pre-specified set-point on the plane and acquire a common non-pre-specified orientation (leaderless orientation consensus). The solution of such problem naturally extends to other typical scenarii of formation control, such as one in which the robots are required to advance in formation, as in a scouting mission. This is because our approach consists in separating the stabilization-on-the-plane and consensus-orientation tasks.

There are many works that provide solutions to such problems [16], specially under a leader-follower approach [6], [10], [12], [14], [17]–[19]. The main advantage of our controller is that it applies to generic directed-topology networks under the assumption that the graph contains a directed spanning-tree, but does not need to correspond to one, as in a leader-follower formulation. In addition, our controller is fully distributed; it does not rely on information on the overall network—cf. [4]. From a control theory viewpoint, we stress that our results hold for any initial conditions, as opposed, *e.g.*, to schemes that ensure semi-global stabilization [9]. As other works on partial consensus (*e.g.*, [1]) we focus on consensus of orientation only.

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II. Problem formulation

We consider the problem of formation control (stabilization and tracking) and orientation consensus for a group of N nonholonomic vehicles modeled by the equations

$$\dot{x}_i = \cos(\theta_i)v_i \tag{1a}$$
$$\dot{y}_i = \sin(\theta_i)v_i \tag{1b}$$
$$\dot{\theta}_i = \omega_i, \qquad i \leq N, \tag{1c}$$

where $(x_i, y_i) =: p_i$ denote the Cartesian coordinates of the center of mass of the ith robot on the plane, θ_i denotes its orientation relative to the abscissae; these variables are measured and are to be controlled. The forward velocity v_i and the angular velocity ω_i constitute the control inputs.

The control goal consists in making the systems acquire a formation around a specified reference point $p_i^* := (x_i^*, y_i^*)$, which may be static or moving on the plane with forward velocity v_r and angular velocity ω_r, both considered piece-wise constant or zero (for set-point stabilization). In addition, it is required that all the robots acquire and maintain a non-pre-specified orientation θ_c. To that end, it is assumed that the robots exchange their orientation measurement with a group of neighbors in an unilateral fashion. In other words, the robots form a directed-graph (digraph) network.

For the robots to achieve a formation we define for each of them an offset $\delta_i := (\delta_{xi}, \delta_{yi})$, relative to p_i^* and the errors $\bar{p}_i := p_i - \delta_i - p_i^*$. Then, the formation control and orientation-consensus control goals consist in ensuring that

$$\lim_{t\to\infty} |\bar{p}_i(t)| = 0, \qquad \lim_{t\to\infty} \theta_i(t) = \theta_c. \tag{2}$$

This problem may be considered as part of a more complex maneuver in which a group of initially scattered robots are first summoned to a predefined rendezvous set-point and acquire a common orientation. In this case p_i^* may be set to a constant. If the robots are required to advance in formation, the reference point p_i^* may be made move forward with (piece-wise) constant velocity v_r and, if required to change orientation, a fictitious orientation-leader, with orientation θ_r generated by $\dot{\theta}_r = \omega_r$, may be incorporated in the network, as long as at least one robot has access to θ_r.

Now, as is customary, the network may be modeled using graph theory. Let $a_{ij} = 1$ if an interconnection from the node j to the node i exists and $a_{ij} = 0$ if otherwise. Then, the so-called Laplacian matrix $L := [\ell_{ij}] \in \mathbb{R}^{N\times N}$, where

$$\ell_{ij} := \begin{cases} -a_{ij} & \forall\, i \neq j \\ \sum_{k\in\mathcal{N}_i} a_{ik}, & \forall\, i = j \end{cases}$$

and $\mathcal{N}_i = \{j \in \{1, \cdots, N\} \,|\, (j, i) \in \mathcal{E}\}$. In general, $a_{ij} \neq a_{ji}$, so L is not symmetric. We assume, however, that the digraph contains a directed spanning tree, so the following statements hold true.

Lemma 1: [26] If the directed graph $\mathcal{G}$ has a rooted spanning tree then its associated Laplacian matrix L has a simple null eigenvalue and $\mathbf{1}_N := \begin{bmatrix}1 & \cdots & 1\end{bmatrix}^\top$ is its associated right eigenvector of dimension N. In addition, all of the remaining eigenvalues lie in the open right-half complex plane. □

Lemma 2: [27] Let $\mathcal{G}$ be a directed graph of order N and $L \in \mathbb{R}^{N\times N}$ be the associated non-symmetric Laplacian matrix. Then, the following statements are equivalent:

1) the graph $\mathcal{G}$ has a spanning tree,
2) for any matrix $Q_L \in \mathbb{R}^{N\times N}, Q_L = Q_L^\top > 0$ and for $\sigma > 0$, there exists matrix $P = P^\top > 0$ such that

$$PL + L^\top P = Q_L - \sigma[P\mathbf{1}_N v_\ell^\top + v_\ell \mathbf{1}_N^\top P], \tag{3}$$

where v_ℓ is the left eigenvector associated to the single zero eigenvalue of L. □

III. Control approach and main results

As is well-known [28], nonholonomic vehicles are not stabilizable asymptotically via smooth control, unless it is time-varying or non-smooth. In this letter we use the approach of so-called δ-persistently-exciting (δ-PE) controllers [23]. These controllers are designed to inject an external time-varying perturbation that is persistently exciting [20], as long as a certain function of the state is away from the origin. The control design is described next.

First, we introduce the error coordinates

$$z_i := \begin{bmatrix} z_{1i} \\ z_{2i} \end{bmatrix} = \begin{bmatrix} -\sin(\theta_i) & \cos(\theta_i) \\ \cos(\theta_i) & \sin(\theta_i) \end{bmatrix} \begin{bmatrix} x_i - \delta_{xi} - p_{xi}^* \\ y_i - \delta_{yi} - p_{yi}^* \end{bmatrix}, \tag{4}$$

which, in compact form, may be written as

$$z_i := \begin{bmatrix} \phi^\perp(\theta_i) \\ \phi^\top(\theta_i) \end{bmatrix} \bar{p}_i, \quad \bar{p}_i := \begin{bmatrix} x_i - \delta_{xi} - p_{xi}^* \\ y_i - \delta_{yi} - p_{yi}^* \end{bmatrix}, \tag{5}$$

where $\phi(\theta_i) := \begin{bmatrix}\cos(\theta_i) & \sin(\theta_i)\end{bmatrix}^\top$ and $\phi^\perp(\theta_i) = \begin{bmatrix}-\sin(\theta_i) & \cos(\theta_i)\end{bmatrix}$. The latter is the annihilator of the former: $\phi^\perp(\theta_i)\phi(\theta_i) \equiv 0$. Then, after (5) and (1), we obtain

$$\begin{bmatrix} \dot{z}_{1i} \\ \dot{z}_{2i} \end{bmatrix} = \begin{bmatrix} -z_{2i}\omega_i \\ z_{1i}\omega_i + v_i \end{bmatrix} - \begin{bmatrix} \phi^\perp(\theta_i) \\ \phi^\top(\theta_i) \end{bmatrix} \dot{p}_i^*, \tag{6}$$

where, for stabilization, we set $\dot{p}_i^* = 0$ whereas for tracking, we define $\dot{p}_i* := \phi(\theta_i)v_r$, with v_r piece-wise constant. In either case, since $\phi^\perp\phi = 0$ and $\phi^\top\phi = 1$,

$$\begin{bmatrix} \dot{z}_{1i} \\ \dot{z}_{2i} \end{bmatrix} = \begin{bmatrix} -z_{2i}\omega_i \\ z_{1i}\omega_i + v_i - v_r \end{bmatrix} \tag{7a}$$
$$\dot{\theta}_i = \omega_i. \tag{7b}$$

Since the matrix in (4) is globally invertible, we have that $z_i = 0$ if and only if $\bar{p}_i = 0$, so the formation control problem boils down to ensuring that $z_i \to 0$ and is decoupled from that of achieving orientation consensus, that is, that $\theta_i \to \theta_j \to \theta_c$ for all i, $j \leq N$. Now, the consensus objective alone is achieved by the common control law

$$\omega_i = -k_w \sum_{j\in\mathcal{N}_i} a_{ij}(\theta_i - \theta_j), \quad k_w > 0, \tag{8}$$

under the assumption that the graph contains a directed spanning tree.

On the other hand, to achieve, separately, the stabilization objective, we see that the equations (7a) may be assimilated to those of a stable (oscillatory) system with input $v_i - v_r$. Hence, for any $\omega_i \neq 0$, the control law

$$v_i = -k_{2i}z_{2i} + (k_{1i} - 1)\omega_i z_{1i} + v_r, \qquad k_{1i},\ k_{2i} > 0, \tag{9}$$

makes $z_i \to 0$. To see this more clearly note that the closed-loop system takes the form

$$\begin{bmatrix} \dot{z}_{2i} \\ \dot{z}_{1i} \end{bmatrix} \begin{bmatrix} -k_{2i} & k_{1i}\omega_i \\ -\omega_i & 0 \end{bmatrix} \begin{bmatrix} z_{2i} \\ z_{1i} \end{bmatrix},$$

which has a structure familiar in adaptive control—cf. [23]. Hence, it may be showed that if $k_{2i} > 0$, $z_{2i} \to 0$ (see the proof of our main result below). Moreover, if ω_i depended only on t, it would be sufficient to make ω_i persistently exciting (PE) [22], *i.e.*, to ensure that there exist μ and $T > 0$ such that

$$\int_t^{t+T} |\omega_i(s)| ds \geq \mu \quad \forall\, t \geq 0,$$

to ensure that $z_{1i} \to 0$. Making ω_i be PE, however, means that θ_i cannot converge to a constant value, which is part of the orientation-consensus objective. In summary, persistency of excitation is needed to attain the formation control objectives, while it is required that ω_i converge to zero to achieve orientation consensus. To overcome this conundrum, we endow the angular-motion consensus control law (8) with an oscillatory perturbation that persists as long as $|z_{1i}| \neq 0$. That is, with a δ-persistently exciting function [23].

A. Formation stabilization

Let $v_r = 0$, that is, consider the problem of stabilizing all the robots at around a fixed set-point p_i^*.

Proposition 1 (Formation stabilization): Consider a network of autonomous vehicles modeled as in (1), communicating over a directed-graph containing a directed spanning tree, in closed loop with the controller

$$v_i = (k_{1i} - 1)\omega_i z_{1i} - k_{2i} z_{2i}, \quad k_{1i},\, k_{2i} > 0 \tag{10a}$$

$$\omega_i = -k_w \sum_{j \in \mathcal{N}_i} a_{ij}(\theta_i - \theta_j) + f_i(t)\beta_i(z_i), \quad k_w > 0, \tag{10b}$$

where

- for each $i \leq N$, $f_i : \mathbb{R}_{\geq 0} \to \mathbb{R}$ is twice continuously differentiable, bounded with bounded derivatives, and $\dot{f}_i$ is persistently exciting, that is, there exist T, $\mu > 0$ such that
$$\int_t^{t+T} |\dot{f}_i(s)| ds \geq \mu \quad \forall\, t \geq 0;$$
- $\beta_i : \mathbb{R}^2 \to \mathbb{R}$ is twice continuously differentiable and satisfies $\beta_i(z_i) = 0$ if and only if $z_i = 0$.

Then, the formation set-point stabilization and orientation consensus objectives (2) are fulfilled, while $v_i \to 0$ and $\omega_i \to 0$. □

Proof: Replacing the control laws (10) in the kinematics equations (7), with $v_r = 0$, we obtain the closed-loop system

$$\dot{z}_{1i} = -\omega_i z_{2i} \tag{11a}$$

$$\dot{z}_{2i} = k_{1i}\omega_i z_{1i} - k_{2i} z_{2i} \tag{11b}$$

$$\dot{\theta}_i = -k_w \sum_{j \in \mathcal{N}_i} a_{ij}(\theta_i - \theta_j) + f_i(t)\beta_i(z_i). \tag{11c}$$

We start by showing that $z_i \in \mathcal{L}_\infty$, where $z_i := [z_{1i}\ z_{2i}]^\top$. To that end, we use the positive-definite, decrescent Lyapunov function

$$V_z(z) = \frac{1}{2}\sum_{i=1}^{N} [k_{1i} z_{1i}^2 + z_{2i}^2]. \tag{12}$$

Its total derivative along the trajectories of (11a)-(11b) yields

$$\dot{V}_z(z(t)) = -k_{2i} \sum_{i=1}^{N} z_{2i}(t)^2 \leq 0. \tag{13}$$

Integrating on both sides of $\dot{V}_z(z(t)) \leq 0$, from $t_o \geq 0$ to t, we obtain $[k_{1i} z_{1i}(t)^2 + z_{2i}(t)^2] \leq [k_{1i} z_{1i}(t_o)^2 + z_{2i}(t_o)^2]$ which implies that

$$|z_i(t)|^2 \leq \frac{\max\{k_{1i},\, 1\}}{\min\{k_{1i},\, 1\}} |z_i(t_o)|^2. \tag{14}$$

That is, z_i is uniformly globally bounded.

Strictly speaking, the latter holds only on the interval of existence of $t \mapsto \omega(t)$. Say, for a certain $\infty > t_f > t_o$, any $t_o \geq 0$ and for all $t \in [t_o + t_f)$. In the sequel we show that $t_f = +\infty$, that $\omega := [\omega_1\ \cdots\ \omega_N]^\top$ is also uniformly bounded and, therefore, the arguments above hold for all $t \geq t_o \geq 0$.

To that end, we start by rewriting (10b) in multi-variable compact form, *i.e.*,

$$\omega = -L\theta + \alpha(t, z), \tag{15}$$

where $\alpha(t, z) := [\, f_1(t)\beta_1(z_1)\ \cdots\ f_N(t)\beta_N(z_N)\,]^\top$ and $\theta := [\,\theta_1\ \cdots\ \theta_N\,]^\top$. Then, following [27], we introduce the synchronization errors

$$e_\theta = [I_N - \mathbf{1}_N v_\ell^\top]\theta, \tag{16}$$

which correspond to the vector $e_\theta := [e_{\theta 1} \cdots\ e_{\theta N}]^\top$, where $e_{\theta i} := \theta_i - v_\ell^\top \theta$ and $v_\ell^\top \theta$ is a weighted average of θ. That is, consensus is reached if $\theta_i \to v_\ell^\top \theta$ for all $i \leq N$ or $e_\theta \to 0$.

Remark 1: The errors e_θ satisfy

$$\dot{e}_\theta = -Le_\theta + [I_N - \mathbf{1}_N v_\ell^\top]\alpha(t, z). \tag{17}$$

To see this, we note that from (16),

$$\dot{e}_\theta = -L[I_N - \mathbf{1}_N v_\ell^\top]\theta + [I_N - \mathbf{1}_N v_\ell^\top]\alpha(t, z),$$

so (17) follows observing that $L\mathbf{1}_N = 0$. •

Consider now the Lyapunov function $V_{e_\theta} = e_\theta^\top P e_\theta$, where $P = P^\top \in \mathbb{R}^N$ solves (3). Its total derivative along the trajectories of (17) yields

$$\dot{V}_{e_\theta} = -e_\theta^\top [PL + L^\top P] e_\theta + 2e_\theta^\top P[I_N - \mathbf{1}_N v_\ell^\top]\alpha(t, z),$$

so, using (3), we obtain

$$\begin{aligned} \dot{V}_{e_\theta} = &- e_\theta^\top \left[Q_L - \sigma[P\mathbf{1}_N v_\ell^\top + v_\ell \mathbf{1}_N^\top P]\right] e_\theta \\ &+ 2e_\theta^\top P[I_N - \mathbf{1}_N v_\ell^\top]\alpha(t, z). \end{aligned}$$

However, since $v_\ell^\top \mathbf{1}_N = 1$, the second term on the right-hand side of the equation above equals to zero. Indeed, after (16), $P\mathbf{1}_N v_\ell^\top e_\theta = P\mathbf{1}_N v_\ell^\top [I_N - \mathbf{1}_N v_\ell^\top]\theta = P[\mathbf{1}_N v_\ell^\top - \mathbf{1}_N v_\ell^\top]\theta = 0_N$. Thus,

$$\dot{V}_{e_\theta} = -e_\theta^\top Q_L e_\theta + 2e_\theta^\top P[I_N - \mathbf{1}_N v_\ell^\top]\alpha(t, z).$$

Now, let $\Delta := |P[I_N - \mathbf{1}_N v_\ell^\top]|$ where $|\cdot|$ denotes here the induced matrix norm. For any $t_f > 0$, let

$$\|\alpha\|_{t_f} := \sup_{t\in[t_o,t_o+t_f)} |\alpha(t, z(t))|.$$

Then, defining $q_m := \lambda_{\min}(Q_L)$, we have

$$\dot{V}_{e_\theta}(e_\theta(t)) \leq -q_m|e_\theta(t)|^2 + \Delta|e_\theta(t)|\|\alpha\|_{t_f} \tag{18}$$

for all $t \in [t_o, t_o + t_f)$. We show by contradiction that $t_f = +\infty$, so all of the above holds for all $t \geq t_o$.

Assume that $|\omega(t)| \to \infty$ as $t \to t_f < \infty$. Then, since $\theta(t) = \int_{t_o}^t \omega(t)dt$ we have $|\theta(t)| \to \infty$ as $t \to t_f$. On the other hand, $V_{e_\theta}(e_\theta(t)) := \theta(t)^\top[I_N - \mathbf{1}_N v_\ell^\top]^\top[I_N - \mathbf{1}_N v_\ell^\top]\theta(t)$, so $V_{e_\theta}(e_\theta(t)) \to \infty$ as $t \to t_f$. Next, defining $\nu(s) := V_{e_\theta}(e_\theta(s))$, after (18), we have

$$\dot{\nu}(t) \leq \frac{\Delta^2}{2q_m}\|\alpha\|_{t_f}^2. \tag{19}$$

Integrating on both sides of (19), from t_o to t_f, we obtain

$$\lim_{t\to t_f} \nu(t) \leq \frac{\Delta^2}{2q_m}\|\alpha\|_{t_f}^2[t_f - t_o] + \nu(t_o).$$

By assumption $t_f < \infty$, so the right-hand side of the inequality above is finite, whereas

$$\lim_{t\to t_f} \nu(t) = \lim_{t\to t_f} V_{e_\theta}(e_\theta(t)) = +\infty,$$

which is a contradiction. We conclude that $t_f = +\infty$ so (13) holds along trajectories for all $t \geq t_o$ and $z_i(t)$ is uniformly globally bounded. Also, in view of the latter, the continuity of $\alpha(t, \cdot)$ and the uniform boundedness of $\alpha(\cdot, z)$, $t \mapsto \alpha(t, z(t))$ is uniformly bounded, that is, $\|\alpha\|_\infty^2 < \infty$. From this and integrating on both sides of (18) along the trajectories, from t_o to ∞, we obtain that e_θ is uniformly globally bounded.

We show next that ω is also uniformly globally bounded. For this, we note that the Laplacian matrix L admits the decomposition $L = UMU^{-1}$ where U is a matrix whose columns are the right eigenvectors of L, *i.e.*, $U = [\mathbf{1}_N\ U_1]$, $U^{-1} = [v_\ell^\top\ U_1^\dagger]^\top$, and M contains the Jordan blocks corresponding to the eigenvalues of L. The first eigenvalue of M being zero and simple, we also have $L = U_1\bar{M}U_1^\dagger$, where $\bar{M}$ contains all the Jordan blocks of L corresponding to the non-zero eigenvalues. It follows that $L\theta = U_1\bar{M}U_1^\dagger\theta$, so $|L\theta| \leq \lambda_{\max}(L)|U_1U_1^\dagger\theta| = \lambda_{\max}(L)|[I_N - \mathbf{1}_N v_\ell^\top]\theta| = \lambda_{\max}(L)|e_\theta|$. We conclude that $|L\theta(t)|$ is uniformly globally bounded. From this, and the boundedness of f_i and $\alpha(t, z(t))$, it follows that ω in (15) is also uniformly globally bounded, and so is $\dot{\theta} = \omega$.

Finally, for further development, we remark that $\dot{\omega}$ is also globally uniformly bounded, since so are all the terms on the right hand side of

$$\dot{\omega}_i = -k_w \sum_{j\in\mathcal{N}_i} a_{ij}(\omega_i - \omega_j) + f_i(t)\frac{\partial\beta(z_i)}{\partial z_i}\dot{z}_i + \dot{f}_i(t)\beta(z_i). \tag{20}$$

Next, we establish the convergence of z and $L\theta$. This part of the proof relies on a recursive application of Barbălat's Lemma.

First, we integrate on both sides of (13) from t_o to ∞. Since z is globally uniformly bounded we obtain that $z_2 \in \mathcal{L}_2$. Since, moreover, $\dot{z}_2 \in \mathcal{L}_\infty$ we obtain, by [20, Lemma 3.2.5] that $z_2 \to 0$ asymptotically. To establish the convergence of z_1 we proceed by establishing, first, the convergence of $\omega_i z_{1i}$ for any $i \leq N$. This follows from the fact that both terms on the right hand side of

$$k_{1i}\omega_i z_{1i} = k_{2i}z_{2i} - \dot{z}_{2i}$$

—cf. (11b), converge to zero. That $z_{2i} \to 0$ was established above, while the convergence of $\dot{z}_{2i}$ follows from Barbălat's Lemma. Indeed, on one hand, $\ddot{z}_{2i}$ is uniformly globally bounded since so are all terms on the right hand side of

$$\begin{aligned}\ddot{z}_{2i} &= k_{1i}\dot{\omega}_i z_{1i} + k_{1i}\omega_i\dot{z}_{1i} - k_{2i}\dot{z}_{2i}\\ &= k_{1i}\Big[-k_w\sum_{j\in\mathcal{N}_i} a_{ij}(\omega_i - \omega_j) + f_i(t)\frac{\partial\beta(z_i)}{\partial z_i}\dot{z}_i\Big]z_{1i}\\ &\quad + k_{1i}\dot{f}_i(t)\beta(z_i)z_{1i} - k_{1i}\omega_i^2 z_{2i} - k_{1i}k_{2i}\omega_i z_{1i} + k_{2i}^2 z_{2i}.\end{aligned} \tag{21}$$

On the other hand,

$$\lim_{t\to\infty}\int_{t_o}^t \dot{z}_{2i}(\tau)d\tau = \lim_{t\to\infty} z_{2i}(t) - z_{2i}(t_o) = -z_{2i}(t_o).$$

Now, since $|\omega_i z_{1i}| \to 0$ and both ω_i and z_{1i} are uniformly globally bounded, either $z_{1i} \to 0$ or $\omega_i \to 0$. In the first case, the proof is completed. Hence, let us assume that $\omega_i \to 0$ and turn our attention to the term $k_{1i}\dot{f}_i(t)\beta(z_i)z_{1i}$ on the right-hand side of (21). By assumption $\beta_i(z_i) = 0$ if and only if $z_i = 0$ and $\dot{f}_i$ is persistently exciting. Therefore, since $t \mapsto \beta_i(z_i(t))$ is uniformly globally bounded we see that $z_{1i} \to 0$ if and only if so does

$$\begin{aligned}k_{1i}\dot{f}_i(t)\beta(z_i)z_{1i} &= \ddot{z}_{2i} + k_{1i}\omega_i^2 z_{2i} + k_{1i}k_{2i}\omega_i z_{1i} - k_{2i}^2 z_{2i}\\ &\quad -k_{1i}\Big[k_w\sum_{j\in\mathcal{N}_i} a_{ij}(\omega_i - \omega_j) - f_i(t)\frac{\partial\beta(z_i)}{\partial z_i}\dot{z}_i\Big]z_{1i}.\end{aligned}$$

We proceed to establish the convergence of all the terms on the right-hand side of the previous expression, individually. As a matter of fact, this has already been done for the second, third, and fourth terms, since both z_{2i} and $\omega_i z_{1i}$ converge to zero. Furthermore, the convergence of the last term on the right-hand side follows from that of $\dot{z}_i$ and the uniform boundedness of $f_i(\cdot)$ and $\beta_i(z_i(\cdot))$. In addition, by assumption $\omega_i \to 0$, so it is only left to show that $\ddot{z}_2 \to 0$. To that end, we use Barbălat's Lemma once more. First, we observe that, for any $t_o \geq 0$

$$\lim_{t\to\infty}\int_{t_o}^t \ddot{z}_{2i} = \lim_{t\to\infty}\dot{z}_{2i}(t) - \dot{z}_{2i}(t_o) = -\dot{z}_{2i}(t_o). \tag{22}$$

On the other hand, we observe that $t \mapsto \ddot{z}_{2i}(t)$ is uniformly continuous since $z_{2i}^{(3)}$ is uniformly globally bounded. Indeed, so are all the terms bounded on the right-hand side of

$$\begin{aligned}z_{2i}^{(3)} = k_{1i}\Big[&-k_w\sum_{j\in\mathcal{N}_i} a_{ij}(\dot{\omega}_i - \dot{\omega}_j) + \dot{f}_i(t)\frac{\partial\beta(z_i)}{\partial z_i}\dot{z}_i\\ &+ f_i(t)\dot{z}_i^\top\frac{\partial^2\beta(z_i)}{\partial^2 z_i}\dot{z}_i + f_i(t)\frac{\partial\beta(z_i)}{\partial z_i}\ddot{z}_i\end{aligned}$$

$$+ \ddot{f}_i(t)\beta(z_i) + \dot{f}_i(t)\frac{\partial\beta(z_i)}{\partial z_i}\dot{z}_i\Big]z_{1i}$$
$$+ k_{1i}\Big[-k_w\sum_{j\in\mathcal{N}_i} a_{ij}(\omega_i-\omega_j) + f_i(t)\frac{\partial\beta(z_i)}{\partial z_i}\dot{z}_i$$
$$+ \dot{f}_i(t)\beta(z_i)\Big]\dot{z}_{1i} - 2k_{1i}\omega_i\dot{\omega}_i z_{2i} - k_{1i}\omega_i^2\dot{z}_{2i}$$
$$- k_{1i}k_{2i}\dot{\omega}_i z_{1i} - k_{1i}k_{2i}\omega_i\dot{z}_{1i} + k_{2i}^2\dot{z}_{2i}.$$

This completes the proof. ■

B. Formation tracking

Complementary to the formation stabilization and orientation consensus tasks that are ensured by the controller (10), consider now the problem of making a group of robots acquire a formation around a pre-specified rendezvous point, with common orientation, and to advance in formation with piece-wise constant forward velocity v_r. This task is achieved by setting each reference trajectory to $\dot{p}_i^* := \phi(\theta_i)v_r$ and using the linear-motion controller (9), as described in Section III. Then, if in addition the robots are required to change direction, a virtual orientation leader may be incorporated. This is accomplished by modifying (8) into

$$\omega_i = -k_w\Big(\sum_{j\in\mathcal{N}_i} a_{ij}(\theta_i-\theta_j) + b_{ir}(\theta_i-\theta_r)\Big) + f_i(t)\beta_i(z_i). \tag{23}$$

Then, we have the following.

Proposition 2 (Formation tracking): Let v_r and ω_r be piece-wise constant functions of t such that, after a certain $T>0$, $v_r\to 0$ and $\omega_r\to 0$ exponentially. Consider a network of autonomous vehicles modeled as in (1), communicating over a directed-graph containing a directed spanning tree $\mathcal{G}_T$, in closed loop with the controller defined by (9) and (23), with $\dot{\theta}_r=\omega_r$, $b_{ir}\geq 0$ for all $i\leq N$ and $b_{kr}>0$, where k is the index of the root node of $\mathcal{G}_T$. Under the conditions of Proposition 1, (2) holds with $\theta_c=\theta_r$. □

Proof: The closed-loop equations correspond to Eqs. (11a)-(11b) and, in terms of the synchronization errors, $e_\theta = \theta - \mathbf{1}_N\theta_r$, $\dot{e}_\theta = -k_w[L+B]e_\theta + \alpha(t,z) - \mathbf{1}_N\omega_r$, where $B=\text{diag}[b_{ir}]$. Under the above-stated assumptions, $[L+B]$ is a non-singular M-matrix, so for any $Q>0$, there exists a matrix $P>0$ that solves the Lyapunov equation $-P[L+B]-[L+B]^\top P=-Q$ (for details, see Theorem 4.25 in [29]). Then, since $\omega_r\to 0$ exponentially, the proof follows as in that of Proposition 1. ■

IV. Simulation Results

We simulated two experiments involving a group of eight vehicles interconnected as in the directed graph showed in Fig. 1, which contains a directed spanning tree. The first experiment consists in making all robots meeting around the origin, so $x_r=(0,0)$, at the vertices of an octagon and acquiring a common non pre-imposed orientation θ_c. The initial positions on the plane are depicted in Fig. 2 with a '∘' while the initial orientations are set to $\theta(0) := [\pi/2\ 0\ \pi/8\ -\pi/8\ \pi/3\ \pi/5\ -\pi/7\ \pi/6]$. The control gains were set to the same values for all robots, as follows: $k_{1i}=3.5$, $k_{2i}=30$,

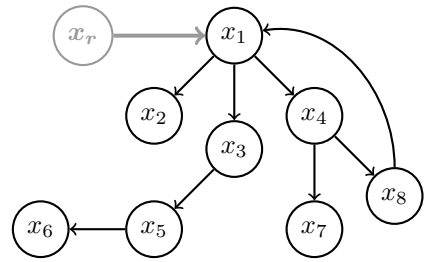

Fig. 1. Connection digraph for the eight communicating agents and one virtual leader $\boldsymbol{x_r}$.

and $k_w=1$. In addition, we used $f_i(t)=\cos(15t)-\sin(35t)$ and $\beta_i(z_i)=22z_{1i}$.

The paths followed by the robots on the plane are also showed in Fig. 2 below, and their final consensual orientation ($\theta_c\approx 35$ [deg]) is illustrated with pointing arrows.

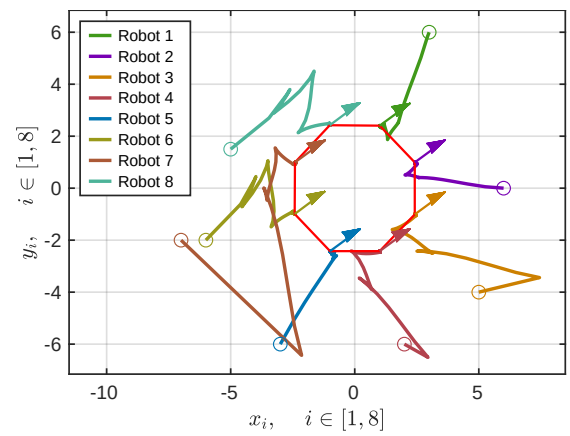

Fig. 2. Vehicles on the phase plane converging to a rendezvous point

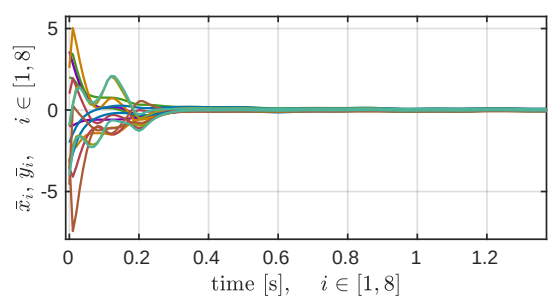

Fig. 3. Systems' error trajectories in Cartesian coordinates

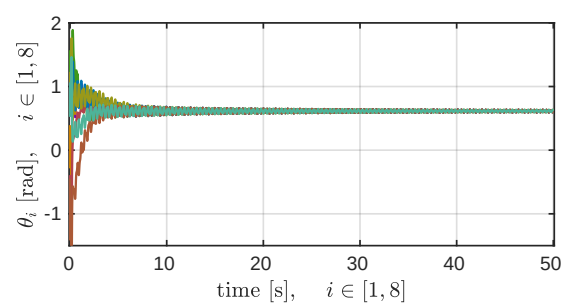

Fig. 4. Systems' orientations achieving consensus

In Figs. 3 and 4 we show the stabilization errors in Cartesian coordinates $\bar{x}_i$ and $\bar{y}_i$, as well as the orientations. The

consensus errors in Cartesian positions rapidly become very small in norm while the orientations take a longer to settle due to the persistent, but vanishing perturbation $f_i(t)\beta_i(z_i)$ in (23), needed to achieve set-point stabilization.

In a second simulated experiment, the robots gather in an octagonal formation and advance in formation along the consensual direction, only that in this case a virtual orientation leader is incorporated. The virtual leader advances with velocity $v_r = 0.05$[m/s] either on straight paths, or makes turns at an angular velocity of $\omega_r = 0.1$[rad/s] for short periods of time. The paths followed by the robots are showed in Fig. 5.

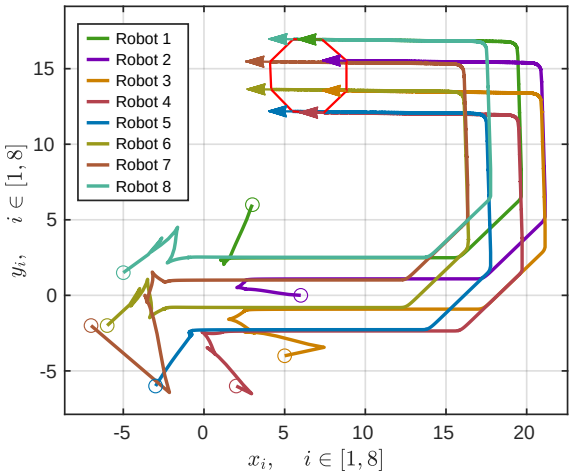

Fig. 5. Systems' paths on the phase plane converging to a formation and moving with piece-wise constant velocities.

V. Conclusions

We presented a simple δ-persistently-exciting formation controller for nonholonomic vehicles. The controller is efficacious to stabilize a group of robots around a rendezvous point with a common orientation, hence for the purpose of executing scouting missions. In that regard, we showed how a simple modification may be performed for tasks in which the robots are required to advance in formation on straight lines with common orientation and performing planned turns.

Further research is carried out to address the general problem of full consensus, in stabilization and orientation, for robots over directed graphs.

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