

Elastic turbulence in polymer solutions: *modelling and numerical simulations*

By

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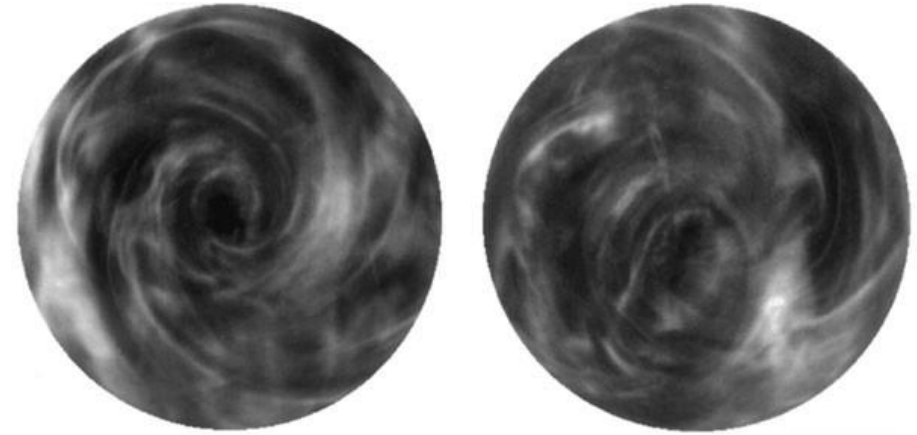
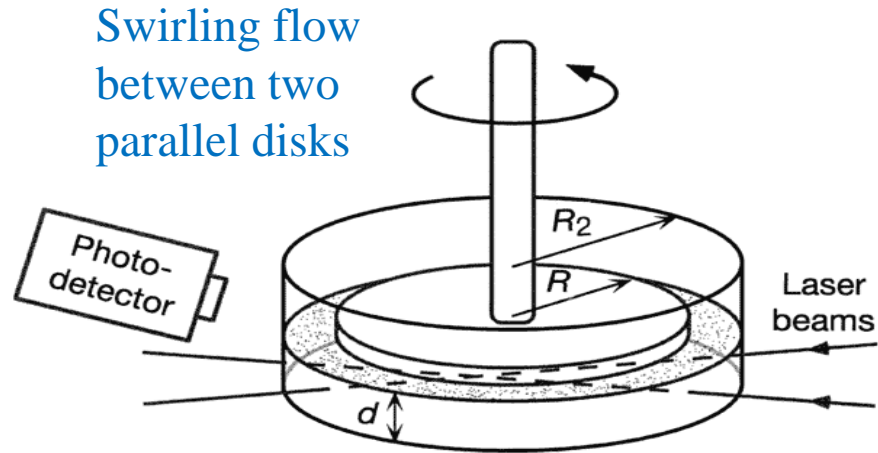


Elastic turbulence:

A chaotic flow that emerges in polymer solutions at low Reynolds number and high Weissenberg number

Purely elastic turbulence (Experiment)

Irregular patterns and spiral-like structures



Wi=13, Re=0.7

$$\text{Reynolds number} = \frac{\text{Viscous time scale}}{\text{Convective time scale}} = \frac{L^2/\nu}{L/U} = \frac{U L}{\nu}$$

$$\text{Weissenberg number} = \frac{\text{Polymer relaxation time}}{\text{Flow time scale}} = \frac{\tau}{T}$$

A. Groisman & V. Steinberg, Nature, 2000

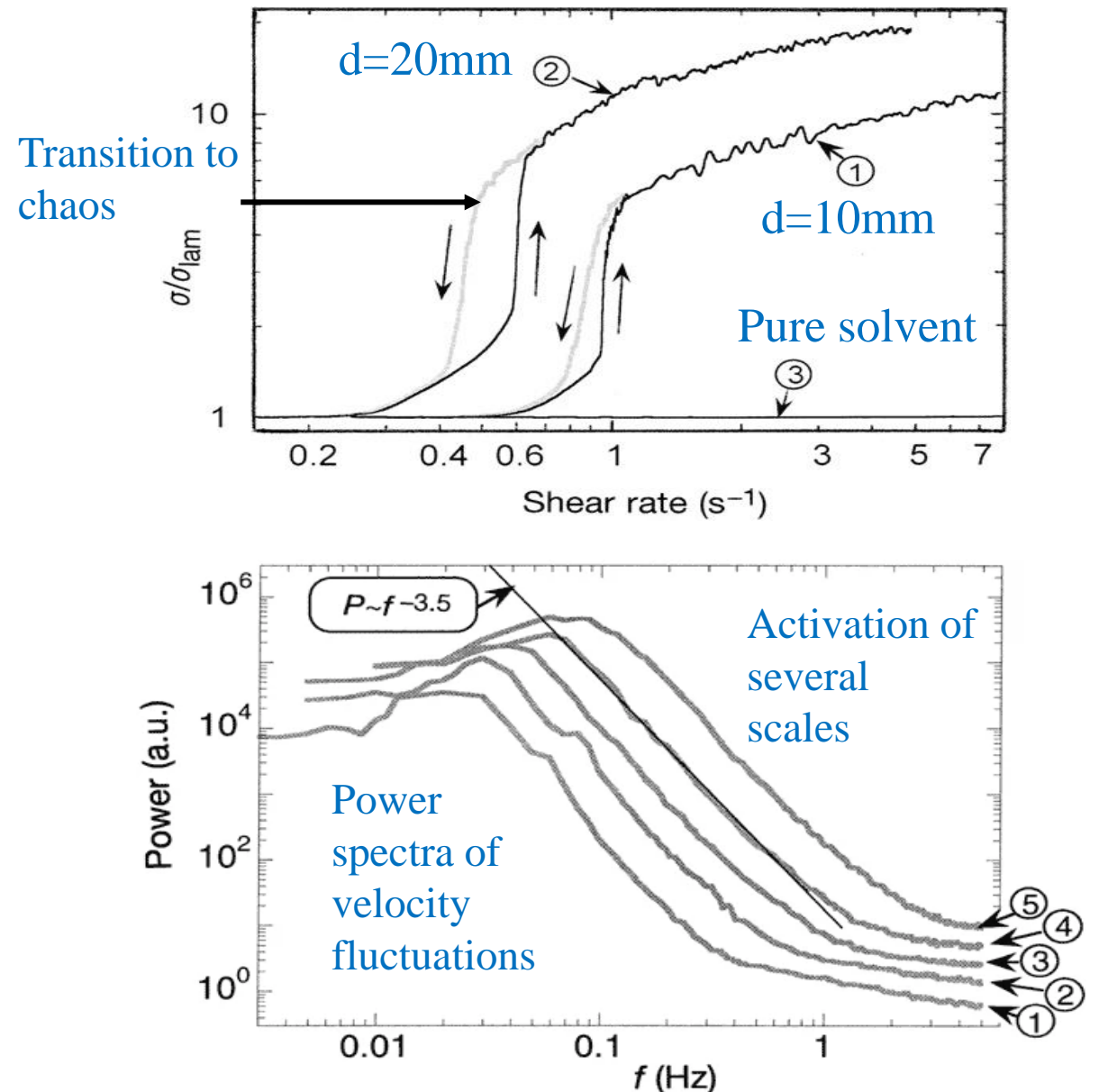
V. Steinberg, Nature, Annu. Rev. Fluid Mech. 2021

S. Datta et al., Phys. Rev. Fluids, 2022

Characteristics of elastic turbulence:

- Activation of several spatial and temporal scales with a steep power law decay in the spectra of the velocity fluctuations
- Increase in the flow resistance compared to the laminar flow at the same Reynolds number
- Enhancement of the mixing rate by orders of magnitude compared to diffusion

Purely elastic turbulence (Experiment)



Applications of elastic turbulence

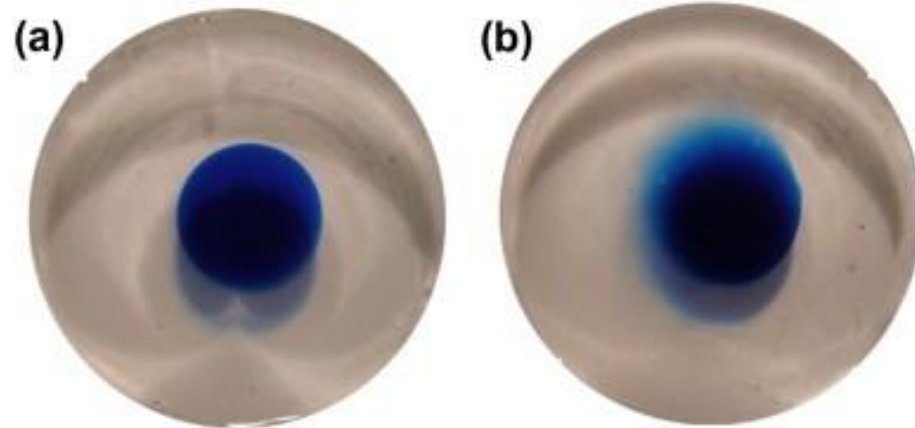
Mixing of a passive scalar (Experiment)

Poole et al., J. Non-Newton. Fluid Mech, 2012

In a pure solvent, $Wi=0$

t = 0 sec

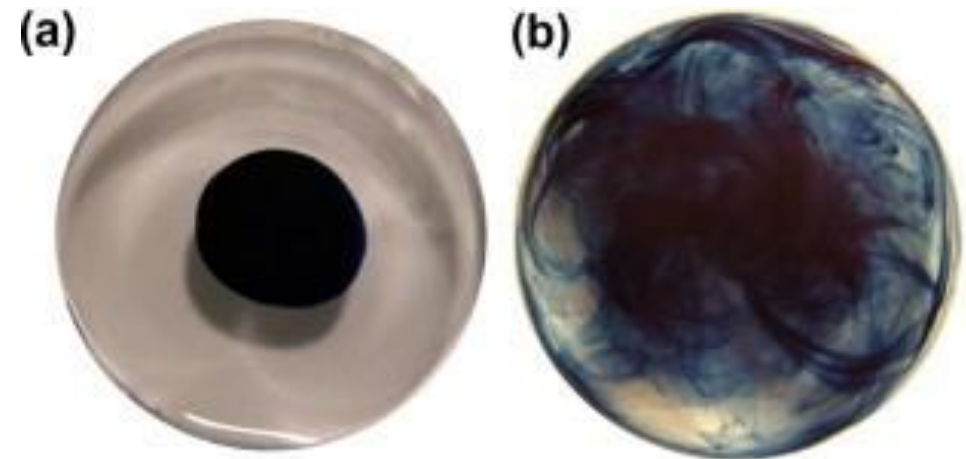
t=120 sec



In the elastic turbulent regime generated by polymers

t=0 sec

t= 30 sec



$Wi= 5.2$

Other applications:

- Increase in the heat transfer efficiency
- Increased efficiency in the formation of emulsions

Traore et al., J. Non-Newton. Fluid Mech, 2015

Governing equations for the viscoelastic fluids

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla p + \nu \nabla^2 \mathbf{u} + \frac{\mu}{\tau} \nabla \cdot \mathbf{C} + \mathbf{f} \quad - \text{Velocity field}$$

$$\frac{\partial \mathbf{C}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{C} = \mathbf{C} \cdot \nabla \mathbf{u} + (\nabla \mathbf{u})^T \cdot \mathbf{C} - \frac{1}{\tau} (\mathbf{C} - \mathbf{I}) \quad - \text{Conformation tensor field}$$

Limitations:

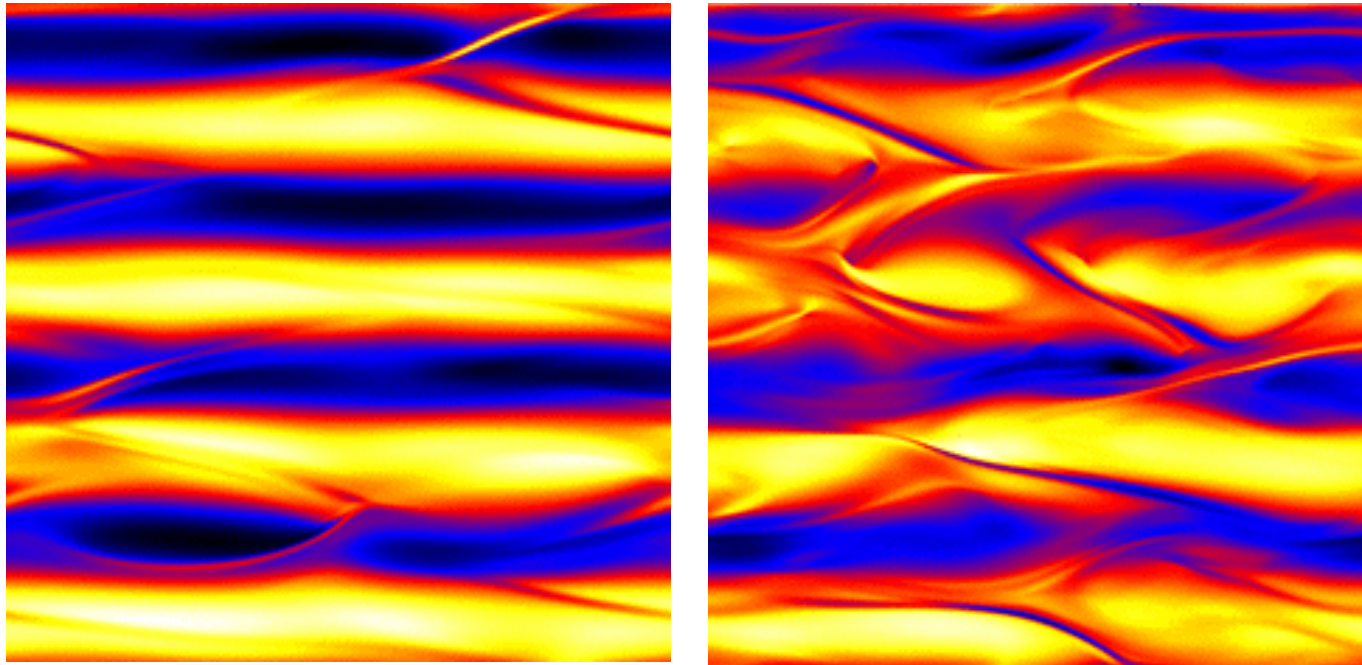
- Loss of positive-definiteness leading to numerical instabilities at high Weissenberg numbers
- Simulations in more realistic geometries to include boundary effects
- Three-dimensional simulations

Numerical treatments:

- Addition of a diffusive term *i.e.* $\kappa(\mathbf{C}, \nabla \mathbf{u}) \nabla^2 \mathbf{C}$
- Use of shock-capturing schemes like *Kurganov-Tadmor scheme*
- Reformulation of the constitutive equations

Numerical simulations : *Kolmogorov forcing*

Contours of the vorticity field



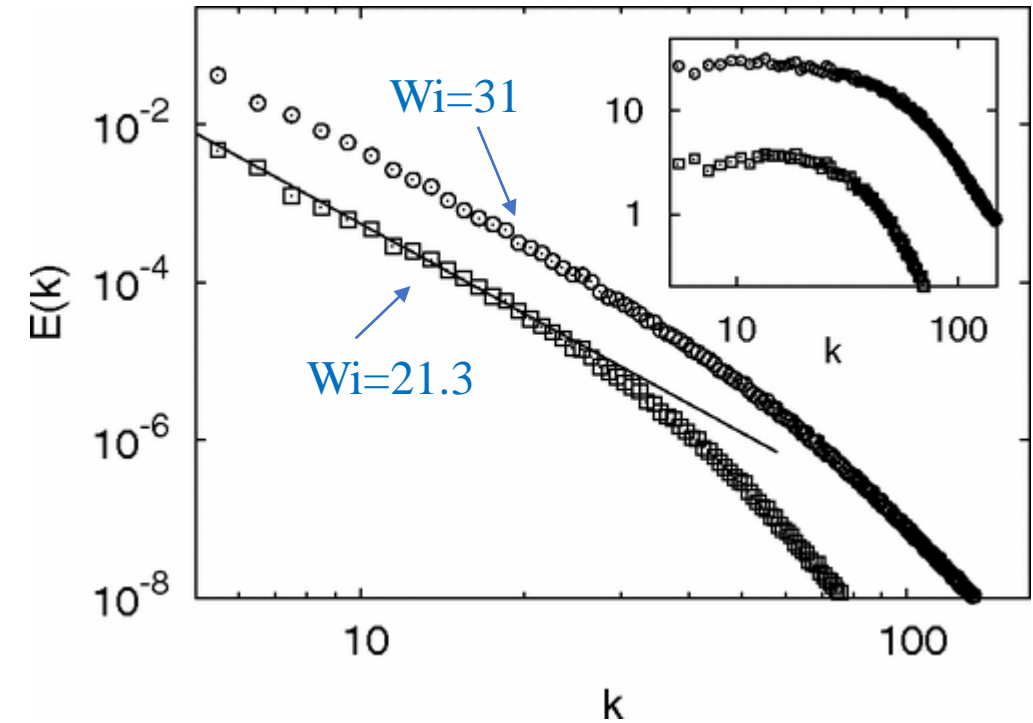
Wi=21.3

Wi=31

Flow transition from regular to irregular pattern

Berti et al., Phys. Rev. E, 2008

Spatial energy spectra



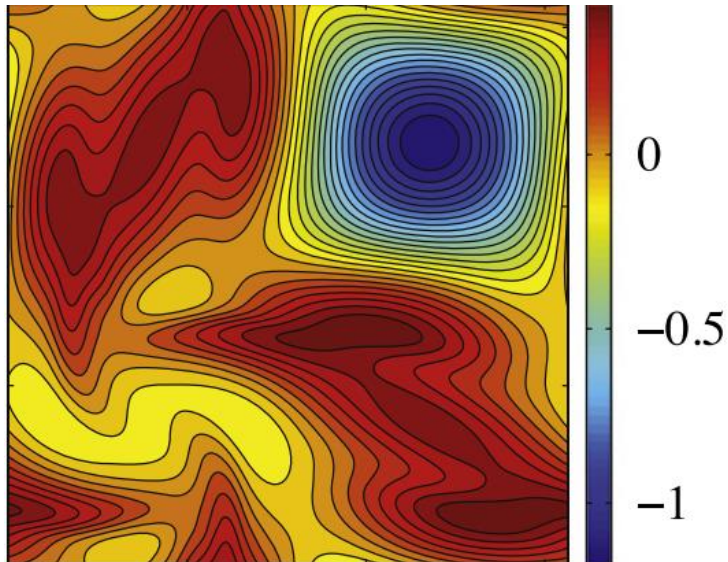
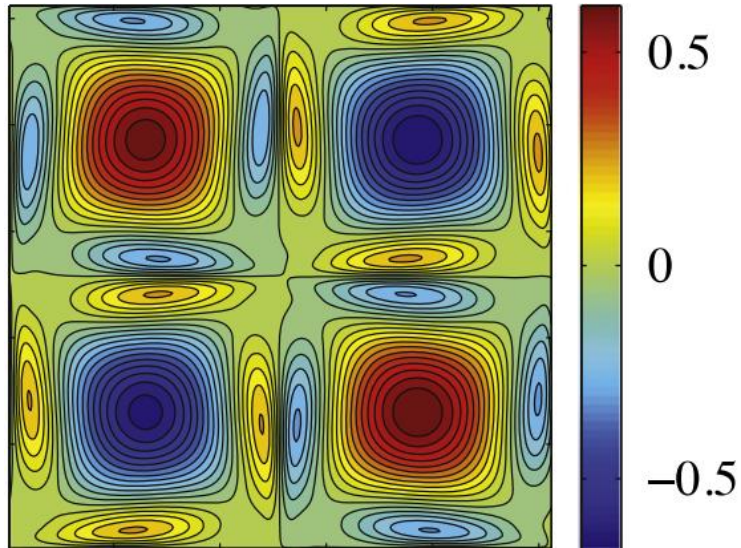
Steep power-law with an exponent ~ 3.8

Two-dimensional simulations

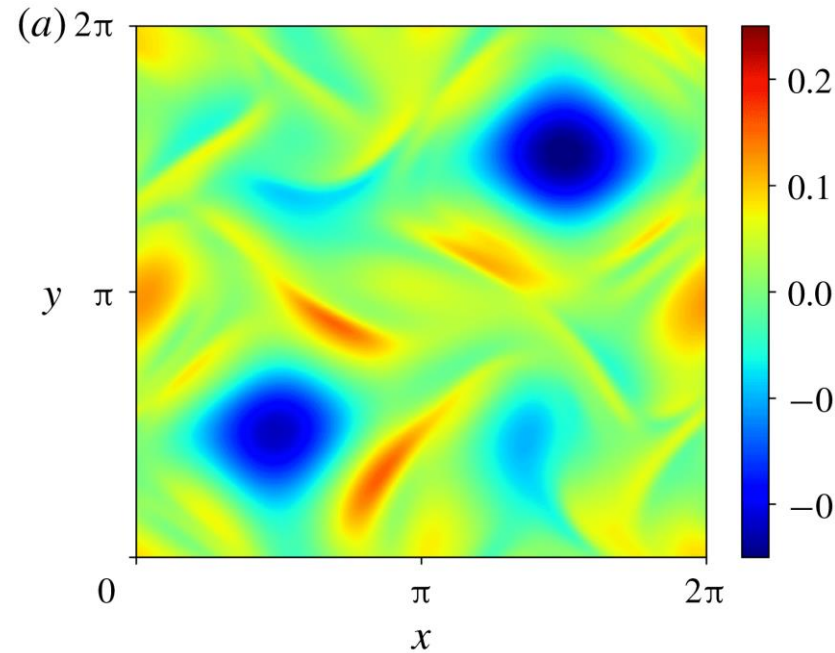
Four-roll mill forcing

Cellular forcing

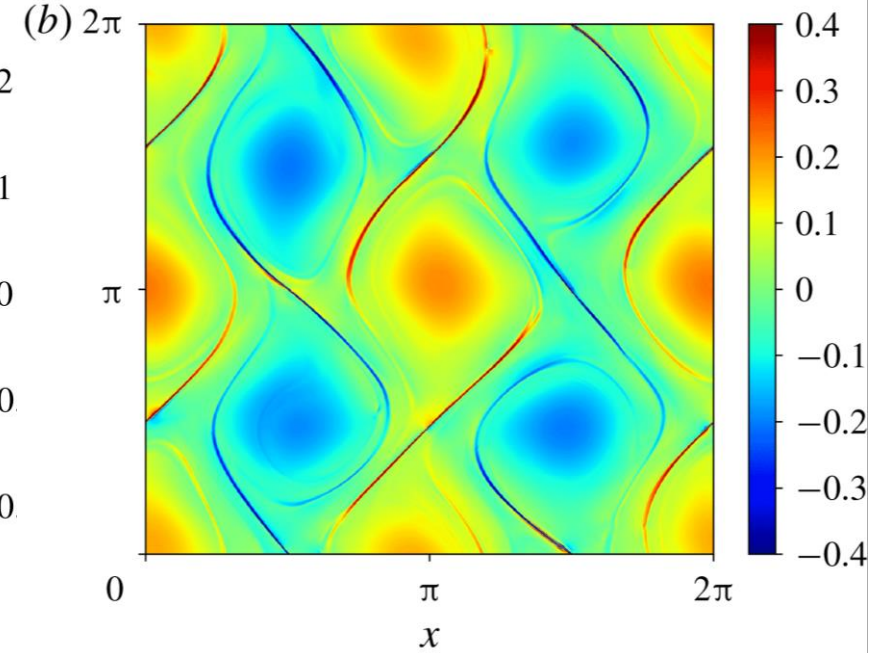
With diffusion



With diffusion



Without diffusion



Two-dimensional simulations

- Addition of a large amount of global diffusion produces unphysical artifacts by smearing out the gradients in the stress field

Thomases et al., Physica D, 2011

A. Gupta & D. Vincenzi, J. Fluid Mech., 2019

Reformulation of the constitutive equations

Log Cholesky decomposition

Symmetric square root decomposition

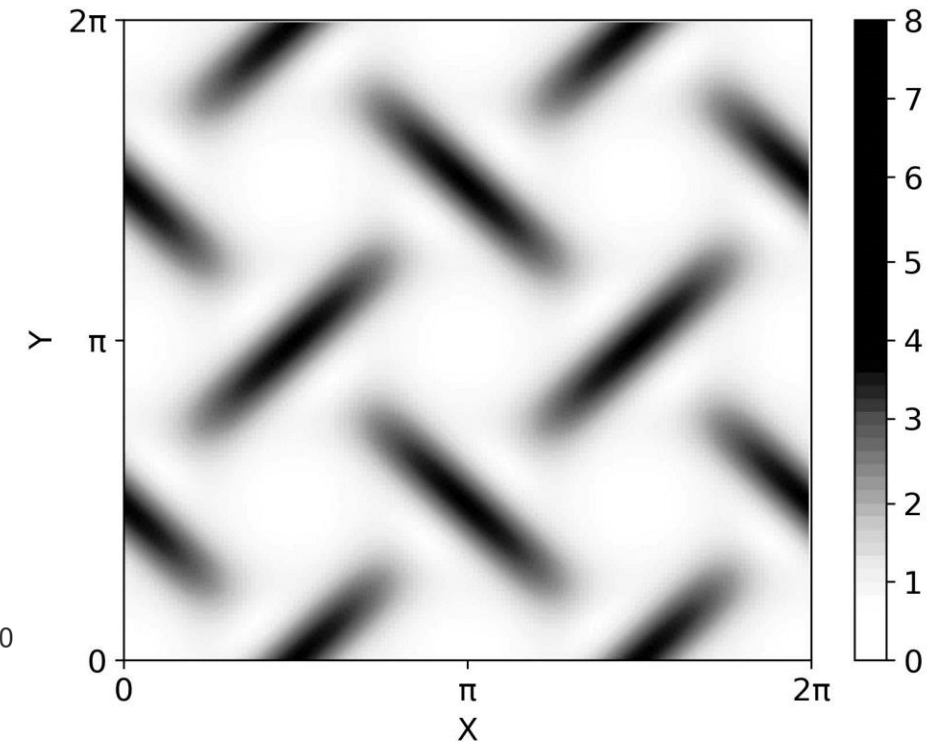
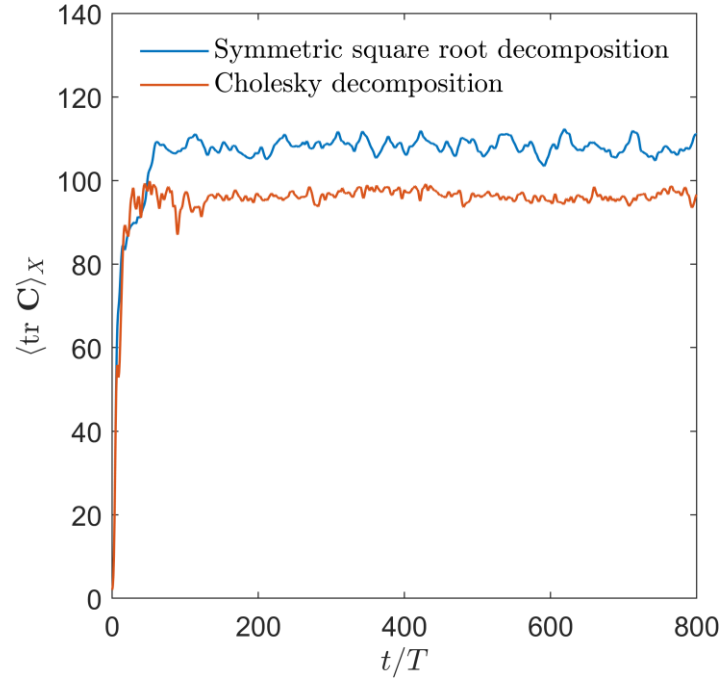
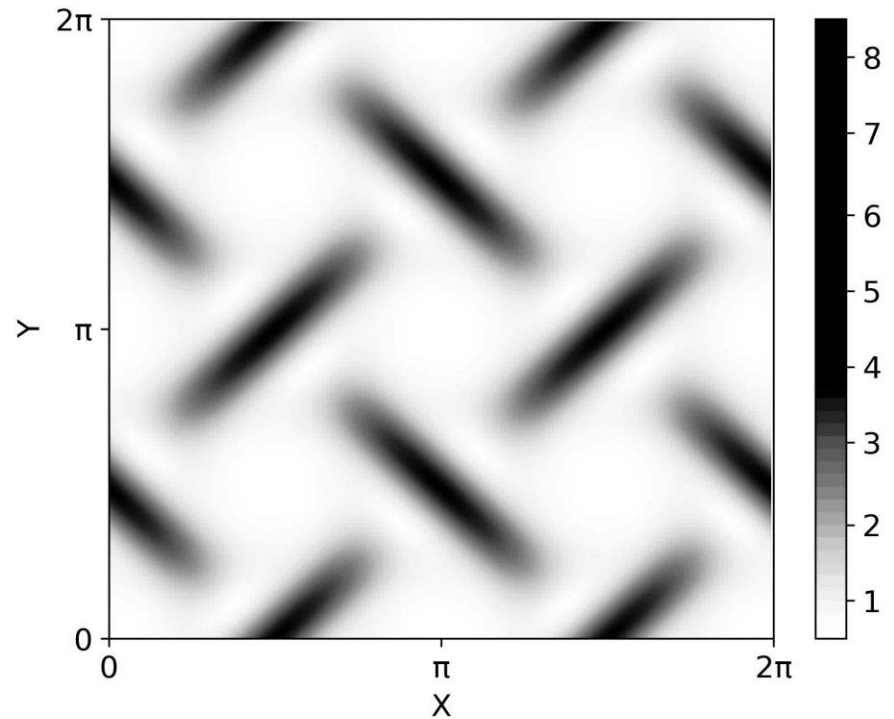
$C = L L^T$, L is the lower triangular matrix

Cellular forcing $C = b b^T$, b is the symmetric square root of C

log(L) transport eqn. is solved

$$f = f_0 k_f [\cos(k_f x) + \cos(k_f y)]$$

b transport eqn. is solved



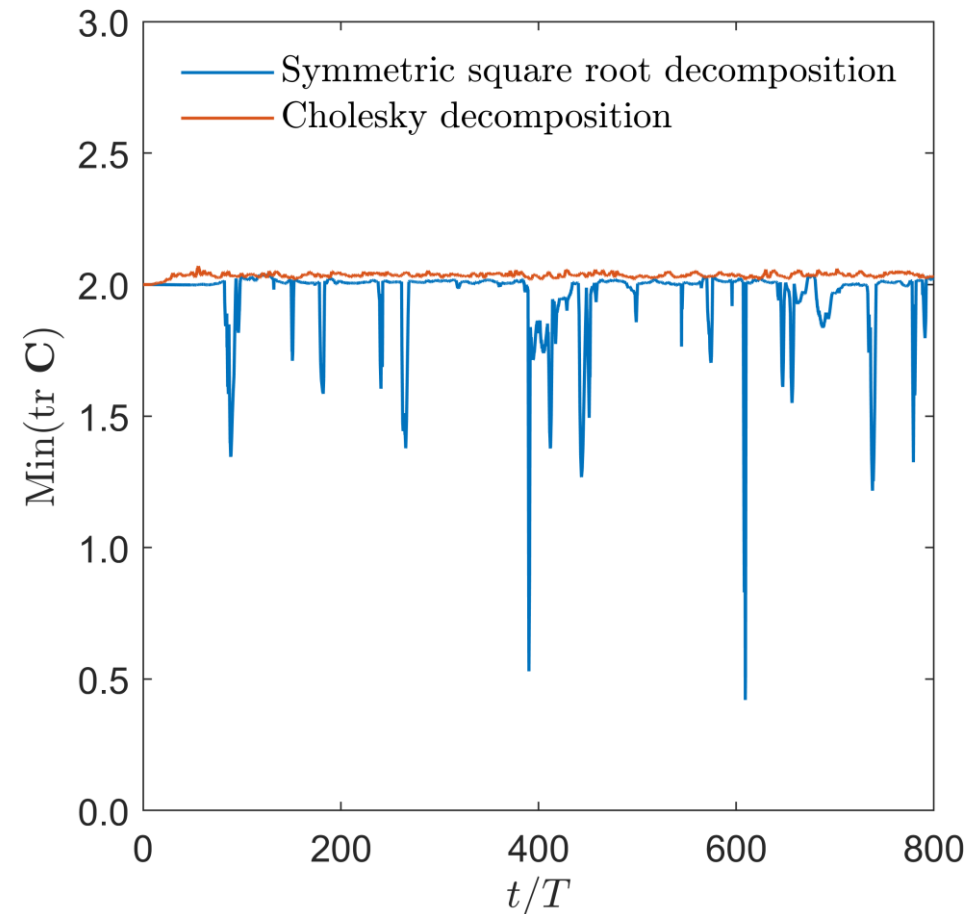
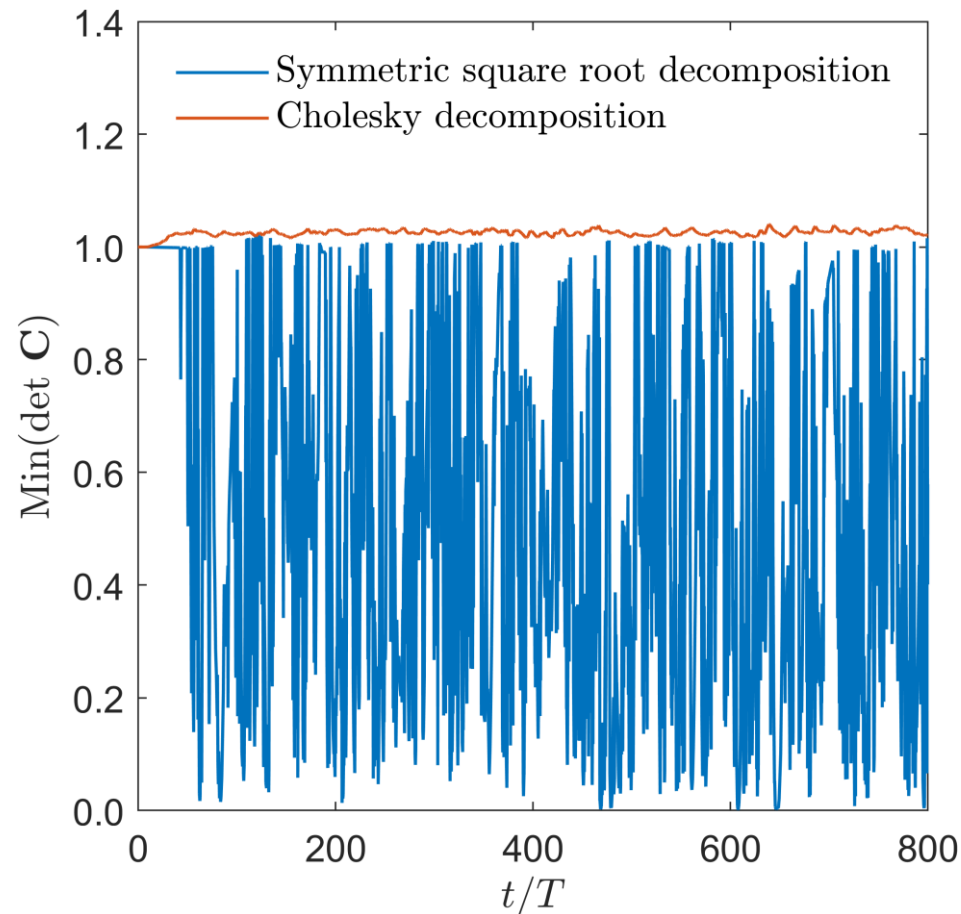
Re=0, Wi=10, no artificial diffusion

Evolution of determinant and trace of \mathbf{C}

An a priori constraint : If $\det(\mathbf{C}) \geq 1$ at $t=0$, it must remain greater than one $\forall t$

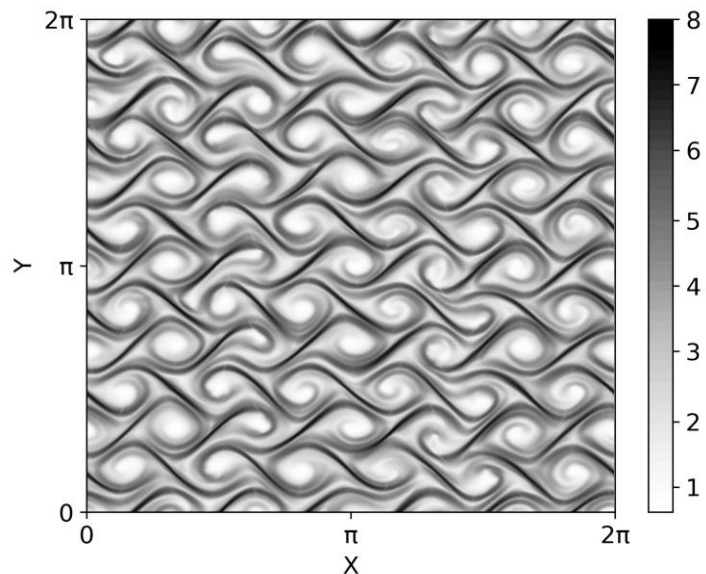
Hu et al., Commun. Math. Sci., 2007

For any symmetric matrix \mathbf{C} : $tr(\mathbf{C}) \geq d \det(\mathbf{C})^{1/d}$; In 2D : $tr(\mathbf{C}) \geq 2$

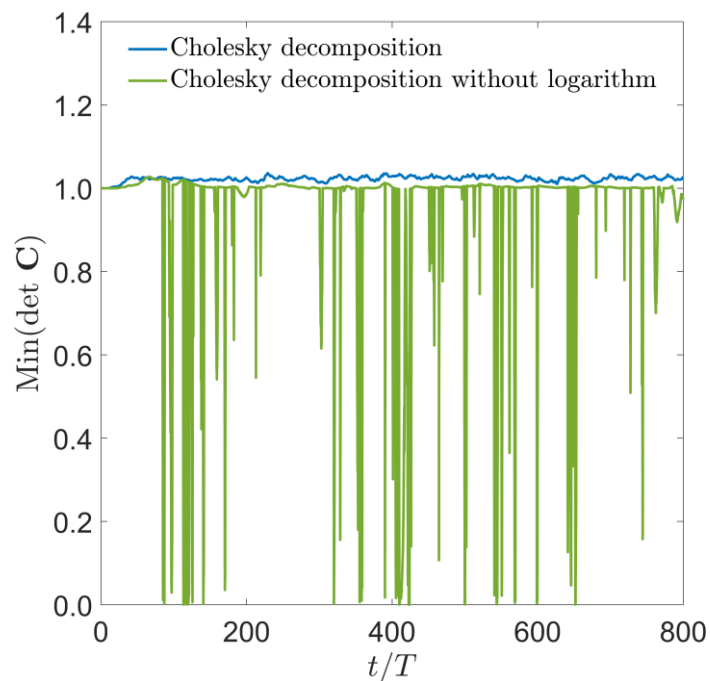
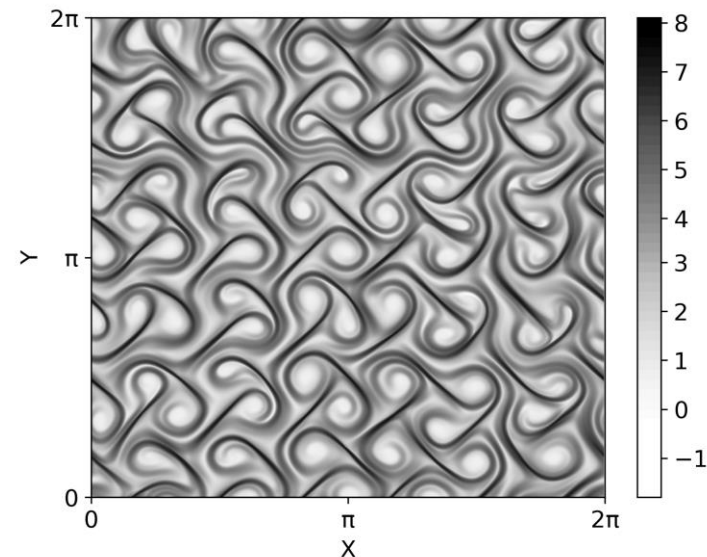


Comparison of large-scale structures at $k_f=6$

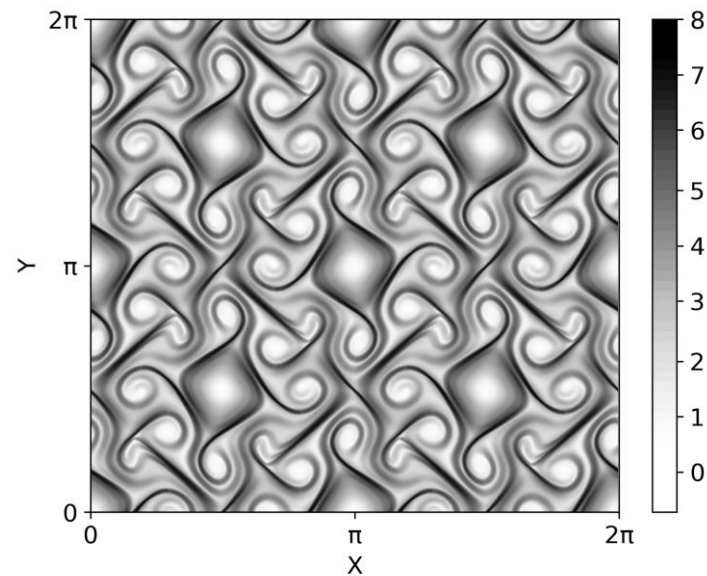
**Log
Cholesky**



**Symmetric
square
root**

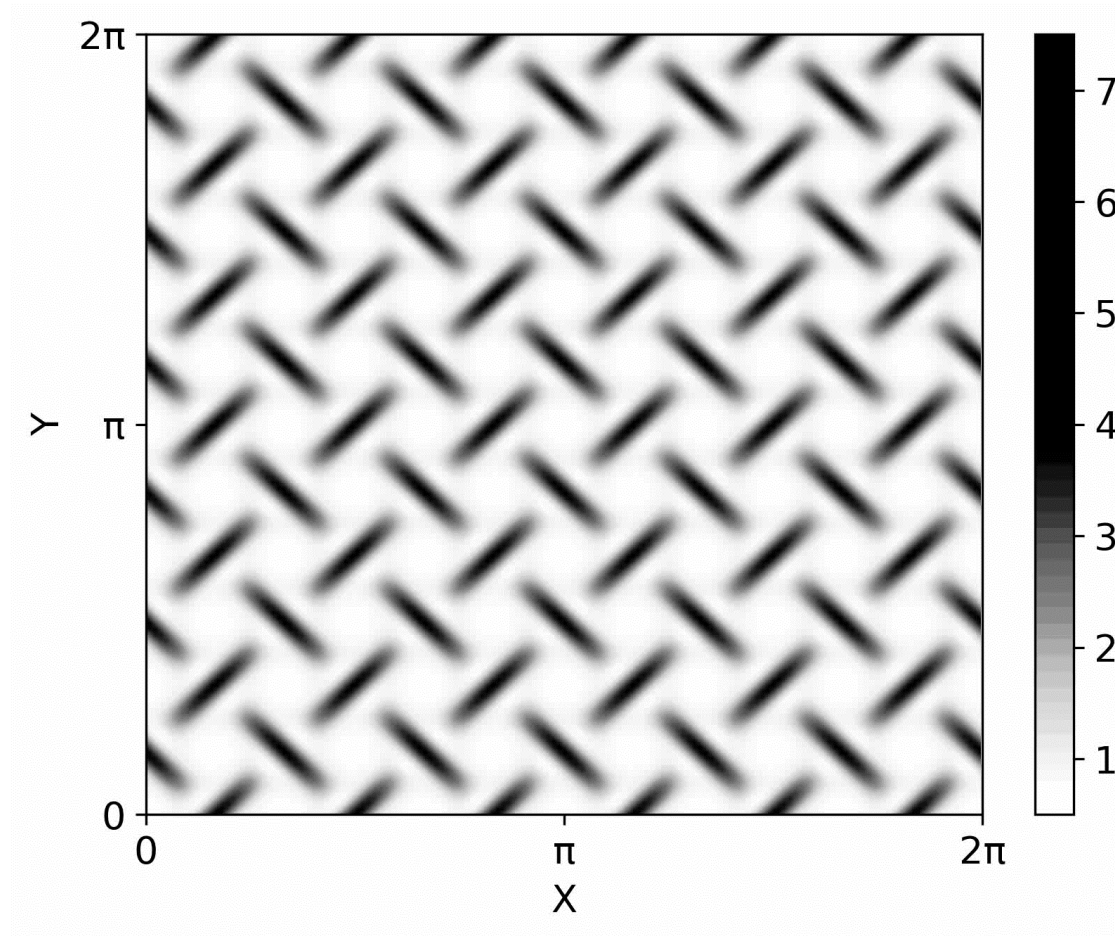


**Cholesky
without
logarithm**



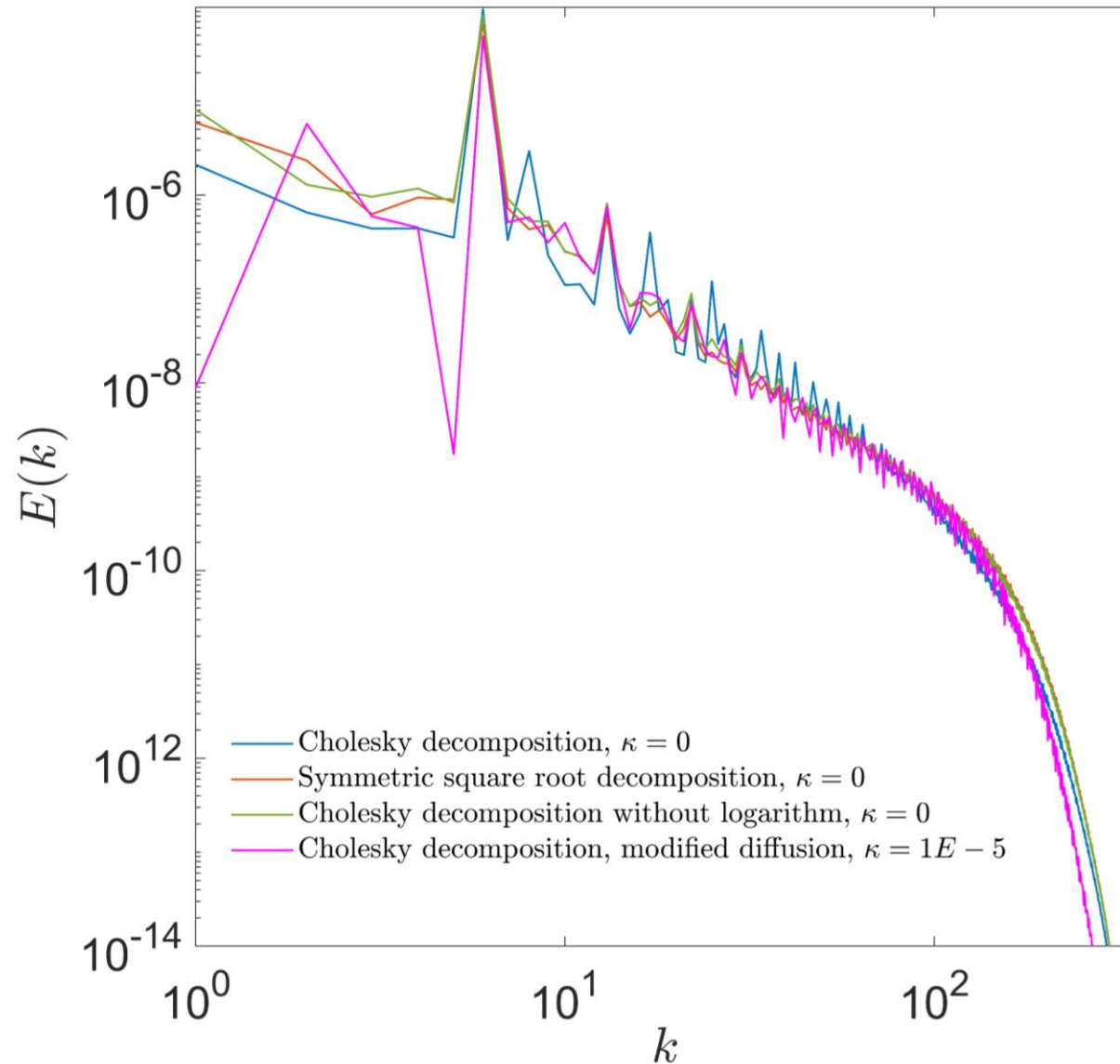
Addition of a modified diffusion $\kappa(\mathbf{C}, \nabla \mathbf{u}) \nabla^2 \mathbf{C}$

Log Cholesky with diffusion



Large-scale structures dominate the stress field

Energy spectra $k_f = 6$



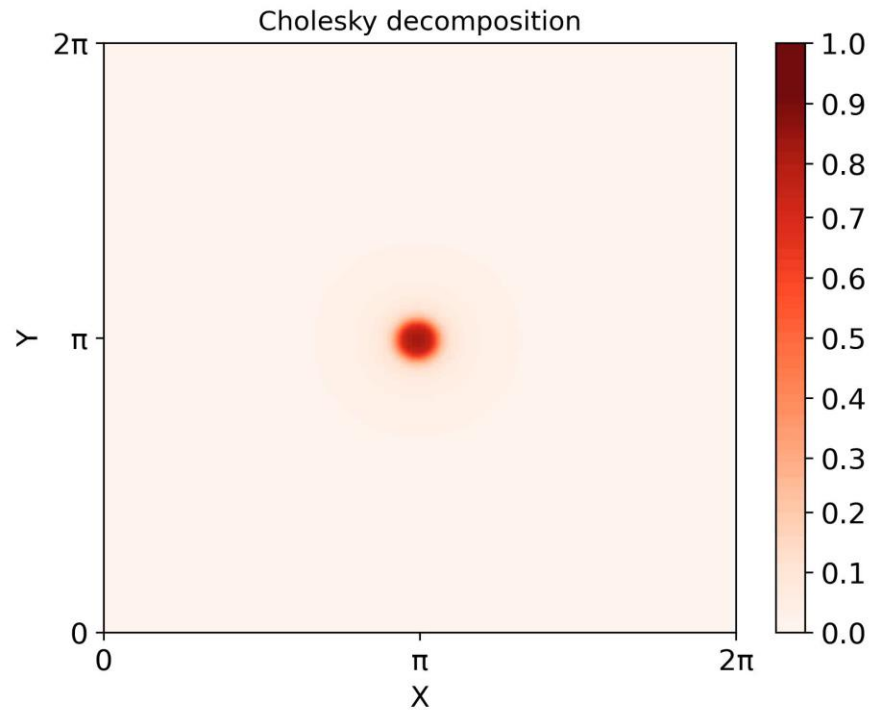
➤ Increase of energy at large-scales ($k < k_f$)

Evolution of a passive scalar field

Log Cholesky decomposition

$C = L L^T$, L is the lower triangular matrix

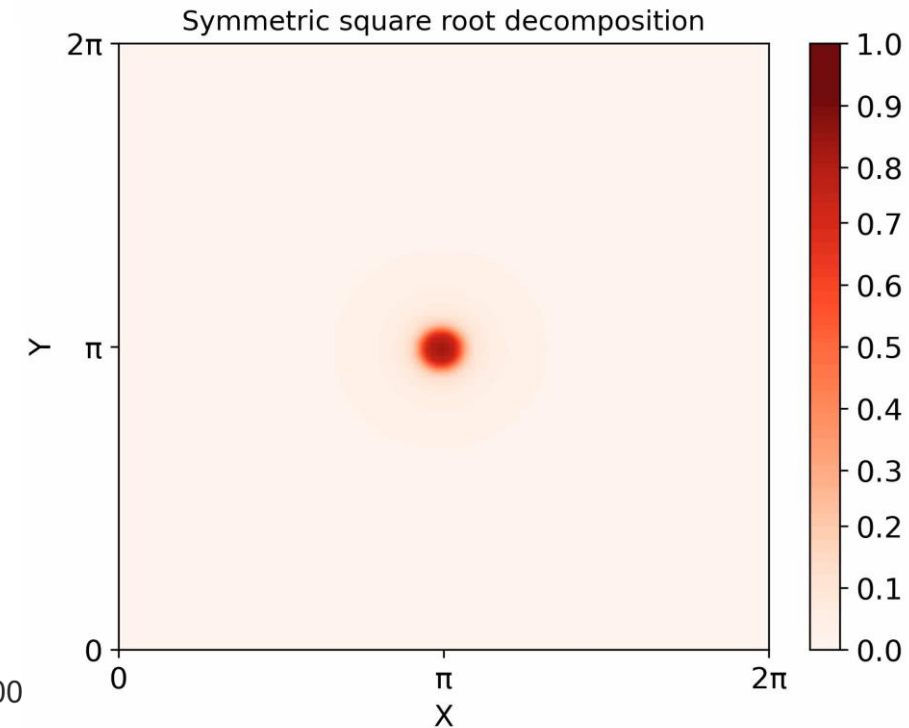
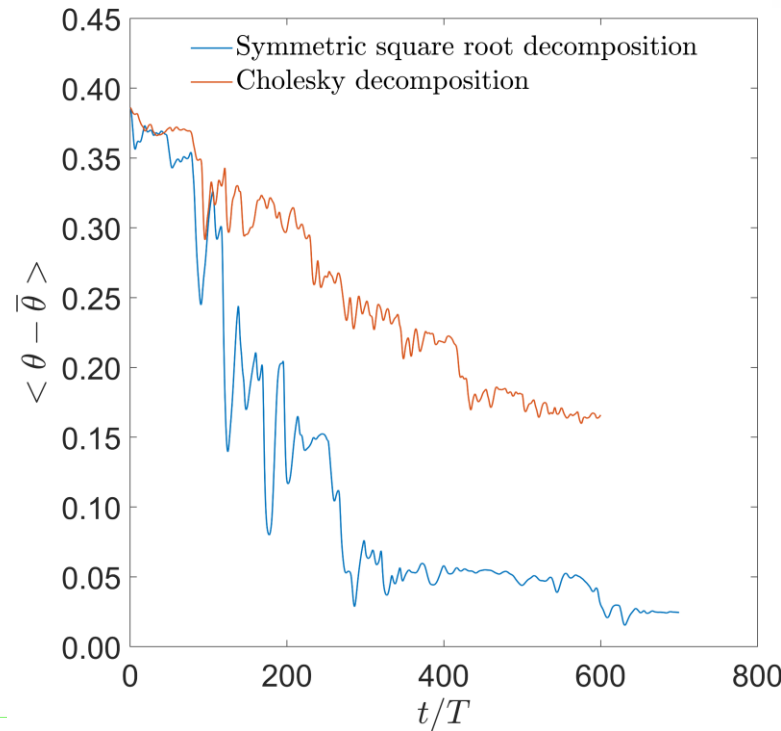
log(L) transport eqn. is solved



Symmetric square root decomposition

$C = b b^T$, b is the symmetric square root of C

b transport eqn. is solved



$$\frac{\partial \theta}{\partial t} + \mathbf{u} \cdot \nabla \theta = \kappa \nabla^2 \theta \text{ -- Passive scalar field}$$

Conclusions:

- The lower bound on the determinant of \mathbf{C} can be used as a criteria to characterise the accuracy of the numerical solutions
- Use of a logarithmic transformation supresses the numerical errors and ensures that the numerical solution satisfies all the necessary criteria for the solution to be accurate
- Addition of any form of diffusion smears out the stress gradients and generates unphysical artifacts

Thank you for your attention!!!