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# Designing recurrent cells to enforce stability in real-time dynamical simulations

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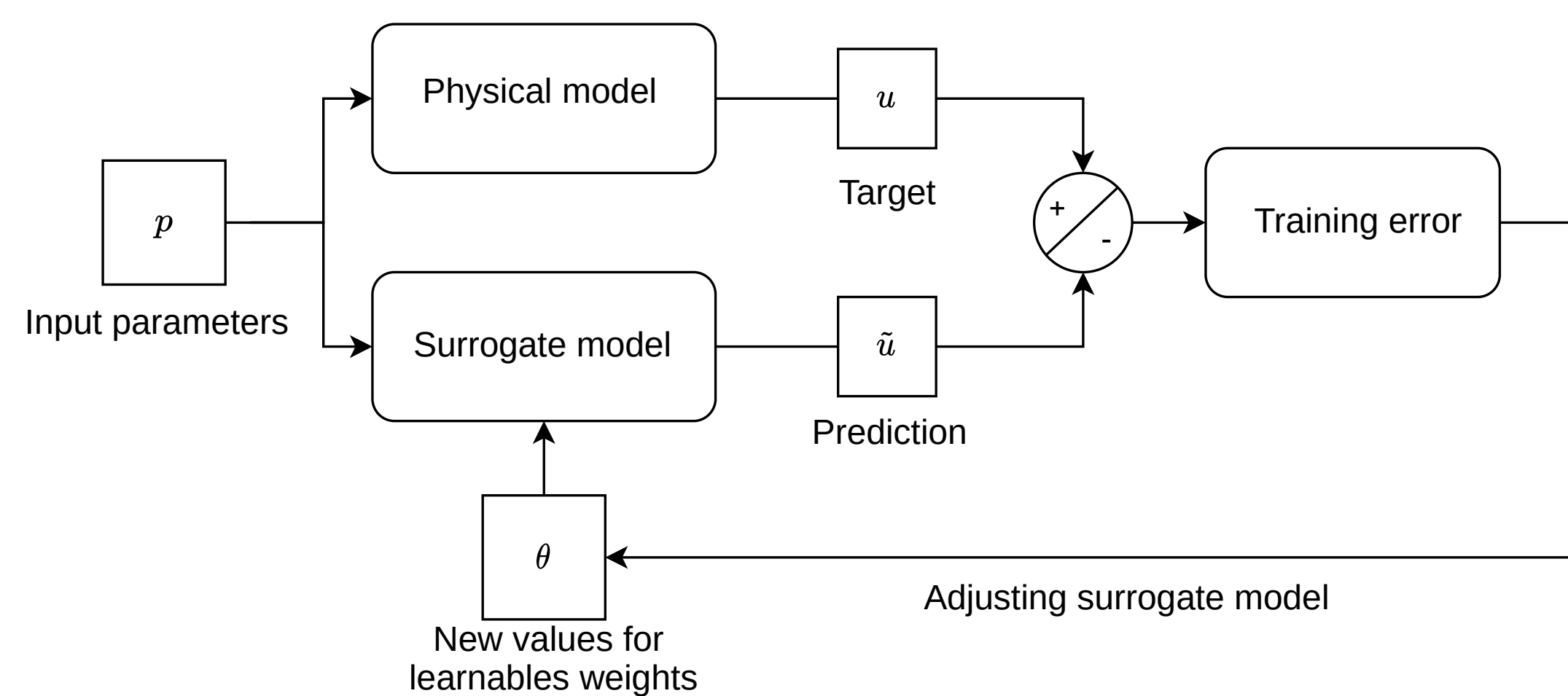
## Context

### Interactive simulation in structural dynamics

- Interactivity  $\Rightarrow$  evolution of mechanical loading not known in advance
- Need to quickly solve the equations of dynamics

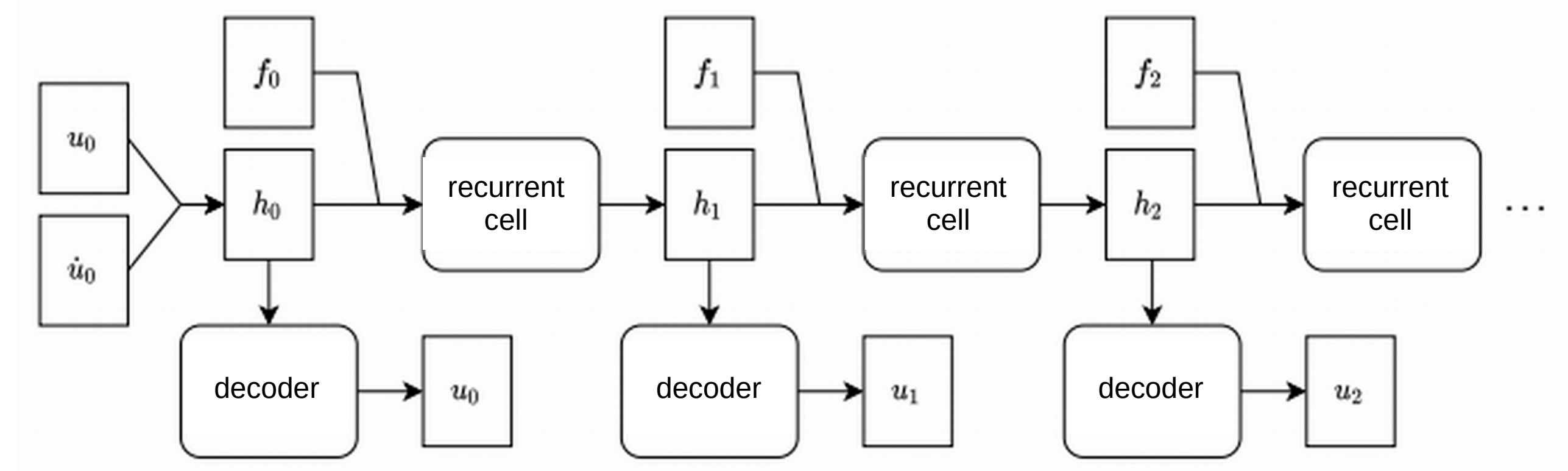
### Surrogate model

- Substitution of the slow physical model by a less expensive model trained using some data generated with the physical model
- « Black box » model, loss of physical sense in favor of speed



### Recurrent Neural Networks as a surrogate model

- Encoding the initial conditions in a low-dimensional latent space
- Time recurrence in the latent space
- Decoding the latent variable at each time step



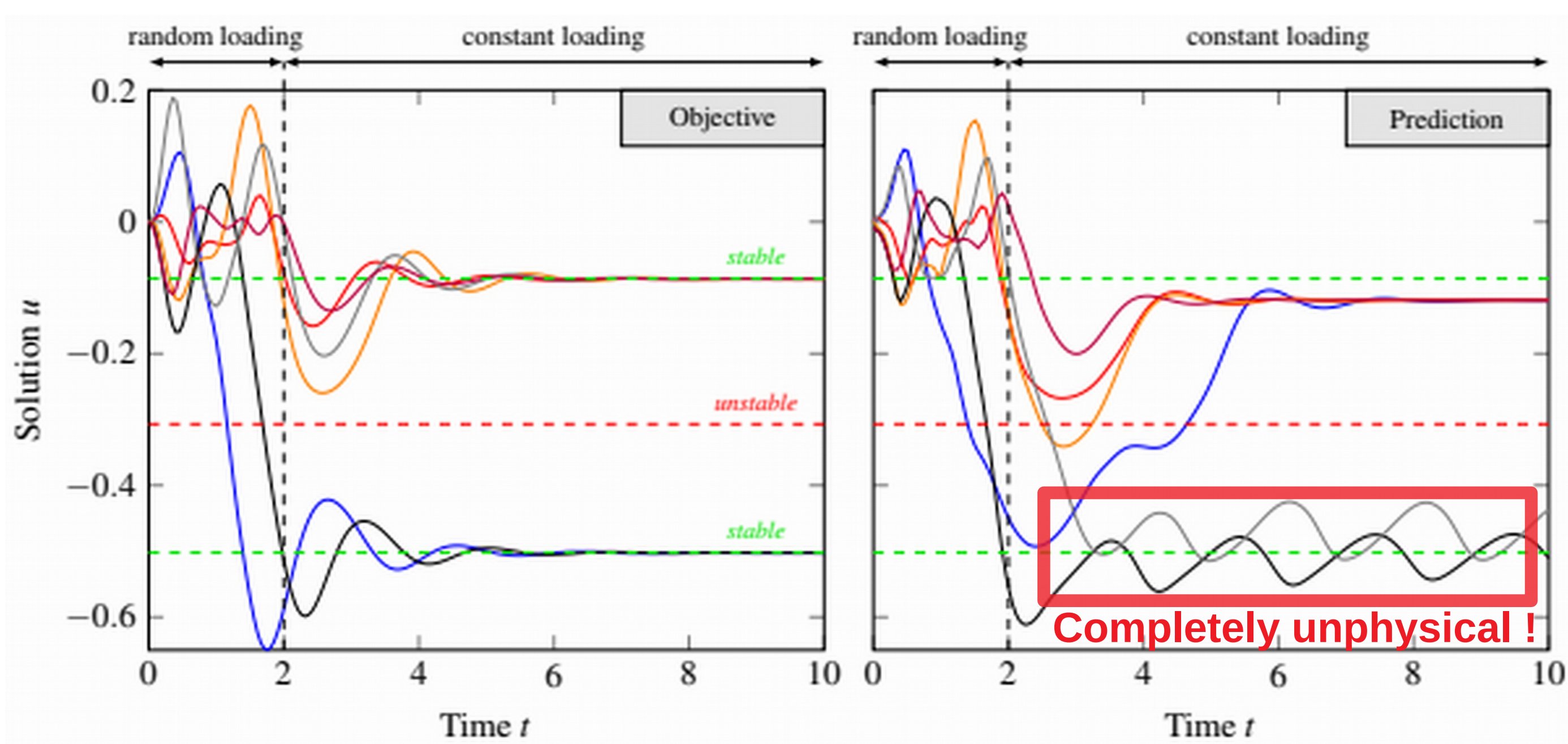
### Recurrent cells

- Classical cells : LSTM, GRU, ResNet ...
- Must be sufficiently permissive while being conducive to gradient training

## Limitations

### Stable orbits sometimes learned unintentionally

- For a dissipative dynamical system: no stable orbits, a converging solicitation must lead to a converging solution



Example of RNN having learned stable orbits instead of one of the two equilibrium points

## Proposed solution

### Recurrent cell architecture structurally incapable of producing stable orbits

- We built a recurrent cell with a learnable internal energy that can never grow without variation of the external solicitation

Continuous cell equation:

$$\begin{cases} E(h, f) = V(h) - h^T W_f f \\ \dot{h} = A(h) \partial_h E \\ \text{under condition } h_2^T A(h_1) h_2 \leq 0 \forall h_1, h_2 \end{cases}$$

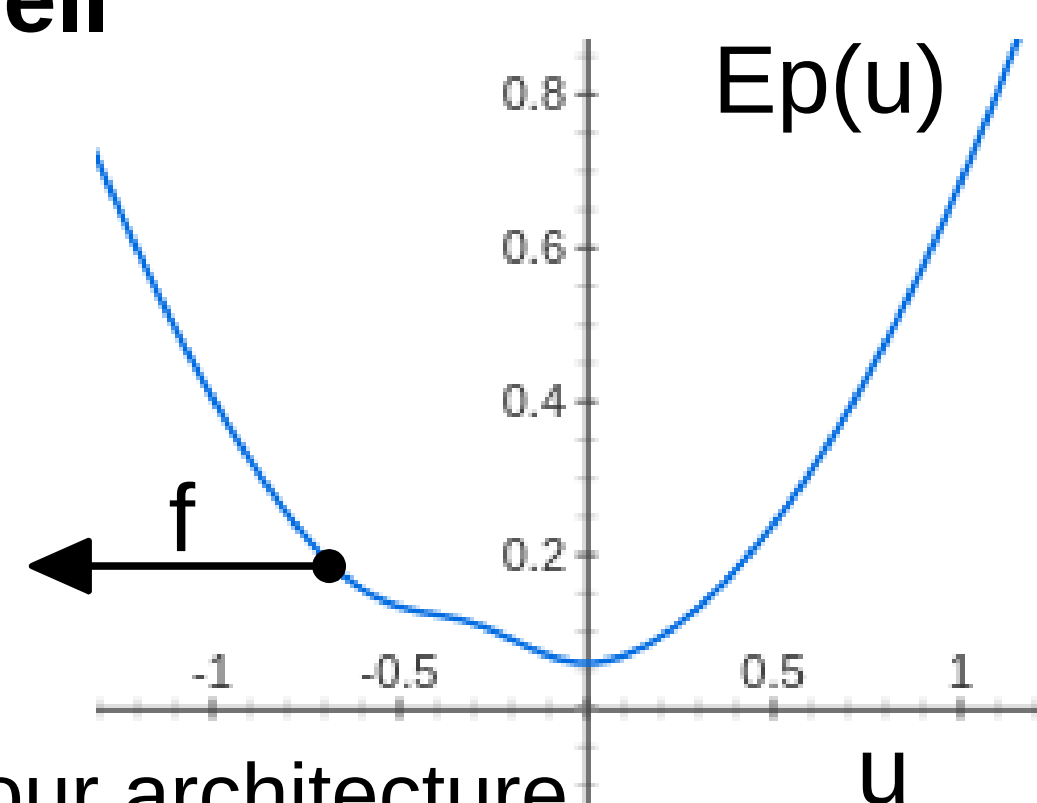
with (for example) :

$$\begin{cases} M(h) = \sum_{i=1}^{n_{pgd}} r n_i^1(h) r n_i^2(h)^T \text{ where } r n_i^1(h) \text{ and } r n_i^2(h) \text{ are arbitrary neural networks} \\ A(h) = -(M(h) + M^T(h))^2 + M(h) - M^T(h) \\ V(h) = \sqrt[n_0]{\prod_{j=1}^{n_0} V_j(h) + 1} - 1 \text{ where } V_1, \dots, V_{n_0} \text{ are learnable quadratics functions} \end{cases}$$

## Experimentations

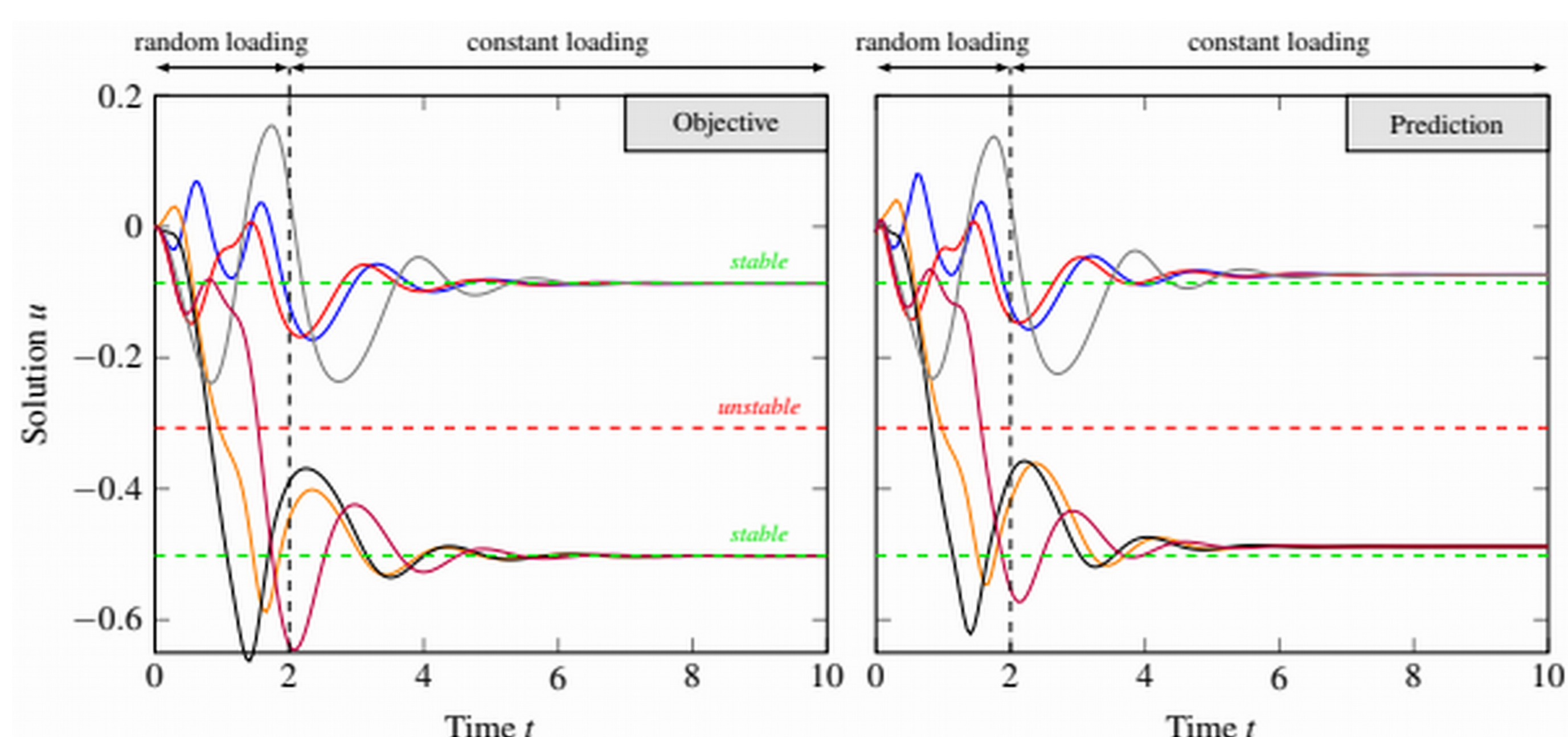
### Material point in a non-convex potential well

- Simple 1 dof test case
- We aim to predict the horizontal displacement  $u(t)$  given oscillating horizontal force  $f(t)$
- One or two possible equilibrium points depending on the horizontal effort applied



### Results

- Both equilibriums can be properly learned by our architecture
- Unlike LSTM and GRU it can never produce stable orbits



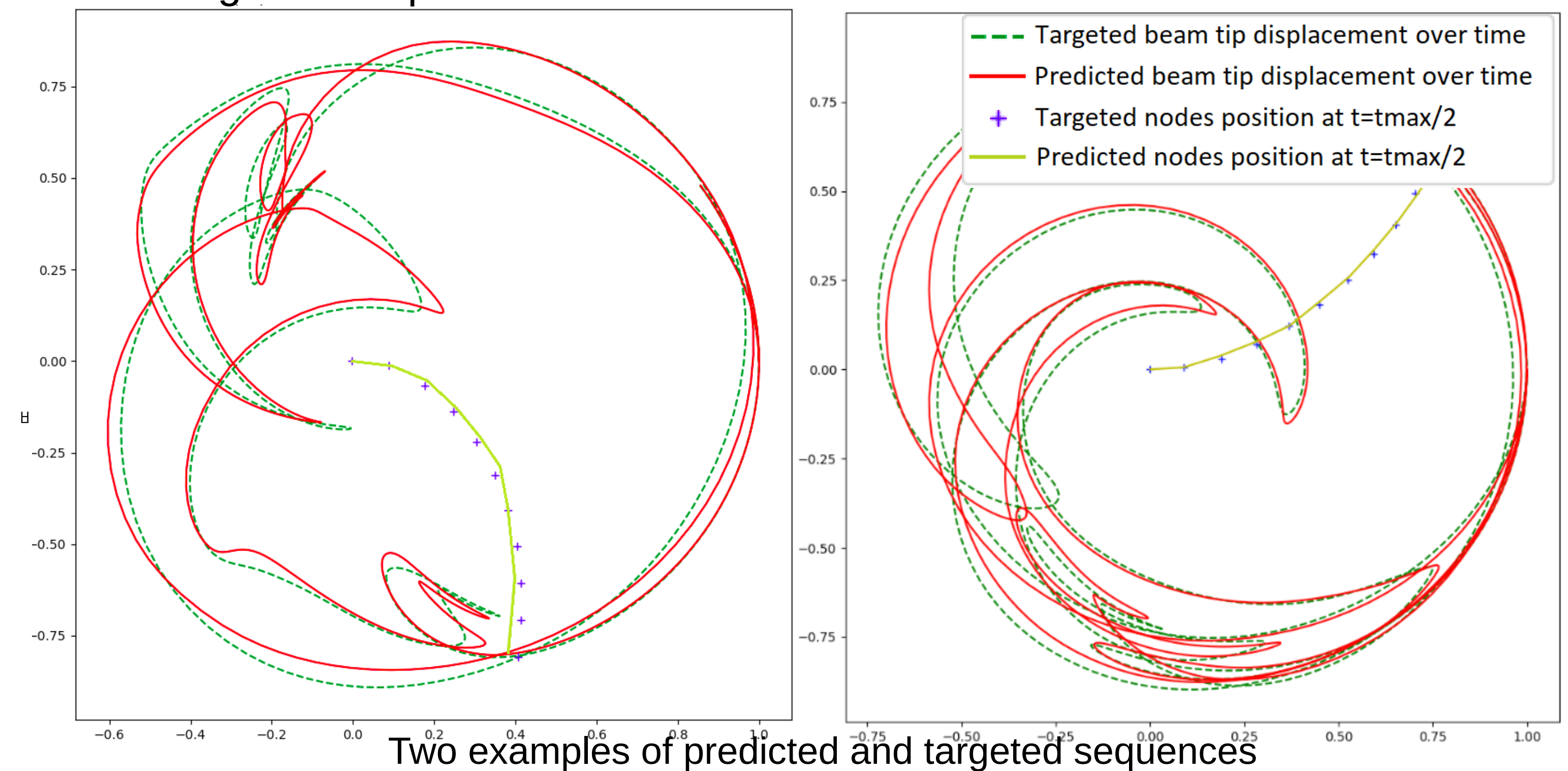
Example of RNN that has correctly learned the two equilibrium points

### Hyperelastic beam in finite transformation

- FEM simulation 30 dofs
- We aim to predict the beam displacement over time given a time sequence of high amplitude oscillating force at beam tip

### Results

- Our architecture is able to predict the beam displacement even for very long time sequences



Two examples of predicted and targeted sequences

## Conclusion and perspectives

- Our architecture has an universal approximator property for hyperelastic structures and a mathematically guaranteed absence of stable orbits
- We were able to train it satisfactorily on two test cases even if the mean error obtained with GRU cells were lower most of the time
- The proposed structure for  $A(h)$  and  $V(h)$  must be improved to increase the representation power of our architecture