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# Transmission Systems to Extend the Workspace of Planar Cable-Driven Parallel Robots

Foroogh Behroozi<sup>1</sup>[0000–0003–4987–8140], Philippe Cardou<sup>1</sup>[0000–0002–8381–3845],  
and Stéphane Caro<sup>3</sup>[0000–0002–8736–7870]

<sup>1</sup> Department of Mechanical Engineering, Robotic Laboratory, Laval University,  
Québec, QC, G1V 0A6, Canada

foroogh.behroozi.1@ulaval.ca, pcardou@gmc.ulaval.ca

<sup>2</sup> Nantes Université, École Centrale Nantes, CNRS, LS2N, UMR 6004, 1, rue de la  
Noe, 44321 Nantes, France  
stephane.caro@ls2n.fr

**Abstract.** In cable-driven parallel robots (CDPRs), each cable is typically driven by one actuator, and one is led to think that the number of actuators necessarily depends on the number of cables. In this paper, we consider that the number of cables depends on the required workspace while the number of actuators depends on the number of degrees of freedom of the robot. When designing a CDPR, one typically starts from the shape of the desired workspace to determine the required number of cables. Thence, he or she adds as many actuators, assuming that each of them drives one cable. Here, the number of actuators is supposed to be equal to one more than the number of degrees of freedom of the CDPR irrespective of the number of cables. A transmission system is designed for the actuators to drive several cables, first theoretically in the form of a transmission matrix, and then mechanically in the form of the corresponding cable-pulley routing. Two examples are proposed with two and three degrees of freedom respectively; both aim at covering a rectangular workspace. The wrench-closure-workspaces of the resulting robots compare favorably to existing CDPRs with more actuators.

**Keywords:** Transmission System · Wrench-Closure-Workspace · Wrench-Feasible-Workspace · Transmission matrix · Schematic design · CDPR.

## 1 Introduction

One of the most commonly adopted architectures for planar Cable-Driven Parallel Robots (CDPRs) aims to cover a rectangular workspace and typically consists of four cables and four actuators (see Fig. 1(a)). As a result, CDPR with four cable and a point-mass end-effector have a degree of redundancy of two. As cables can only exert tension and not compression, an  $n$ -DoF CDPR should generally include at least  $n + 1$  actuated cables to ensure that the end-effector is in a wrench-closure configuration. In the 2-DoF planar CDPR with a point-mass end-effector under consideration, this means that the minimum number of actuators is three, so it has one more than necessary. The additional actuator(s)

can result in a significant cost for the robot, and therefore, we seek to remove one actuator while preserving the rectangular workspace. Our strategy is to keep the same four cables while searching for a transmission system that would allow them to be driven by just three actuators, as shown in Fig. 1(b). A planar 3-DoF CDPR driven by no more than four actuators is also synthesized and studied in this paper.

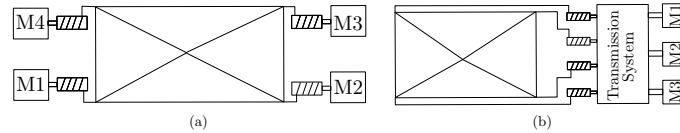


Fig. 1: Planar CDPRs with four cables and a point-mass end-effector: (a) having four actuators; (b) using a transmission system to drive four cables with three actuators

The use of pulleys and additional cables to reduce the number of actuators in CDPRs has been previously explored in the literature [5–8]. The concept of using cable differentials in the design of spatial and planar CDPRs is presented in [6, 7]. Those papers focused on comparing the results of using the same actuator and increasing the number of cables and differentials to determine the coverage of the workspace [6]. In [7], the authors used differentials to actuate several cables of a planar CDPR by an actuator to minimize the number of actuators. Their results showed that using differentials leads to larger workspaces and improved kinetostatic performance with the same number of actuators. They covered the triangular shape of the WCW using three actuated cables.

Cable-pulley transmission systems offer a unique combination of zero-backlash motion, high stiffness, low stiction, and low friction, making them desirable in force and torque control applications [11]. In [2], a mechanism comprising two non-circular pulleys and a constant-length cable was introduced. The authors proposed a differential cable routing method in order to enhance the capabilities of the non-circular pulley mechanism. In [5], Kevac and Filipovic provided an overview of various construction modeling approaches in cable-suspended parallel robots (CSPR). They presented six types of CSPR systems for camera carriers that can cover the workspace using only three actuators instead of the typical four, although they did not specify the percentage of the desired workspace area that can be covered by their proposed concepts.

In [13], an original design for a 3-DoF CDPR that utilizes six cables and three actuators are proposed. The authors employed a parallelogram configuration to simultaneously wind two cables by an actuator, allowing for the fixation of the moving-platform (MP) orientation while performing the translational movement. In [1], the design, modeling, and prototyping of a planar CDPR with infinite rotation capabilities, free from parasitic tilt, and without the need for an additional actuator is presented. The 2-DoF motions of the MP and the internal

degrees of freedom of the embedded mechanism are controlled by a total of three actuators, which are fixed to the frame. The proposed crane robot comprises of a mobile platform (MP) equipped with an embedded mechanism and a transmission module. The MP is connected to the frame via four cables, three of which act in parallel, forming a double parallelogram structure. The 2-DoF motions of the moving platform and the internal degrees of freedom of the embedded mechanism are controlled by a total of three actuators, which are fixed to the frame.

In [9], two novel architectures for planar spring-loaded CDPRs that do not require actuation redundancy are proposed. The authors combine springs with a cable-loop system to eliminate the need for actuator redundancy, allowing for  $N$  actuators to control  $N$ -DoF motion. The proposed method ensures that the cables and springs are kept in tension within a rectangular workspace, but preloading of the springs is required to cover the entire workspace. The authors suggest that appropriate adjustments to the portion of stiffness and preload can increase the workspace.

In [8], a configuration for planar CDPRs that utilizes parallelogram links instead of conventional links is presented. The use of parallelogram links ensures that the cables remain in tension during the robot's movements, thereby enhancing its dexterity and elastic stiffness, as well as the magnitude of its stiffness indices. The authors manipulate cable redundancy to maintain the robot's structure while utilizing only three actuators in the design of the planar CDPRs. In this research, a general formulation for parallelogram links is provided to support the suitable design of planar CDPRs. Although the proposed structure is applied to both fully constrained actuated and redundantly actuated configurations, it is unable to cover the entire rectangular workspace.

In this paper, two novel cable robot architectures are proposed that aim to reduce the number of actuators while preserving a large wrench-closure-workspace (WCW). This architecture is distinct from traditional cable robot architectures, which typically feature a one-to-one correspondence between cables and actuators. In these proposed architectures, a single cable may be linked to multiple actuators (or conversely), resulting in a reduction in the number of actuators required. In general, if  $p$  actuators drive an  $n$ -DoF CDPR that does not rely on gravity or any other external force to keep the cables taut, then  $p \geq n + 1$  is a necessary condition for the moving-platform to be in equilibrium. We resort to a transmission matrix to effectively describe the relationship between actuators and driving cables.

The organization of this paper is as follows. The mathematical foundation of the kinetostatic analysis of  $n$ -DoF  $m$ -cable  $p$ -actuator CDPRs is presented in Section 2. In Section 3, the transmission matrices, WCW, Wrench-Feasible-Workspace (WFW), and architecture of a 2-DoF four-cable three-actuator CDPR are discussed. The transmission matrix for a 3-DoF six-cable four-actuator CDPR is presented in Section 4, along with an illustration of its WCW in various orientations.

## 2 Kinetostatic Analysis

In this section, we present the kinetostatic analysis of the proposed  $n$ -DoF  $m$ -cable  $p$ -actuator planar C DPR. The equations of equilibrium of the end-effector can be written as Eq. (1) as reported in [3].

$$\mathbf{W}(\mathbf{p})\mathbf{t} + \mathbf{w}_e = \mathbf{0}_n \quad , \quad \mathbf{t} > \mathbf{0}_m, \quad (1)$$

Where  $\mathbf{W}(\mathbf{p}) \in \mathbb{R}^{n \times m}$  is the wrench matrix of the cable robot when its end-effector is located at  $\mathbf{p}$ ,  $\mathbf{w}_e$  is the external wrench and  $\mathbf{t}$  is tension vector. The WCW is the set of poses of the platform where any wrench that can be generated at the platform by tightening the cables [4]. Roberts et al. [10] showed that a given pose lies inside the WCW (i.e., has wrench closure) if and only if one can find a vector  $\mathbf{t}_\perp > \mathbf{0}_m$  in the nullspace of  $\mathbf{W}$ , where  $>$  indicates a componentwise strict inequality.

In our case, however, not all the tensions found in the nullspace of  $\mathbf{W}$  can be generated from  $p < m$  motors. Let us define matrix  $\mathbf{T} \in \mathbb{R}^{m \times p}$ , which represents the linear transmission between actuators and winches. The matrix  $\mathbf{T}$  maps actuator torques  $\boldsymbol{\tau}$  into cable tensions  $\mathbf{t} \in \mathbb{R}^{m \times 1}$ , namely,

$$\mathbf{t} = \mathbf{T}\boldsymbol{\tau}. \quad (2)$$

To determine whether there exists a solution  $(\mathbf{T}, \boldsymbol{\tau})$  to equations (2) and for external wrench  $\mathbf{w}_e = \mathbf{0}$ , we must identify a positive vector  $\mathbf{t}$  that lies simultaneously within the nullspace of the wrench matrix  $\mathbf{W}(\mathbf{p})$  and in the range<sup>1</sup> of matrix  $\mathbf{T}$ . To accomplish this, we recast equations (2) into matrix form, which yields:

$$\begin{bmatrix} \mathbf{W} & \mathbf{0}_{n \times p} \\ \mathbf{I}_{m \times m} & -\mathbf{T} \end{bmatrix} \begin{bmatrix} \mathbf{t} \\ \boldsymbol{\tau} \end{bmatrix} = \begin{bmatrix} \mathbf{0}_n \\ \mathbf{0}_m \end{bmatrix} \quad (3)$$

where  $\mathbf{I}_{m \times m}$  is the  $m \times m$  identity matrix and  $\mathbf{0}_{n \times p}$  is the  $n \times p$  zero matrix. The solution of Eq. (3) amounts to computing the nullspace of the  $(m+n) \times (m+p)$  matrix and verifying whether  $\mathbf{t} > \mathbf{0}_n$ . As  $p = n + 1$ , the number of columns is one more than the number of rows and this matrix has a rank of  $m + n$  unless one of  $\mathbf{W}$  and  $\mathbf{T}$  is rank-deficient. To compute this nullspace symbolically, let us modify it by adding two variable vectors  $\mathbf{z}_1 \in \mathbb{R}^m$  and  $\mathbf{z}_2 \in \mathbb{R}^p$ , as the first row of the matrix of Eq. (3). This yields the square matrix

$$\mathbf{A} = \begin{bmatrix} \mathbf{z}_1^T & \mathbf{z}_2^T \\ \mathbf{W}_{n \times m} & \mathbf{0}_{n \times p} \\ \mathbf{I}_{m \times m} & -\mathbf{T}_{m \times p} \end{bmatrix} \quad (4)$$

To calculate the cable tensions, we can take the partial derivative of the determinant of Eq. (4). If we let  $\text{adj}(\mathbf{A})$  and  $\det(\mathbf{A})$  be the adjoint matrix and determinant of  $\mathbf{A}$  respectively, then, by definition,

<sup>1</sup> The range (also called column space or image) of a  $m \times n$  matrix  $\mathbf{T}$  is the span (set of all possible linear combinations) of its column vectors.

$$\mathbf{A}\text{adj}(\mathbf{A}) = \det(\mathbf{A})\mathbf{1}_{(m+n+1)\times(m+n+1)}. \quad (5)$$

By knowing that  $\mathbf{t}_\perp$  is the first column of  $\text{adj}(\mathbf{A})$ , the WCW of the proposed CDPR is the set of poses where:

$$\mathbf{t}_\perp = \frac{\partial}{\partial \mathbf{z}_1} \det(\mathbf{A}) > \mathbf{0}_m, \quad (6)$$

Since  $\mathbf{t}_\perp$  can be computed in closed form, leaving the pose parameters as variables and setting the entries of  $\mathbf{t}_\perp$  equal to zero one at the time yields the potential boundaries of the WCW. To our knowledge, this method of tracing the WCW for CDPRs where  $\mathbf{T} \neq \mathbf{I}_{m \times m}$  has not been reported before.

### 3 A 2-DoF Four-Cable Three-Actuator CDPR

Consider the case of the 2-DoF CDPR shown in Fig. 1b, where a particle end-effector is to cover a workspace that is as close as possible to the encompassing rectangle.

#### 3.1 Determination of the transmission system $\mathbf{T}$ of a 2-DoF CDPR

The CDPR workspace depends on the value of the transmission matrix  $\mathbf{T}$ . This value was obtained through trial and errors, while following a few guidelines. First, we should pick  $\mathbf{T}$  so as to maximise the area of the corresponding WCW. Second,  $\mathbf{T}$  should be as sparse as possible in order to simplify the mechanical implementation of the transmission system. Third, we hypothesized that the transmission should abide at least partly by the same symmetry found in the CDPR geometry for the WCW area to be maximised. Fourth, we hypothesized that restricting ourselves to entries of  $\mathbf{T}$  that were either -1, 0 or 1 would still allow us to maximise the WCW area. Indeed, the scale of a row of  $\mathbf{T}$  has no effect on the WCW, as long as its sign remains the same. The same goes for the scale of a column of  $\mathbf{T}$ , which can always be adjusted by changing the input torque of a motor. This is the rationale behind this hypothesis. Optimal  $\mathbf{T}$  matrices were found, which maximize the WCW of the CDPR, and each of them yields the same WCW:

$$\mathbf{T} = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & -1 & 0 \\ 1 & 0 & -1 \end{bmatrix} \quad \text{and} \quad \mathbf{T} = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \quad (7)$$

The value of the element  $T_{i,j}$  of  $\mathbf{T}$  is  $T_{i,j} = 1$  when the  $i^{\text{th}}$  cable is connected to the  $j^{\text{th}}$  actuator directly and is  $T_{i,j} = -1$  when the  $i^{\text{th}}$  cable is connected to the  $j^{\text{th}}$  actuator in reverse.  $T_{i,j} = 0$  when the  $i^{\text{th}}$  cable is not connected to the  $j^{\text{th}}$  actuator. The second actuator needs to connect to cables one and three, while the third actuator connects to cables two and four. When the second actuator

winds the first cable, it has to unwind the third cable at the same rate. Also, the relation between cable numbers two and four is the same, but with the third actuator.

Having defined  $\mathbf{T}$ , we turn our attention to the computation of its associated WCW. This is done by following the method detailed in section 2, which results in the WCW shown in Fig. 2(a). As shown, in this figure the WCW of this robot covers the whole rectangle formed by the fixed cable attachment. In Fig. 2(b), the WCW of a similar CDPR proposed in [8] is shown, which could not cover some parts of the rectangular shape workspace. Using Maple2018, we obtained symbolic expressions of the workspace through the four inequalities  $\mathbf{t}_\perp = \frac{\partial}{\partial \mathbf{z}_1} \det(\mathbf{A}) > \mathbf{0}_4$ . The workspace boundaries in the Cartesian plane take the form  $\mathbf{t}_\perp(x, y) = \mathbf{0}_4$  are non-polynomial in  $x$  and  $y$ .

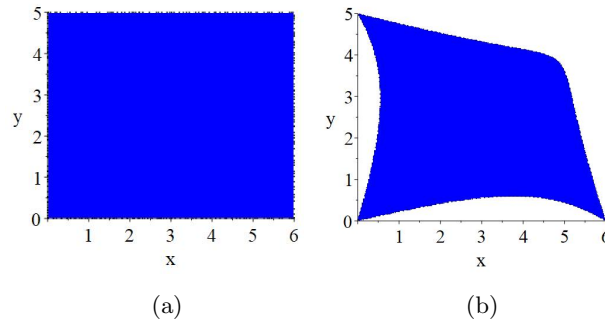


Fig. 2: (a) WCW of the proposed 2-DoF four-cable three-actuator CDPR, (b) WCW of the planar CDPR proposed in [8]

The WFW is the set of platform poses where all the wrenches of a given set, can be balanced with tension forces in the cables such that the cable tensions are within prescribed limits [4]. To determine the area where all cable tensions are within a specified range, the WFW of the manipulator is determined.

We trace the WFW with the values of drum radius  $R_1 = R_2 = R_3 = 25\text{mm}$ , where the tension limits are set to 0.5N and 20N (Fig. 3). We see that the WFW is approximately rectangular, but its size shrinks as the interval of external forces widens.

Having confirmed that the proposed CDPR architecture allows wrench closure over the full rectangle formed by its fixed attachment points with three actuators, we set out to propose a mechanical realization of this theoretical design.

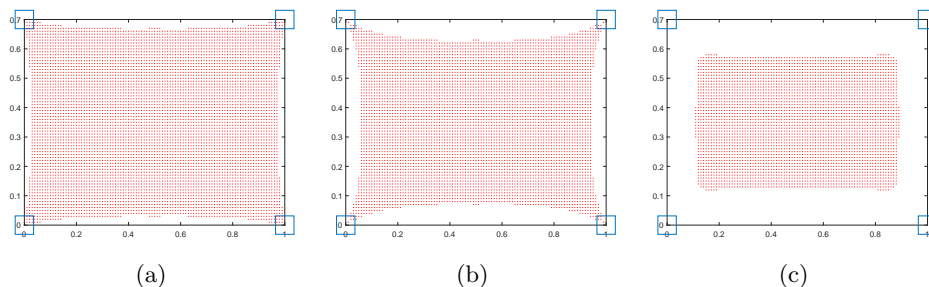


Fig. 3: WFW of the proposed CDPR (Case 1) considering different external forces: (a) WFW of proposed CDPR (Case 1) with square external force 0.5N with the origin center; (b) WFW of proposed CDPR (Case 1) with square external force 2.5N with the origin center; (c) WFW of proposed CDPR (Case 1) with square external force 5N with the origin center.

### 3.2 Embodiment of the Transmission System in a 2-DoF Planar CDPR

For the implementation of the proposed transmission matrix, we decide to use cable-pulley transmission systems to distribute the power of three actuators among four cables. Two mechanical implementations of the transmission matrices are schematised in Fig. 4. The schematic corresponding to the transmission matrix given in Eq. (7) is presented in Fig. 4(a). As shown in Fig. 4(a), we use two moving three-level pulleys, which directly control the cables in the workspace. The first actuator helps to drive all moving pulleys, while the second and third actuators are used to drive two cables at the same time.

Another schematic that represents a possible embodiment of the transmission matrix given in Eq. (7) is proposed in Fig. 4(b). As shown in Fig. 4(b), we use two moving two-level pulleys to keep cables one and four in tension in the workspace. The first actuator controls the force applied by these two-level pulleys.

To ensure that the cable routings of Fig. 4 correspond to the transmission matrices of Eq. (7), a static analysis of the schematic in Fig 4(a) is done, which yields the relationship presented in Eq. (8) between the motor torques and cable tensions.

$$\mathbf{T}' = \begin{bmatrix} \frac{1}{2R_1} & \frac{1}{2R_2} & 0 \\ \frac{1}{2R_1} & 0 & \frac{1}{2R_3} \\ \frac{1}{2R_1} & -\frac{1}{2R_2} & 0 \\ \frac{1}{2R_1} & 0 & -\frac{1}{2R_3} \end{bmatrix} \quad (8)$$

In this matrix,  $\mathbf{R}_i$  is the radius of the drum corresponding to the  $i^{th}$  actuator. If all the drums have the same radius, then  $R_1 = R_2 = R_3 = R$ , and we have  $\mathbf{T} = 2R\mathbf{T}'$ .



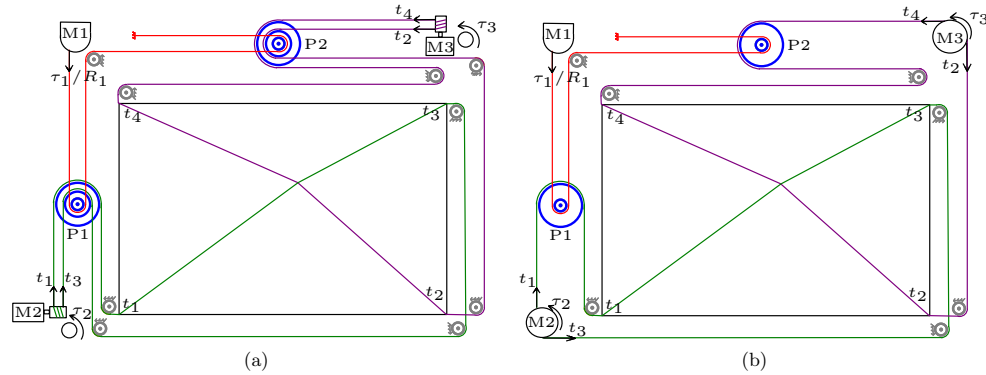


Fig. 4: Proposed schematic of three actuators 2-DoF four cables CDPR: (a) Case 1; (b) Case 2

#### 4 A 3-DoF Six-Cable Four-Actuator CDPR

The most common task of a 3-DoF planar CDPR is probably to move its platform over a rectangular area. The standard 3-DoF planar CDPR consists of four cables driven independently by as many actuators, attached at the vertices of the said rectangle and at the other ends to the vertices of a rectangular moving-platform. Such an arrangement was proposed in [3, 12], and is shown in Fig. 5a. One important problem with this architecture is that its translational workspace shrinks dramatically as the platform deviates from its reference orientation shown in Fig. 5(a). To resolve this, we propose to use a transmission system in 3-DoF planar six-cable CDPR with four actuators as shown in Fig. 5(b). This architecture uses two more cables than that found in [3, 12], but, importantly, it has same number of actuators. The problem becomes that of finding an appropriate transmission system that would allow large rotations of the moving platform.

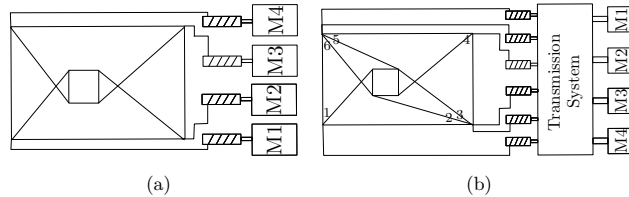


Fig. 5: Using transmission system in 3-DoF six-cable four-actuator CDPRs

#### 4.1 Determination of the Transmission System for 3-DoF CDPR through $\mathbf{T}$ Matrix

The WCW of the CDPR is directly related to the choice of its  $\mathbf{T}$  matrix. A trial and error method was applied, and guidelines outlined in Section 3.1 were followed again for this 3-DoF CDPR. This led to the following  $\mathbf{T}$  matrices:

$$\mathbf{T} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ -1 & 0 & 0 & 1 \\ 0 & 0 & -1 & 1 \\ 0 & -1 & 0 & 1 \end{bmatrix}, \quad \text{and} \quad \mathbf{T} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 1 \\ 0 & 0 & -1 & 1 \\ 0 & -1 & 0 & 1 \end{bmatrix}. \quad (9)$$

Considering the first transmission matrix proposed in Eq. (9), one actuator should drive all cables for the purpose of keeping them in tension. The first actuator should wind the first cable while unwinding the fourth cable at the same rate thus forming a cable loop. The same goes for actuators 2 and 3 with cable pairs (2, 6) and (3, 5), respectively. In the second matrix of Eq. (9), the number of connections between actuators and cables is reduced by taking advantage of the opposition between cables. Indeed, motor 4 keeps in tension only cables 4-6, but since these cables form loops with cables 1-3, all the cables are tensioned by this motor.

With both transmission matrices defined in Eq. (9), the constant-orientation WCWs were computed for nine different angles of the moving platform. The resulting WCWs are traced in Fig. 6. The results are the same for both transmission matrices. As shown, although these transmission matrices do not allow to cover the complete rectangle formed by the workspace defined by the fixed attachment points, the area covers a wide range of angles.

#### 4.2 Embodiment of the Transmission System of the 3-DoF Planar CDPR

In order to implement the proposed transmission matrix for the 3-DoF four-actuator six-cable planar CDPR, we adopt an approach similar to that adopted for the 2-DoF planar CDPR in Subsection 3.2. A cable-pulley transmission system is used to distribute the power of the four actuators among the six cables. The schematic for the implementation of the transmission matrix outlined in Eq. (9) is presented in Fig. 7. One actuator is used to control the tension in the other cables, while the remaining actuators drive two cables in different directions. A system comprising two suspended three-level pulleys is used to directly control the cables within the workspace. The statics analysis of the mechanism can be performed by calculating the sums moments on the actuator drums and the sums of forces on the moving pulleys similar to what was outlined in Subsection 3.2.

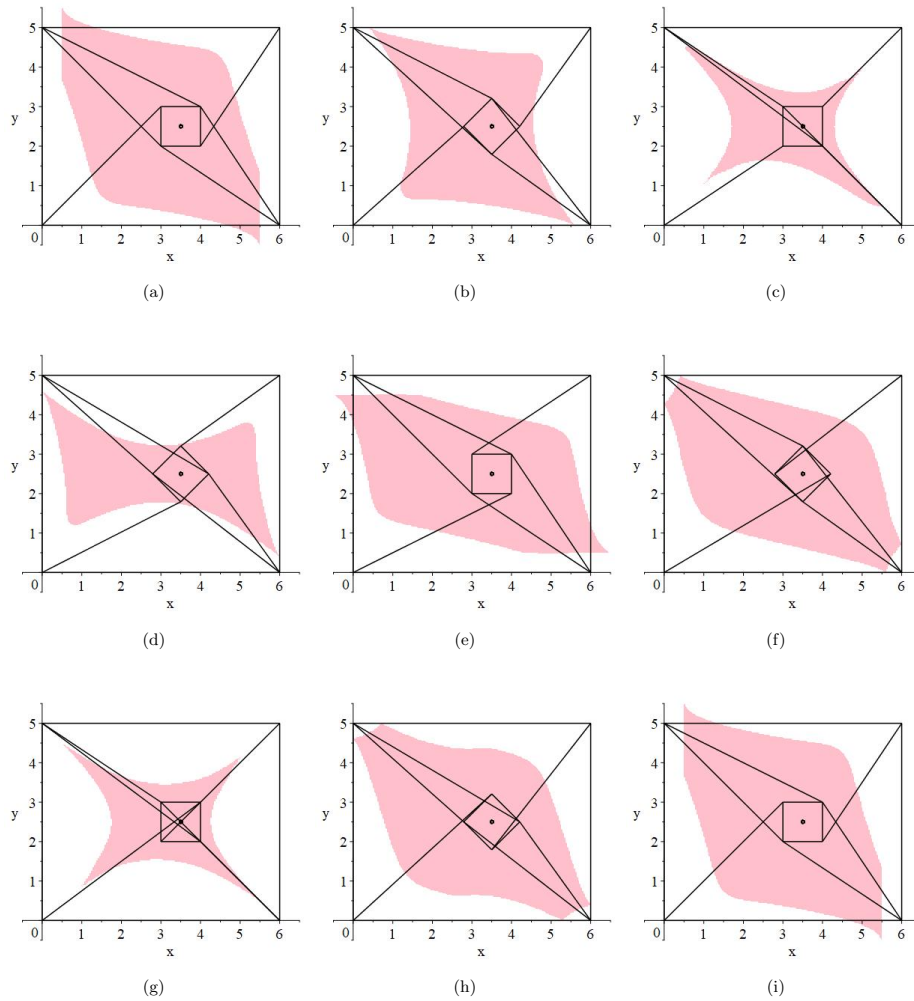


Fig. 6: WCW of the 3-DoF six-cable four-actuator CDPR for nine MP orientations: (a) 0°; (b) 45°; (c) 90°; (d) 135°; (e) 180°; (f) 225°; (g) 270°; (h) 315°; (i) 360°.

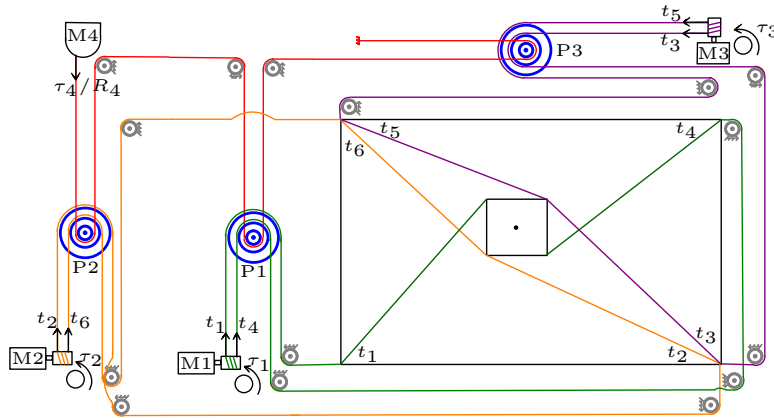


Fig. 7: Proposed schematic of four actuators 3-DoF six-cable CDPR

### 4.3 Comparison Results

In Fig. 8, the WCW of the classic 3-DoF CDPR with four actuated cables is traced in eight different orientations. In this version, we note a fairly good coverage of the workspace except for moving-platform orientations of  $90^\circ$  and  $270^\circ$ , where there is no workspace. On the other hand, the WCW is shown in Fig. 6 for the proposed 3-DoF six-cable four-actuator, we observe slightly smaller workspaces in most orientations. We note also that there is a WCW in all tested orientations for the proposed architecture. This implies that the proposed six-cable CDPR may theoretically perform full rotations when the end-effector is in the central region of its workspace while the classic 3-DoF planar CDPR would be limited to  $180^\circ$  continuous rotations.

## 5 Conclusions

In this research, we proposed two planar CDPRs with fewer actuators than cables by using appropriate transmission systems. The presented architectures hold advantages over other classic CDPRs that are available in the literature. It shows that using the transmission system in a 2-DoF CDPR allows it to cover a rectangular WCW with one less actuator than the classic four-actuator version. Also, using a transmission system in a 3-DoF CDPR with six cables and four actuators allows for a fairly large WCW that extends over  $360^\circ$  range of possible rotations of the moving-platform. Other researchers had proposed CDPRs with fewer actuators than cables in the past [9, 7, 8]. What sets this work apart from theirs is the design method that was followed. Instead of starting with the selection of mechanical components for the transmission system, we focused first on the choice of the transmission matrix mapping the actuator torques onto the cable tensions. Only once a suitable transmission matrix had been found did we try to implement it with machine components—cables and

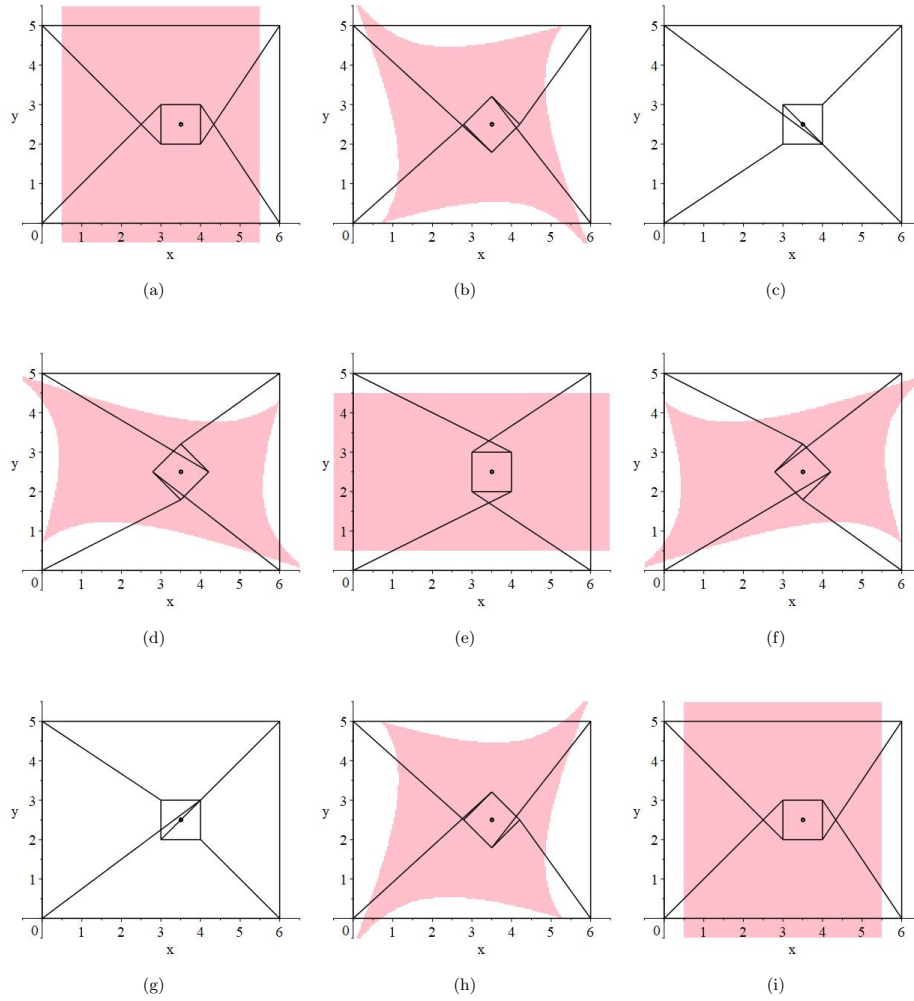


Fig. 8: The WCW of the classic version of 3-DoF four-cable four-actuator CDPR in different end-effector angles (a)  $0^\circ$ ; (b)  $45^\circ$ ; (c)  $90^\circ$ ; (d)  $135^\circ$ ; (e)  $180^\circ$ ; (f)  $225^\circ$ ; (g)  $270^\circ$ ; (h)  $315^\circ$ ; (i)  $360^\circ$ .

pulleys were used, but gears, belts, and other standard mechanical components could have been used as well. Transmission matrices for two and three DoF are presented and two CDPR schematics for 2-DoF and a schematic for 3-DoF planar CDPRs are proposed to cover a rectangular WCW. For specific three different external force sets, the WFW was found for the 2-DoF CDPR. The next step of this research consists in automating the choice of the transmission matrix to expand the WCW while simplifying its matrix as sparse as possible to keep the mechanical implementation.

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