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Approximate and Commonsense Reasoning: From Theory to Practice

Didier Dubois and Henri Prade

Institut de Recherche en Informatique de Toulouse (IRIT), Université Paul Sabatier
118 route de Narbonne, 31062 Toulouse Cedex, France
Email: {dubois, prade}@irit.irit.fr

Abstract. This paper provides an overview of present trends in approximate and commonsense reasoning. The different types of reasoning, which can be covered by this generic expression, take place when the available information is either incomplete, or inconsistent, or pervaded with uncertainty, or imprecise and qualitative. The conclusions which are then obtained are usually plausible but uncertain. Yet, approximate or commonsense reasoning is useful in practical problems such as prospect evaluation, diagnosis, forecasting and decision tasks, where better information cannot be got. Classical logic is insufficient for handling these types of reasoning. Different ideas of orderings play a role in these reasoning processes: plausibility orderings between interpretations or situations which are unequally uncertain, similarity orderings with respect to prototypical situations or cases, preference orderings between acts or situations when the problem is a matter of choice. These orderings can be encoded using purely ordinal scales, or scales with a richer structure (when it is meaningful and compatible with the quality of the available information). This general idea of ordering provides a kind of unification between the different reasoning modes and somewhat typifies approximate and commonsense reasoning. Advances in default reasoning, inconsistency handling, data fusion, updating, abductive reasoning, interpolative reasoning, and decision issues in relation with Artificial Intelligence research, are briefly reviewed. Open questions and directions for future research which seem especially important for the development of practical applications are pointed out. The paper is largely based on authors' research experience, and as such, presents a rather personal view, which may not be exempt from some biases.

1 - What is Approximate and Commonsense Reasoning?

Approximate reasoning, inexact reasoning, uncertain reasoning, plausible reasoning, commonsense reasoning are expressions which have been used, with slightly different intended meanings, in the Artificial Intelligence area for about twenty years in relation with the activities of distinct research trends such as nonmonotonic reasoning, reasoning under uncertainty, and fuzzy logic. Up to a few noticeable exceptions, research on these three topics has been developing along separate roads and often with different prospects. Most of their respective results are still currently presented in specialized and well-identified workshops and conferences. Nonmonotonic reasoning focuses on reasoning under incomplete information and commonsense knowledge using symbolic approaches based on logical machineries. Reasoning under uncertainty uses numerical models, especially probability theory and causal Bayesian networks, while the mainstream of fuzzy logic concentrates on the handling of fuzzy rules. Viewed in that way, the concerns of the three schools seem rather different. Moreover, most of the works on nonmonotonic logics are theoretically oriented. Numerical uncertainty research is much interested in computational issues on the probabilistic

side, and in the development of other representation frameworks such as belief function theory or possibility theory. Fuzzy logic is especially known for its applications to rule-based control. Apart these three communities, there are still other, more recent, research groups devoted to human-like reasoning issues, like case-based reasoning whose concerns are practically oriented. However all these approaches have more in common than it seems at first glance.

Noticeably, all these schools are interested in formalizing aspects of reasoning which go beyond classical deductive reasoning. In a way or in another, they can be seen as attempts at providing more rigorous basis for different types of problems empirically handled by expert systems, in order to overcome the limitations of these inference systems. Nonmonotonic reasoning aims at offering a proper treatment of rules having implicit exceptions. Uncertainty approaches propose theoretically founded uncertainty calculi based on different representation principles regarding the modelling of partial ignorance, or the use of independence assumptions. Fuzzy logic rather increases the representation capabilities of usual rule-based systems by allowing for the introduction of properties whose satisfaction is a matter of degree, in the condition or conclusion parts of the rules. Moreover the idea of ordering is present in all these approaches. The existence of an ordering between the more or less plausible/normal states of the world, or equivalently between the more or less exceptional situations which can be encountered, underlies nonmonotonic logics. In numerical approaches, this ordering is explicitly reflected by the measures of uncertainty. Fuzzy set membership degrees encode orderings whose interpretation may differ according to the application, as pointed out in the following.

It seems that there are three main distinct notions which are naturally a matter of degree in approximate or commonsense reasoning: uncertainty, similarity and preference. Approximate reasoning plays an important role in three classes of applications: reasoning under uncertainty, classification and data analysis, and decision-making problems. Interestingly enough, these three directions, that have been investigated by many researchers, actually correspond and/or exploit three semantics which have been proposed for fuzzy set membership grades, respectively in terms of uncertainty, similarity and preference. Indeed, considering the degree of membership $\mu_F(u)$ of an element u in a fuzzy set F , defined on a referential U , one can find in the literature, three interpretations of this degree:

- degree of uncertainty: this interpretation is the one at work in possibility theory (Zadeh, 1978; Dubois and Prade, 1988) where fuzzy sets are used to represent imprecise, uncertain or linguistically expressed pieces of information. $\mu_F(u)$ is then the degree of possibility that a parameter x has value u , given that all that is known about it is that " x is F ". F then describes the more or less plausible values of x ;
- degree of similarity: $\mu_F(u)$ is the degree of proximity of u from prototype elements of F . Historically, this is the oldest semantics of fuzzy set membership grades (Bellman, Kalaba and Zadeh, 1966). This view is particularly suitable in classification, clustering, regression analysis and the like, where the problem is that of abstraction from a set of data. It is also at work in fuzzy control techniques, where the similarity degrees between the current situation and the prototypical ones described in the condition parts of the rules, are the basis for the interpolation mechanism between the conclusions of the rules. Besides, similarity plays a crucial role in case-based reasoning. It can be also used in decision (see Section 2.6);

- degree of preference: this interpretation is closely connected with decision analysis. Then a fuzzy set F represents a flexible constraint restricting a set of more or less preferred objects (or values of a decision variable x) and $\mu_F(u)$ represents an intensity of preference in favor of object u , or the feasibility of selecting u as a value of x . This view is the one later put forward by Bellman and Zadeh (1970). μ_F may be then thought as a utility function. Approximate reasoning is then concerned with the propagation of preferences when several constraints (which may be fuzzy) relate the variables. Examples of applications are in design and scheduling problems where it is natural to express preferences about characteristics of the object to be realized, or about due dates.

If/then rules often provide a convenient format for expressing pieces of knowledge. However the accurate representation of rules using classical logic is not an obvious matter, especially if the rule may have exceptions, or involve fuzzy terms in its linguistic expression. In fact, the intended use and meaning of rules may be very different according to the cases and should be properly understood when representing rules. Clearly, rules may express preference, uncertainty, or similarity. When they are decision-oriented, rules are of the general form "if <situation i > then <decision i >". Obviously, a graded set of recommended decisions may also appear in the conclusion part of the rule. The idea of similarity may be also at work in the condition part of the rules, which is then of the form, "the more the state of the world corresponds to <situation i >, the more recommended is <decision i >". Rules expressing uncertainty are more oriented towards reasoning tasks. They are of the form "if p_i is true then q_i is true with certainty λ_i " (where the level of certainty is expressed in the framework of some uncertainty calculus), or they involve some probability, or possibility, distribution δ_i in their conclusion part as in the rule "if p_i is true, then the possible values of x are restricted by δ_i ". If the condition part involves similarity, it leads to rules of the form "the more x is A_i , the more certain q_i " where A_i is a gradual property whose truth is a matter of degree. There exist also purely gradual rules, which are of the form "the more X is A_i , the more Y is B_i " (or equivalently "the less Y is B_i , the less X is A_i ") which provide a qualitative description of relations between variables X and Y in terms of the gradual properties A_i and B_i ; such a rule does not involve any uncertainty by itself. Other rules do not express a restriction in their conclusion part on the possible values of a variable, but rather assert that the possibility/feasibility of some values is guaranteed, as in the rules "the more X is A , the more possible Y is B " and "the more X is A , the larger the set of possible values for Y ". In these "possibility rules", asserting that the value of Y belongs to a set B do not prevent from having other values out of B possible also, it is why the conclusions of several possibility rules fired by the same situation, have to be combined disjunctively. This contrasts with gradual rules, or rules with conclusions pervaded with uncertainty whose conclusions have to be combined conjunctively. This points out that a proper understanding of the intended meaning of rules is very important in approximate reasoning. All the above-mentioned types of rules can be represented in the framework of fuzzy set or possibility theory; see (Dubois and Prade, 1992).

After this brief survey of different types of approximate or commonsense knowledge, where the notions of preference, uncertainty and similarity¹ are present, an overview of the main types of reasoning is now presented.

2 - The Main Paradigms

2.1 - Default Reasoning

Default reasoning is at the core of the knowledge-based systems enterprise. The problem is the handling of the presence of (possibly hidden) exceptions in the rule-base of an expert system. The kind of plausible reasoning that is involved here can be summarized as follows: how to automatically derive plausible conclusions about an incompletely described situation, on the basis of generic knowledge describing what is the normal course of things. For instance, in a medical expert system, generic knowledge encodes what the physician knows about the relationships between symptoms and diseases, and the situation at hand is a given patient on which some test results are available, and plausible inference is supposed to perform a diagnosis task. More generally, this kind of problem can be cast in the setting of taxonomic reasoning, where generic knowledge describe the links between classes and subclasses, and some factual evidence provides an incomplete description of an instance to be classified. The particularity of the problem is that the generic knowledge encoded as a set of rules is pervaded with uncertainty due to the presence of exceptions. Solving this problem in a satisfactory way presupposes that three requirements be met, as emphasized in (Dubois and Prade, 1994a)

- i) *The necessity of a clear distinction between factual evidence and generic knowledge.* This distinction is fundamental and has been explicitly acknowledged in the expert systems literature at the implementation level (facts versus rules). The generic rules encode a background knowledge that is used to jump to conclusions that the only consideration of the available factual evidence would not allow. Clearly, accounting for the arrival of a new piece of evidence does not produce the same effect as the arrival of a new rule or the modification of a rule. The arrival of a new piece of evidence does not affect the generic knowledge, but modifies the reference class of the case under study. On the contrary the introduction of a new rule causes a revision of the generic knowledge.
- ii) *The need for representing partial ignorance in an unbiased way.* There are three extreme epistemic attitudes with regard to a proposition p : on the basis of current evidence and background knowledge one can be sure that p is true, sure that p is false, or the truth-value of p can be unknown. The third situation corresponds to partial ignorance, and its representation should not depend on the count of situations in which p is true, since this count can depend on how these situations are described, i.e., is language-dependent.
- iii) *The inference at work cannot be monotonic.* A plausible reasoning system is expected not to be cautious, namely to go beyond the conclusions strictly entailed by the incomplete evidence. This is done by assuming that the particular situation

¹ Let us mention the idea of *permission* as a fourth basic notion which may appear in commonsense knowledge. As the three other notions, permission might be a matter of degree.

under study is as normal as possible, so that it is possible to jump to adventurous, but plausible conclusions. The price paid by this kind of deductive efficiency is that such conclusions may be canceled upon the arrival of new evidence, when the latter tells us that the current situation is not so normal. This is in obvious contradiction with the monotonicity property of classical logic that forbids conclusions to be retracted when new axioms come in.

Classical logic can neither provide plausible conclusions when information is incomplete nor leave room for implicit exceptions in rules. Thus classical logic fails to satisfy requirement iii) and representation of generic knowledge by universally quantified formulas does not allow for exceptions. The solutions proposed by the expert systems literature were either based on the propagation of certainty coefficients (like in MYCIN and PROSPECTOR), or based on an explicit handling of the reasons for uncertainty at the control level. However these solutions were partially ad hoc, and exception handling in rule-based systems has motivated further, better founded streams of work, namely Bayesian networks and nonmonotonic reasoning. While the first of these approaches could be safely developed due to the strong probabilistic tradition, the second line of research proved to be more adventurous, but eventually fruitful. Although Bayesian approach provides a debatable representation of partial ignorance (point (ii) above, see Dubois, Prade and Smets (1995) for a detailed discussion), it turns out that many lessons from the Bayesian net literature are worth being learned, in order to solve the exception-tolerant inference problem while remaining in the tradition of logic, especially the handling of contexts by means of conditional probability.

In the last ten years, many works in nonmonotonic reasoning have concentrated on the determination of natural properties for a nonmonotonic consequence relation, likely to achieve a satisfactory treatment of plausible reasoning in the presence of incomplete information (Gabbay, 1985; Kraus et al., 1990; Gärdenfors and Makinson, 1994). Besides, Pearl (1988) has suggested that Adams (1975)' logic of infinitesimal probabilities was a good basis for nonmonotonic reasoning, and indeed the core properties of a nonmonotonic consequence relation are present in this logic. These properties constitute the basis of the inference system P (P for preferential) proposed by Kraus, Lehmann and Magidor (1990), which provides a very cautious inference system. In order to get a less conservative inference, Lehmann (see Lehmann and Magidor, 1992) and Pearl (1990) have proposed to add a property, first suggested by Makinson, called rational monotony, and a particular entailment (named "rational closure entailment" (Lehmann and Magidor, 1992)) has been defined which satisfies rational monotony. Remarkably enough, Adams' logic of infinitesimal probabilities or equivalently, system P, can be expressed in terms of conditional objects (Dubois and Prade, 1994b). A conditional object qlp can be seen as a purely symbolic counterpart of the conditional probability $\text{Prob}(qlp)$ (Goodman et al., 1991). Thus it shows that probabilities do not play a crucial role in the modelling of preferential entailment, since no probability degree, infinitesimal or not, are necessary with conditional objects. Only the conditional structure is important. It can be easily handled in terms of a 3-valued semantics much simpler than the preferential semantics (Kraus et al., 1990).

The logic of system P, or equivalently of conditional objects, has also the merit of displaying the difference between two modes of belief revision: evidence focusing

and knowledge expansion that can be defined as follows, where K stands for the knowledge base storing generic knowledge and E gathers the factual evidence:

- *Evidence focusing*: a new piece of evidence p arrives and makes the available information on the case at hand more complete. Then E is changed into $E \cup \{p\}$ (supposedly consistent). K remains untouched. But the plausible conclusions from K and $E \cup \{p\}$, i.e., r such that $K \models r \mid E \wedge p$, may radically differ from those derived from K and E, where we use a conditional object notation (see Dubois and Prade (1994b) for the technical definition of \models); it means equivalently that the nonmonotonic consequence relation $E \wedge p \vdash r$ can be derived from the conditional knowledge in K using the rules of system P (Kraus et al., 1990).
- *Knowledge expansion*: it corresponds to adding new generic rules tainted with possible exceptions. Insofar as the new knowledge base is consistent (see Lehmann and Magidor (1992), and Dubois and Prade (1994) for the definition of the consistency of a conditional knowledge base) it is clear that due to the monotonicity of inference \models , all plausible conclusions derived from K can still be derived from K' since if K is a subset of K' and $K \models r \mid E$ then $K' \models r \mid E$. But more conclusions may perhaps be obtained by K'.
- *Knowledge revision*: it encompasses the situation when the result of adding new generic rules to K leads to an inconsistency. In that case some mending of the knowledge base must be carried out in order to recover consistency. Preliminary results along this line are in Boutilier and Goldszmidt (1993).

The distinction between focusing and expansion cannot be made at all in revision theories that represent cognitive states by sets of formulas in propositional logic, such as Gärdenfors (1988) theory.

In possibility theory "p generally entails q" is understood as " $p \wedge q$ is a more plausible situation than $p \wedge \neg q$ ". It defines a constraint of the form $\prod(p \wedge q) > \prod(p \wedge \neg q)$ that restricts a set of possibility measures \prod . Thus, a set K of generic knowledge statements of the form "p_i generally entails q_i", is equivalent to a collection of constraints $\{\prod(p_i \wedge q_i) > \prod(p_i \wedge \neg q_i), i = 1, n\}$ which define (if they are consistent) a family of possibility measures. It is shown in (Dubois and Prade, 1995a) that the entailment of a statement "generally q in context p" (i.e., $\prod(p \wedge q) > \prod(p \wedge \neg q)$) understood as a consequence of the set constraints modelling K which holds for any possibility measure, is precisely equivalent to the entailment of system P. Rather than working with a family of possibility measures, we can select a particular one which provides a "faithful" representation of K, which can be computed as follows. For each interpretation ω of the language, the maximal possibility degree $\pi(\omega) = \prod(\{\omega\})$ is computed, that obeys the set of possibilistic constraints representing K. This is done by virtue of the principle of minimal specificity (or commitment) that assumes each situation as a possible one insofar as it has not been ruled out. Then each generic statement is turned into a material implication $\neg p_i \vee q_i$, to which a weight $N(\neg p_i \vee q_i)$ is attached where N is measure of necessity associated with the less specific possibility distribution π . It comes down, as shown in Benferhat et al. (1992) to rank-ordering the generic rules giving priority to the most specific ones, as done in Pearl (1990)'s system Z. It offers a convenient framework for implementing "rational closure" (Lehmann and Magidor, 1992) which is thus captured. Possibilistic logic does not allow for a direct encoding of pieces of generic knowledge such as "birds fly" under the form of a pair of a classical formula and a weight. However, it

provides a target language in which plausible inference from generic knowledge can be achieved in the face of incomplete evidence, once the weights are computed as said above.

Generally speaking, a possibilistic knowledge base K is a set of pairs (p,s) where p is a classical logic formula and s is a lower bound of a degree of necessity ($N(p) \geq s$). It can be viewed as a stratified deductive data base where the higher s , the safer the piece of knowledge p . Reasoning from K means using the safest part of K to make inference, whenever possible. Denoting $K_\alpha = \{p, (p,s) \in K, s \geq \alpha\}$, the entailment $K \vdash (p,\alpha)$ means that $K_\alpha \vdash p$. K can be inconsistent and its inconsistency degree is $\text{inc}(K) = \sup\{\alpha, K \vdash (\perp,\alpha)\}$ where \perp denotes the contradiction. In contrast with classical logic, inference in the presence of inconsistency becomes non-trivial. This is the case when $K \vdash (p,\alpha)$ where $\alpha > \text{inc}(K)$. Then it means that p follows from a consistent and safe part of K (at least at level α). This kind of syntactic non-trivial inference is sound and complete with respect to the above defined preferential entailment. Moreover adding p to K and nontrivially entailing q from $K \cup \{p\}$ corresponds to revising K upon learning p , and having q as a consequence of the revised knowledge base. This notion of revision is exactly the one studied by Gärdenfors (1988) at the axiomatic level. See Dubois, Lang & Prade, 1994 for details.

2.2 - Abductive Reasoning

Abductive reasoning is viewed as the task of retrieving plausible explanations of available observations on the basis of causal knowledge. Rules relating causes and manifestations are pervaded with exceptions and uncertainty. Sometimes it is possible to assess, at least qualitatively, the level of certainty with which a cause or a set of causes entail a manifestation. Then a rather simple approach can be proposed (under the hypothesis that there is only one failure at a time in the systems) which takes advantage of the level of certainty for rank-ordering the plausible explanation. See Cayrac et al. (1994) for instance. More generally the idea of parsimonious covering (Peng and Reggia, 1990) which look for explanations involving a minimal set of causes should be used. Lastly let us mention the fuzzy set approach first introduced by Sanchez, which does not deal with uncertainty strictly speaking but rather consider with which intensity a manifestation can be observed when a cause is present.

Assumption-based Truth Maintenance Systems (De Kleer, 1986) cope with incomplete information by explicitly handling assumptions under which conclusions can be derived. To this end some literals in the language are distinguished as being assumptions. Possibilistic logic offers a tool for reasoning with assumptions. It is based on the fact that in possibilistic logic a clause $(\neg h \vee q, \alpha)$ is semantically equivalent to the formula with a symbolic weight $(q, \min(\alpha, t(h)))$ where $t(h)$ is the (possibly unknown) truth value of h . The set of environments in which a proposition p is true can thus be calculated by putting all assumptions in the weight slots, carrying out possibilistic inference so as to derive p . The subsets of assumptions under which p is true with more or less certainty can be retrieved from the weight attached to p . This technique can be used to detect minimal inconsistent subsets of a propositional knowledge base and can be applied to diagnosis problems (see Benferhat et al., 1994). Diagnosis problems can be related to nonmonotonic reasoning also in the sense that, an abnormal exceptional state of affairs corresponding to a failure mode is found to be inconsistent with the usual course of things describing the generic

behavior of the system to be diagnosed (consistency-based approach). See (Console and Torasso, 1991).

2.3 - Reasoning under Inconsistency

As already said, inconsistency can be encountered in different reasoning tasks, in particular:

- when reasoning with exception-tolerant generic knowledge, where the knowledge base includes default rules and instantiated facts, and later a new information is received that contradicts a plausible conclusion derived from the previous knowledge base;
- in abductive reasoning, for instance in model-based diagnosis, when observations conflict with the normal functioning mode of the system and the hypothesis that the components of the system are working well; this leads to diagnose what component(s) fail(s);
- when several consistent knowledge bases pertaining to the same domain, but coming from n different experts, are available. For instance, each expert is a reliable specialist in some aspect of the concerned domain but less reliable on other aspects. A straightforward way of building a global base Σ is to concatenate the knowledge bases K_j provided by each expert. Even if K_j is consistent, it is rather unlikely that $K_1 \cup K_2 \cup \dots \cup K_n$ will be consistent also.

This subsection briefly discusses the treatment of inconsistency caused by the use of multiple sources of information. Reasoning under inconsistent pieces of information, although it requires to go out of the framework of classical logic in order to avoid triviality, has been considered for a long time in the literature by philosophers (e.g., Rescher and Manor, 1970), but is also of interest in combining knowledge bases (e.g., Baral et al., 1992). Then the syntactic appearance of the knowledge base is of primary importance (Nebel, 1991) since it is semantically inconsistent. In such problems, it is interesting to consider that knowledge bases are all stratified, namely that each formula in the knowledge base is associated with its level of certainty corresponding to the layer to which it belongs. The use of priorities among formulas has been shown to be very important to appropriately revise inconsistent knowledge bases (Fagin et al., 1983). In particular, Gärdenfors (1988) has proved that any revision process that satisfies natural requirements is implicitly based on priority ordering. In the context of merging several knowledge bases, the introduction of priorities between pieces of information in Σ can be explained by the two following scenarios:

- Each consistent knowledge base K_j , issued from a source of information, is "flat" (i.e., without any priority between their elements). But we have a total pre-ordering between the sources of information according to their reliability. In this case merging different sources of information lead to a prioritized knowledge base Σ , where the certainty level of each formula reflects the reliability of the source. A particular case is when each piece of information in Σ is supported by a different source.
- All sources of information are equally reliable (and thus have the same level of reliability), but inside each consistent knowledge base K_j there exists a preference

relation between pieces of information given by an expert, who rank-orders them according to their level of certainty. Here again, the combination of the different sources of information gives an uncertain knowledge base, provided that the scales of uncertainty used in each knowledge base K_i are commensurate.

The most elementary form of non-trivial entailment from an inconsistent prioritized knowledge base is possibilistic logic. On the basis of the stratified structure of the knowledge base, many other types of entailment can be proposed; see e.g., (Benferhat et al., 1995; Elvang-Goransson et al., 1993). These entailments are more powerful/adventurous than the possibilistic logic entailment which only consider the consistent part of the bases which is above the level of inconsistency. Several of these entailments are based on the notion of consistent argument (in favor of a conclusion) whose strength depends on the layer of the least certain formulas involved in the argument. An argumentative entailment may then be proposed which allows for the production of consequences for which the strongest argument pro is stronger than the strongest argument against. We may also think of attaching, to each formula in the base, a weight which reflects to what extent there exist arguments that support both the formula and its negation; such "paraconsistency" weight can then be propagated. See Benferhat et al. (1995) for details.

2.4 - Data Fusion

In numerical settings, the problem of combining pieces of evidence issued from several sources of information can be encountered in various fields of application, particularly in i) sensor fusion, i.e., when pieces of information coming from different sensors are to be aggregated, ii) multiple source interrogation systems where each of the sources can provide precise, imprecise or uncertain information about values of interest, iii) expert opinion pooling, when different individual statements have to be synthesized. Our basic claim is that there cannot be a unique mode of combination, which would be satisfactory in any situations, even when the framework for representing information is chosen.

Various combination problems exist, especially, i) preference aggregation versus information aggregation and ii) the combination of information coming from parallel sources versus the revision of already available information. In the preference aggregation problem it makes sense to find the opinion of the "average man" in an homogeneous group of individuals, to look for trade-offs between preferences, while on the contrary, if the information aggregation is a matter of truth and reliability, logical combinations are natural candidates. In this latter case conjunctive combinations apply when all the sources are reliable, while disjunctive combinations deal with the case of unreliable sources hidden in a group of other reliable ones. Obviously weighted logical combinations may be considered in particular when the sources are not equally reliable. Averaging operations in information aggregation can be justified when the set of sources can be viewed as a single random source producing different inputs. In that case, indeed, the set of data to be fused can be interpreted as standard statistics. For instance several successive measurements from a single sensor can be viewed as the result of a random experiment. Then the discrepancies between the sources can be explained in terms of random variability. However in the case of unique measurements issued from distinct sensors, or in the case of expert opinions, it is not clear that averaging combination modes make sense. Besides, the case of

merging information from *parallel* sources should be distinguished from the problem of belief revision where sources do not play a symmetrical role. In the first situation, all the sources provide information simultaneously, while in the revision process there is a chronological ordering between the source which represents the present state of belief and the source which issues the new information. In each case the pooling obeys different requirements, for instance belief revision is generally not commutative; see Dubois and Prade (1994c).

2.5 - Interpolative Reasoning

Similarity is the basic tool in at least three cognitive tasks: classification, case-based reasoning and interpolation. In classification tasks, objects are put in the same class insofar as they are indistinguishable with respect to suitable criteria. Similarity is meant to describe indistinguishability, and an important limiting case is obtained using equivalence relations leading to the partitioning of a set of objects. Classification based on equivalence relations is done in the theory of rough sets (Pawlak, 1991). Case-based reasoning (Kolodner, 1993) exploits the similarity between already solved problems and a new problem to be solved in order to build up a solution to this new problem. When this solution to a new problem is obtained by adapting solutions to already solved problems, the reasoning methodology then comes close to a matter of interpolation, whereby the value of a partially unknown function at a given point of a space is estimated by exploiting the proximity of this point to other points for which the value of the function is known. Although interpolative inference is part of usual commonsense reasoning tasks, it has been seldom considered as amenable to logical settings, because it fundamentally relies on a gradual view of proximities that is absent from classical logic. In contrast, uncertain reasoning, which also involves gradual notions, has received a logical treatment. Results in nonmonotonic reasoning show that some form of uncertain reasoning can be captured by equipping the set of interpretations with an ordering structure expressing plausibility (Lehmann and Magidor, 1992; Shoham, 1988). It may be thus tempting to model interpolative reasoning by equipping a set of logical interpretations with a proximity structure.

This kind of investigation has been started by Ruspini (1991) with a view to cast fuzzy patterns of inference such as the generalized modus ponens of Zadeh (1979) into a logical setting. Indeed in the scope of similarity modeling, a basic reasoning pattern can be expressed informally as follows,

p is close to being true; p approximately implies q — q is not far from being true
 where "close", "approximately", and "not far" refer to the similarity relation S , while p and q are classical propositions. This pattern expresses an *extrapolative syllogism*, and is in accordance with the generalized modus ponens of Zadeh. An example of situation where this type of inference pattern looks natural is the following. Consider the expert advice in finance: "if you have saved more than 10.000 \$ (p) then you should invest 50 % of your capital". Suppose you have 9.500 \$ (p'). Using classical logic, $p' \not\equiv p$, and thus $p' \wedge (p \rightarrow q) \not\equiv q$. But in practice people would not wait to reach the 10.000 \$ threshold and would start investing some percentage of their savings, all the closer to 50 % as these savings amount to near 10.000 \$. In that case the similarity stems from the metric structure equipping the monetary scale.

This type of reasoning is at work in fuzzy control applications, albeit without clear logical foundations. Klawonn and Kruse (1993) have shown that a set of fuzzy rules can be viewed as a set of crisp rules along with a set of similarity relations. Moreover an interpolation-dedicated fuzzy rule "if is A then Y is B" can be understood as "the more x is A the more Y is B" and the corresponding inference means that if $X = x$ and $\alpha = \mu_A(x)$ then Y lies in the level cut B_α . When two rules are at work, such that $\alpha_1 = \mu_{A_1}(x)$, $\alpha_2 = \mu_{A_2}(x)$, then the conclusion $Y \in (B_1)_{\alpha_1} \cap (B_2)_{\alpha_2}$ lies between the cores of B_1 and B_2 , i.e., on ordered universes, an interpolation effect is obtained. It can be proved that Sugeno's fuzzy reasoning method for control can be cast in this framework (Dubois, Grabisch, Prade, 1994). More generally interpolation is clearly a kind of reasoning based on similarity (rather than uncertainty) and it should be related to current research on similarity logics (Dubois et al., 1995). More generally similarity relations and fuzzy interpolation methods should impact on current research in case-based reasoning.

The long term perspective of such a line of research could be to provide logical foundations to some forms of "fuzzy logic", and also case-based reasoning where similarity plays a basic role. The idea would be to start from a set of conditional statements of the form "p is not far from implying q", that forms a conditional similarity-oriented knowledge base given by a domain expert, and to reconstruct an underlying, "least committed", similarity measure, using the characteristic axioms of a similarity-based inference machinery, by analogy to the treatment of conditional knowledge bases in nonmonotonic reasoning.

2.6 - Decision and Artificial Intelligence

Decision theory has been mainly developed in economy and in operations research for a long time. It is only recently that decision under uncertainty is become a topic of interest in Artificial Intelligence, especially among people interested in planning. However we may foresee a richer complementarity between the two fields. In this section, we point out two examples illustrating this view.

Recently, Gilboa and Schmeidler (1992) have advocated a similarity-based approach to decision where a case is described taking inspiration from case-based reasoning, as a triple (situation, act, result) and where a decision-maker's utility function u assigns a numerical value $u(r)$ to a result r . When faced with a new situation s_0 , the decision-maker is supposed to choose an act which maximizes a counterpart of classical expected utility used in decision under uncertainty, namely

$$U_{s_0, M}(a) = \sum_{(s, a, r) \in M} S(s_0, s) \cdot u(r)$$

where S is a non-negative function which estimates the similarity of situations, here the similarity of the current situation s_0 against already encountered ones stored in the repertory set M .

It is worth noticing that this similarity-based utility looks like Sugeno's computation of the command to perform in a fuzzy controller. Indeed both expressions use an interpolation mechanism, but for solving a different problem. A set of rules "if X is $A^{(i)}$ then $Y = b^{(i)}$ " $i = 1, n$ à la Sugeno can be equivalently viewed as a set of pairs (situation, result) = $(a^{(i)}, b^{(i)})$ equipped with a similarity relation S , provided that $\forall i, A^{(i)} = \{a^{(i)}\} \circ S$. In Sugeno's approach the notion of result and utility of result

are not distinguished, or if we prefer all the "results" $b^{(i)}$ have the same utility. What plays the role of an action in the sense of Gilboa and Schmeidler, is here the fact itself of applying the set of fuzzy rules, which explains why there is no maximization in Sugeno's approach. The similarity-based utility is perhaps more akin to the fuzzy vote procedure proposed in Bensana et al. (1988) for selecting a (scheduling) decision b in a situation x from a set of rules "if X is $A^{(i)}$ then $Y = b^{(i)}$ with weight $w^{(i)}$ " ($w^{(i)}$ can be viewed as the utility of the result of the rule), by maximizing an index of the form $U(b) = \sum_{i: b=b^{(i)}} \mu_{A^{(i)}}(x) * w^{(i)}$.

Another line of research is to design a logical machinery able to compute the best decision in a given situation, according to some normative theoretical framework. Possibility theory offers such a framework where both inference and decision under uncertainty can be captured. Indeed a counterpart to von Neumann and Morgenstern' expected utility theory has been proposed in the framework of possibility theory. The existence of a utility function, representing a preference ordering among possibility distributions (on the consequences of decision-maker's actions) that satisfies a series of axioms pertaining to decision-maker's behavior, has been established (Dubois and Prade, 1995b). The obtained utility is a generalization of Wald's criterion, which is recovered in case of total ignorance; when ignorance is only partial, the utility takes into account the fact that some situations are more plausible than others. Mathematically, the qualitative utility is nothing but the necessity measure of a fuzzy event in the sense of possibility theory (a so-called Sugeno integral). The possibilistic representation of uncertainty, which only requires a linearly ordered scale, is qualitative in nature. Only max, min and order-reversing operations are used on the scale. The axioms express a risk-averse behavior of the decision maker and correspond to a pessimistic view of what may happen. The proposed qualitative utility function is currently used in flexible constraint satisfaction problems under incomplete information.

It can also be used in association with possibilistic logic, which is tailored to reasoning under incomplete states of knowledge. A crucial point in decision theory is to make a clear difference between knowledge about the world and decision maker's preferences. We have seen that default or uncertain conditional knowledge can be represented in the framework of possibilistic logic. Similarly, more or less preferred states associated with different levels of priority can be also represented by possibilistic formulas, since necessity measures capture both the ideas of certainty and priority. Indeed a constraint is less prioritary in as much it is possible to violate it; this idea has been extensively used when extending the Constraint Satisfaction Problem framework to flexible constraints having different levels of priority (Dubois, Fargier and Prade, 1994). Thus we build two possibilistic logic bases, one for the knowledge about the world and one for the preferences. Possible decisions correspond to literals that can be fixed to true or false by the decision-maker. Then, we are looking for the decision(s) which are such that, when added to the knowledge base describing what is known about the world, it entails that the preferred states are satisfied. In possibilistic logic this entailment becomes a matter of degree and corresponds to the pessimistic view, risk-averse point of view captured by the qualitative utility function built in the framework of possibility theory. An optimistic point of view would only require the consistency of the knowledge base to which the decision is added, with the set of formulas expressing the preferences.

2.7 - Updating

One of the most challenging problems in databases and knowledge-based systems is that of modifying a knowledge base under the arrival of a new piece of information. Basically, if the new information contradicts the contents of the knowledge base, there exists several ways of restoring consistency, unless strict guidelines are supplied that lead to a unique solution. An important distinction has been drawn by Katsuno and Mendelzon (1991) between revising a knowledge base and updating it. In revision, the new information is meant to improve our cognitive state regarding a given situation; what was plausibly thought as being true may actually be false. In updating, the new information is meant to inform the knowledge base that something has changed in the actual world; what was thought to be true may no longer be true because things have changed.

In order to discuss the problem of updating a knowledge base describing the behaviour of an evolving system, it would be fruitful to unify three points of view on this problem: the point of view of the system analyst who describes the evolution of a system via a transition graph between states, the point of view of formal philosophers who have laid bare the postulates of rational updating, and the point of view of database research, where the update is achieved at the syntactic level by means of transition constraints (e.g., Cholvy, 1994). Such transition constraints partially determine a transition graph between states of the system. Such a graph can be viewed as a possibilistic Markov chain (Dubois et al., 1995). The same observation can be derived from Katsuno and Mendelzon's postulates, except that the obtained transition graph underlies an inertia property.

3 - Concluding Remarks — Going From Theory to Practice

In this paper, different forms of reasoning which can be considered as approximate or commonsense reasoning have been surveyed. The role played by the ideas of uncertainty, preference and similarity has been emphasized. These notions are naturally a matter of degree. However it seems reasonable to use in practice models which are as qualitative as possible, i.e., to use ordinal scales for graded uncertainty, preference or similarity (except if the available data allows for a less qualitative representation). Possibility theory and fuzzy sets can be used in this spirit.

Although considerable progress have been made in the last twenty years in the modelling of approximate and commonsense reasoning, important issues still need to be investigated before developing applications on a large scale. Some of these directions of research have already been pointed out in the main part of the paper. Let us briefly mention two others.

Concerning reasoning with rules having exceptions: all the considered approaches suffer from the same limitations regarding the blocking of property inheritance problem (a subclass cannot inherit any property of a superclass as soon as the subclass is already exceptional with respect to one property of the superclass). A possible way for overcoming this problem is to add pieces of conditional independence information of the form "in the context α , accepting β has no influence on accepting γ ". This kind of information can be represented in the possibility theory framework by adding constraints which are of the same form as the constraints modelling default rules; see (Benferhat et al., 1994), and (Delgrande and Pelletier, 1994) for a related proposal. Expressing independence information may be the right way to get

conclusions which are in agreement with intuitions. Another important issue for practical applications is the validation of knowledge bases: this applies for any kind of non-standard knowledge base: default conditional base, fuzzy rules base, etc.

References

- Adams E.W. (1975) The logic of conditionals. D. Reidel, Dordrecht.
- Baral C., Kraus S., Minker J., Subrahmanian V.S. (1992) Combining knowledge bases consisting in first order theories. *Computational Intelligence*, 8(1), 45-71.
- Bellman R., Kalaba L., Zadeh L.A. (1966) Abstraction and pattern classification. *J. Math. Anal. & Appl.*, 13, 1-7.
- Bellman R., Zadeh L.A. (1970) Decision making in a fuzzy environment. *Management Science*, 17, B141-B164.
- Benferhat S., Dubois D., Lang J., Prade H. (1994) Hypothetical reasoning in possibilistic logic: basic notions, applications and implementation issues. In: *Between Mind and Computer* (P.Z. Wang, K.F. Lo, eds.), World Scientific Publ., 1-29.
- Benferhat S., Dubois D., Prade H. (1992) Representing default rules in possibilistic logic. Proc. 3rd Inter. Conf. on Principles of Knowledge Representation and Reasoning (KR'92), Cambridge, MA, Oct. 26-29, 673-684.
- Benferhat S., Dubois D., Prade H. (1994) Expressing independence in a possibilistic framework and its application to default reasoning. Proc. ECAI'94, 150-154.
- Benferhat S., Dubois D., Prade H. (1995) How to infer from inconsistent beliefs without revising? Proc. IJCAI'95, Montréal, Canada, Aug. 20-25, 1449-1455.
- Bensana E., Bel G., Dubois D. (1988) OPAL: A multi-knowledge-based system for industrial job-shop scheduling. *Int. J. Prod. Res.*, 26, 795-819.
- Boutilier C., Goldszmidt M. (1993) Revision by conditional beliefs. Proc. AAAI'93, July 11-15, 649-654.
- Cayrac D., Dubois D., Haziza M., Prade H. (1994) Possibility theory in "fault mode effect analyses" — A satellite fault diagnosis application—. Proc. of the IEEE World Cong. on Computational Intelligence, Orlando, FL, June 26-July 2, 1176-1181.
- Cholvy L. (1994) Database updates and transition constraints. *Int. J. Intelligent Systems*, 9, 169-180.
- Console L., Torasso P. (1991) A spectrum of logical definitions of model-based diagnosis. *Computational Intelligence*, 7(3), 133-141.
- De Kleer J. (1986) 'An assumption-based TMS' and 'Extending the ATMS'. *Artificial Intelligence*, 28, 127-196.
- Delgrande J., Pelletier F.J. (1994) A formal to relevance. Proc. 1994 Fall Symp., New Orleans, Louisiana, Nov. 4-6, 1994, AAAI Press, 40-43.
- Dubois D., Dupin de Saint-Cyr F., Prade H. (1995) Updating, transition constraints and possibilistic Markov chains. In: *Advances in Intelligent Computing — IPMU'94* (B. Bouchon-Meunier et al., eds.), LNCS, Vol. 945, Springer Verlag, Berlin, 263-272.
- Dubois D., Esteva F., Garcia P., Godo L., Prade H. (1995) Similarity-based consequence relations. In: *Symbolic and Quantitative Approaches to Reasoning and Uncertainty* (Proc. ECSQARU'95, 1995) (C. Froidevaux, J. Kohlas, eds.), Springer Verlag, 171-179.
- Dubois D., Fargier H., Prade H. (1994) Propagation and satisfaction of flexible constraints. In: *Fuzzy Sets, Neural Networks and Soft Computing* (R.R. Yager, L.A. Zadeh, eds.), Van Nostrand Reinhold, New York, 166-187.
- Dubois D., Grabisch M., Prade H. (1994) Gradual rules and the approximation of control laws. In: *Theoretical Aspects of Fuzzy Control* (H.T. Nguyen et al., eds.), Wiley, New York, 147-181.
- Dubois D., Lang J., Prade H. (1994) Possibilistic logic. In: *Handbook of Logic in Artificial Intelligence and Logic Programming*, Vol. 3 (D.M. Gabbay et al., eds.), Oxford University Press, 439-513.
- Dubois D., Prade H. (1988) *Possibility Theory*. Plenum Press, New York.
- Dubois D., Prade H. (1992) Fuzzy rules in knowledge-based systems. In: *An Introduction to Fuzzy Logic Applications in Intelligent Syst.* (R.R. Yager, L.A. Zadeh, eds.), 45-68.
- Dubois D., Prade H. (1994a) Non-standard theories of uncertainty in knowledge representation and reasoning. Proc. 4th Inter. Conf. on Principles of Knowledge

- Representation and Reasoning (J. Doyle et al., eds.), Bonn, 1994, 634-645. Revised version in: *The Knowledge Engineering Review*, 9, 1994, 399-416.
- Dubois D., Prade H. (1994b) Conditional objects as nonmonotonic consequence relationships. *IEEE Trans. on Systems, Man and Cybernetics*, 24(12), 1724-1740.
- Dubois D., Prade H. (1994c) Possibility theory and data fusion in poorly informed environments. *Control Engineering Practice*, 2(5), 811-823.
- Dubois D., Prade H. (1995a) Conditional objects, possibility theory and default rules. In: *Conditionals : From Philosophy to Computer Sciences* (G. Crocco et al., eds.), Oxford University Press, 311-346.
- Dubois D., Prade H. (1995b) Possibility theory as a basis for qualitative decision theory. *Proc. 14th Inter. Joint Conf. on Artificial Intellig. (IJCAI'95)*, Montréal, 1924-1930.
- Dubois D., Prade H., Smets P. (1995) Representing partial ignorance. *IEEE Trans. on Systems, Man and Cybernetics*, to appear.
- Elvang-Goransson M., Krause P., Fox J. (1993) Dialectic reasoning with inconsistent information. *Proc. 9th Conf. on Uncertainty in Artificial Intelligence* (D. Heckerman, A. Mamdani, eds.), 114-121.
- Fagin R., Ullman J.D., Vardi M.Y. (1983) On the semantics of updates in database. *Proc. 2nd ACM SIGACT-SIGMOD Symp. on the Principles of Databases Systems*, 352-365.
- Gabbay D.M. (1985) Theoretical foundations for non-monotonic reasoning in expert systems. In: *Logics and models of Concurrent Systems* (K.R. Apt, ed.), 439-457.
- Gärdenfors P. (1988) *Knowledge in Flux*. The MIT Press, Cambridge, MA.
- Gärdenfors P., Makinson D. (1994) Nonmonotonic inference based on expectations. *Artificial Intelligence*, 65, 197-245.
- Gilboa I., Schmeidler D. (1992) Case-based decision theory. Discussion Paper No. 994, Northwestern University. Revised, 1993.
- Goodman I.R., Nguyen H.T., Walker E.A. (1991) *Conditional Inference and Logic For Intelligent Systems*. North-Holland, Amsterdam.
- Katsuno H., Mendelzon A.O. (1991) On the difference between updating a knowledge base and revising it. *Proc. 2nd Conf. on Principles of Knowledge Representation and Reasoning (KR'91)* (J. Allen et al., eds.), Cambridge, MA, 387-394.
- Klawonn F., Kruse R. (1993) Equality relations as a basis for fuzzy control. *Fuzzy Sets and Systems*, 54, 147-156.
- Kolodner J. (1993) *Case-Based Reasoning*. Morgan and Kaufmann, San Francisco, CA.
- Kraus S., Lehmann D., Magidor M. (1990) Nonmonotonic reasoning, preferential models and cumulative logics. *Artificial Intelligence*, 44, 167-207.
- Lehmann D., Magidor M. (1992) What does a conditional knowledge base entail? *Artificial Intelligence*, 55(1), 1-60.
- Nebel B. (1991) Belief revision and default reasoning: Syntax-based approaches. *Proc. 2nd Inter. Conf. on Principles of Knowledge Representation and Reasoning* (J. Allen et al., eds.), Cambridge, MA, April 22-25, 114-121.
- Pawlak Z. (1991) *Rough Sets — Theoretical Aspects of Reasoning about Data*. Kluwer Academic Publ., Dordrecht.
- Pearl J. (1988) *Probabilistic Reasoning in Intelligent Systems*. Morgan & Kaufmann.
- Pearl J. (1990) System Z: a natural ordering of defaults with tractable applications to default reasoning. *Proc. 3rd Conf. on the Theoretical Aspects of Reasoning About Knowledge (TARK'90)*, Morgan and Kaufmann, San Mateo, CA, 121-135.
- Peng Y., Reggia (1990) *Abductive Inference Models for Diagnostic Problem-Solving*. Springer Verlag, New York.
- Rescher N., Manor R. (1970) On inference from inconsistent premises. *Theory and Decision*, 1, 179-219.
- Ruspini E. (1991) On the semantics of fuzzy logic. *Int. J. Approx. Reasoning*, 5, 45-88.
- Sanchez E. (1977) Solutions in composite fuzzy relations equations — Application to medical diagnosis in Brouwerian logic. In: *Fuzzy Automated and Decision Processes* (M.M. Gupta et al., eds.) North-Holland, 221-234.
- Shoham Y. (1988) *Reasoning about Change*. The MIT Press, Cambridge, MA.
- Zadeh L.A. (1978) Fuzzy sets as a basis for a theory of possibility. *Fuzzy Sets & Systems*, 1, 3-28.
- Zadeh L.A. (1979) A theory of approximate reasoning. In: *Machine Intelligence*, Vol. 9 (J.E. Hayes, D. Michie, L.I. Mikulich, eds.), Elsevier, New York, 149-194.