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1 Investigating the typicality of the dynamics
2 leading to extreme temperatures in the
3 IPSL-CM6A-LR model

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15 **Abstract**

16 Determining the underlying mechanisms leading to extreme events in
17 dynamical systems is a challenging task. Under mild hypotheses, large
18 deviations theory predicts that as one increases the threshold defining an
19 extreme, dynamical trajectories which reach the extreme will look more
20 and more like one another: they converge towards a typical, i.e. most
21 probable, one called the instanton. In this paper, we use a 2000-year sim-
22 ulation of the IPSL-CM6A-LR model under a stationary pre-industrial
23 climate to test this prediction on the case of hot extremes. We investi-
24 gate whether the physical mechanisms leading to extreme temperatures
25 at four locations in Europe are more similar with increasing extreme
26 temperatures. Our results show that most physical variables exhibit
27 the expected convergence towards a most probable trajectory, with
28 some geographical and temporal variations. In particular, we observe
29 the presence of a cut-off low in some trajectories, which suggests the
30 existence of multiple pathways leading to extreme temperatures. These
31 findings confirm the relevance of instanton dynamics in understanding
32 the physical mechanisms driving extreme events in climate models.

Keywords: extreme events, large deviations theory, instanton, typicality, atmospheric dynamics

1 Introduction

Extreme weather events can have tremendous impacts on societies and ecosystems (Pörtner et al, 2022). Among them, heatwaves have been the focus of extensive attention due to their increasing frequency with global warming (Seneviratne et al, 2021). Their impacts include adverse health effects, increased power consumption, infrastructure damages, forest fires, droughts and crop failures (Koppe et al, 2004; Zuo et al, 2015; Yaghmaei, 2020). In the last decade, extensive research has been conducted to better understand the dynamics leading to heatwaves (Perkins, 2015; Horton et al, 2016; Domeisen et al, 2022a). The general scenario combines specific atmospheric synoptic conditions and anomalously low soil moisture, which can interact to further increase the intensity of the event.

Heatwaves in the midlatitudes are usually associated with a slow moving, sometimes called quasi-stationary, high-amplitude Rossby wave (Petoukhov et al, 2013). This structure is often embedded in a hemispheric pattern of wave patterns 5-7, which can trigger extreme heat and rainfalls simultaneously at different places (Coumou et al, 2014; Kornhuber et al, 2020; Di Capua et al, 2021). The mechanisms and causes of the amplification of such wave patterns are still discussed, especially their dependence on climate change (Screen and Simmonds, 2014; Petoukhov et al, 2016; Kornhuber et al, 2017; Mann et al, 2017, 2018; Kornhuber and Tamarin-Brodsky, 2021). Over the heatwave region, an anticyclone builds up — a situation called 'blocked' — at mid- and upper-level troposphere in conjunction with a change in the jet stream's climatological path, towards a large poleward meridional meander. The anticyclone sustains the poleward advection of warm air along its western flank, adiabatic warming by subsidence and clear skies at its center. Those conditions favor warming by short-wave insolation, especially at the peak of the seasonal cycle. Close to the ground, a positive feedback loop is initiated as anomalously dry soils favor the partition of incoming solar energy into sensible rather than latent heat, which enhances surface evaporation and may reinforce the anticyclonic structure (Hirschi et al, 2011; Miralles et al, 2012, 2014; Rasmijn et al, 2018; Dirmeyer et al, 2021).

These mechanisms all played a role in the record-breaking heatwaves of 2003 in western Europe (García-Herrera et al, 2010) and 2010 in Russia (Dole et al, 2011; Otto et al, 2012; Trenberth and Fasullo, 2012; Di Capua et al, 2021). The same mechanisms are also present in the exceptionally intense heatwave that occurred in Western North America in June 2021. Previous records of temperature were broken by up to 5°C (Philip et al, 2021) making it one of the most intense heatwaves ever recorded (Thompson et al, 2022). Due to its

74 exceptional nature, this event triggered extensive research. Dry soils (Zhang
75 et al, 2021, 2022) likely combined with an intense omega blocking anticyclone
76 resulting from a wave breaking event, associated with southern excursion of
77 the polar vortex (Overland, 2021; Neal et al, 2022), and interacting with an
78 atmospheric wave emanating from the tropical Pacific (Bartusek et al, 2022).
79 Some authors also suggested the role of latent heat release through moisture
80 advection by an unusual atmospheric river through the North Pacific (Qian
81 et al, 2022; Lin et al, 2022; Mo et al, 2022). Lucarini et al (2022) however show
82 using a long simulation in a pre-industrial climate that this event was *typical*
83 with respect to other intense events simulated by the model at this location.

84 Extreme Value Theory (EVT) has been used to determine statistical
85 models for maxima (or minima) of climate variables of interest (typically tem-
86 perature or precipitation) (Ghil et al, 2011). EVT is based on a convergence
87 principle of the probability distribution of maxima or peaks-over-threshold
88 (Coles et al, 2001). It allows to compute return values corresponding to very
89 large return periods (i.e. longer than the period of observations), even in non
90 stationary contexts (Cheng et al, 2014). In practice, EVT has been an efficient
91 framework to estimate statistical variations on short lived extremes (e.g. the
92 highest daily temperature). In principle multi-variate EVT provides a frame-
93 work to investigate events that combine several variables (e.g. temperature,
94 precipitation and the atmospheric circulation) (Tawn, 1990), but such analyses
95 are not designed to investigate long lasting events, for which the time persis-
96 tence is a key factor in the extreme (although using the so-called extremal
97 index partially alleviates this issue, e.g. Moloney et al (2019)).

98 Large Deviations Theory (LDT) is however the key statistical frame-
99 work employed in statistical physics (Touchette, 2009) and has begun to gain
100 momentum in climate sciences (Galfi et al, 2021). In contrast to EVT, it pro-
101 vides asymptotic laws at the leading exponential order for extremes of sums
102 of random variables. One of its applications is the Freidlin-Wentzell theory of
103 large deviations in random dynamical systems (Freidlin and Wentzell, 1987).
104 This theory studies the dynamics leading to extremes by describing it as
105 the optimal noise pushing the system in the direction leading to extremes.
106 In particular, it predicts that, under mild hypotheses, the dynamics leading
107 to extremes of any observable concentrates around a single most probable
108 trajectory, usually called the *instanton* (Chetrite and Touchette, 2015; Demat-
109 teis et al, 2019a; Grafke and Vanden-Eijnden, 2019). This framework can be
110 extended to systems without any explicit stochasticity, where it amounts to
111 finding the "optimal" initial conditions (Dematteis et al, 2019b; Lucarini et al,
112 2022) which will lead the dynamics to reaching extremes of an observable under
113 the hypothesis that the observable admits a large deviations principle at its
114 right (or left) tail.

115 In this paper, we expand the work of Galfi and Lucarini (2021), Galfi
116 et al (2021) and Lucarini et al (2022) who pioneered the use of LDT to study
117 extreme events in geophysical systems. We also investigate the prediction of
118 LDT concerning the concentration of trajectories leading to extremes of an

119 observable around a most probable one in a dynamical system. We address the
 120 question whether the dynamics leading to extreme 2m air temperature events
 121 is *typical* in a long simulation of a climate model.

122 As a case study, we take the 2000 years pre-industrial control run of
 123 the IPSL-CM6A-LR Earth System Model (ESM) (Boucher et al, 2020). We
 124 examine the dynamic characteristics of trajectories leading to heatwaves as
 125 their duration and strength vary. We evaluate how typical the heatwaves are
 126 by examining the key dynamic factors that contribute to their formation.
 127 Lastly, we study how the dynamics change when the location from which the
 128 heatwaves are observed is altered.

129 This paper is organized as follows: in Section 2, we detail the mathemat-
 130 ical paradigm of large deviations theory in dynamical systems and make our
 131 assumptions explicit. In Section 3, we introduce the methodology employed to
 132 isolate the dynamics leading to extremes and the normalized variance metrics
 133 used to measure convergence. Section 4 presents the results of the analysis,
 134 focusing first on one observable and then on three observables at different loca-
 135 tions. Finally, the discussion of the results and the conclusions drawn from our
 136 analysis are presented in section 5.

137 2 Mathematical preliminaries

138 The principle of convergence of trajectories around a most probable one for
 139 extreme events, the so-called instanton, can be presented either in systems with
 140 an explicit stochasticity — where it is equivalent to finding an "optimal" noise
 141 to perturb the system (Freidlin and Wentzell, 1987; Chetrite and Touchette,
 142 2015; Dematteis et al, 2019a; Grafke and Vanden-Eijnden, 2019) — or with
 143 a deterministic evolution — where it is equivalent to finding the "optimal"
 144 initial conditions (Dematteis et al, 2019b; Lucarini et al, 2022). There is no
 145 explicit stochasticity in the climate model we study, therefore we follow here
 146 the presentation by Lucarini et al (2022).

147 We consider a chaotic dynamical system evolving continuously in time. Let
 148 $x(t) \in \mathbb{R}^d$ be the state vector at time t , where d is the number of dimensions
 149 needed to describe the system. We assume that x evolves according to the
 150 following ordinary differential equation

$$\frac{dx}{dt} = b(x) \quad (1)$$

151 where $b : x \mapsto b(x) \in \mathbb{R}^d$ defines the dynamics of the system. In the fol-
 152 lowing we always assume that the transients have died out and all trajectories
 153 considered belong to the attractor of the system. We assume that there is a
 154 unique physical invariant measure μ on this attractor. We are interested in the
 155 statistics of observables computed on the attractor. Observables are smooth
 156 functions of phase-space variables $F : \mathbb{R}^d \rightarrow \mathbb{R}$ which have a physical interest.

157 Our observables will be of the following type:

$$F_r(x_t) = \frac{1}{r} \int_{-r/2}^{r/2} f(x_{t+t'}) dt' \quad (2)$$

158 where f is also an observable and $r \in \mathbb{R}$ is the size of the rolling mean
 159 window. In the following, f is the daily mean temperature over a specific area.
 160 For $q \in \mathbb{R}$, we define the set Ω_q as the set of points x on the attractor such that
 161 $F_r(x) \in [q, q + dq]$. In other words, we are interested in the states of the system
 162 for which the value of the observable F_r is close to q , where q will correspond
 163 to extreme quantiles of the distribution of F_r . We now explicitly assume that
 164 F_r admits a large deviations principle, i.e. the set Ω_q is exponentially rare with
 165 respect to the measure μ with increasing q . Then, one can write:

$$\mathbb{P}(F_r(x_t) = q) = \mu(\Omega_q) \asymp \exp\left(-\min_{x \in \Omega_q} I(x)\right) \quad (3)$$

166 where \asymp means that the ratio of the logarithms of both sides tends to unity
 167 when $q \rightarrow +\infty$. Here q plays the role of the large deviations parameter even
 168 though it does not appear explicitly as such in Eq. (3). The functional I is
 169 called the rate function and is given by the Legendre transform (Dematteis
 170 et al, 2019b):

$$I(x) = \max_p (\langle p, x \rangle - S(p)) \quad (4)$$

171 of $S(p) = \log \mathbb{E}_\mu(e^{\langle p, x \rangle})$, the cumulant generating function of x under the
 172 measure μ with $\langle \cdot, \cdot \rangle$ the inner product in \mathbb{R}^d .

173 When $q \rightarrow +\infty$ in Eq. (3), the probability is exponentially dominated by
 174 the point \hat{x} which minimizes the rate function: $\hat{x} = \underset{x \in \Omega_q}{\operatorname{argmin}} I(x)$. The minimizer
 175 \hat{x} represents physically the state with the maximum likelihood of realization.
 176 Under the stated conditions, the probability measure accumulates near \hat{x} : this
 177 *most likely* of the *least likely* states is usually called the *instanton*.

178 The instanton formalism gives powerful results about the paths in the phase
 179 space leading to extreme events in dynamical systems. If the observable admits
 180 a large deviations principle, then the paths leading to any extreme are domi-
 181 nated exponentially in probability by one unique path. One should note that
 182 in general the uniqueness is not fully guaranteed because the rate function I
 183 may have several minima. The intuition behind this result is the following:
 184 because extremes of an observable are rare (i.e. have low probability), the sys-
 185 tem has few paths in the phase space to reach those extremes. If the system
 186 had many paths to reach the extremes, then these would not be rare. This
 187 idea of uniqueness of limit behaviors in a dynamical system is reminiscent of
 188 the uniqueness of the limit distribution in EVT (Coles et al, 2001).

189 This mathematical derivation suggests that if we examine the extreme daily
 190 temperatures at a particular location, the paths that lead to these extremes
 191 are to converge around a single path. We investigate this prediction in the
 192 following. To do so, we employ the instanton filtering procedure introduced
 193 by Grafke et al (2013). It consists of averaging independent events x taken

194 from a long simulation of the dynamical system for which the value of the
 195 observable $F_r(x)$ is close to an extreme level. According to the theory presented
 196 above, this procedure leads — up to a good approximation — to the instanton
 197 reaching the extreme level of the observable F_r . In other words, we investigate
 198 the typical state $\mathbb{E}[x \mid F_r(x) = q]$ conditional on $F_r(x) = q$ where q is a given
 199 extreme level of the distribution of F_r .

200 Contrary to [Lucarini et al \(2022\)](#), we do not consider events above a certain
 201 level $\mathbb{E}[x \mid F_r(x) \geq q]$. The rationale of this choice is twofold. First we want
 202 to investigate the convergence properties for different values of q for a fixed
 203 number of independent events in each case. Second, large deviations theory
 204 predicts that for sufficiently extreme values of q the events for which $F_r(x) \sim q$
 205 are exponentially favored compared to the ones with $F_r(x) > q$, but we cannot
 206 a priori rule out the possibility of a non linear response of the dynamics $\mathbb{E}[x \mid$
 207 $F_r(x) = q]$ with respect to q when reaching extremes. Therefore, the dynamical
 208 mechanisms needed to reach high temperatures may be different than the ones
 209 to reach *very* high temperatures.

210 3 Data and methods

211 We use the output of the pre-industrial control run of the IPSL-CM6A-LR
 212 ([Boucher et al, 2020](#)) model in the context of the CMIP6 intercomparison
 213 project ([Eyring et al, 2016](#)). This simulation is 2000 years long and represents
 214 a stationary climate with a CO₂ concentration corresponding to pre-industrial
 215 levels. The model has an horizontal atmospheric resolution of 2.5° in longi-
 216 tude and 1.3° in latitude. During the 2000 years, the global mean 2 meter air
 217 temperature of the Earth slightly drifts by 0.25K but we will assume that this
 218 drift can be neglected when it comes to studying extremes in the midlatitudes,
 219 because we are interested in deviations that can exceed several K. We use the
 220 sea level pressure (SLP), upper level soil moisture (SM), air temperature at
 221 2m (T2M), air temperature at 850hPa (T850), geopotential height at 500hPa
 222 (Z500) and the meridional component of the wind at 250hPa (V250). These
 223 variables are used with a daily frequency. Due to a technical issue, approxi-
 224 mately 1/4 of the data for the Z500 variable are missing. In the following, we
 225 therefore present results for Z500 only for the dates available in our data set.
 226 We have checked that when restricting to the period when we have the Z500
 227 variable available for the other variables do not affect the results. We therefore
 228 present the results for those other variables over the whole 2000 years.

229 In the following, we consider four observables derived from T2M: T2M at
 230 three grid points situated in Southern (38-39°N, 5.25-3.75°W), Western (49-
 231 50°N, 1.25-3.75°E) and Northern Europe (59-60°N, 13.75-16.25°E) and T2M
 232 averaged over a region extending in Western and Central Europe (46-53.5°N,
 233 0-25°E). These observables are named respectively S, W, N and WCE. We
 234 want to investigate the highest values reached by these observables, therefore
 235 we restrict the analysis to the three months of the meteorological summer:
 236 June, July and August (JJA). Except specified explicitly, we do not consider

237 detrended or deseasonalized variables. To investigate how the dynamics can
 238 change for longer events, we consider the extremes of the observable after
 239 applying a rolling mean window of $r = 1, 5$ and 15 days. The size of these
 240 windows were chosen to investigate both short and long lasting heatwaves
 241 events.

242 We consider the time series of one given observable F_r among S, W, N and
 243 WCE regions during the summer months for a given rolling mean window r .
 244 For a quantile q_α of a given order α of the empirical distribution of F_r , we
 245 select the pool of the $n = 50$ independent events x_i for which the values of
 246 their observable $F_r(x_i)$ are the closest from the value of the quantile q_α . In
 247 other words, we find the dates of the n -nearest neighbors of the quantile q_α
 248 of the observable. These events are searched over any of the days in the JJA
 249 months. To ensure that these events are independent one from another, we
 250 impose that for a nearest neighbor to be chosen, it must be separated by more
 251 than 15 days from any nearest neighbor already present in the pool. The choice
 252 of this timescale was made with regards to the typical chaotic timescale of the
 253 atmosphere (around 10 days). Our procedure is equivalent to defining the set
 254 Ω_q of section 2 as:

$$\Omega_{q_\alpha} := \Omega_\alpha = \{x \mid F_r(x) \in [q_\alpha - \eta, q_\alpha + \eta]\}$$

255 for η as small as possible to ensure that the number of elements in Ω_α
 256 is $\#\Omega_\alpha = n$. We consider the quantiles of order $\alpha = 0.75, 0.95, 0.99$ and
 257 0.999 of the empirical distributions of observables S, W, N and WCE. These
 258 quantiles are arbitrary and are chosen to ensure homogeneity between the
 259 different observables for which the extreme values may be very different.

260 Figure 1 presents the histograms of the empirical distributions of the four
 261 observables (rows) for the different rolling mean windows (columns). The ver-
 262 tical lines in color present the quantiles for the different orders considered. The
 263 horizontal lines of the same colors present the spread η of the observables for
 264 the points found with our procedure. For the smallest orders ($0.75, 0.95$ and
 265 0.99), the spread is barely visible, which means that the values of the observ-
 266 able for points in Ω_α are very close to the value of the quantile. For the order
 267 0.999 , the spread is bigger but still small compared to the standard deviation
 268 of the full empirical distribution.

269 For each value of the α -th order quantile, a given observable F_r and a
 270 rolling mean window r , we therefore consider fields (SLP, SM, T2M, T850,
 271 Z500 and V250) $\psi_{\alpha,r,F_r}(\phi, \theta, t, m)$ that have four components: the longitude ϕ
 272 and latitude θ , the time t — where the time is expressed relative to the day
 273 t when the observable is such that $F_r(x_t) \simeq q_\alpha$ —, and the number m among
 274 the points in Ω_α ($m = 1, 2, \dots, n$). In the following we skip the α, r, F_r indices
 275 for simplicity, but note that the quantities introduced are always relative to
 276 a given value of this triplet. For any field ψ , we use the term composite to
 277 denote the average \widehat{A}_ψ over the points in Ω_α and the rolling mean window r :

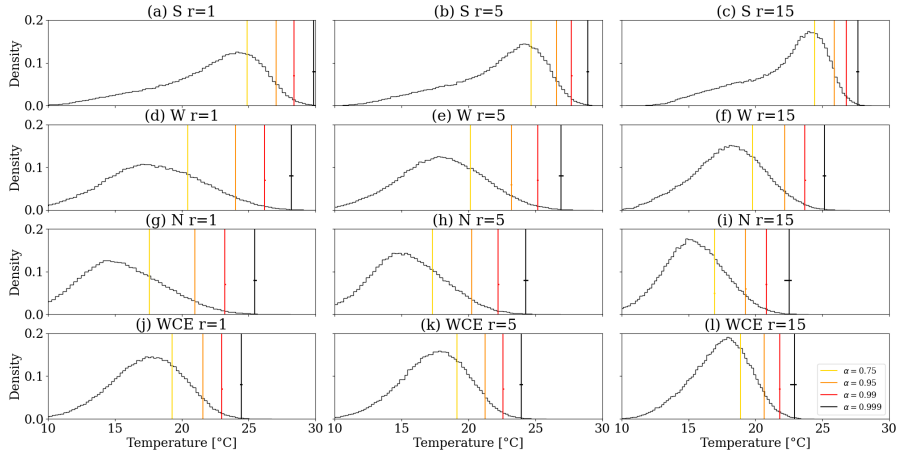
8 *Typicality extreme heatwaves*

Fig. 1 Histograms of the observables and associated quantiles. In rows: observables derived from the T2M variable at three grid points situated in Southern Europe (S: 38–39°N, 5.25–3.75°W), Western Europe (W: 49–50°N, 1.25–3.75°E) and Northern Europe (N: 59–60°N, 13.75–16.25°E) and over one grid box extending in the North-West Europe (NWE, 46–53.5°N, 0–25°E). In columns: rolling mean windows for the computation of the observable of $r = 1, 5$ and 15 days. The vertical bars show the value of the quantiles of order α of the time series for $\alpha = 0.75, 0.95, 0.99$ and 0.999 . The horizontal lines show the spread of the observables for the points in Ω_α .

$$\widehat{A}_\psi(\phi, \theta) = \frac{1}{n} \sum_{m=1}^n \frac{1}{r} \sum_{t=-r/2}^{r/2} \psi(\phi, \theta, t, m) \quad (5)$$

To measure the clustering of these points, we define two normalized variances:

$$\widehat{V}_\psi(\phi, \theta) = \text{Var}_m \left[\frac{1}{r} \sum_{t=-r/2}^{r/2} \psi(\phi, \theta, t, m) \right] / \text{Var}_{\psi, clim}(\phi, \theta) \quad (6)$$

and:

$$\widetilde{V}_\psi(\phi, \theta, t) = \text{Var}_m [\psi(\phi, \theta, t, m)] / \text{Var}_{\psi, clim}(\phi, \theta) \quad (7)$$

278 where $\text{Var}_{\psi, clim}(\phi, \theta)$ is the climatological variance of the field ψ considered
 279 over the whole summer (JJA). Both of these variances are normalized in the
 280 sense that they are compared to the variance observed at a specific location
 281 over the whole summer. In the following, they are thus expressed in percentage.
 282 The lower the value of this percentage, the smaller the variance between the
 283 points in Ω_α either over the rolling mean period (\widehat{V}_ψ) or at a specific time (\widetilde{V}_ψ).
 284 If we were to select n days randomly over the full data set, we could expect to
 285 find a variance close to $\text{Var}_{\psi, clim}(\phi, \theta)$. Therefore, the closer $\widetilde{V}_\psi(\phi, \theta, t)$ is to
 286 one (or 100%), the less specific is the dynamics of the averaged field $\widehat{\psi}(\phi, \theta, t) =$
 287 $\frac{1}{m} \sum_{m=1}^n \psi(\phi, \theta, t, m)$ at (ϕ, θ, t) . The closer it is to zero, the more concentrated

288 are the n points in Ω_α . For \widehat{V}_ψ the situation is different because when the size
 289 of the rolling window r increases, the variance \widehat{V}_ψ spontaneously decreases.
 290 Nonetheless, it is still possible to compare the values of \widehat{V}_ψ at specific locations
 291 when the order α of the quantile changes for a fixed r . Finally, we emphasize
 292 that \widehat{V}_ψ is not equal to the average of \widetilde{V}_ψ over the rolling mean window r .

293 In order to have simple diagnosing metrics, we also average the \widetilde{V}_ψ variance
 294 spatially:

$$\langle \widetilde{V}_\psi(t) \rangle = \sum_{\phi, \theta} \widetilde{V}_\psi(\phi, \theta, t) \cos(\theta) / \sum_{\phi, \theta} \cos(\theta) \quad (8)$$

295 over either the North-Hemisphere ($\theta \in [22.5^\circ\text{N}, 90^\circ\text{N}]$ and $\phi \in [0^\circ\text{E}, 360^\circ\text{E}]$)
 296 or the Euro-Atlantic sector ($\theta \in [22.5^\circ\text{N}, 70^\circ\text{N}]$ and $\phi \in [80^\circ\text{W}, 50^\circ\text{E}]$). For
 297 simplicity, we drop the index ψ in the following where it is not ambiguous.

298 4 Results

299 4.1 Extreme temperatures in Western Europe

300 In this section we present the results for the extremes of T2M at the grid point
 301 in Western Europe (observable W). Figure 2 presents the composite maps of
 302 anomalies of T2M and Z500 over the Euro-Atlantic sector for the $n = 50$
 303 points for which the values of their observable is the closest to the quantile
 304 and over the rolling window $r = 5$ days for the four quantiles considered.
 305 The anomalous T2M values and their spatial extension increase as the α -th
 306 order quantile increases. As α increases, the deviation in Z500 also increases,
 307 reaching a maximum of 160m at the center of the high-pressure system located
 308 above North-Western Europe. There is also a warm anomaly in Eastern North-
 309 America and a cold anomaly in Western Russia. Figure A1 in annex shows
 310 the same results over the whole North Hemisphere. Anticyclonic anomalies are
 311 present all over the Hemisphere, with a distinct wave number 4 hemispheric
 312 pattern at mid-troposphere for the highest quantiles. These structures coincide
 313 with warm anomalies at the ground.

314 Figure 3 shows in contours the composite T2M field and in colors the nor-
 315 malized variance \widehat{V} . For all values of α , the lowest values of the normalized
 316 variance are located in Western Europe, i.e. around the location where the
 317 observable is computed. When α increases, the normalized variance decreases:
 318 for example, the normalized variance exceeds 60% in the North Atlantic region
 319 for $\alpha = 0.75$, and it decreases to less than 30% for $\alpha = 0.999$. Figure 4
 320 presents the same analysis for Z500. The extension of regions of highest vari-
 321 ance also decreases when increasing the order α . The regions with the lowest
 322 variance is again centered at the location where the observable is computed.
 323 For $\alpha = 0.999$ for example, a large region of very low normalized variance
 324 ($\widehat{V} < 10\%$) embraces most of Western Europe. The decrease of variance is
 325 however not uniform, with high variance remaining downstream of the anticy-
 326 clonic region and a smaller localized region west of the Iberian peninsula. We

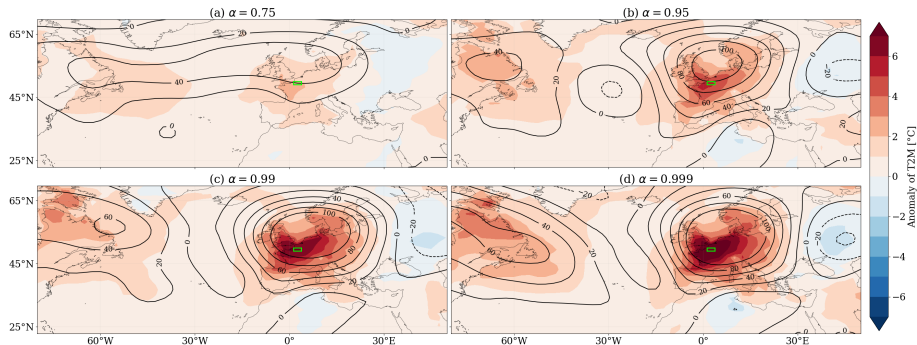


Fig. 2 Composite maps \hat{A} of anomalies of T2M and Z500 for increasing α -th order of the quantile of the W observable's empirical distribution. The figure is computed for a rolling mean window of $r = 5$ days. Colors: anomaly of air temperature at 2m (T2M, [K]). Contours: anomaly of geopotential height at 500hPa (Z500, [m]). Anomalies are computed with respect to the JJA average. The green box displays the location where the observable is computed.

327 come back to this latter structure below. We finally remark that on Fig. 4, for
 328 the highest quantiles the composite Z500 field shows that the large anticyclone
 329 over Western Europe is not centered just above the location of the observable,
 330 but is rather situated to its south-east.

331 Figures A2 and A3 in annex display the same analysis over the whole
 332 North-Hemisphere. The reduction of variance is seen over remote regions of the
 333 atmosphere, for example with a region of low variance ($\hat{V} < 20\%$) in the North-
 334 Western Pacific for $\alpha = 0.999$ for Z500 (Fig. A3 panel (d)). We emphasize that
 335 for all those maps, the number of points in Ω_α is always the same ($n = 50$ for
 336 T2M, $n \sim 30$ for Z500 due to the missing data). The outcome shown implies
 337 that the dynamics leading to extreme temperatures concentrates within a large
 338 geographical region.

339 So far we have presented the results for a rolling window of $r = 5$ days.
 340 The results for $r = 1$ day and $r = 15$ days for the composite maps are similar
 341 and are thus not shown here. However, as can be seen in figures A4 and A5 in
 342 annex for $r = 1$ day and in figures A6 and A7 in annex for $r = 15$ days, the
 343 results for the variance strongly depend on r . As we mentioned earlier, this
 344 is expected in so far as averaging temporally naturally reduces the variance
 345 \tilde{V} . We however still observe in those figures a reduction of the variance when
 346 α increases.

347 Figure 5 presents composites (colors) and zones of high variance ($\hat{V} >$
 348 50% , hatches) for the other variables: anomaly of soil moisture (SM, panel(a)),
 349 anomaly of sea-level pressure (SLP, panel (b)), air temperature at 850hPa
 350 (T850, panel(c)) and meridional wind speed at 250hPa (V250, panel (d)). The
 351 threshold 50% for discriminating between "high" and "low" variance is chosen
 352 arbitrarily to highlight differences between regions. This figure is drawn for
 353 $r = 5$ days and $\alpha = 0.999$.

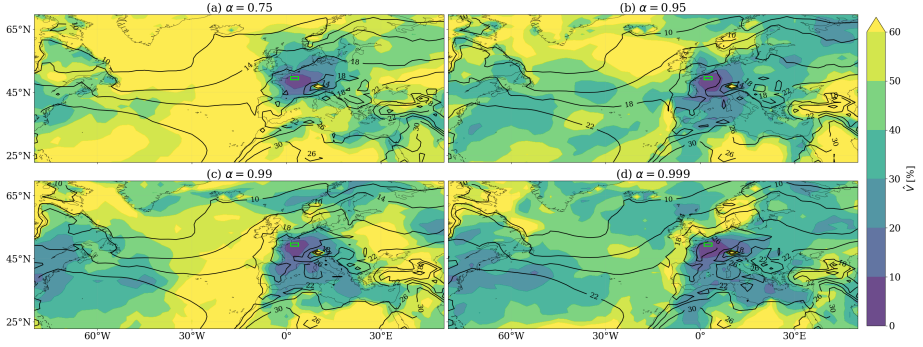


Fig. 3 Composite \hat{A} (contours) [°C] and normalized variance \hat{V} (colors) of the T2M field for increasing α -th order of the quantile of the W observable's empirical distribution. The figure is computed for a rolling mean window of $r = 5$ days. The normalized variance is computed after averaging over the rolling window and is expressed in %. The green box displays the location where the observable is computed.

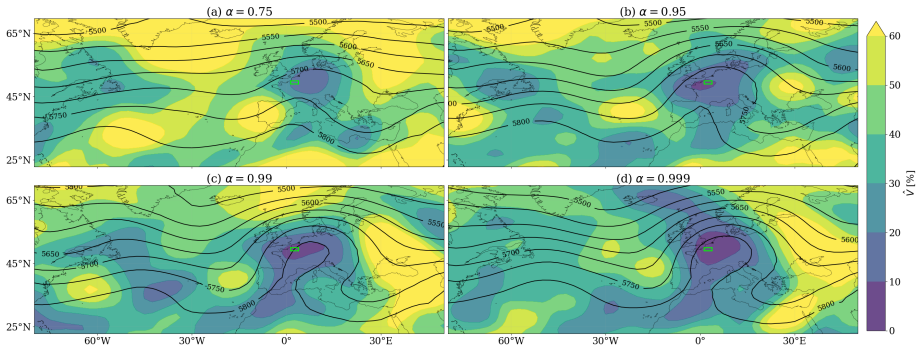


Fig. 4 Composite \hat{A} (contours) [m] and normalized variance \hat{V} (colors) of the Z500 field for increasing α -th order of the quantile of the W observable's empirical distribution. The figure is computed for a rolling mean window of $r = 5$ days. The normalized variance is computed after averaging over the rolling window and is expressed in %. The green box displays the location where the observable is computed.

354 Panel (a) shows the large anomalously dry soils over most of Western
 355 and Central Europe. The small variance over this region demonstrate the key
 356 importance of local dry soils for reaching very high temperatures. The conver-
 357 gence is stronger to the east of the location of the observable, which agrees
 358 with the results of Zschenderlein et al (2019). This pattern is similar to the
 359 one found by Faranda et al (2022) using the SPEI index (Beguería et al, 2014).
 360 We also note that when checking at $t = -15d$ (i.e. 15 days before the event,
 361 not shown), the anomaly is still strong ($\simeq 4kg/m^2$) and the variance still small
 362 ($\tilde{V} < 30\%$), which supports our preceding statement for the role of dry soils.
 363 Contrary to other fields (Fig. 5 panels (b), (c) and (d)), the region of low

364 variance is concentrated around the observable and do not extend over large
 365 regions.

366 Panel (b) shows a large positive anomaly of SLP north-east of the observ-
 367 able location and the low-level thermal low associated with the heatwave in
 368 Western Europe. The normalized variance is low over these regions. The posi-
 369 tive anomaly situated above the Western Atlantic ocean may not be a relevant
 370 feature of the typical dynamics because the variance is high at this place,
 371 contrary to the anomalous low pressure over Greenland. Panel (c) shows the
 372 composite map of T850. The large intrusion of warm air from the south is a
 373 key feature of the dynamics. Even though the variance is low over mainland
 374 Western Europe, we detect a region of high variance extending from the west
 375 of the Iberian peninsula to Ireland and southern Norway and Sweden, i.e. to
 376 the western flank of the anticyclonic structure (Fig. 4 panel (d)). This tongue
 377 of high variance is located at the highest gradient of T850. It is therefore dif-
 378 ficult to decide between the two following explanations for this feature: either
 379 different dynamical mechanisms between the points in Ω_α (e.g. advection of
 380 warm air from the tropics) or a slightly displaced anticyclone which, combined
 381 with the strong gradients, would display such a strong variance tongue.

382 Finally, panel (d) shows the dynamics in the upper troposphere with the
 383 meridional wind speed at 250hPa. The situation is characterized by a strongly
 384 meridional circulation west of the observable, which is coherent with the anti-
 385 cyclonic situation presented in figure 2. The regions of high variance also
 386 coincide with the regions with the highest gradients of V250. The one situated
 387 above the Balkans and the Black Sea regions suggests the presence of an arm
 388 of the jet stream oriented to the North. We checked this explanation using the
 389 zonal wind speed at 250hPa (not shown) and found that the synoptic situa-
 390 tion over the event corresponds indeed to a splitting of the jet caused by the
 391 large blocked anticyclonic situation over Western Europe. This explanation is
 392 coherent with the double jet dynamics associated with heatwaves (Rousi et al,
 393 2022).

394 Figure 6 presents the same analysis over the entire North Hemisphere.
 395 For soil moisture (panel (a)), the only region outside the Euro-Atlantic sector
 396 where there are important anomalies and low variance is the western coast
 397 of the USA. This region is also characterized by positive T2M temperature
 398 anomalies as can be seen in figure A1. Panel (b) shows a good agreement among
 399 points in Ω_α for the low-level low over Greenland and the Ural. We also notice
 400 a positive SLP anomaly over the North-Pacific, but this region is associated
 401 with high variance. Apart from the high variance tongue in Western Europe,
 402 and high variance in the Arctic regions, panel (c) shows a good agreement
 403 between points in Ω_α for the T850 field, including over most of the Atlantic
 404 and Pacific oceans. Finally, panel (d) shows a wave pattern 6-7 extending over
 405 the entire North-Hemisphere upper-troposphere. It should be noted, however,
 406 that the anomaly is stronger and the variance is smaller in the upstream area
 407 compared to the downstream area of the observable. Specifically, there is little
 408 structure visible above the Western Pacific region. This situation may be the

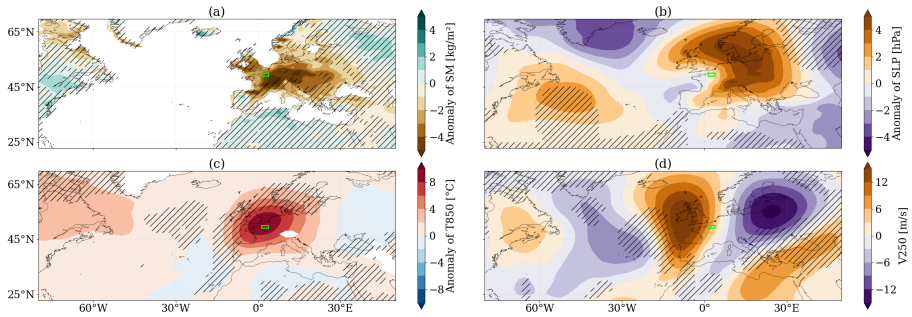


Fig. 5 Composite maps \hat{A} for SM, SLP, T850 and V250 for the quantile of order $\alpha = 0.999$ of the W observable's empirical distribution. The hatched areas correspond to $\hat{V} > 50\%$. The figure is computed for a rolling mean window of $r = 5$ days. Anomaly of (a) soil moisture (SM), (b) sea-level pressure (SLP) and (c) temperature at 850hPa (T850) with respect to their average over the summer (JJA), and (d) meridional wind speed at 250hPa (V250). The green box displays the location where the observable is computed.

409 result of either an hemispheric quasi-stationary pattern (Coumou et al, 2014;
 410 Kornhuber et al, 2020) or a transient Rossby wave packet (Fragkoulidis et al,
 411 2018).

412 The analyses presented so far support the instanton interpretation of
 413 extreme events: the higher the value of the quantile q_α , the stronger the con-
 414 centration of trajectories reaching this quantile around a single trajectory.
 415 To check whether our visual inspection is correct, we present in figure 7 the
 416 temporal evolution of the normalized variance $\langle \tilde{V}(t) \rangle$ averaged over both the
 417 Euro-Atlantic sector (plain lines) and the entire North-Hemisphere (dashed
 418 lines) from $t = -15$ to $t = +15$ days with respect to the event for all the
 419 variables and all the rolling windows r . The general picture drawn above is
 420 validated by the results presented in this figure: overall, the higher the value
 421 of the order α for the quantiles of the observable empirical distribution, the
 422 smaller the value of the normalized variance $\langle \tilde{V}(t) \rangle$. This result holds when
 423 averaging over either only the Euro-Atlantic sector — where the observable
 424 is computed and the synoptic situation is the most relevant for the event —
 425 or the entire North-Hemisphere. However, when we focus on the details of the
 426 different panels, we see that this behavior is less clear for some variables. The
 427 convergence is clear for T2m (panels (a), (b) and (c)) and T850 (panels (m),
 428 (n) and (o)) for which the normalized variance $\langle \tilde{V}(t) \rangle$ is a decreasing function
 429 of α for almost all values of t . For dynamical variables such as Z500, we see
 430 that the convergence is stronger for higher values of the rolling window (com-
 431 pare for example panel (f) and panel (d)). This could be explained by the fact
 432 that a long extreme needs a persistent anticyclonic circulation, which is more
 433 likely to be "typical" than for a short extreme. For the SM and SLP vari-
 434 ables the situation is the most blurred. Higher values of α broadly correspond
 435 to smaller values of the normalized variance $\langle \tilde{V}(t) \rangle$, but the ordering of the
 436 quantiles changes with both r and t .

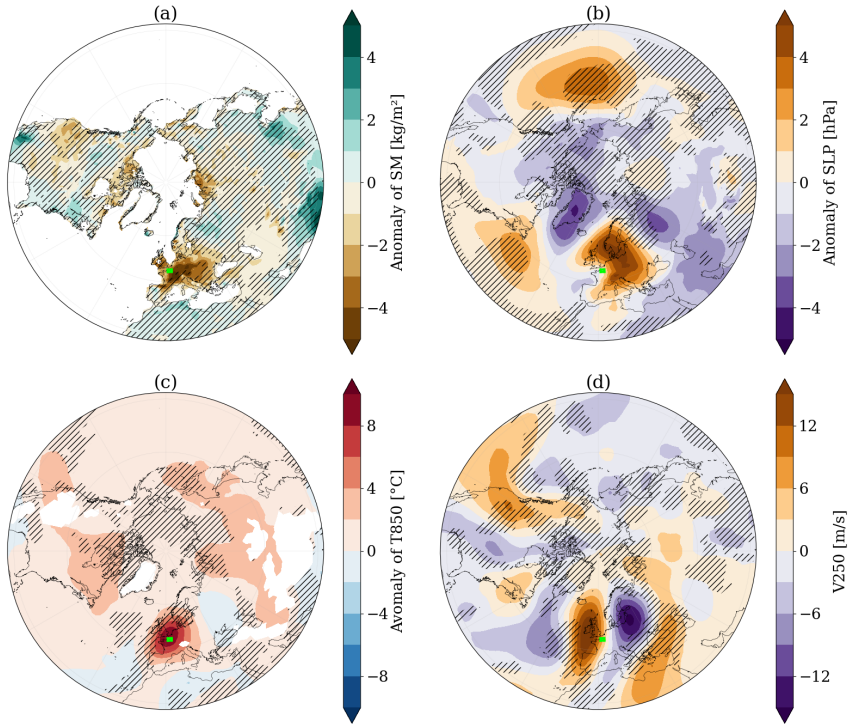


Fig. 6 Composite maps \hat{A} for SM, SLP, T850 and V250 for the quantile of order $\alpha = 0.999$ of the **W** observable's empirical distribution. The hatch areas correspond to $\hat{V} > 50\%$. The figure is computed for a rolling mean window of $r = 5$ days. Anomaly of (a) soil moisture (SM), (b) sea-level pressure (SLP) and (c) temperature at 850hPa (T850) with respect to their average over the summer (JJA), and (d) meridional wind speed at 250hPa (V250). The green box displays the location where the observable is computed.

437 In this section we analyzed the dynamics $\mathbb{E}[x | F_r(x) = q]$ conditional on
 438 the value reached by an observable F_r . We demonstrated the concentration of
 439 the trajectories as the value q of the observable reaches extremes. We can now
 440 sketch the mechanisms associated with extreme temperatures at the grid point
 441 considered in Western Europe (observable W). The mechanisms summarized in
 442 the reviews of Perkins (2015), Horton et al (2016) and Domeisen et al (2022a)
 443 are present. To ensure very high temperatures, one needs dry soils, a large mid-
 444 troposphere anticyclone centered slightly to the South-East of the location of
 445 the extreme and an upper level Rossby wave train of 6-7 wave number. This
 446 situation ensures both the advection of warm air from the south at the west
 447 flank of the anticyclone, subsidence and associated adiabatic heating at the
 448 center of the anticyclone, and clear skies caused by the high-pressure system
 449 that allows for more radiative heating of the lowest layer of the atmosphere
 450 in conjunction with reduced water evaporation. We also note that the dates
 451 at which these extremes are reached are less dispersed around the peak of the

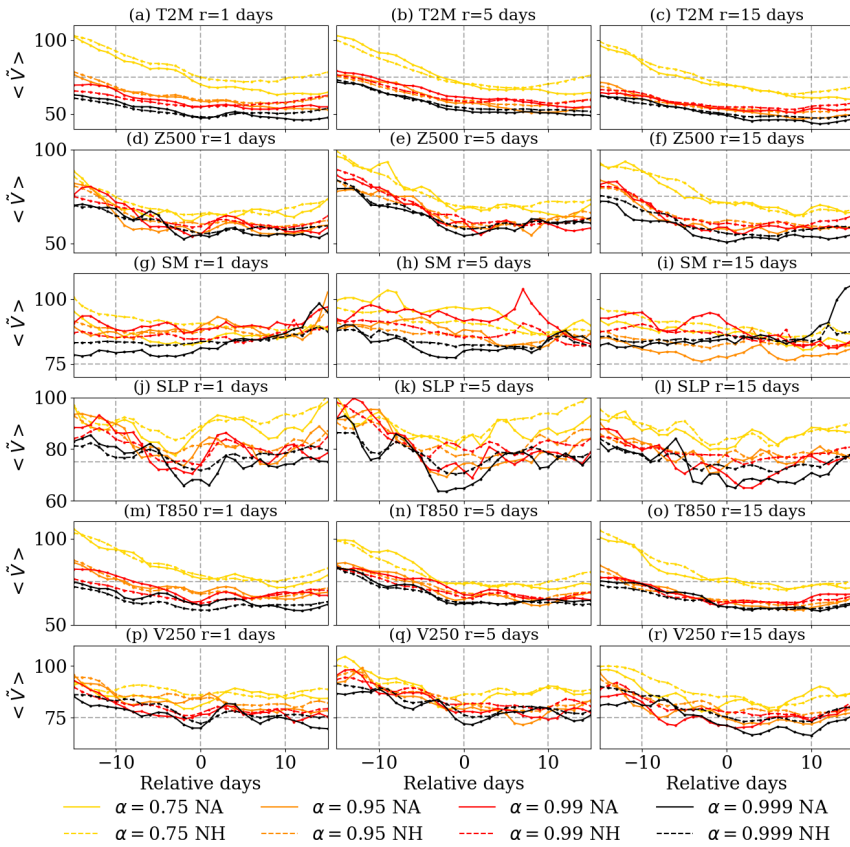


Fig. 7 Evolution of the normalized variance $\langle \tilde{V}(t) \rangle$ averaged over the Euro-Atlantic sector (plain lines) and the North-Hemisphere (dashed lines) for the different variables (**W** observable). The normalized variance is expressed in %. The colors show the α -th order quantile of the observable empirical distribution. The time is expressed relative to the day when the observable is such that $F_r(x_t) \simeq q_\alpha$. The gray dashed line shows the 75% level.

452 T2M seasonality when α increases: the standard deviation in the calendar days
 453 of the points in Ω_α goes from 23.1 days for $\alpha = 0.75$ to 15.5 days for $\alpha = 0.999$.

454 In figure 4 there is a small isolated region of high variance situated west
 455 of the Iberian peninsula for all quantiles. We investigated this discrepancy by
 456 looking at individual events. It turns out that the synoptic dynamics at the
 457 mid-troposphere associated with some of the high temperature events are char-
 458 acterized by the presence of a cut-off low around this location. Table A1 in
 459 appendix presents the percentage of such events. The presence of a cut-off low
 460 was investigated in a semi-objective way by looking at the existence of an iso-
 461 lated minimum of the stream function at 500hPa located within $[-30^\circ\text{S}, +5^\circ\text{N}]$
 462 and $[-40^\circ\text{W}, +0^\circ\text{E}]$ from the location where the observable **W** is computed and

not embedded in the upper-level jet (250hPa) (Muñoz et al, 2020). We however note that it is sometimes difficult to distinguish between a true cut-off low and a large meander of the jet caused by a deep, almost isolated, low pressure system over the Atlantic. The percentage of cut-off lows vary between 15 to 30%, with an average around 20%, depending on the quantiles and the size of the rolling mean window considered but without any clear trend (the differences may be due to the limited sample size). These cut-offs are not visible on the averaged maps (e.g. Fig. 4) not only because they represent only 20% of the events but also because their characteristic size is of the same order as the variance in the location of their center. Therefore even when considering only events with a cut-off low, they tend to be averaged out. The fact that even for very high quantiles there is a substantial amount of events with such a cut-off (around 20%) is in contradiction with the unique path hypothesis presented in section 2 only if one assumes that the rate function I has a single minimum. It therefore suggests that there may be at least a bi-modality in the typical dynamical paths to reach extremes for the W observable. We however note that if there is indeed two minima, the convergence of trajectories that we showed above using all trajectories suggests that they are close to one another in the phase space.

4.2 Results for the other locations

In this section we present the same analysis applied to the three other observables: T2M at two grid-points situated in the south (observable S) and north of Europe (observable N), and T2M averaged over a large area in Western and Central Europe (WCE observable). The results for the evolution of their normalized variance $\langle \tilde{V}(t) \rangle$ is presented in figures A8, A9 and A10 respectively in annex. As above, the general picture of decreasing variance with increasing α is still valid but there are substantial inter-location variations. We note for example that the dynamical signal represented by the variables Z500, SLP and V250 is much clearer for locations farther to the North (Fig. A8 vs Fig. A9 panels (d), (e), (f) for example). In the contrary, soil moisture plays a bigger role for the locations situated farther to the south (Fig. A8 vs Fig. A9 panels (g), (h), (i)). This suggests that extremes of temperature at places situated to the south are more "local" in the sense that they require less large scale organized circulation to be reached, which is coherent with the results of Sousa et al (2018) who showed that a ridge situation better described the occurrence of heatwaves in southern Europe than the blocked situation as in northern Europe. This interpretation however needs to be validated at other locations. We also note that these dynamical differences between lower and higher latitudes may be related the skewness differences in the distribution of the summer temperature (Fig. 1).

The observable over a large area (observable WCE) has a similar behavior to the W and N observables, with a stronger concentration of trajectories for dynamical variables than for soil moisture. We also note that the absolute values of the normalized variances for this observable are smaller than

507 the one for the observable W which is situated around the same latitudes.
508 It suggests that the typicality of the dynamics leading to anomalies of T2M
509 over a large geographical area is stronger than for a localized observable. This
510 seems reasonable in so far as it is less likely to have anomalies over a larger
511 than a smaller area situated inside the larger one, hence the smaller number of
512 synoptic conditions which can lead to an extreme for an extended observable.

513 Figure 8 presents the composite maps and the normalized variance maps
514 for the S observable for the order $\alpha = 0.999$ and a rolling mean window of
515 $r = 5$ days. The situation over the North-Hemisphere is presented in annex
516 in figure A11. The synoptic situation is characterized by a large anticyclone
517 centered south-east of the observable (panel(b)) and a positive SLP anomaly
518 extending from Algeria to Northern France (panel (d)). The upper-level circu-
519 lation displays a short wavelength Rossby wave (panel (f), wave number 6-7).
520 We however note that its amplitude is smaller than for the W observable. As
521 previously, this dynamics leads to high T2M and T850 values and it has been
522 anticipated by dry soils over most of southern Europe and northern Africa.
523 The variance is lowest close to the location where the observable is computed.
524 We note a large region of high variance downstream of the observable for V250
525 (panel(f)), but upstream for SLP and Z500 (panels (b) and (d)). As for the
526 W observable, we see a high variance region at the west of the maximum gra-
527 dients of T850 (panel (e)). The percentage of cut-offs associated with these
528 events (Table A1) is between 15 and 30%, similar to the W observable.

529 Figure 9 presents the same analysis for the N observable. The situation
530 over the North-Hemisphere is presented in annex in figure A12. The synoptic
531 situation is characterized by an anticyclone centered south of the observable
532 (panel (b)), positive anomalies of SLP north of the observable (panel (d))
533 and a large amplitude Rossby wave at 250hPa (panel (f)). The soil moisture
534 situation (panel (c)) is almost the opposite of the one of the observable S,
535 with a moist southern Europe and a dry northern Europe. The dry soils over
536 Northern Europe, as for the region of anomalous T2M (panel (a)), extends over
537 a large region encompassing Northern Europe and the Scandinavian and Baltic
538 areas. This is in opposition to the extension of the anomaly in Fig. 8 panel (a),
539 which is confined to the Iberian peninsula. This support the idea of "local"
540 extremes in the south compared to more extended ones in the north. We also
541 note the region of low variance associated with strong negative anomaly of
542 soil moisture east of the observable (panel(c)) (Zschenderlein et al, 2019). The
543 percentage of cut-offs is similar to the W and S observables but when looking
544 at individual events we remark that a majority (>50%) of these cut-offs are
545 embedded in a so-called modon structure (Butchart et al, 1989) with a blocking
546 high above Scandinavia and a symmetric low above the eastern Mediterranean,
547 splitting the jet into two branches. Again, for the reasons explained above,
548 these structures are averaged out on composite maps.

549 Figure 10 presents the results for the WCE observable. The situation over
550 the North-Hemisphere is presented in annex in figure A13. Contrary to the
551 observables S, W and N, WCE has a large spatial extension as it encompasses

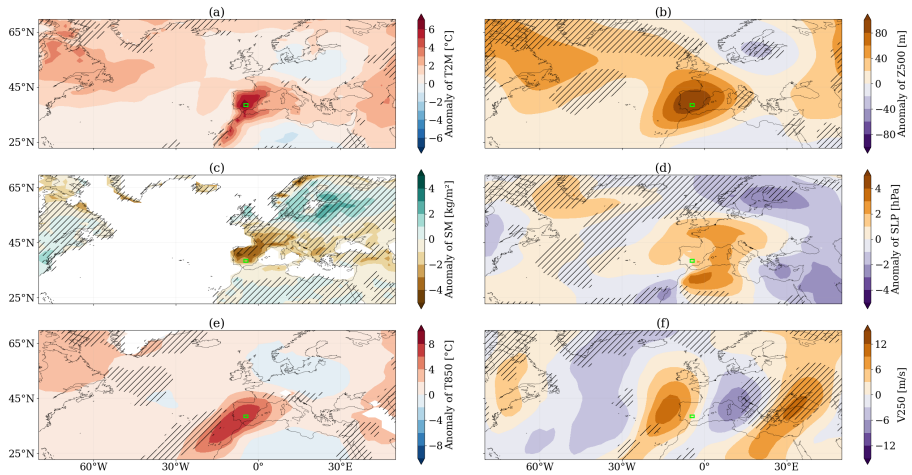


Fig. 8 Composite maps \hat{A} for T2M, Z500, SM, SLP, T850 and V250 for the quantile of order $\alpha = 0.999$ of the S observable's empirical distribution. The hatch areas correspond to $\hat{V} > 50\%$. The figure is computed for a rolling mean window of $r = 5$ days. Anomaly of (a) air temperature at 2m (T2M), (b) geopotential height at 500hPa (Z500), (c) soil moisture (SM), (d) sea-level pressure (SLP) and (e) temperature at 850hPa (T850), and (f) meridional wind speed at 250hPa (V250). The anomalies are computed with respect to their average over the summer (JJA). The green box displays the location where the observable is computed.

552 most of West and Central Europe. In this case the synoptic situation is char-
 553 acterized by an anticyclone centered just above the observable (panel (b)),
 554 contrary to the precedent ones. The upper-level circulation looks very similar
 555 to the previous ones, with a short wavelength Rossby wave (panel (f)), wave
 556 number 6-7). The soil moisture anomalies (panel (c)) extend over a large region
 557 of Northern, Central and Eastern Europe and this feature is consistent across
 558 points in Ω_α . We also note a region of high variance above southern Sweden for
 559 T850 (panel (e)), which is a feature similar to what was found for observable
 560 W (cf. Fig. 5 panel (c)). This feature could reflect different advecting dynam-
 561 ics of warm air above the boundary layer. The percentage of cut-offs is much
 562 higher than for the other observables (around 50%). As for the N observable
 563 most of these cut-offs are embedded in a modon structure.

564 5 Discussion and conclusion

565 The application of large deviations theory to dynamical systems (Freidlin and
 566 Wentzell, 1987; Grafke and Vanden-Eijnden, 2019; Dematteis et al, 2019b)
 567 predicts a concentration of trajectories leading to extremes for any observable:
 568 this is the so-called instanton hypothesis. In this paper we have investigated
 569 this prediction using air temperature at 2m (T2M) at four locations in Europe
 570 in the IPSL-CM6A-LR model (Boucher et al, 2020) pre-industrial control run
 571 as the observables of interest. Using the 2000-year simulation, we employed

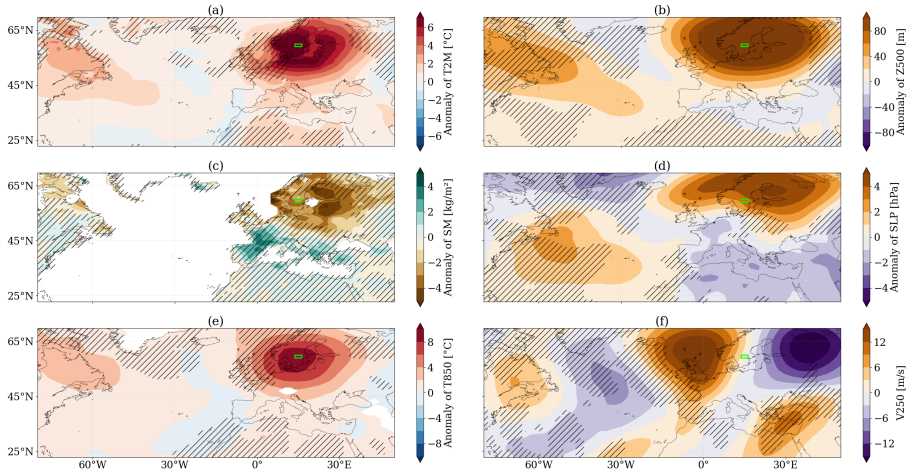


Fig. 9 Composite maps \hat{A} for T2M, Z500, SM, SLP, T850 and V250 for the quantile of order $\alpha = 0.999$ of the N observable's empirical distribution. The hatch areas correspond to $\hat{V} > 50\%$. The figure is computed for a rolling mean window of $r = 5$ days. Anomaly of (a) air temperature at 2m (T2M), (b) geopotential height at 500hPa (Z500), (c) soil moisture (SM), (d) sea-level pressure (SLP) and (e) temperature at 850hPa (T850), and (f) meridional wind speed at 250hPa (V250). The anomalies are computed with respect to their average over the summer (JJA). The green box displays the location where the observable is computed.

572 an instanton filtering procedure (Grafke et al, 2013) consisting in averaging
 573 trajectories which reach a similar extreme value of these observables.

574 We have shown that the variance between trajectories reaching the extreme
 575 decreases as the level of extremeness of the observable increases. In other words,
 576 the more intense the extreme of T2M, the more likely that the trajectories
 577 reaching this extreme all look the same. We demonstrated this convergence
 578 on all variables considered: air temperature at 2m (T2M), geopotential height
 579 at 500hPa (Z500), upper level soil moisture (SM), sea-level pressure (SLP),
 580 temperature at 850hPa (T850) and meridional wind speed at 250hPa (V250)
 581 for a grid point observable in western Europe (observable W). Remarkably, the
 582 variance decreases even at places far from the location of the observable, which
 583 suggests a hemispheric dynamics leading to very intense heatwaves. We also
 584 showed a stronger decrease of variance for higher values of the rolling mean
 585 window r , indicating that the typicality is more easily reached when looking
 586 at longer time averages as suggested by Lucarini et al (2022).

587 The instanton dynamics found with our analysis is coherent with the
 588 mechanisms identified by previous literature for heatwaves dynamics in mid-
 589 latitudes (Perkins, 2015; Horton et al, 2016; Domeisen et al, 2022a). In the
 590 IPSL-CM6A-LR model with a pre-industrial CO_2 level, very high air tempera-
 591 ture at 2m are reached by a combination of dry soils, a large mid-troposphere
 592 anticyclone and an upper level Rossby wave of 6-7 wave number. This situa-
 593 tion ensures both the advection of warm air from the south at the west flank of

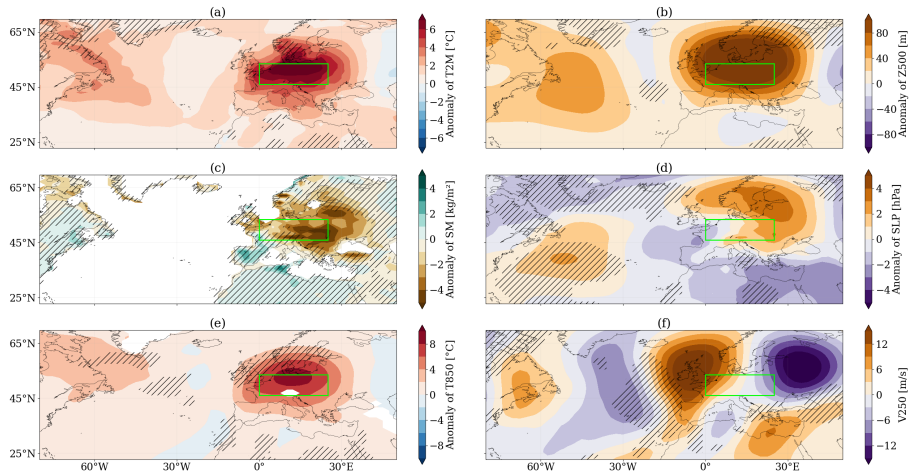


Fig. 10 Composite maps \hat{A} for T2M, Z500, SM, SLP, T850 and V250 for the quantile of order $\alpha = 0.999$ of the WCE observable's empirical distribution. The hatch areas correspond to $\hat{V} > 50\%$. The figure is computed for a rolling mean window of $r = 5$ days. Anomaly of (a) air temperature at 2m (T2M), (b) geopotential height at 500hPa (Z500), (c) soil moisture (SM), (d) sea-level pressure (SLP) and (e) temperature at 850hPa (T850), and (f) meridional wind speed at 250hPa (V250). The anomalies are computed with respect to their average over the summer (JJA). The green box displays the location where the observable is computed.

594 the anticyclone over the whole troposphere, subsidence, associated adiabatic
 595 heating, and clear skies which favors the radiative heating of the lowest layers
 596 of the atmosphere in combination with reduced water evaporation.

597 We investigated the instanton hypothesis for three other observables: T2M
 598 at two grid points situated in the South (observable S) and North (observable
 599 N) of Europe and T2M averaged over an area covering most of Western and
 600 Central Europe (observable WCE). We found a similar convergence mecha-
 601 nism of different trajectories, but with some discrepancies. The convergence
 602 of variance is much stronger for observable N than observable S for dynamical
 603 variables (SLP, Z500 and V250), and also stronger for observable WCE
 604 than observable W for most variables. We showed, that the global dynamics
 605 described above is similar for reaching extremes at these locations. Overall
 606 we found that the instanton hypothesis is consistent with our results, but the
 607 convergence is stronger for observable farther to the north, observables com-
 608 puted on extended spatial locations and for longer extremes (higher values of
 609 the rolling mean window r). We also investigated the local discrepancies on
 610 the convergence of variance and found that a substantial amount of dynamical
 611 paths (around 20% for the grid point observables) are associated to the pres-
 612 ence of cut-off lows. This suggests that the rate function I may have several
 613 minima and therefore that there may be a multi-modality in the dynamical
 614 paths reaching very intense hot events. This is however not clear what are the

615 precise effects - either dynamical or thermodynamical - of the presence of these
616 cut-offs on the intensity of the observed heatwaves.

617 The observed discrepancies in the decrease of the variance for some vari-
618 ables at certain location however do not allow to disprove the instanton
619 hypothesis. Indeed, with an order $\alpha = 0.999$ for the quantile of the observable's
620 empirical distribution over the JJA months and 2000 summers, it corresponds
621 to roughly choosing $n = 50$ heatwaves with a return time around 40 years. This
622 amount of data is greater than what has been recorded through observations
623 since the start of the satellite era. Nonetheless, this may not be sufficient to
624 ensure full convergence and this could explain why the variance remain high at
625 certain locations. A much longer data set would be needed to investigate the
626 dynamics leading to more extreme temperatures, especially daily records. For
627 such very intense events, the bi-modality suggested here may disappear. Even
628 if our analysis suggests that very high extremes are reached by a strengthen-
629 ing of the mechanisms leading to high extremes, it cannot be completely ruled
630 out that the mechanisms can change if one wants to reach even higher val-
631 ues. One possible explanation could be the release of latent heat from tropical
632 air in conjunction with an atmospheric river over the North Atlantic, which
633 seems to be one of the reinforcing structures of the 2021 North-Western Amer-
634 ica heatwave (Qian et al, 2022; Lin et al, 2022; Mo et al, 2022). To the best
635 of our knowledge, such a dynamics has never been observed for heatwaves in
636 Western Europe.

637 In this paper we did not study long term potential precursors of heatwaves
638 such as anomalous sea surface temperature (SST) patterns or large-scale modes
639 of climate variability (Domeisen et al, 2022a). Anomalous SST patterns are
640 known to be present in observed heatwaves (Black and Sutton, 2007; Duchez
641 et al, 2016; McKinnon et al, 2016), and a convergence of the surface oceanic
642 dynamics to reach extreme land temperature is therefore likely in climate
643 models. It may also be the case for large-scale modes of climate variability such
644 as El Nino-Southern Oscillation (ENSO) (Martija-Díez et al, 2021), Atlantic
645 Multidecadal Oscillation (AMO) or Pacific Decadal Oscillation (PDO). These
646 processes are nonetheless suggested by the large regions of low variance for
647 T2M above the Atlantic and the Pacific oceans found with our analysis.

648 One may wonder how the paradigm of the typical dynamics leading to
649 heatwaves is coherent with the presence of different clusters of heatwaves
650 demonstrated by several studies on empirical data (e.g. Stefanon et al (2012);
651 Gibson et al (2017); Wang et al (2018); Keellings and Moradkhani (2020); Mon-
652 dal and Mishra (2021)). As we have shown here, the typical dynamics highly
653 depends on the observable considered. It may therefore be possible that the
654 typical dynamics leading to extremes of neighboring grid points is very sim-
655 ilar and changes dramatically when crossing relevant physical barriers, such
656 as mountains (Lucarini et al, 2022). Moreover, if one considers less extreme
657 values of the distribution of grid point observables, it is possible that the typ-
658 icality is not reached for this observable but it may be reached for another
659 observable encompassing a broader region. One could therefore compare the

660 event observed to the typical events for the observable which maximizes its
661 spatiotemporal rarity as proposed by Cattiaux and Ribes (2018) in the context
662 of attribution of extreme events.

663 Despite the lack of data for extremely rare events, the framework of large
664 deviations theory offers a significant simplification by predicting that study-
665 ing such events is equivalent to studying a single trajectory, assuming that
666 the rate function has only one minimum. Therefore, this theoretical frame-
667 work is encouraging for gaining predictive power on the dynamics leading to
668 extreme events and it may explain why summer heatwaves are among the
669 most predictable meteorological extremes on subseasonal timescales (Vitart
670 and Robertson, 2018; Vitart et al, 2019; Domeisen et al, 2022b). Here we stud-
671 ied the extremes of air temperature close to the surface. For other variables
672 of interest our preceding statement of the validity of the instanton hypothesis
673 should also be tested. More generally, this paper documents a method to study
674 the dynamics leading to extreme events in non-equilibrium physical systems.
675 Our framework suggests a connection between the statistical method of study-
676 ing extreme events and the in-depth examination of specific events through
677 case studies.

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680 **Declarations**

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- 686 • **Ethics approval :** Not applicable.
- 687 • **Consent to participate:** Not applicable.
- 688 • **Consent for publication:** Not applicable.
- 689 • **Availability of data and materials:** The control run of the IPSL model is
690 available upon request.
- 691 • **Code availability:** The main results of this work were obtained using Python.
692 The scripts are available upon request.
- 693 • **Authors’ contributions:** RN did the data analysis. All the authors con-
694 tributed to writing and reviewing the article.

695 Appendix A Supplementary information

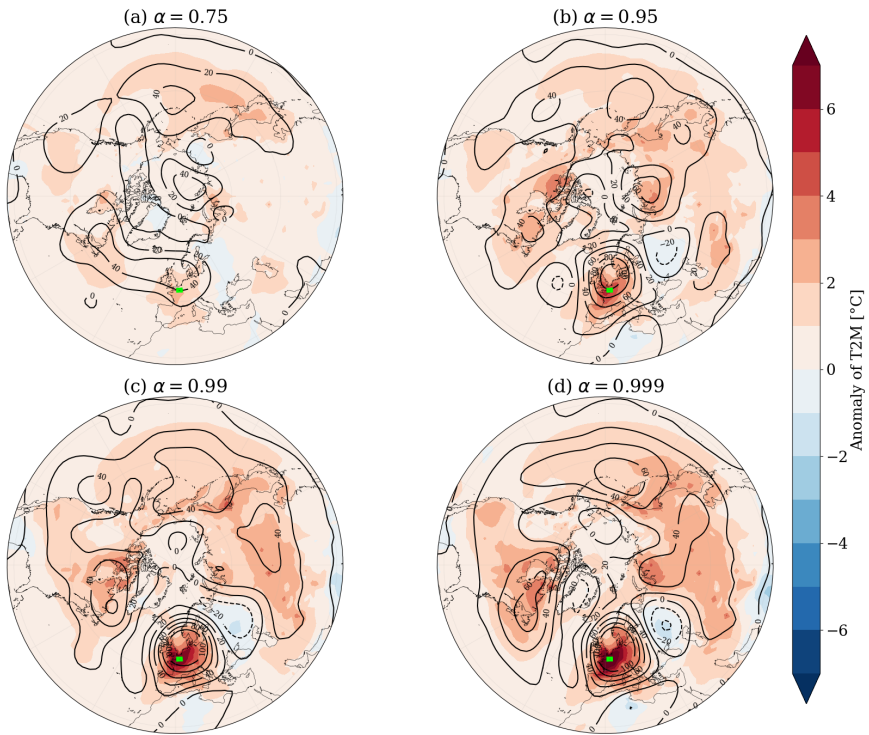


Fig. A1 Composite maps \hat{A} of anomalies of T2M and Z500 for increasing α -th order of the quantile of the W observable's empirical distribution. The figure is computed for a rolling mean window of $r = 5$ days. Colors: anomaly of T2M [K]. Contours: anomaly of Z500 [m]. Anomalies are computed with respect to the JJA average. The green box displays the location where the observable is computed.

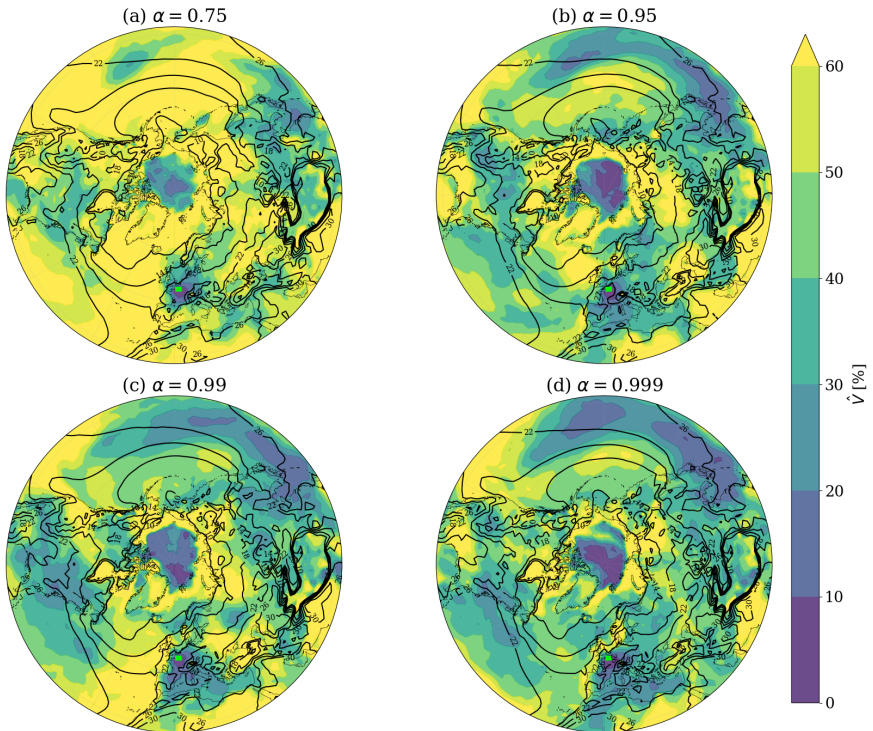


Fig. A2 Composite \hat{A} (contours) [$^{\circ}\text{C}$] and normalized variance \hat{V} (colors) of the T2M field for increasing α -th order of the quantile of the W observable's empirical distribution. The figure is computed for a rolling mean window of $r = 5$ days. The normalized variance is computed after averaging over the rolling window and is expressed in %. The green box displays the location where the observable is computed.

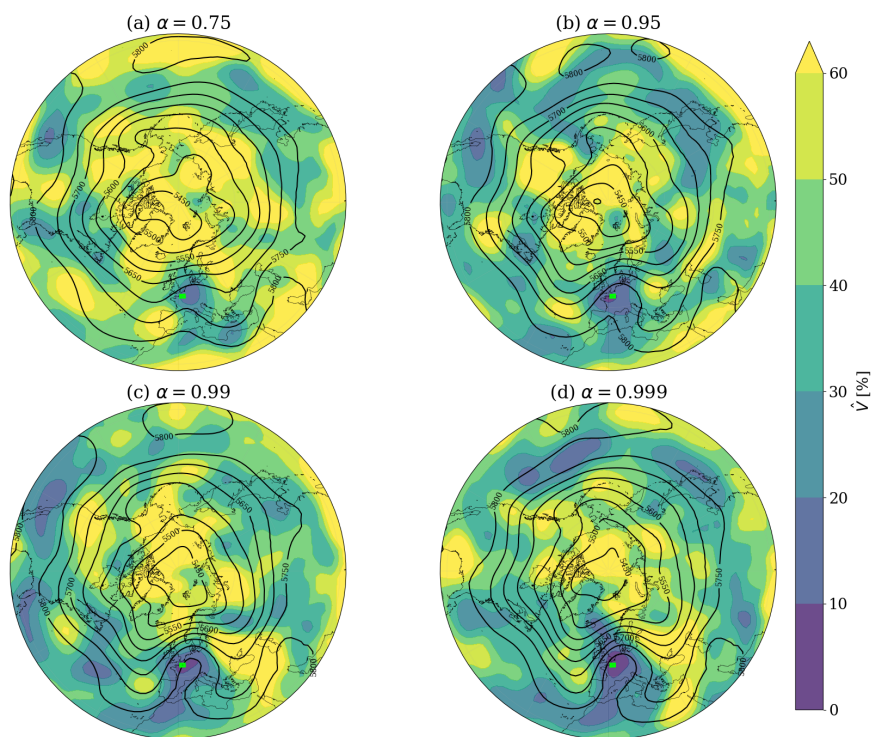


Fig. A3 Composite \hat{A} (contours) [m] and normalized variance \hat{V} (colors) of the Z500 field for increasing α -th order of the quantile of the W observable's empirical distribution. The figure is computed for a rolling mean window of $r = 5$ days. The normalized variance is computed after averaging over the rolling window and is expressed in %. The green box displays the location where the observable is computed.

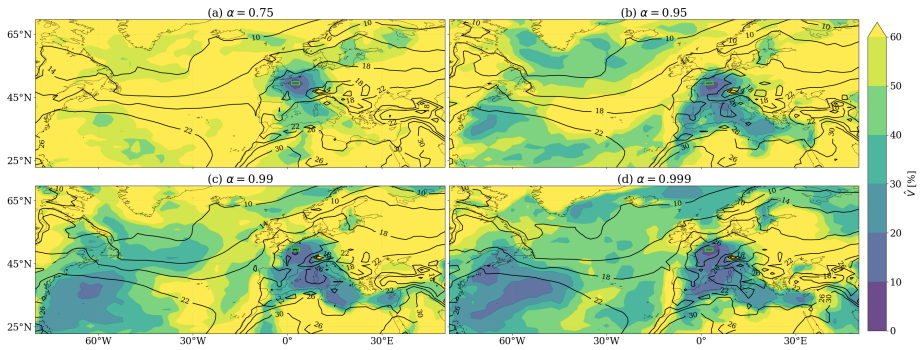


Fig. A4 Composite \hat{A} (contours) [$^{\circ}\text{C}$] and normalized variance \hat{V} (colors) of the T2M field for increasing α -th order of the quantile of the W observable's empirical distribution. The figure is computed for a rolling mean window of $r = 1$ day. The normalized variance is computed after averaging over the rolling window and is expressed in %. The green box displays the location where the observable is computed.

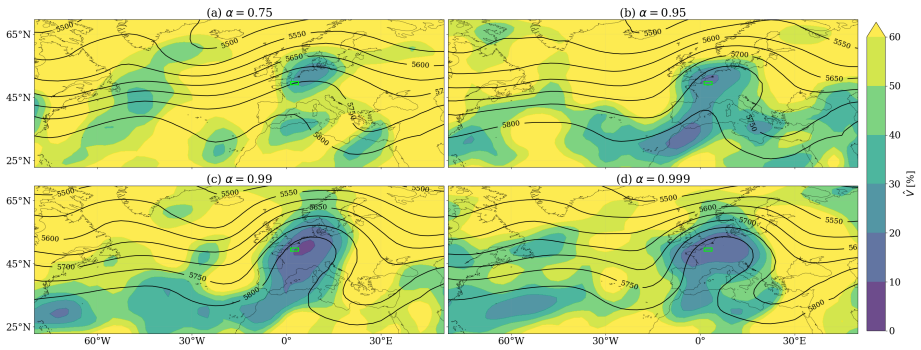


Fig. A5 Composite \hat{A} (contours) [m] and normalized variance \hat{V} (colors) of the Z500 field for increasing α -th order of the quantile of the W observable's empirical distribution. The figure is computed for a rolling mean window of $r = 1$ day. The normalized variance is computed after averaging over the rolling window and is expressed in %. The green box displays the location where the observable is computed.

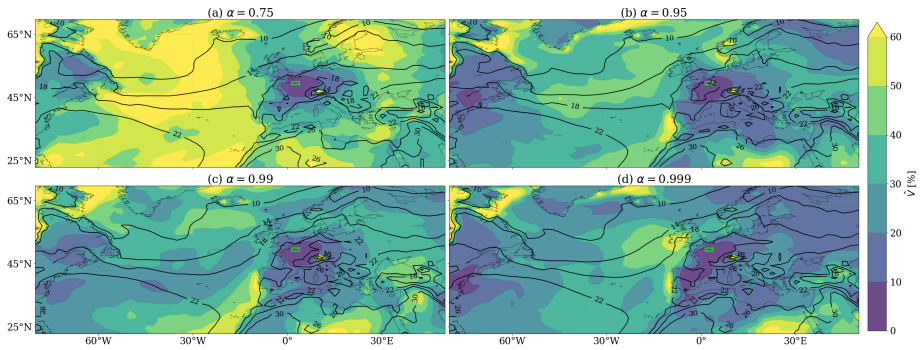


Fig. A6 Composite \hat{A} (contours) [$^{\circ}\text{C}$] and normalized variance \hat{V} (colors) of the T2M field for increasing α -th order of the quantile of the W observable's empirical distribution. The figure is computed for a rolling mean window of $r = 15$ days. The normalized variance is computed after averaging over the rolling window and is expressed in %. The green box displays the location where the observable is computed.

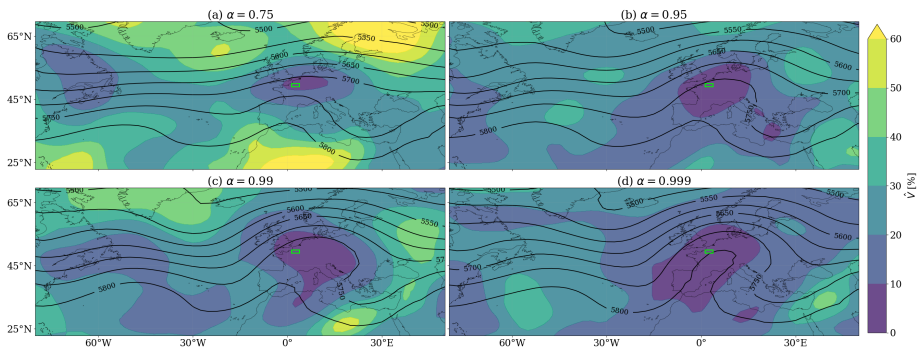


Fig. A7 Composite \hat{A} (contours) [m] and normalized variance \hat{V} (colors) of the Z500 field for increasing α -th order of the quantile of the W observable's empirical distribution. The figure is computed for a rolling mean window of $r = 15$ days. The normalized variance is computed after averaging over the rolling window and is expressed in %. The green box displays the location where the observable is computed.

30 *Typicality extreme heatwaves*

Observable	r=1			r=5			r=15		
	0.95	0.99	0.999	0.95	0.99	0.999	0.95	0.99	0.999
S	22%	30%	22%	22%	16%	24%	24%	22%	16%
W	22%	16%	20%	14%	30%	28%	16%	22%	24%
N	28%	26%	14%	20%	20%	20%	16%	32%	20%
WCE	56%	44%	50%	38%	60%	48%	42%	32%	58%

Table A1 Percentage of cut-off lows in the dynamics leading to extremes.

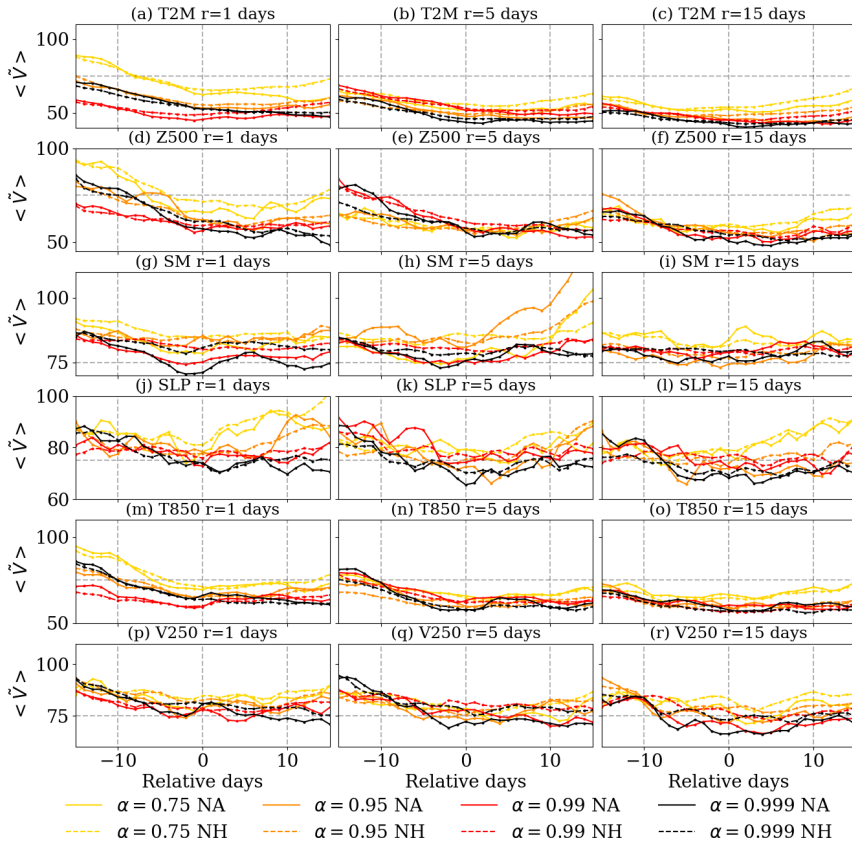


Fig. A8 Evolution of the normalized variance $\langle \tilde{V}(t) \rangle$ averaged over the Euro-Atlantic sector (plain lines) and the North-Hemisphere (dashed lines) for the different variables (S observable). The normalized variance is expressed in %. The colors show the α -th order quantile of the observable's empirical distribution. The time is expressed relative to the day when the observable is such that $F_r(x_t) \simeq q_\alpha$. The gray dashed line shows the 75% level.

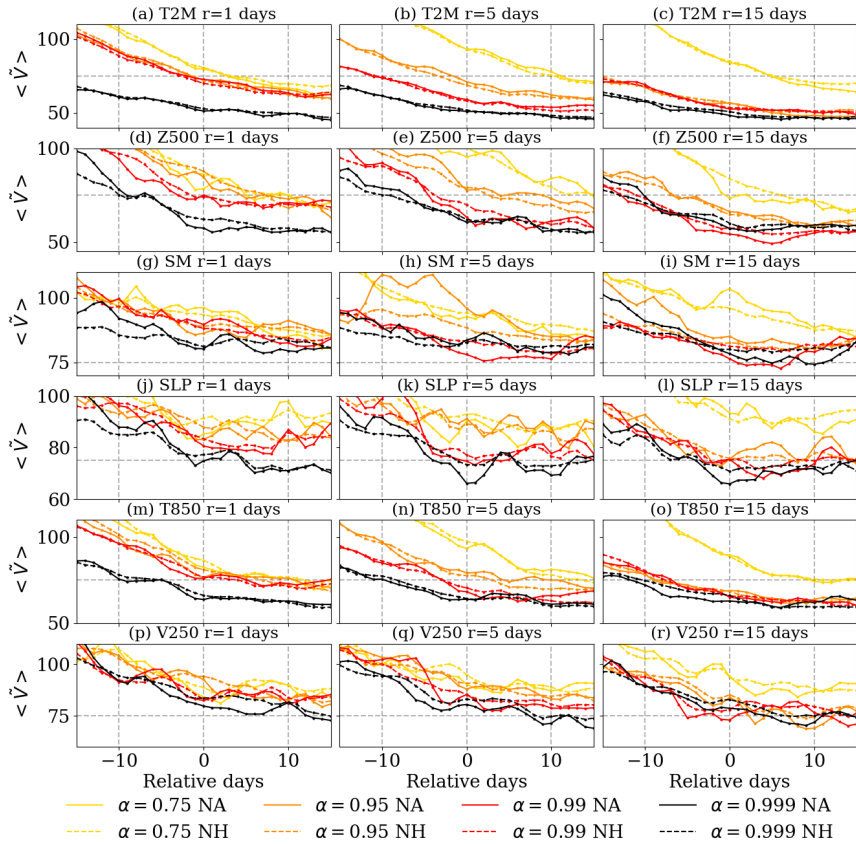


Fig. A9 Evolution of the normalized variance $\langle \tilde{V}(t) \rangle$ averaged over the Euro-Atlantic sector (plain lines) and the North-Hemisphere (dashed lines) for the different variables (N observable). The normalized variance is expressed in %. The colors show the α -th order quantile of the observable's empirical distribution. The time is expressed relative to the day when the observable is such that $F_r(x_t) \simeq q_\alpha$. The gray dashed line shows the 75% level.

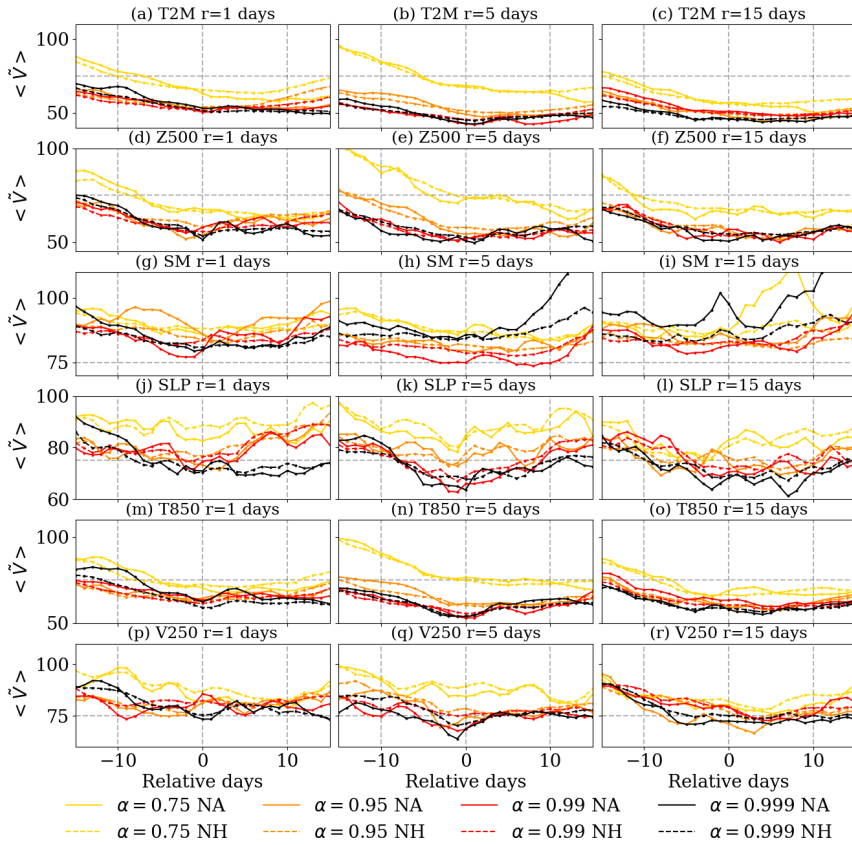


Fig. A10 Evolution of the normalized variance $\langle \tilde{V}(t) \rangle$ averaged over the Euro-Atlantic sector (plain lines) and the North-Hemisphere (dashed lines) for the different variables (WCE observable). The normalized variance is expressed in %. The colors show the α -th order quantile of the observable's empirical distribution. The time is expressed relative to the day when the observable is such that $F_r(x_t) \simeq q_\alpha$. The gray dashed line shows the 75% level.

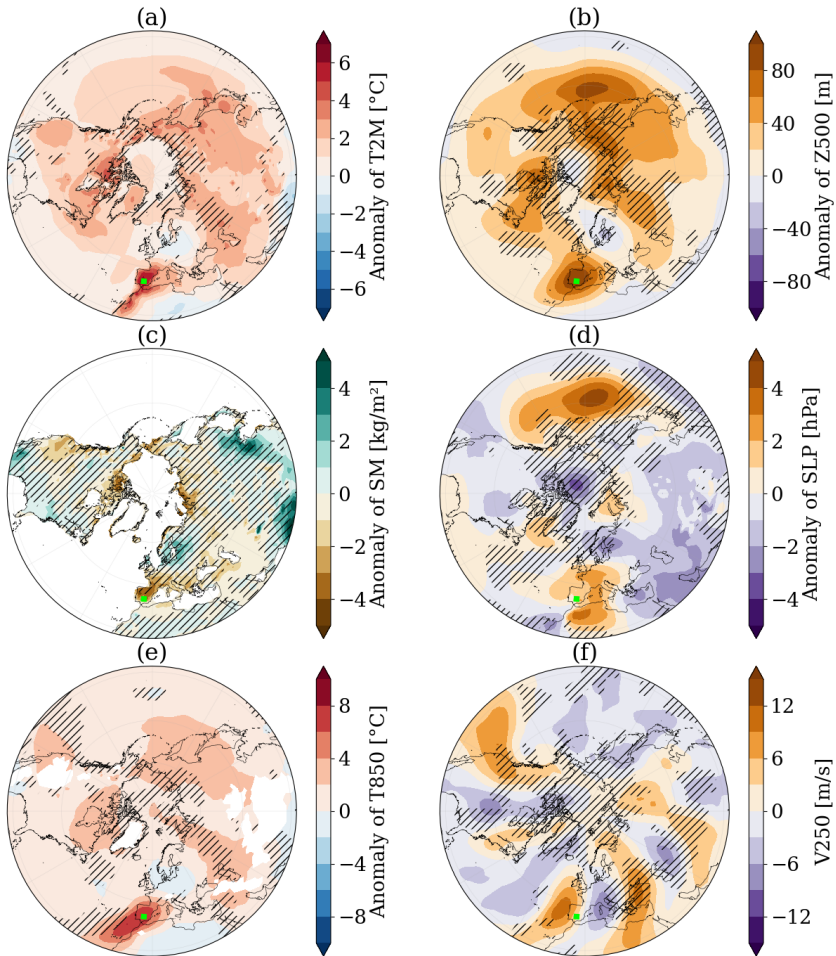


Fig. A11 Composite maps \hat{A} for T2M, Z500, SM, SLP, T850 and V250 for the quantile of order $\alpha = 0.999$ of the S observable's empirical distribution. The hatch areas correspond to $\hat{V} > 50\%$. The figure is computed for a rolling mean window of $r = 5$ days. Anomaly of (a) air temperature at 2m (T2M), (b) geopotential height at 500hPa (Z500), (c) soil moisture (SM), (d) sea-level pressure (SLP) and (e) temperature at 850hPa (T850), and (f) meridional wind speed at 250hPa (V250). The anomalies are computed with respect to their average over the summer (JJA). The green box displays the location where the observable is computed.

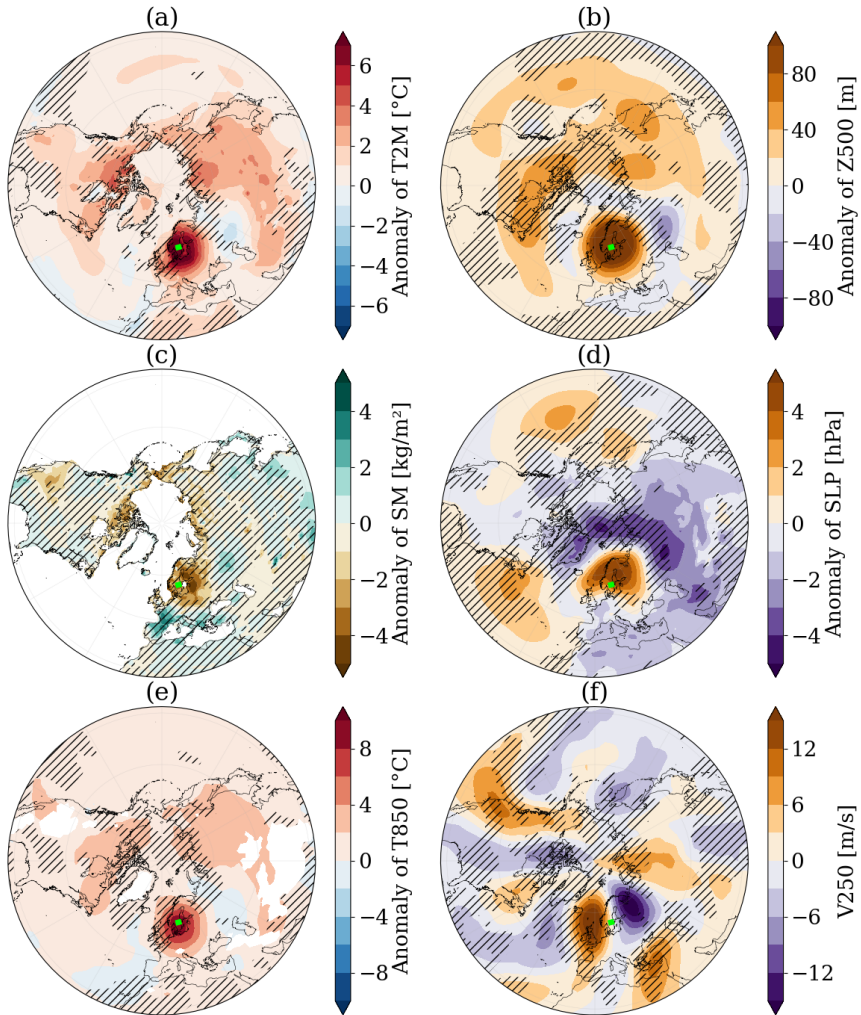


Fig. A12 Composite maps \hat{A} for T2M, Z500, SM, SLP, T850 and V250 for the quantile of order $\alpha = 0.999$ of the N observable's empirical distribution. The hatch areas correspond to $\hat{V} > 50\%$. The figure is computed for a rolling mean window of $r = 5$ days. Anomaly of (a) air temperature at 2m (T2M), (b) geopotential height at 500hPa (Z500), (c) soil moisture (SM), (d) sea-level pressure (SLP) and (e) temperature at 850hPa (T850), and (f) meridional wind speed at 250hPa (V250). The anomalies are computed with respect to their average over the summer (JJA). The green box displays the location where the observable is computed.

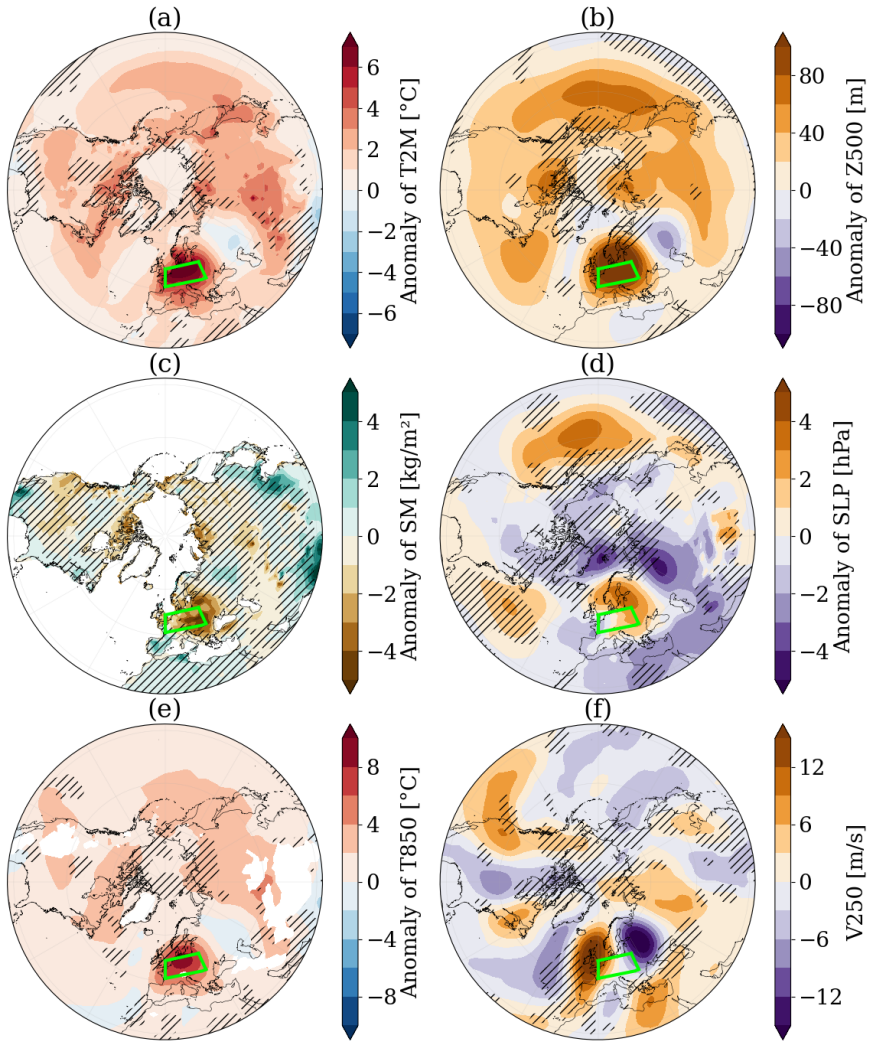


Fig. A13 Composite maps \hat{A} for T2M, Z500, SM, SLP, T850 and V250 for the quantile of order $\alpha = 0.999$ of the WCE observable's empirical distribution. The hatch areas correspond to $\hat{V} > 50\%$. The figure is computed for a rolling mean window of $r = 5$ days. Anomaly of (a) air temperature at 2m (T2M), (b) geopotential height at 500hPa (Z500), (c) soil moisture (SM), (d) sea-level pressure (SLP) and (e) temperature at 850hPa (T850), and (f) meridional wind speed at 250hPa (V250). The anomalies are computed with respect to their average over the summer (JJA). The green box displays the location where the observable is computed.

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