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Quantum optics to reconstruct the monetary state of global economy

Antonino Claudio Bonan

Abstract Once the whole economic system is treated as a quantum system in an abstract space-like frame, a question arises: how do data help in reconstructing its states? Optical-like elements can do the job, then tomography is addressed by suitable quantum circuits and algorithms. A path is proposed for this, assembling ideas from quantum linear optics which can be adapted for an implementation in global economical statistics.

Keywords Econophysics · Quantum optics · Quantum state tomography

1 Introduction

An approach considers global economy as a quantum system [1], in a frame accounting for the fact that its actions happen within a natural and human environment [2]. Here, firstly, it is adopted in order to possibly use statistical data for a tomographic reconstruction [3] of its current or past states.

There are some paths to do that, suggested by the logic of quantum gates and circuits (as is done for other reasons in [4]), which brings along its difficulties and tricks. Any such a path has then to be implemented for global economy, by being transposed from typical physical realizations of quantum circuits, or at least from elements which are relevant for the circuits to be used. In particular, an intuitive transposition is suggested by quantum optics [5], with global monetary configurations playing the role of photons.

In Sec. 2, a quantum optical interpretation is provided for the abstract geometry and dynamics of the flow of the monetary field through different media. Sec. 3 depicts the corresponding fundamental elements in global economy, to be used in

optical-like thought devices which have matrix representation. In Sec. 4, a tomographic algorithm is outlined for reconstructing quantum states. As suggested in Sec. 5, every ingredient of such a recipe assumes one to think in terms of an abstract geometry and some particular quantum circuits. In Sec. 6, some key issues and trial ansatz of the method are suggested for possible applications. Sec. 7 explains how this approach is a prelude to represent general dynamics of global economy.

2 Quantum optical interpretation of global money

In physics, optics often entails the idea that any change of dynamics explicitly induced by matter just lies in the interfaces between different media through which electromagnetic field propagates, while inside each medium it is like only the electromagnetic field comes into play and the interaction by matter is parameterized by a complex version of the propagation speed [6]. In an approach considering economy as dynamics of quantum fields, this amounts to classifying any subsystem as a medium, according to the speed at which money is transferred. The economic subjectivity is just the field changing monetary dynamics in any interface between different subsystems.

Suitable economical counterparts of the spatial dimensions in which optics resides are given by allocations of subjects, their roles in society and their niche in nature. These dimensions are three too, so that the polarization of the monetary field is parameterized by a 2-dimensional complex Jones vector, a third component being constrained by a conventional gauge [7]. In absence of net sources or sinks of money, this makes induced oscillations of economic charges (i.e. ethical-social values) be perpendicular to the direction of propagation of any monetary signal. Within media, such oscillations only affect money; within interfaces, they can be actually detected by economical subjectivity, possibly be-

side their transient longitudinal components.

Polarization is what has to be measured, when one wants to test Fourier modes (plane waves) in an electromagnetic field. In terms of economy, one detects oscillations of ethical-social values being induced perpendicularly to the direction of propagation of money in the abstract frame.

Qubits are the quantum counterpart of 2-dimensional complex Jones vectors, while linear operators acting on them represent optical-like devices acting upon polarizations and are parameterized as combinations of Pauli matrices. Put another way, qubits represent free photons according to their transverse independent polarizations, with approximately instantaneous interactions driven by reversible unitary transformations localized in interfaces. For global economy, reversibility and unitarity relate to the ansatz that dynamic effects due to the environment are not dealt with, as the system is considered closed (including economy and environment in one whole system). Ultimately, this means that quantum logic gates involving qubits have the features to approximately represent global monetary processes.

3 Useful optical-like devices represented as matrices

The classical reflection of light is the change in direction of the ray and so is represented by a rotation matrix

$$R(\theta) = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix},$$

depending on the angle $\theta/2$ of incidence (or equivalently, the orientation of the reflecting plane with respect to the ray), which is the only parameter because of symmetry arguments constraining the process on a plane. In terms of electromagnetic waves [6], if the speed of light is increased beyond an interface acting as a mirror (i.e. $v_2 > v_1$), then the polarization of light and the amount of increase in its speed (through the critical angle θ_c and the Brewster angle $\theta_B \leq \theta_c$) are keys to classically determine the phase advance δ due to reflection. This could be made explicit by Fresnel equations for the two polarizations with electric field being normal or parallel to the plane of incidence, respectively denoted by "s" and "p" subscripts. In the opposite situation $v_1 > v_2$, the critical angle does not exist and simply $\delta_s = \delta_p = \pi$. Henceforth, the latter kind of mirrors is considered.

The flow of money can approximately undergo similar instantaneous transformations, for corresponding conditions in the abstract frame.

It is worth noting by the way that $R(-\theta)$ formally equals $R^{-1}(\theta)$, i.e. it takes from a state modified by a mirror hit at an angle $\theta/2$ back to the original state. Furthermore, one can show the formal relation $R(-\pi/4) = -R^3(\pi/4)$.

Mirrors are generalized by beam splitters, for which another

parameter ϕ comes into play, taking transmission into account beyond reflection: the parameter $\pi - \phi$ is just the change of phase for electromagnetic waves, which would be fixed to 0 for the ones being purely reflected by an interface beyond which the speed of light is decreased. This general device is represented as an unitary transformation, by a matrix

$$B(\theta, \phi) = \begin{pmatrix} \cos \theta & -e^{i\phi} \sin \theta \\ e^{-i\phi} \sin \theta & \cos \theta \end{pmatrix}.$$

The names "mirror" and "beam splitter" actually remind what happens as the observables are two independent directions of propagation, *not* of polarization. Such a distinction becomes irrelevant, as long as first there is another device (the so called polarizing beam splitter) which just separates the two eigenstates of polarizations into two orthogonal directions of propagation. Otherwise, this may be an abuse of language. In particular, a matrix acting on polarization states and formally written as $R(\theta) = B(\theta, 0)$ may not represent an actual mirror. Yet it can be formally recast as a rotation $R_y(\theta)$ by an angle 2θ around the y axis in the Bloch sphere. A rotation by an angle 2θ around the x axis in the Bloch sphere is represented by

$$R_x(\theta) = \begin{pmatrix} \cos \theta & -i \sin \theta \\ -i \sin \theta & \cos \theta \end{pmatrix}$$

and is formally the same matrix as $B(\theta, \pi/2)$ but may not represent a particular beam splitter because it does not act on directions of propagation.

A symmetric beam splitter is another particular device, represented for directions of propagations by

$$B_s = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix} = B(\pi/4, -\pi/2),$$

after which any incident power equally splits into transmitted and reflected parts; one formally gets $R_x(\pi/2) = -B_s^2$.

A symmetric beam splitter can also be used in order to check for the quantum nature of the field, through a Hong-Hou-Mandel experiment [8]: couples of identical photons tend to be detected together after passing the device, instead of being equally scattered in a classical fashion, as their initial relative delay goes to zero. In the case of economy such a test suggests that, after the corresponding agents, concurrent money does not scatter but converges in any subsequent direction. This suggestion is confirmed by the perception that the dispersion of monetary fluxes is usually controlled and limited by specific splitting actions, whereas its appearance is thought as a possible minor result due to defects.

A rotation by an angle 2θ around the z axis in the Bloch sphere can be written as follows:

$$R_z(\theta) = \begin{pmatrix} e^{i\theta} & 0 \\ 0 & e^{-i\theta} \end{pmatrix} = R_y(\pi/4) R_x(\theta) R_y(-\pi/4).$$

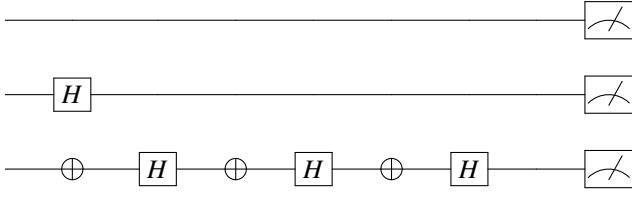


Fig. 1 From top to bottom, respectively: circuit representations of the measurements for $\sigma_{3,1,2}$, giving $n_{3,1,2}^{(\pm)}$ as binary counts.

In particular, $R_z(\pi/2) = iS$ where

$$S = \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix} = -iR_y(\pi/4)R_x(\pi/2)R_y(-\pi/4)$$

is the so-called phase gate.

An useful element of quantum circuits is the Hadamard gate, which is represented by a matrix

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} = \sigma_1 R_y(\pi/4)$$

acting on polarization states, where the gate $NOT = \sigma_1$ formally equals $-iB_s^2$ and is one among the Pauli matrices

$$\sigma_0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

All Pauli matrices can be obtained after unitarily transforming σ_3 , so that the corresponding eigenspaces refer to changes of basis with respect to the original one: $\sigma_{1,2} = U_{1,2}^\dagger \sigma_3 U_{1,2}$ where one can choose $U_1 = H$ and $U_2 = H\sigma_1 H\sigma_1 H\sigma_1$.

4 An algorithm for state tomography

The density matrix of the monetary state for global economy can be written as follows:

$$\rho = \frac{1}{2}(\sigma_0 + r_1\sigma_1 + r_2\sigma_2 + r_3\sigma_3) = \frac{1}{2} \begin{pmatrix} 1+r_3 & r_1 - ir_2 \\ r_1 + ir_2 & 1-r_3 \end{pmatrix}.$$

According to a tomographic point of view [9], in order to statistically reconstruct ρ , a fairly simple way is to count the numbers $n_{3,1,2}^{(\pm)}$ of respective outcomes ± 1 for the measurements after the following three operations (see fig. 1):

- directly detect the state without any gate before, so that $n_3^{(\pm)}$ counts outcomes of measuring σ_3 , whose eigenvalues are respectively just ± 1 ;
- precede such a detection by a Hadamard gate, so that $n_1^{(\pm)}$ respectively counts ± 1 as binary outcomes after measuring σ_1 ;
- precede the same detection by a triple sequence of a Hadamard gate and a conversion of the polarization, so that $n_2^{(\pm)}$ respectively counts ± 1 as binary outcomes after measuring σ_2 .

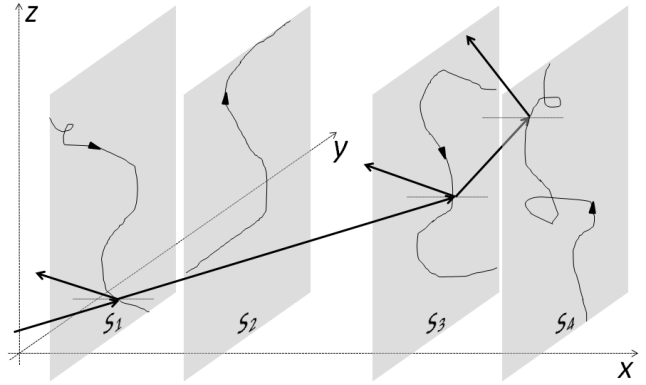


Fig. 2 In the abstract 3-dimensional spacetime given by allocations x , roles y and niches z , any agent's history is a trajectory in the corresponding surface ($S_{1,2,\dots}$) with fixed x . The flow of money (bold arrows) may affect different agents (on $S_{1,3,4}$ in the picture) or not (on S_2). When it does, it interacts like light does at an optical element (with reflection as on $S_{1,3,4}$ and possible refraction as on S_3), with or without deflection of the refracted ray according to whether the subsequent propagation speed is respectively changed or not. Each element is the current snapshot of an interface between two optical-like media for the money flux, provided it makes this change happen.

An acceptable estimate of ρ then comes from these components of a vector within the Bloch sphere:

$$r_{1,2,3} = \frac{n_{1,2,3}}{\max\left(1, \sqrt{n_1^2 + n_2^2 + n_3^2}\right)},$$

where

$$n_{1,2,3} = \frac{n_{1,2,3}^{(+)} - n_{1,2,3}^{(-)}}{n_{1,2,3}^{(+)} + n_{1,2,3}^{(-)}}.$$

5 From geometrical frame to optical-like tomography

The space-like coordinates of the reference system for economic actions may be thought of in various ways. According to a simple point of view they are just three, as anticipated:

- economic subjects correspond to numbers on an ordered axis, e.g. like alphanumeric codes; economic actions concern money being allocated on this x axis;
- each subject corresponds to a natural or legal person, whose role in society may be established or vary by actions; such a role is considered as a single y axis, whose order is given e.g. by levels of a societal ladder;
- each person lives in a niche, i.e. a framework where physical and human limits come into play beyond allocations of money (x) and social roles (y); therefore, after simplifying the niche as one-dimensional, a z axis is introduced whose order is a matter of choice (for instance, a direction along a road connecting various resources and subjects).

Such a frame allows one to envision what each circuit element may be, inspired by quantum optics and adapted to monetary dynamics in global economy. The units of the abstract axes provide the metric and then the angles of beams with respect to interfaces.

Any agent realizes the history of an economic subject and is therefore characterized by a trajectory in a slice with fixed x and variable (y, z) . In the abstract frame, any given transfer of money can meet a time series of agents. Each corresponding slice in the series has to be identified as an instantaneous optical element which deflects or transmits money (see fig. 2). The resulting optical-like media freely evolve according to the coupled dynamics of money and economic subjects, unless constraints are juxtaposed e.g. by societal and currency regimes within geographical boundaries. However, in order to estimate the instantaneous macrostate of money, an approach can build on what is statistically known about agents at the current time: their (x, y, z) coordinates and the way they possibly interact with the money flux as it impinges on them.

In terms of economy, a model of mirrors with negligible dissipation is given by non-profit services of money deposit. Profit would be the wealth left to the service and would be represented by the energy left while rays cross the mirror, with a dissipation that is proportional to the thickness and then to the delay due to the crossing. Mirrors hit at an angle $\theta/2 = \pi/4$ represent services whose intervention resides halfway (in an abstract spatial sense) between the act of depositing money and the event of getting it back by the same agent. Yet, more generally, the withdrawing agent may not be the same depositing: in such a case, the reflected money is sent by the service to another agent being on its actual trajectory, while the condition of the depositing one is not good for withdrawing. This is just a way to spread monetary wealth, through e.g. the banks managing savings (provided their profit margin is neglected or treated as a purely external factor for the process).

An economical counterpart of a beam splitter is an agent which exerts a power over money in a sense, by both letting it pass and deviating it away, towards other owners or further processes. In particular, symmetric beam splitters equally transmit and reflect money.

Polarizing beam splitters are useful in order to convert between states of polarization and states of direction in quantum optical circuits. They can represent economical agents which selectively transmit or reflect money according to its polarization between two orthogonal directions, i.e. two independent induced directions of oscillation for ethical-social values. A model for their implementation is given by binary regulatory constraints, which direct money according to its provable intended use. Inverse applications of polarizing beam splitters correspond to complete transductions of two independent directions of oscillation into a single out-

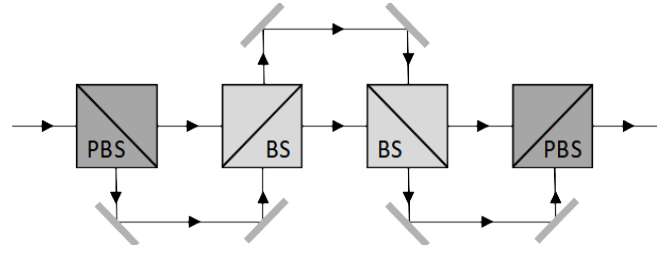


Fig. 3 A configuration for a *NOT* gate, with input from the left and output to the right, where each *BS* is a symmetric beam splitter and each *PBS* is a polarizing beam splitter, while mirrors hit at an angle $\pi/4$ are depicted by thin grey rectangles.

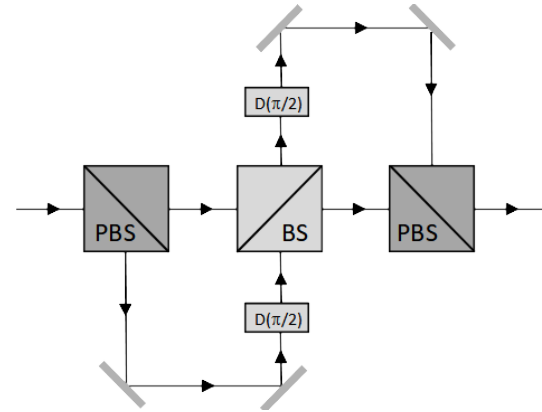


Fig. 4 A configuration for a Hadamard gate, with input from the left and output to the right; in addition to devices as in Fig. 3, each $D(\pi/2)$ is a $\pi/2$ retardation in phase of the second polarization component only (and is represented by S^+).

put monetary flow.

The relations $\sigma_1 = -iB_s^2$ and $H = S^+B_sS^+$, where S^+ is just a $\pi/2$ retardation in phase of the second polarization component only, suggest that suitable configurations implementing the *NOT* gate and the *H* gate are respectively the ones in Fig. 3 and Fig. 4 [10]. This is explicitly proved in the Appendix and is enough for what is required by the circuits in Fig. 1, where the final measurements consist in detecting polarization states at the end of the circuits. One should actually note that polarizing beam splitters can be just left at the beginning and the end of sequences, instead of being replicated for every single gate.

6 Issues in reconstructing global monetary states

As a matter of principle, one can think of populating a global inventory with services of money deposits (represented by mirrors), redistributing agents (beam splitters) and regulatory constraints (polarizing beam splitters). According to Fig. 1, among all monetary flows possibly intercepting these, three are selected for tomography: those which are not affected by any agent of the inventory (to get n_3), those which just interact with any single system like the one in Fig. 4 (to get n_1),

those which intercept a triple sequence of two systems like the ones in Fig. 3 and Fig. 4 (to get n_2).

The trajectory of monetary quanta in the abstract space cannot be exactly detected, because one should trace back from observations in the real economy to the coordinates of involved agents and anyway because of the quantum uncertainty principle. It may be even more difficult to add the recognition of their polarization, as moreover one should detect *induced* oscillations of ethical-social values; if the matter is to distinguish any intended use of money according to the oscillation it is expected to induce, then this is just a proxy. In particular, any regulatory constraint miming a polarizing beam splitter should distinguish the relative orientation of oscillations with respect to the propagation of money in the abstract space, so that two sources of uncertainty add up. This reflects the difficulties met by institutions as they try to direct monetary fluxes according to expected effects. Yet polarizing beam splitters are at the beginning and end of the circuits in Fig. 3 and Fig. 4, so that their suggested use for tomography needs to trust institutions and statistical reports (official or unofficial ones, choosing what is considered as reliable) about the flux of money and the induced transverse effects in a representation of the three abstract coordinates. Money actually flows in wavepackets, that are superpositions of modes: single-frequency monetary quanta are not actually observable. Like other quantum fields, their dynamics is to be viewed in terms of cross sections, depending on frequency. The latter is proportional to the amount of global wealth carried by money, with a factor like the Planck constant to be empirically quantified [2]. This is why statistics should also estimate the various contributions by money to global wealth, beyond approximately detecting its abstract trajectories and induced oscillations.

Under the very restrictive assumption that money flows along x axis (i.e. involving socially equal agents and being confined to a single niche), an obvious choice for two independent linear polarizations just regards oscillations of values in y and z . For instance, if the world were a global village without hierarchy, then money would anyway induce social and natural oscillations of values. Sometimes the first happen (money brings a temporary variation of rights to the subject it is interacting with), sometimes the second ones do (as before, but regarding natural needs); statistics estimate their relative weight. One may think of a regulatory constraint, which distinguishes between the two cases and correspondingly directs money into two different channels. Symmetric redistributing agents may be fed by both channels or their combinations, possibly preceded or followed by delays. For instance, forcing one quarter-period delay for the oscillations of rights before and one after such an agent, while letting oscillations of needs without retardation, the central part in Fig. 4 is implemented.

Among the main tasks carried out by a large-scale public

administration, there is the one of directing investments for a planned mobilisation of rights and duties or goods and needs. This is a kind of realization for a polarizing beam splitter. Its corresponding inverse task is to control the flux of money among agents through taxation, gathering it for later global uses. Here taxation is intended in a broad sense, i.e. it includes all contributions by agents to the administration of global economy, apart from the strictly subjective ones.

A rough but quite reasonable ansatz is that self-employed or unemployed agents implement symmetric beam splitters, in that they equally and spontaneously split money to manage induced oscillations of rights and needs. Whenever a social perturbation occurs, needs may be unchanged but there is a delay for both input and output of rights. This perturbation possibly takes the form of a contract before and a remuneration after the interaction characterizing the agent. Of course, in order to comply with the very restrictive assumption above, it should not force any significant hierarchy.

After tax collection, economic policies by public administrations implement measurements of polarization through subsequent investments: it is estimated by the relative monetary weight of promoting rights (while controlling fulfillment of duties) and the acquisition or the production of goods (while addressing needs) for the people.

Hence there would be a chance for an affordable reconstruction of the global monetary state by tracking selected effects of policies, yet in a very restrictive set of hypotheses. Of course, actual economy does not work through national policy only, at least because administrations are mainly tied to the international monetary governance and dynamics of resources are often worldwide.

When money is invested by an administration but is not actually expended, it is ready to repay debts or for future investments. The former offset each other in the world economy and are then neglected as global amounts, while the latter may go to govern the game of goods and demands (polarization denoted by the $(+)$ index) or the game of right and duties (according to the $(-)$ index). Single acts of direct reinvestment policy give intentional nudges to global economic dynamics, so that quantum states veer to the wanted balance of $n_3^{(+)}$ and $n_3^{(-)}$, while obviously competing with unintended dynamics. The same happens when remainders come to the administration from more complex chains of agents, such as the ones related to $n_{1,2}^{(\pm)}$.

An ansatz is to consider the expenditure for maintaining public administration as the only way to estimate the $(-)$ polarization, since such an administration is just based upon the governance of rights and duties, while the game of goods and demands is the core of the global market. Furthermore, actual polarization may be indistinguishable among remainders coming from different chains (public investments are typically oriented no matter which chains money is collected

from):

$$\frac{n_1^{(-)}}{n_1^{(+)} + n_1^{(-)}} = \frac{n_2^{(-)}}{n_2^{(+)} + n_2^{(-)}} = \frac{n_3^{(-)}}{n_3^{(+)} + n_3^{(-)}} = \varepsilon.$$

This ratio is roughly the fraction of total fiscal revenue devoted to expenditure for public institutions. The further extreme ansatz of taking Italy as a good sample for the world economy is considered here, just to give an idea of the method. Then [11] one obtains $\varepsilon \approx 6 \cdot 10^{10} / 1.1 \cdot 10^{12} \approx 5.5\%$, so that $n_{1,2,3} \approx 89\%$, which provides $\vec{r} \approx (0.58, 0.58, 0.58)$.

If a separation is forced among polarizations after different chains, then the $r_{1,2,3}$ may not be all equal. For instance, in the case of an economy where reinvestments for maintaining the public administration weigh on it and its employees only, $n_3^{(-)} = 0$ and the same estimate

$$\frac{n_1^{(-)}}{n_1^{(+)} + n_1^{(-)}} = \frac{n_3^{(-)}}{n_3^{(+)} + n_3^{(-)}} \approx 5.5\%$$

would give $\vec{n} \approx (0.89, 0.89, 1)$ and then $\vec{r} \approx (0.55, 0.55, 0.62)$.

7 Outlook towards general dynamics of global economy

Tomography can be applied to estimate the quantum state of subjectivity too, just like the one of the matter interacting with photons in optics (in this case, the Hilbert space has typically dimensions $d > 2$ and so qudits are considered, even with need for a machine learning algorithm in order to face exponential scaling [12][13]). It can be used to identify the dynamical process as well [14]: one gets an estimate of the Hamiltonian operator, which is used to predict the unitary evolution of global economy as a closed system.

Turning to the dynamics of economy as an open system, reduced from its environment, this can be expressed by a Lindblad-like equation in a correlation picture [15]. State and process tomography (the former as a diagnosis, the latter as evolving prognosis [16]) can provide the necessary input for that.

Research could collect better data for quantum state tomography of the global economy, apply it for monetary and subjective fields, prognose evolution to be actively changed by policy, outline economical scenarios correlated to the human and natural environment.

Appendix: Explicit proof of Fig. 3 and Fig. 4

Any optical-like interaction between money and agents occurs on a plane of the abstract space with spatial orthogonal coordinates ξ and χ . So four independent observables are suitable to characterize the quantum states upon which gates act: two directions of polarization (leading to the eigenstates

$|h\rangle$ and $|v\rangle$) and two directions of propagation (leading to the eigenstates $|\xi\rangle$ and $|\chi\rangle$, with conventionally $\xi > 0$ rightwards and $\chi > 0$ downwards). All devices of the interaction are represented by 4×4 matrices acting on corresponding column vectors $(a \ b \ c \ d)^T$, whose components are the coefficients in $a|\xi\rangle|h\rangle + b|\xi\rangle|v\rangle + c|\chi\rangle|h\rangle + d|\chi\rangle|v\rangle$. As input, consider a kind of generic horizontal state $a|\xi\rangle|h\rangle + b|\xi\rangle|v\rangle$. A polarizing beam splitter

$$PBS = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

sends it to a state $a|\xi\rangle|h\rangle + b|\chi\rangle|v\rangle$, because the component along $|h\rangle$ is transmitted and the one along $|v\rangle$ is reflected to the orthogonal direction. A couple of mirrors $R^2(2(\pi/4)) = R(\pi)$, hit at angles $\pi/4$ and acting on the subsequent vertical component, leads to the state $a|\xi\rangle|h\rangle - b|\chi\rangle|v\rangle$. After applying a symmetric beam splitter BS that just acts upon the two entries regardless of any distinction between propagation and polarization, one obtains $[(a - ib)/\sqrt{2}]|\xi\rangle|h\rangle + [(ia - b)/\sqrt{2}]|\chi\rangle|v\rangle$. A couple of mirrors $R(-\pi)$ sends it to $[(a - ib)/\sqrt{2}]|\xi\rangle|h\rangle + [(b - ia)/\sqrt{2}]|\chi\rangle|v\rangle$. After applying a BS rotated by $R(-\pi/2)$, one gets $b|\xi\rangle|h\rangle - a|\chi\rangle|v\rangle$. A couple of mirrors $R(\pi)$ leads to $b|\xi\rangle|h\rangle + a|\chi\rangle|v\rangle$. A final polarizing beam splitter $PBS^{-1} = PBS$ sends the state to $b|\xi\rangle|h\rangle + a|\xi\rangle|v\rangle$, that is just the result of converting polarization without changing the direction of propagation. This is why a *NOT* gate is configured by the sequence in Fig. 3. Again consider the generic horizontal input $a|\xi\rangle|h\rangle + b|\xi\rangle|v\rangle$, splitted by a PBS . A couple of mirrors $R(\pi)$, hit at angles $\pi/4$ and acting on the subsequent vertical component, sends the state to $a|\xi\rangle|h\rangle - b|\chi\rangle|v\rangle$. A quarter period delay of the same component leads to $a|\xi\rangle|h\rangle - ib|\chi\rangle|v\rangle$. After applying a BS , one gets $[(a + b)/\sqrt{2}]|\xi\rangle|h\rangle + [i(a - b)/\sqrt{2}]|\chi\rangle|v\rangle$. After applying another quarter period delay of the subsequent vertical component, one obtains $[(a + b)/\sqrt{2}]|\xi\rangle|h\rangle + [(b - a)/\sqrt{2}]|\chi\rangle|v\rangle$. A couple of mirrors $R(-\pi)$, acting on the vertical component, leads to the state $[(a + b)/\sqrt{2}]|\xi\rangle|h\rangle + [(a - b)/\sqrt{2}]|\chi\rangle|v\rangle$. A final $PBS^{-1} = PBS$ sends it to $[(a + b)/\sqrt{2}]|\xi\rangle|h\rangle + [(a - b)/\sqrt{2}]|\xi\rangle|v\rangle$, that is just what results from the application of an Hadamard gate. This justifies the sequence in Fig. 4.

Conflict of interest

The author declares that he has no conflict of interest.

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