DRlib: a C++ library for marked Gibbs point processes simulation and inference

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Purpose

DRlib is a new **C++** library for modeling, simulation and inference of marked point processes. The aim is to complete existing tools such as the **spatstat** library in **R** [1] with reliable and efficient **C++** code allowing intensive Bayesian MCMC based inference for marked Gibbs point processes. This proposal has its roots in the **MPPLIB** library developed mainly by [2] where exact simulation algorithms for marked point processes were translated into **C++** in order to perform massive simulation studies. The current version of the library is freely available at the following address ***https://gitlab.univ-lorraine.fr/labos/iecl/drlib***.

Interacting point process

The points are situated in W a compact region of $\mathbb{R}^2$. The marks are described by the probability space $(M, \mathcal{M}, \nu_M)$.

Unit Intensity Homogeneous Poisson Point Process with independent marks : independence - completely random patterns
. the number of points is a Poisson r.v. $\sim \mathcal{P}(\nu(W))$ with $\nu(W)$ the Lebesgue measure of W
. points are distributed uniformly in W while marks are associated to each points i.i.d. w.r.t. ν_M

Interactions can be specified via a probability density w.r.t. the previous process.

Strauss process : repulsive patterns Its probability density is

$$p(\mathbf{x}) \propto \beta^{n(\mathbf{x})} \gamma^{s_r(\mathbf{x})}$$

with $n(\mathbf{x})$ the number of points, $s_r(\mathbf{x}) = \sum_{i,j=1,i<j}^{n(\mathbf{x})} \mathbf{1}\{d(x_i, x_j) \leq r\}$ the number of pairs of points closer than the prefixed r. The model parameters are $\beta > 0$ and $\gamma \in]0, 1]$.

Area interaction process : repulsive or clustered patterns Its probability density is

$$p(\mathbf{x}) \propto \beta^{n(\mathbf{x})} \gamma^{a_r(\mathbf{x})}$$

with $n(\mathbf{x})$ the number of points, $a_r(\mathbf{x}) = \nu\left[\bigcup_i n(\mathbf{x}) b(x_i, r)\right]/\pi r^2$ the volume of the union of balls centred on the points of configuration and of radius r. The model parameters are $\beta, \gamma > 0$.

A **practical marked extension**: discs with random radius

Simulation

Problem: the normalising constant of the previous models is not always available in analytical closed form.
Adopted solution: use MCMC methods - simulate a Markov chain whose unique equilibrium distribution is the distribution of the point process of interest.
Algorithms: spatial birth-and-death processes, Metropolis-Hastings dynamics, Gibbs samplers, perfect algorithms

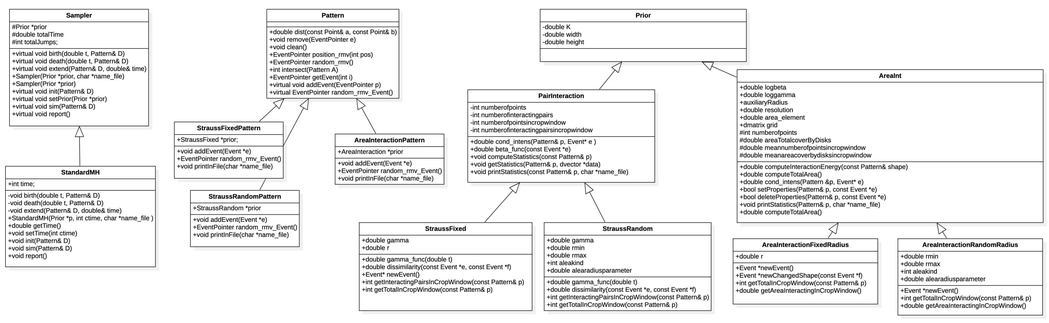
Key ideas: add or remove a point from the current configuration till equilibrium is reached. The construction of the acceptance probability for the proposed transition guarantees convergence properties of the simulation algorithms [3, 4]

Posterior sampling

Problem: sampling the posterior is difficult because the normalising constant ratio does not simplify
Adopted solution: ABC Shadow, SSA algorithms [5, 6].
Key ideas: approximate the behaviour of Markov chain that has the equilibrium distribution the posterior of interest.
Algorithm description: fix δ and m. Assume the observed pattern is $\mathbf{y}$ and the current state is θ_0.

1. Generate $\mathbf{x}$ according to $p(\mathbf{x}|\theta_0)$.
2. For $k = 1$ to m do
   . Generate a new candidate ψ following the density
   $U_\delta(\theta \to \psi) = \frac{1}{V_\delta} \mathbb{1}_{b(\theta,\delta/2)}\{\psi\}$ with V_δ vol. of the ball $b(\theta, \delta/2)$.
   . The new state $\theta_k = \psi$ is accepted with probability
   $\alpha_s(\theta_{k-1} \to \theta_k) = min\{1, \frac{f(\mathbf{y}|\theta_k)p(\theta_k)}{f(\mathbf{y}|\theta_{k-1})p(\theta_{k-1})} \frac{f(\mathbf{x}|\theta_{k-1})}{f(\mathbf{x}|\theta_k)}\}$
   otherwise $\theta_k = \theta_{k-1}$.
3. Return θ_m.
4. If another sample is needed, go to step 1 with $\theta_0 = \theta_n$.

Implementation and classes diagram



The C++ implementation builds basic classes following the mathematical principles of the problem on hand. For example:

- **Event, Pattern, Prior** classes to build configurations of objects and point processes models
- **Sampler, StandardMH** classes to build simulation dynamics and the Metropolis Hastings algorithm
- **ABCShadow** implements the posterior sampling

From the basis structure, the following derived classes implement the corresponding models:

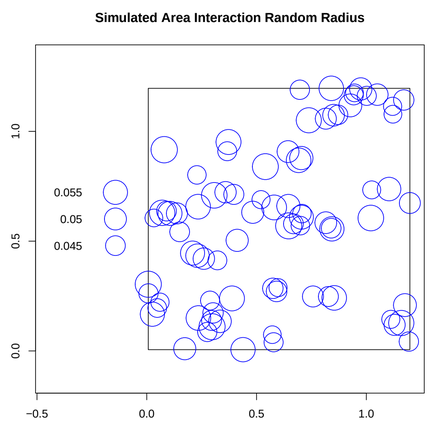
StraussFixedRadius : unmarked Strauss

StraussRandomRadius : marked Strauss - each point has a different range

AreaInteractionFixedRadius : unmarked Area Interaction

AreaInteractionRandomRadius : marked Area Interaction - each point occupies a different territory

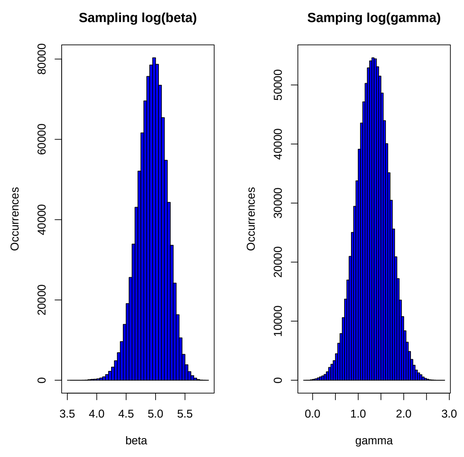
Sample Area Int model - random radius



A sample realisation of the Area Interaction model - random radius
. $\log(\beta)$=5.0; $\log(\gamma)$=1.4
. radius $\sim$ Uniform(0.04,0.06)
. grid resolution(side)=0.005

Observed sufficient statistics computed by averaging 1000 realisations:
. mean number of points=55.61
. a(r)=-$\frac{surface\ covert\ by\ disks}{\pi(rmin+rmax)/2}$=-50.26

Posterior sampling model parameters



Posterior sampling of the model parameters ($\log\beta$,$\log\gamma$) with the observed sufficient statistics
nt = 55.61 a(r) = -50.26
with :
r $\in$ [0.04,0.06]
$\log(\beta) \in$ [3.5,5.5]
$\log(\gamma) \in$ [-5.0,0.0]
mh_time = 400
time_mh_theta = 100
nbiter_theta = 1000000

Next challenges

- superposition of interacting point processes multiple interactions
- inference from patterns with an increased number of points
- increase the number of dimensions
- communicate with **R spatstat** routines

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