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# DRlib: a C++ library for marked Gibbs point processes simulation and inference



Didier Gemmerlé<sup>1</sup>, Radu S. Stoica<sup>2</sup>, Christophe Reype<sup>2</sup>

<sup>1</sup> Université de Lorraine, CNRS, IECL, F-54000 Nancy, France

<sup>2</sup> Université de Lorraine, CNRS, IECL, Inria, F-54000 Nancy, France  
{didier.gemmerle,radu-stefan.stoica,christophe.reype}@univ-lorraine.fr

## Purpose

**DRlib** is a new **C++** library for modeling, simulation and inference of marked point processes. The aim is to complete existing tools such as the **spatstat** library in **R** [1] with reliable and efficient **C++** code allowing intensive Bayesian MCMC based inference for marked Gibbs point processes. This proposal has its roots in the **MPPLIB** library developed mainly by [2] where exact simulation algorithms for marked point processes were translated into **C++** in order to perform massive simulation studies. The current version of the library is freely available at the following address ***https://gitlab.univ-lorraine.fr/labs/iecl/drlib***.

## Interacting point process

The points are situated in  $W$  a compact region of  $\mathbb{R}^2$ . The marks are described by the probability space  $(M, \mathcal{M}, \nu_M)$ .

**Unit Intensity Homogeneous Poisson Point Process with independent marks : independence - completely random patterns**

- . the number of points is a Poisson r.v.  $\sim \mathcal{P}(\nu(W))$  with  $\nu(W)$  the Lebesgue measure of  $W$
- . points are distributed uniformly in  $W$  while marks are associated to each points i.i.d. w.r.t.  $\nu_M$

Interactions can be specified via a probability density w.r.t. the previous process.

**Strauss process : repulsive patterns** Its probability density is

$$p(\mathbf{x}) \propto \beta^{n(\mathbf{x})} \gamma^{s_r(\mathbf{x})}$$

with  $n(\mathbf{x})$  the number of points,  $s_r(\mathbf{x}) = \sum_{i,j=1, i < j}^n \mathbf{1}\{d(x_i, x_j) \leq r\}$  the number of pairs of points closer than the prefixed  $r$ . The model parameters are  $\beta > 0$  and  $\gamma \in ]0, 1]$ .

**Area interaction process : repulsive or clustered patterns** Its probability density is

$$p(\mathbf{x}) \propto \beta^{n(\mathbf{x})} \gamma^{a_r(\mathbf{x})}$$

with  $n(\mathbf{x})$  the number of points,  $a_r(\mathbf{x}) = \nu[\bigcup_i n(\mathbf{x})b(x_i, r)]/\pi r^2$  the volume of the union of balls centred on the points of configuration and of radius  $r$ . The model parameters are  $\beta, \gamma > 0$ .

A **practical marked extension**: discs with random radius

## Simulation

**Problem:** the normalising constant of the previous models is not always available in analytical closed form.

**Adopted solution:** use MCMC methods - simulate a Markov chain whose unique equilibrium distribution is the distribution of the point process of interest.

**Algorithms:** spatial birth-and-death processes, Metropolis-Hastings dynamics, Gibbs samplers, perfect algorithms

**Key ideas:** add or remove a point from the current configuration till equilibrium is reached. The construction of the acceptance probability for the proposed transition guarantees convergence properties of the simulation algorithms [3, 4]

## Posterior sampling

**Problem:** sampling the posterior is difficult because the normalising constant ratio does not simplify

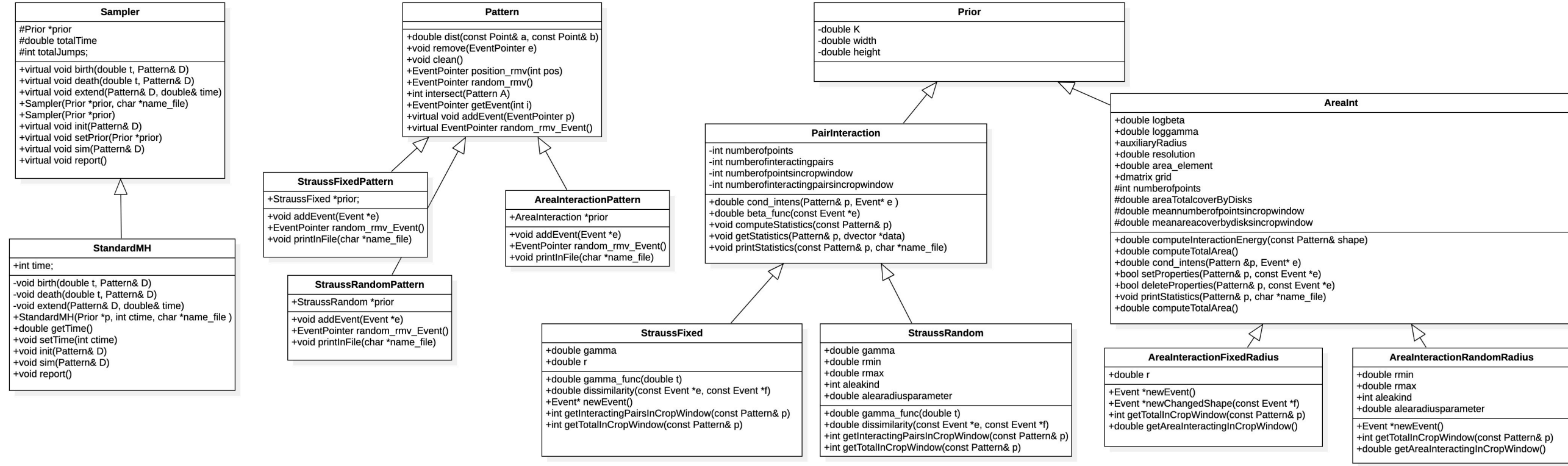
**Adopted solution: ABC Shadow, SSA algorithms** [5, 6].

**Key ideas:** approximate the behaviour of Markov chain that has the equilibrium distribution the posterior of interest.

**Algorithm description:** fix  $\delta$  and  $m$ . Assume the observed pattern is  $\mathbf{y}$  and the current state is  $\theta_0$ .

1. Generate  $\mathbf{x}$  according to  $p(\mathbf{x}|\theta_0)$ .
2. For  $k = 1$  to  $m$  do
  - . Generate a new candidate  $\psi$  following the density  $U_\delta(\theta \rightarrow \psi) = \frac{1}{V_\delta} \mathbb{1}_{b(\theta, \delta/2)}\{\psi\}$  with  $V_\delta$  vol. of the ball  $b(\theta, \delta/2)$ .
  - . The new state  $\theta_k = \psi$  is accepted with probability  $\alpha_s(\theta_{k-1} \rightarrow \theta_k) = \min\{1, \frac{f(\mathbf{y}|\theta_k)p(\theta_k)}{f(\mathbf{y}|\theta_{k-1})p(\theta_{k-1})} \frac{f(\mathbf{x}|\theta_{k-1})}{f(\mathbf{x}|\theta_k)}\}$  otherwise  $\theta_k = \theta_{k-1}$ .
3. Return  $\theta_m$ .
4. If another sample is needed, go to step 1 with  $\theta_0 = \theta_n$ .

## Implementation and classes diagram

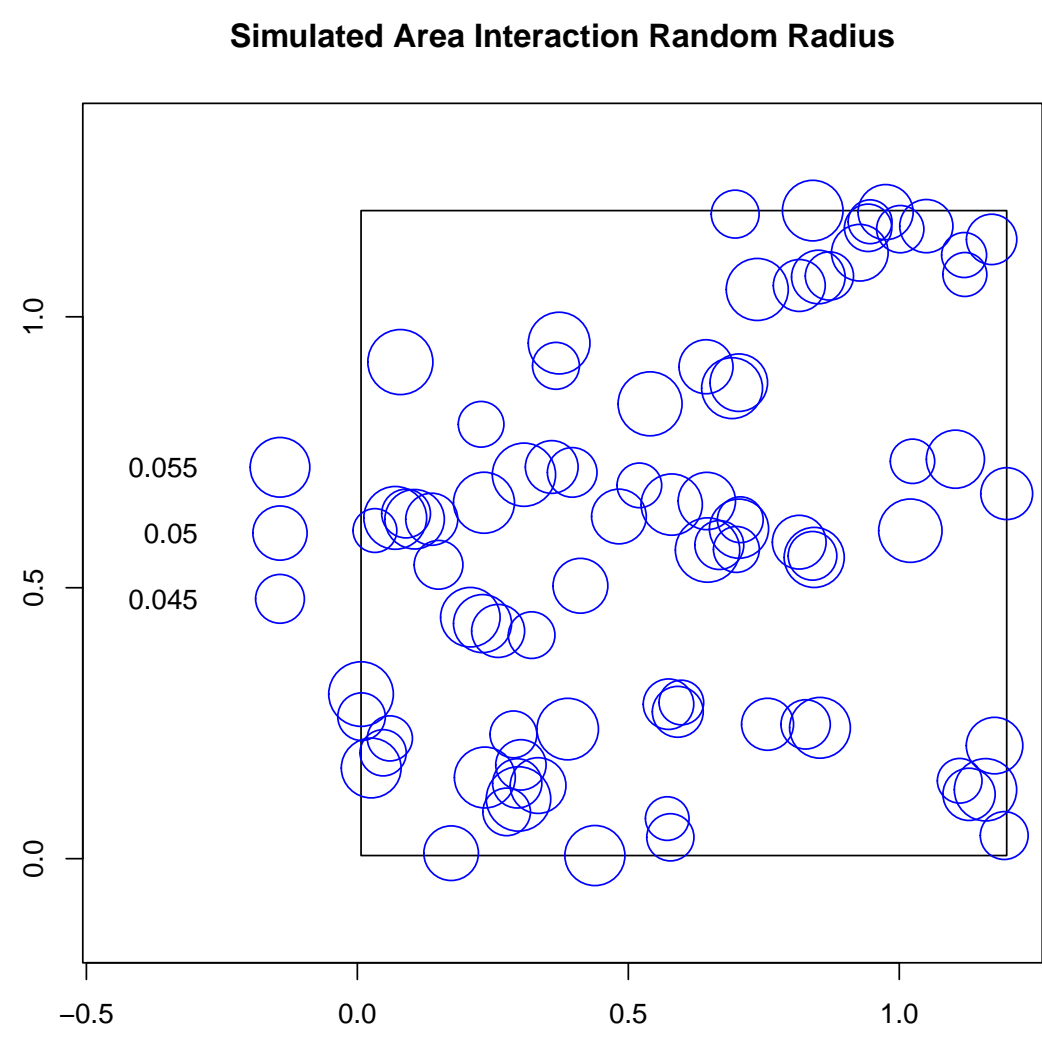


The C++ implementation builds basic classes following the mathematical principles of the problem on hand. For example:

- **Event, Pattern, Prior** classes to build configurations of objects and point processes models
- **Sampler, StandardMH** classes to build simulation dynamics and the Metropolis Hastings algorithm
- **ABCShadow** implements the posterior sampling

**AreaInteractionFixedRadius** : unmarked Area Interaction  
**AreaInteractionRandomRadius** : marked Area Interaction - each point occupies a different territory

## Sample Area Int model - random radius



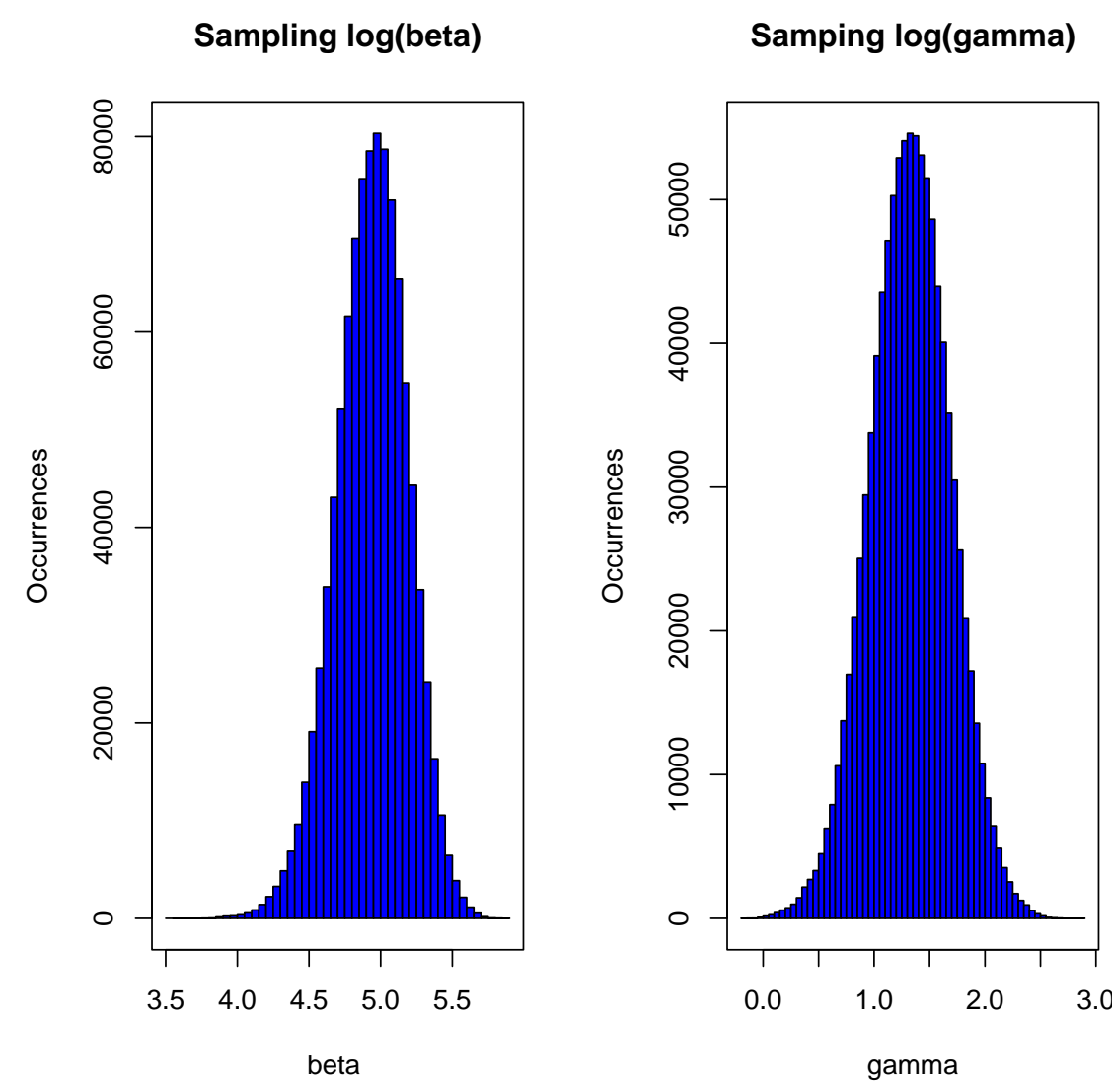
A sample realisation of the Area Interaction model - random radius

- .  $\log(\beta)=5.0$ ;  $\log(\gamma)=1.4$
- . radius  $\sim \text{Uniform}(0.04,0.06)$
- . grid resolution(side)=0.005

Observed sufficient statistics computed by averaging 1000 realisations:

- . mean number of points=55.61
- .  $a(r)=-\frac{\text{surface covered by disks}}{\pi(rmin+rmax)/2}=-50.26$

## Posterior sampling model parameters



Posterior sampling of the model parameters  $(\log \beta, \log \gamma)$  with the observed sufficient statistics  $nt = 55.61$   $a(r) = -50.26$

with :  
 $r \in [0.04, 0.06]$   
 $\log(\beta) \in [3.5, 5.5]$   
 $\log(\gamma) \in [-5.0, 0.0]$   
mh\_time = 400  
time\_mh\_theta = 100  
nbiter\_theta = 1000000

## Next challenges

- superposition of interacting point processes multiple interactions
- inference from patterns with an increased number of points
- increase the number of dimensions
- communicate with **R spatstat** routines

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