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## Parameter estimation for a degradation-based imperfect maintenance model with different information levels

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In this article, technological or industrial equipment that are subject to degradation are considered (e.g. a heat exchanger in power plants). These units undergo maintenance actions which reduce their degradation level. In this work, a degradation model is considered with imperfect maintenance effect. The underlying degradation process is a Wiener process with drift (see e.g. Kahle (2019)). The maintenance effects are described with an Arithmetic Reduction of Degradation ( $ARD_1$ ) model. This model is inspired by the  $ARD_1$  model studied in Mercier and Castro (2019) with a Gamma process. The system is regularly inspected and the degradation levels are measured.

Let  $\{X(t)\}_{t \geq 0}$  be the underlying degradation process without maintenance actions so that  $X(t) = \mu t + \sigma B(t)$  where  $B(t)$  is a standard Brownian motion.  $\{Y(t)\}_{t \geq 0}$  is the degradation process of the maintained system.  $\forall j \in \{1, \dots, k\}$ ,  $\tau_j$  is the  $j^{\text{th}}$  maintenance time and  $Z_j$  is the jump in the degradation level due to maintenance, observed at time  $\tau_j$ . The  $ARD_1$  model assumes that the effect of a maintenance is to reduce the degradation level of a quantity proportional to

the amount of degradation accumulated since the last maintenance. Therefore, for  $t \in [\tau_j, \tau_{j+1}[$ ,  $Y(t)$  is defined as

$$Y(t) = X(t) - \rho X(\tau_j) \quad (1)$$

Thus, the observed jumps are

$$Z_j = -\rho(X(\tau_j) - X(\tau_{j-1})) \quad (2)$$

where  $\rho$  is the maintenance efficiency parameter.

In the studies of degradation models with imperfect maintenance, degradation levels are generally observed just before maintenance actions (see e.g. Salles et al. (2020)). Here four different observation schemes are considered so that degradation levels can be observed between maintenance actions as well as just before or just after maintenance times. In each scheme, degradation levels between successive maintenance actions are observed. At maintenance times, in the first observation scheme, degradation levels just before and just after each maintenance action are observed. In the second scheme, degradation levels just after each maintenance action are not observed but are observed just before. On the contrary, in the third scheme, degradation levels just before the maintenance actions are not observed but are ob-

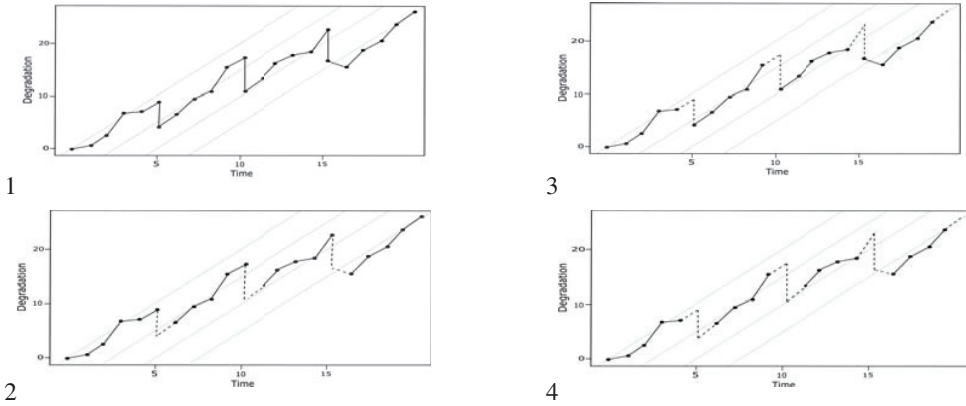


Fig. 1. The degradation levels according to the four different observation schemes

served just after. Finally, in the fourth observation scheme, the degradation levels are not observed neither just before nor just after the maintenance action.

In figure 1, four trajectories of the degradation process are represented according to the four observation schemes. Three maintenance actions are performed periodically. Every point represents an observed degradation level and the missing observations are defined by the dotted lines.

This work proposes a method to estimate the model parameters under the different observation schemes. For each scheme, a general expression of the likelihood is

$$L(\theta) = \left[ \prod_j \prod_i f_{\Delta Y_{j,i}}(\Delta y_{j,i}) \right] \prod_j f_{Z_j^m | \mathcal{H}_{\tau_j}^m}(z_j^m) \tag{3}$$

where  $\theta = (\mu, \sigma^2, \rho)$ ,  $f_X$  is the density of  $X$ ,  $\Delta y_{j,i}$  are the increments of degradation and  $\mathcal{H}_{\tau_j}^m$  is the history of the observed degradation process for the  $m^{th}$  observation scheme. The increments densities are easy to write as the increments are independent and normally distributed. The difficulty consist in computing the distributions of the observed jumps  $Z_j^m$  conditionally to the past. These distributions are found by using the  $ARD_1$  model's definition and the Wiener process properties.

The maximum likelihood estimators are derived for each scheme. Several situations are studied in

order to assess the impact of different features on the estimation quality. Among them, the number of observations between successive maintenance actions, the number of maintenance actions, the maintenance efficiency parameter and the location of the observations are considered. These situations are used to assess the estimation quality and compare the observation schemes through an extensive simulation and performance study.

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