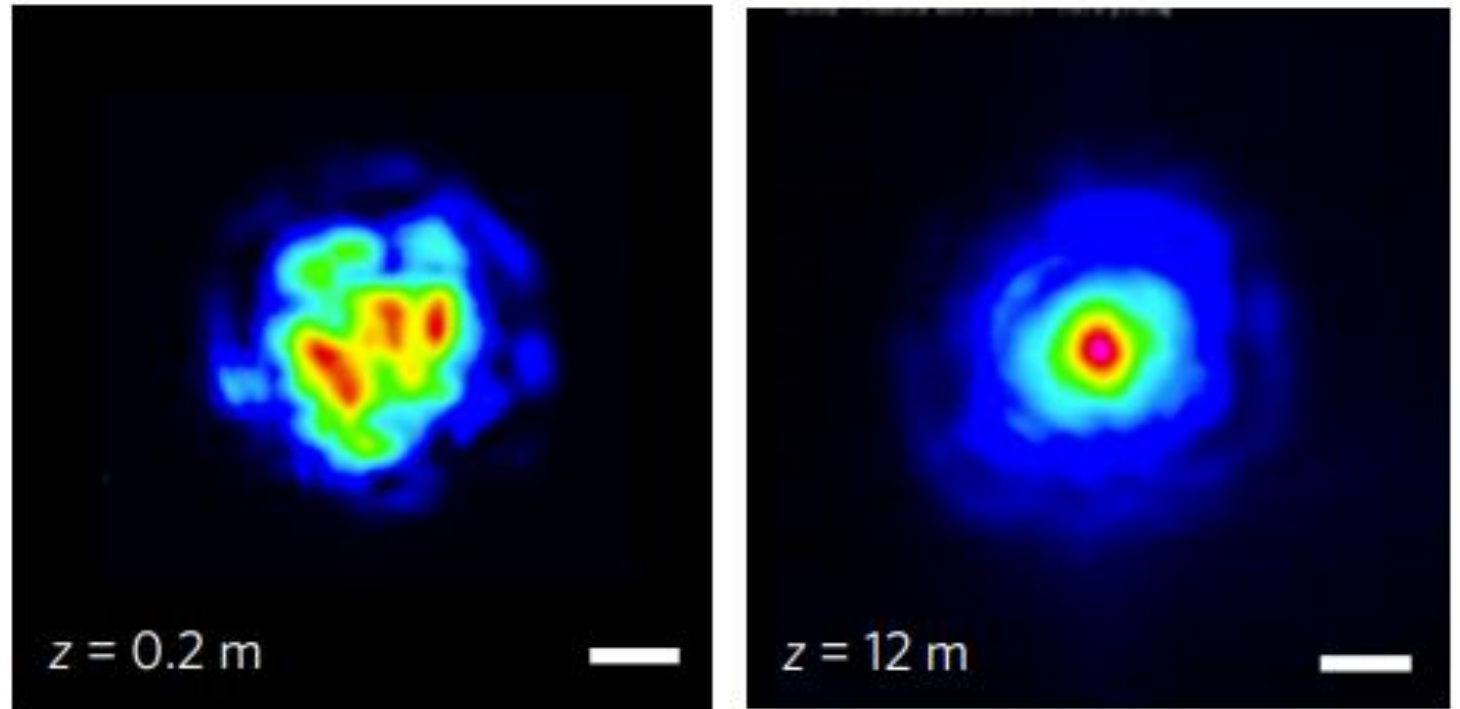


Classical wave thermalisation in chaotic multimode optical fibre

Lucas Zanaglia^{a,b}

C. Michel^{a,b}, V. Doya^{a,b}, A. Picozzi^c,
F. Mortessagne^{a,b}, J. Garnier^d

- a. Université Côte d'Azur
- b. Institut de Physique de Nice
- c. Laboratoire Interdisciplinaire Carnot de Bourgogne
- d. Ecole Polytechnique



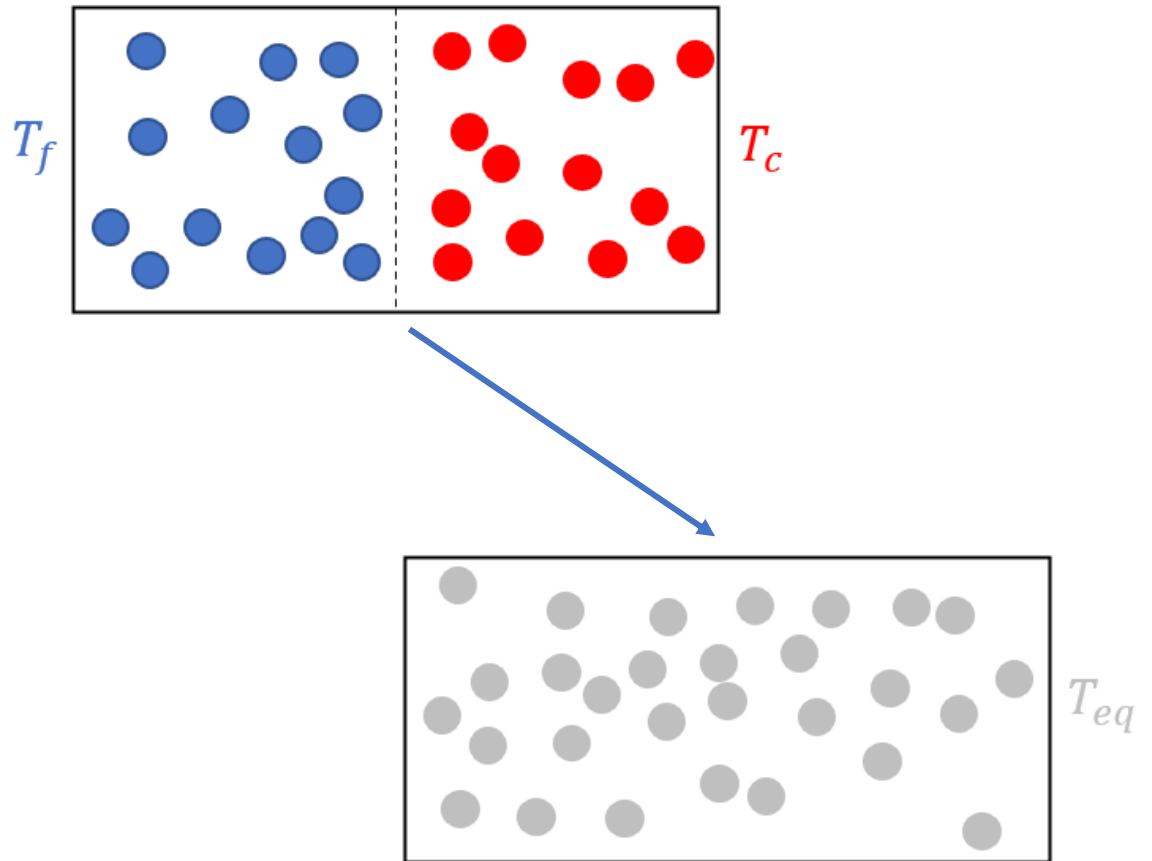
Krupa et al, "Spatial beam self-cleaning in multimode fibres", Nature Photonics 11 (2017)

Thermalisation in non-linear optics

- Wave turbulence :
Interactions between waves in non-linear systems
Non-equilibrium thermodynamic description

Thermalisation in non-linear optics

- Wave turbulence :
Interactions between waves in non-linear systems
Non-equilibrium thermodynamic description
- Gas thermalisation:
Increasing disorder/entropy
Collisions (molecular chaos) → irreversibility



Thermalisation in non-linear optics

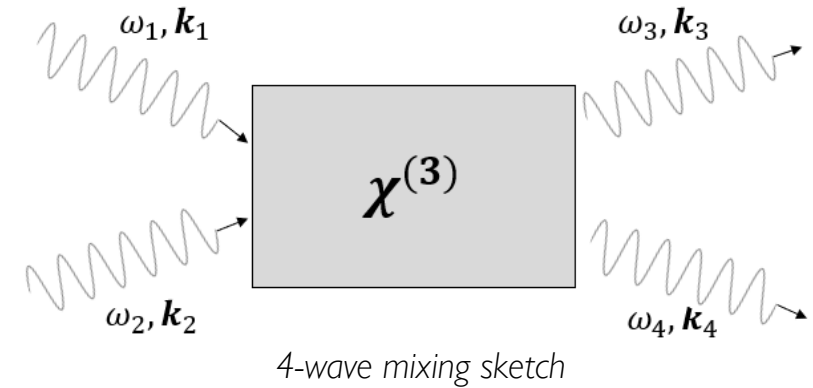
- Thermalisation in non-linear optics :

Non-linear interaction  evolution toward thermodynamic equilibrium

4-wave mixing

Conservation : energy and phase matching : $\omega_1 + \omega_2 = \omega_3 + \omega_4$

$$\mathbf{k}_1 + \mathbf{k}_2 = \mathbf{k}_3 + \mathbf{k}_4$$



Thermalisation in non-linear optics

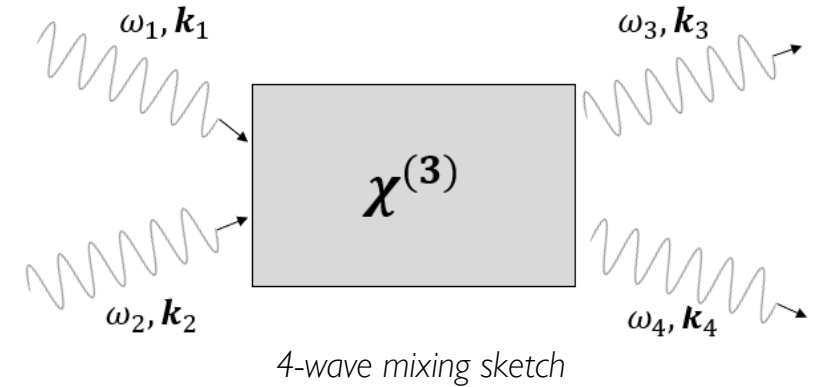
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Thermal equilibrium in an optical system

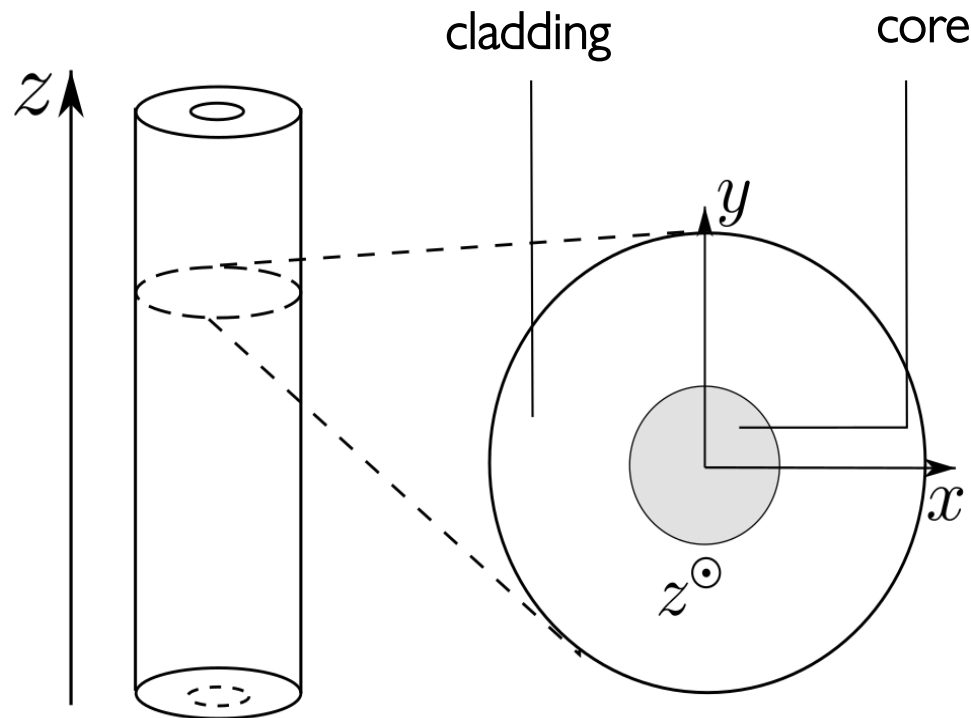
- Some quantities are conserved (number of photons/optical power N , linear energy E , ...)

Maximise entropy \mathcal{S} with the constraints of conservations
(Lagrange multipliers)



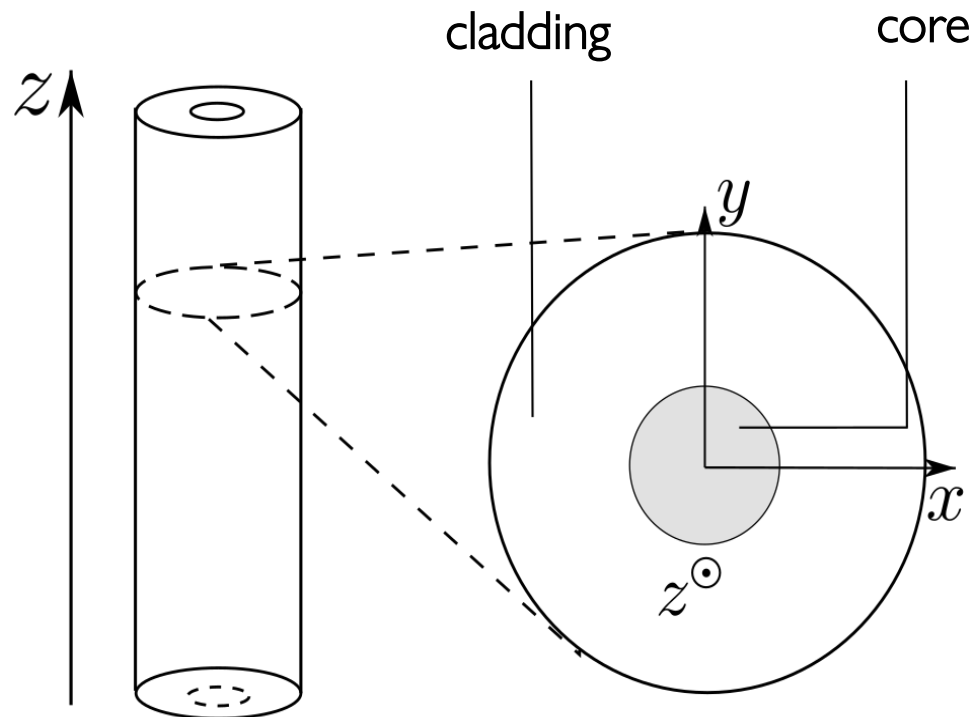
Equilibrium state/distribution

Multimode optical fibres (MMF)



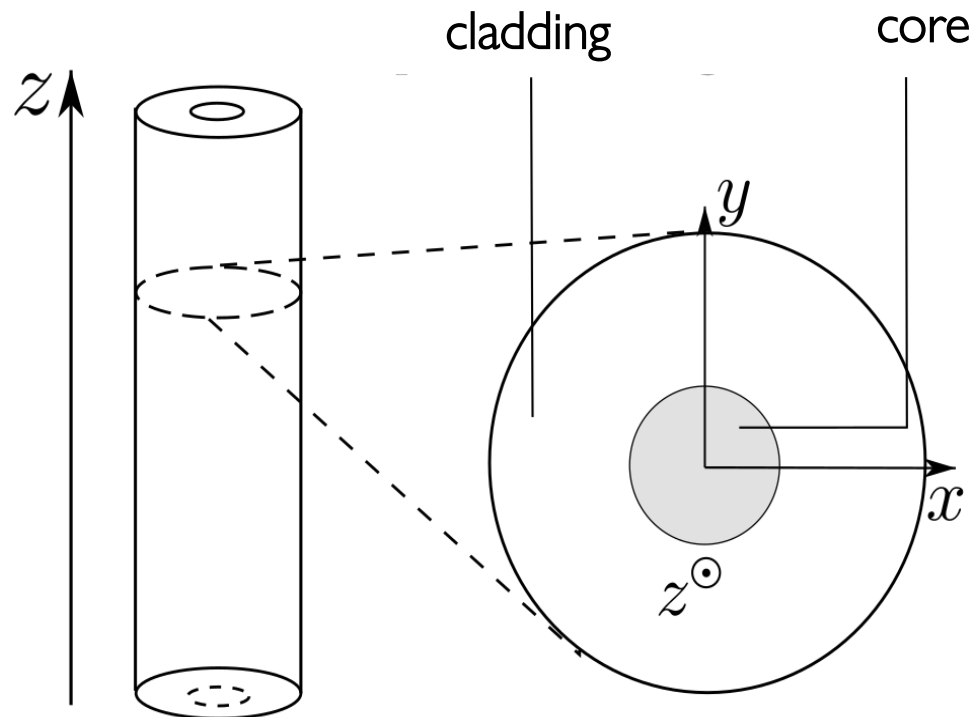
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($n_{cl} < n_{co}$)
- Centrosymmetric silica :
Lowest non-linear order : $\chi^{(3)}$

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Propagation constant β_m
Transverse amplitude distribution $u_m(\mathbf{r})$

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Transverse amplitude distribution $u_m(\mathbf{r})$
- Why MMFs ?
 - ➔ Light confinement
 - ➔ Large propagation distances

Theoretical model : Non-linear Schrödinger eq.

Helmholtz

$$\partial_z^2 E(\mathbf{r}, z) + \Delta_{\mathbf{r}} E(\mathbf{r}, z) + \tilde{n}^2(\mathbf{r}) k_0 E(\mathbf{r}, z) = 0$$

Theoretical model : Non-linear Schrödinger eq.

- Hypotheses :
 - Paraxial approximation
 - Weak non-linear Kerr effect
 $\tilde{n}(\mathbf{r})^2 \approx n(\mathbf{r})^2 + 2n_{c0}n_2|E|^2$

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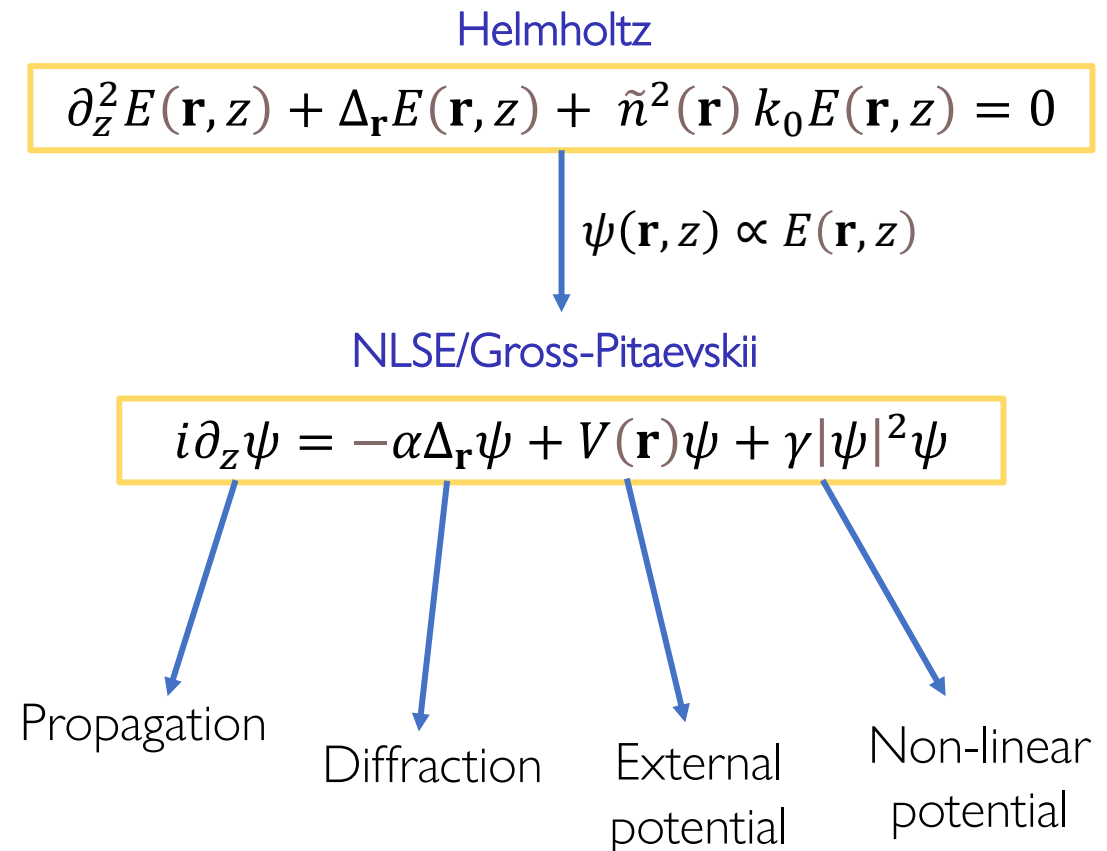
$$\psi(\mathbf{r}, z) \propto E(\mathbf{r}, z)$$

NLSE/Gross-Pitaevskii

$$i\partial_z \psi = -\alpha \Delta_{\mathbf{r}} \psi + V(\mathbf{r})\psi + \gamma |\psi|^2 \psi$$

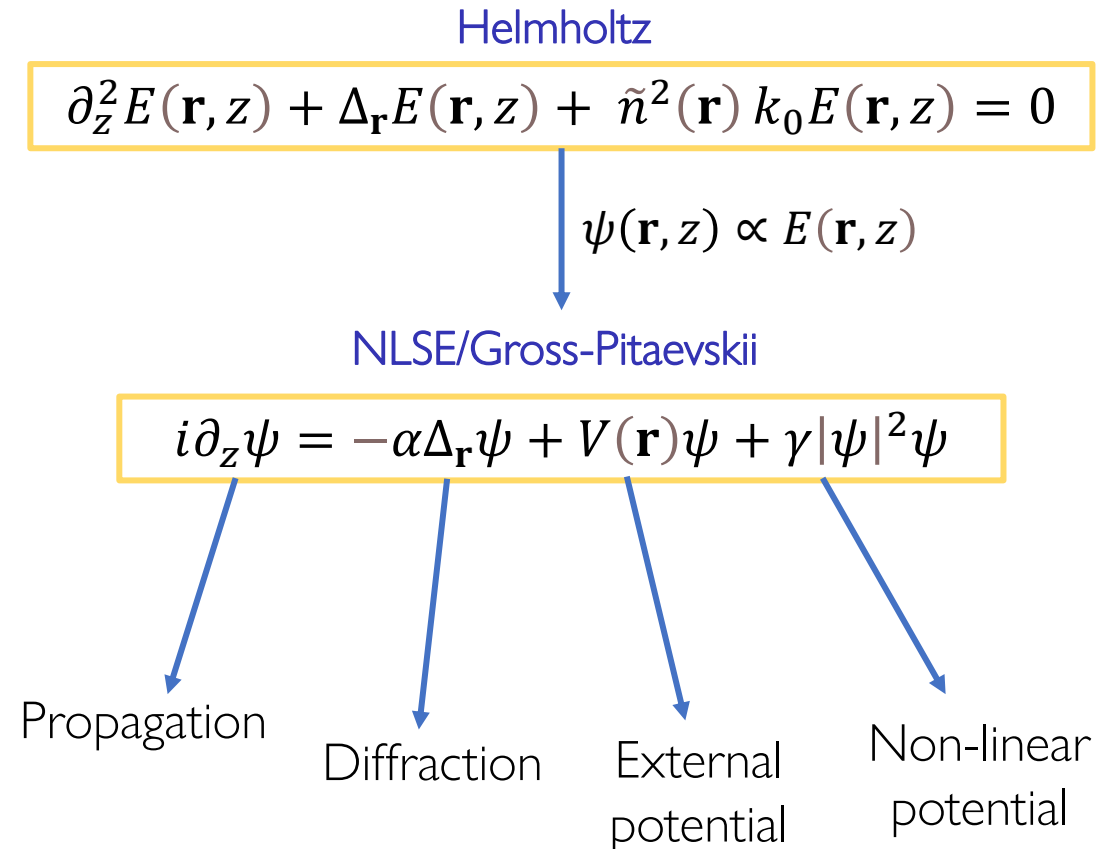
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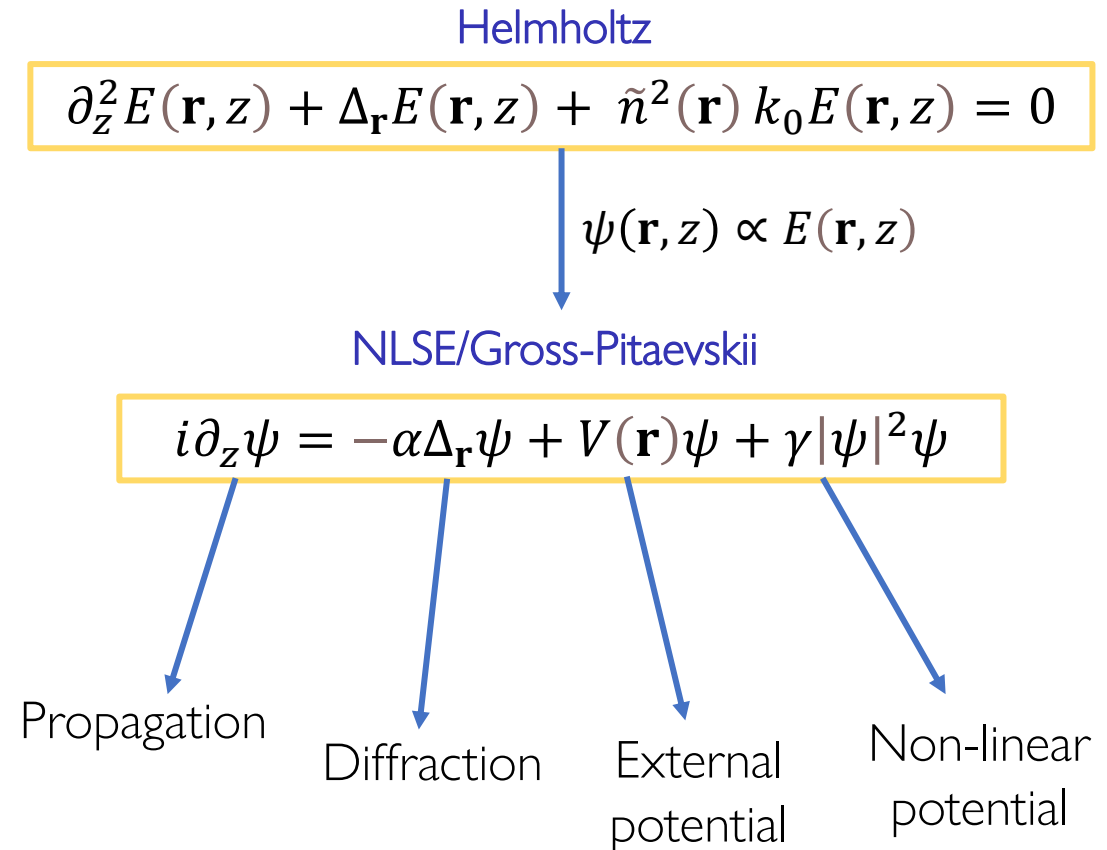
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 ➔ Hamiltonian system



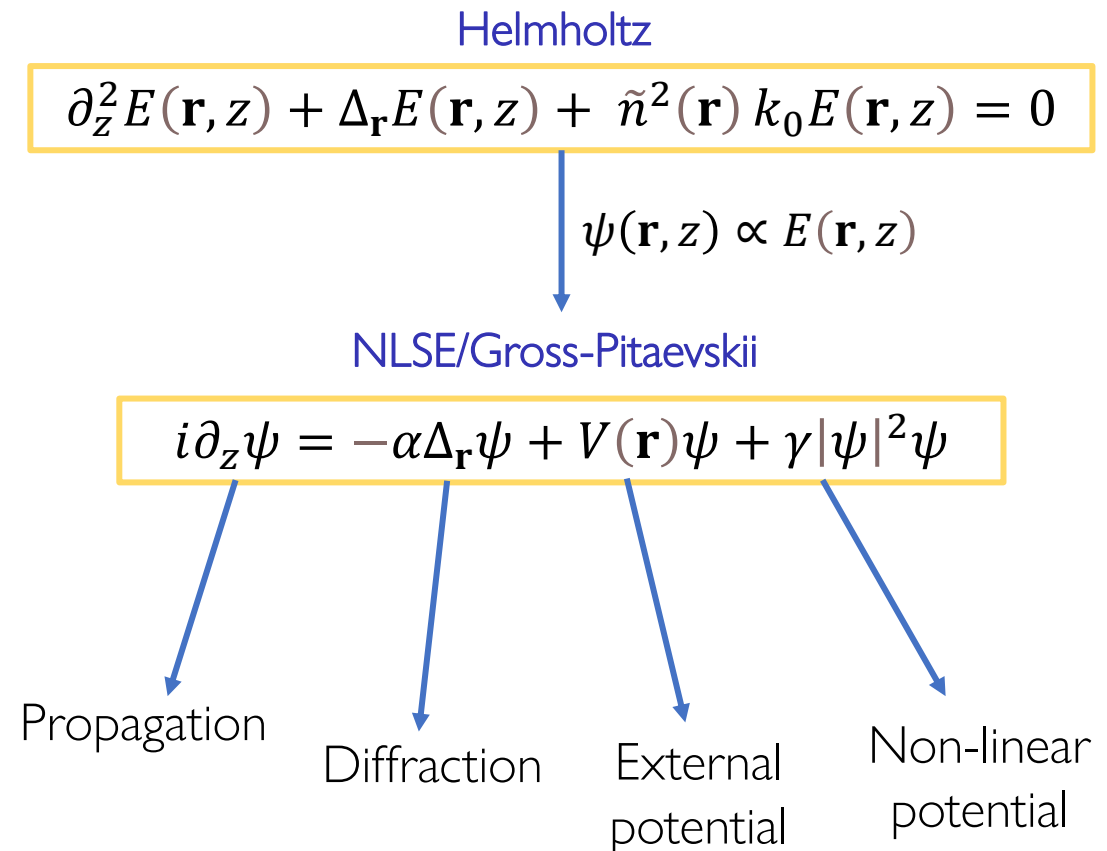
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→ An equation giving the evolution of the amount of power in each mode (spectrum)

Discrete kinetic equation for the spectrum

- NLS solution (weak non-linearity) : $\psi(\mathbf{r}, z) = \sum_m c_m(z) u_m(\mathbf{r}) e^{-i \beta_m z}$

(u_m, β_m) : eigenvectors/values of linearised NLS
 c_m : complex amplitude of the mode m

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 c_m : complex amplitude of the mode m

$$\partial_z c_m = \beta_m c_m + \gamma \sum_{p,q,s} W_{mpqs} c_m c_p^* c_q c_s^* e^{-i\Delta\beta_{mpqs} z}$$

$$n_m = \langle c_m(z) c_m^*(z) \rangle : \text{power in the mode } m$$

- Important parameters :

➔ Modal overlap (4 modes) :

$$W_{mpqs} = \int u_m(\mathbf{r}) u_q^*(\mathbf{r}) u_p(\mathbf{r}) u_s^*(\mathbf{r}) d\mathbf{r}$$

➔ Phase-matching/Quasi-resonances (4 modes) :

$$\Delta\beta_{mpqs} = \beta_m + \beta_q - \beta_p - \beta_s$$

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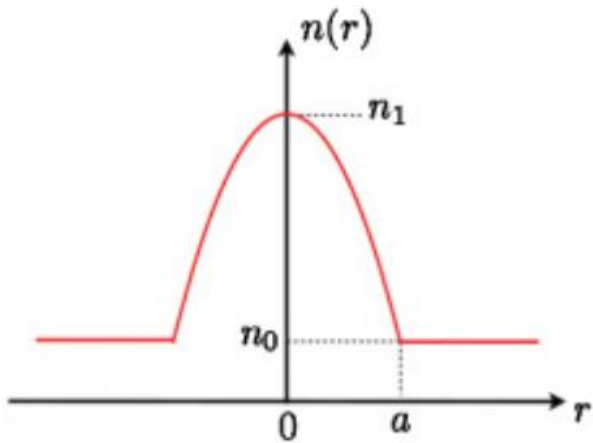
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- Maximising entropy under constraints ➔ Rayleigh-Jeans (RJ) equilibrium :

$$n_m^{eq} = \frac{T}{\beta_m - \mu}$$

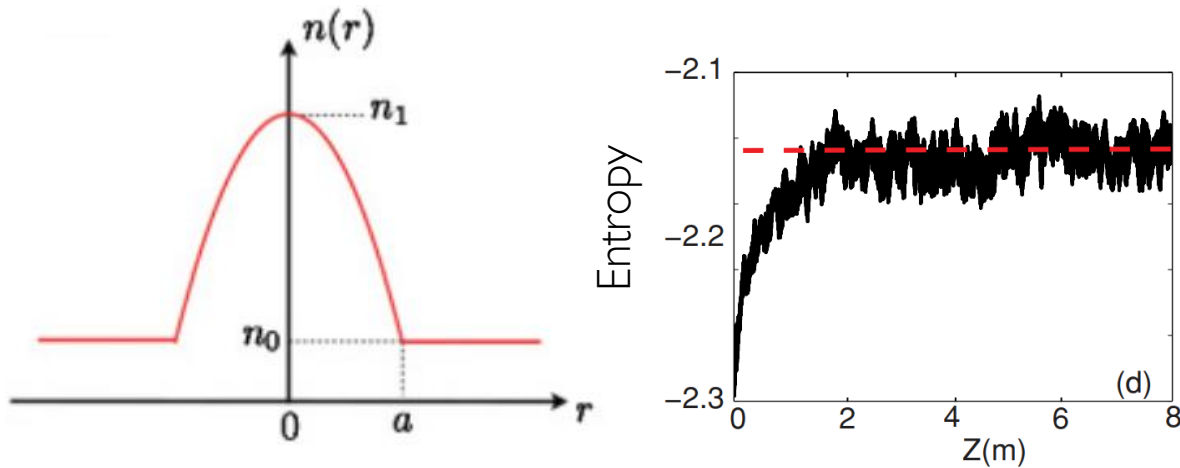
Studies of graded-index (GRIN) MMF

P. Aschieri et al., "Condensation and thermalization of classical optical waves in a waveguide", Physical Review A 83 (2011)



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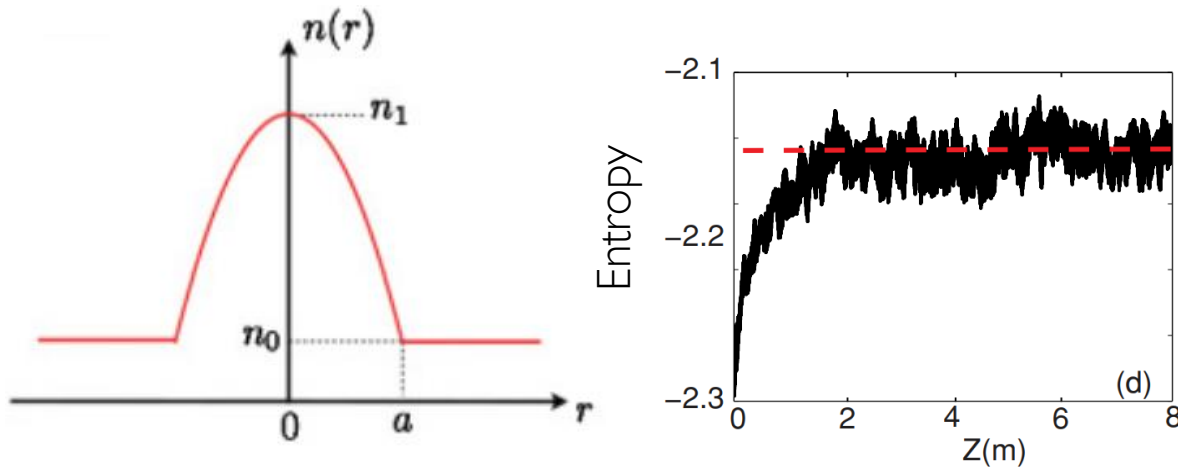
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- Numerical simulation in GRIN fibre
Weak non-linear regime
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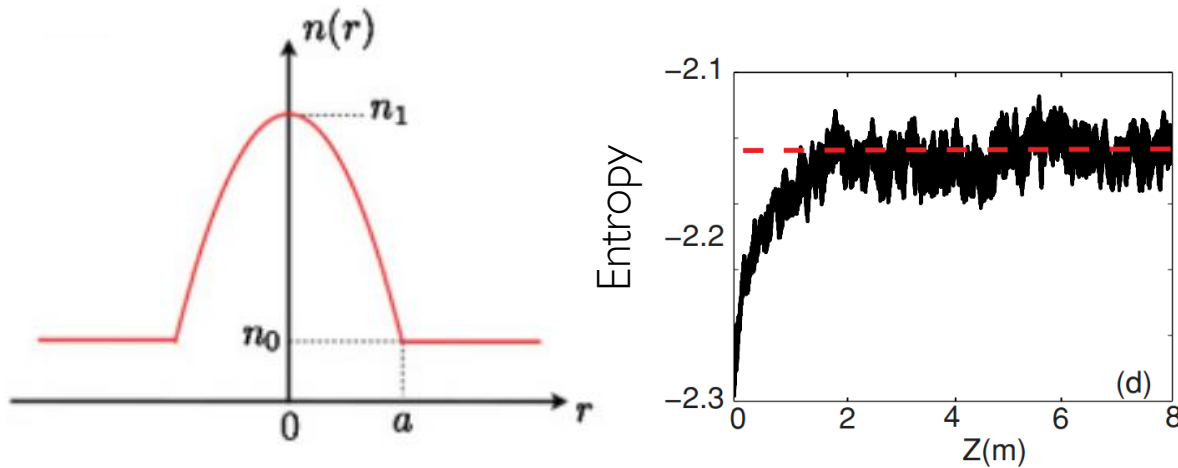
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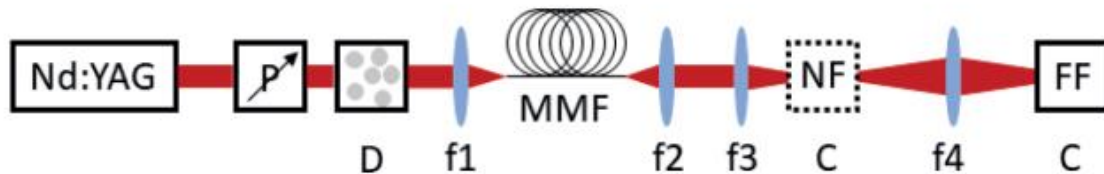
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K. Baudin, et al. "Rayleigh-Jeans condensation of classical light : Observation and thermodynamic characterization" , *PRL*, 125, (2020)



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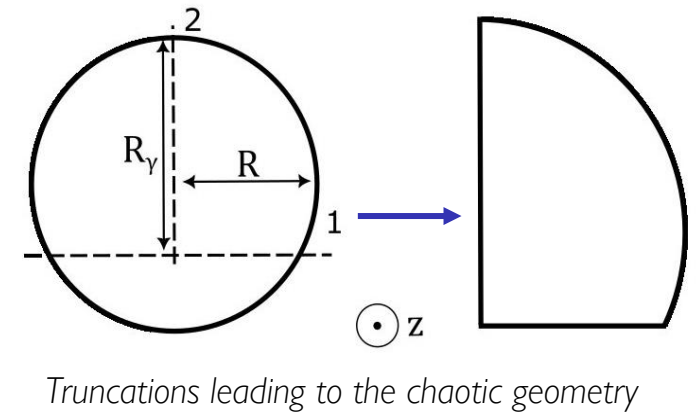
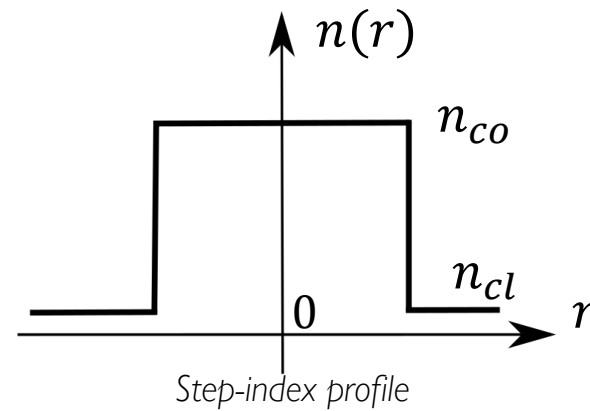
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Is there a way to improve thermalisation in more common fibres ?

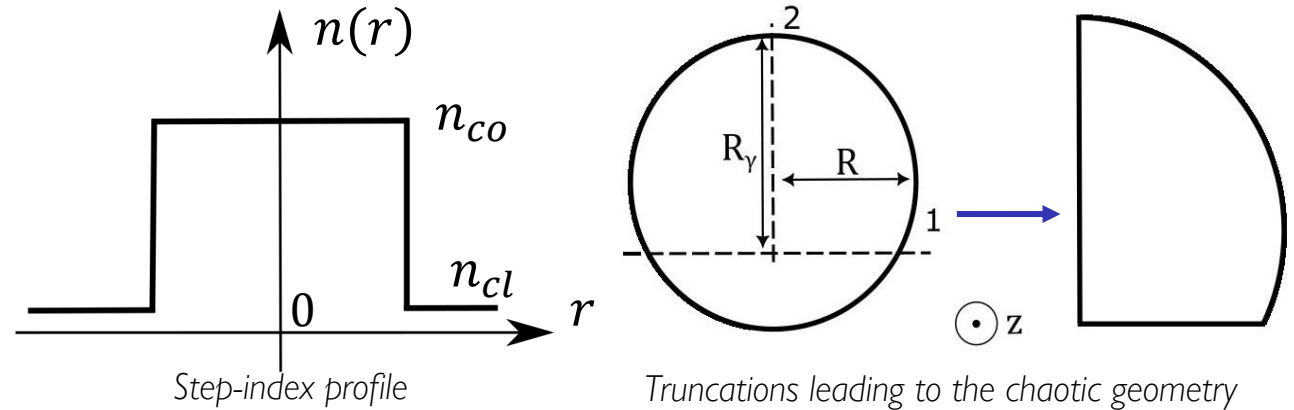
Study of step-index chaotic MMF

- Chaotic step-index fibre
Core index : $n_{co} = 1.475$
Cladding index: $n_{cl} = 1.45$
2 truncations of the transverse section

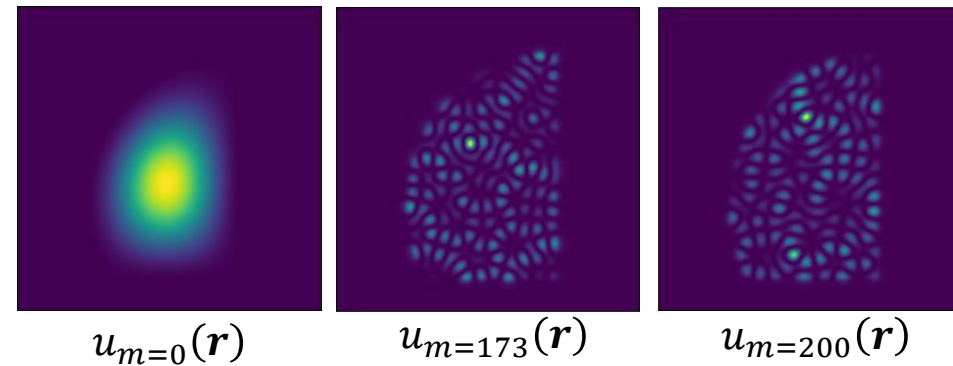


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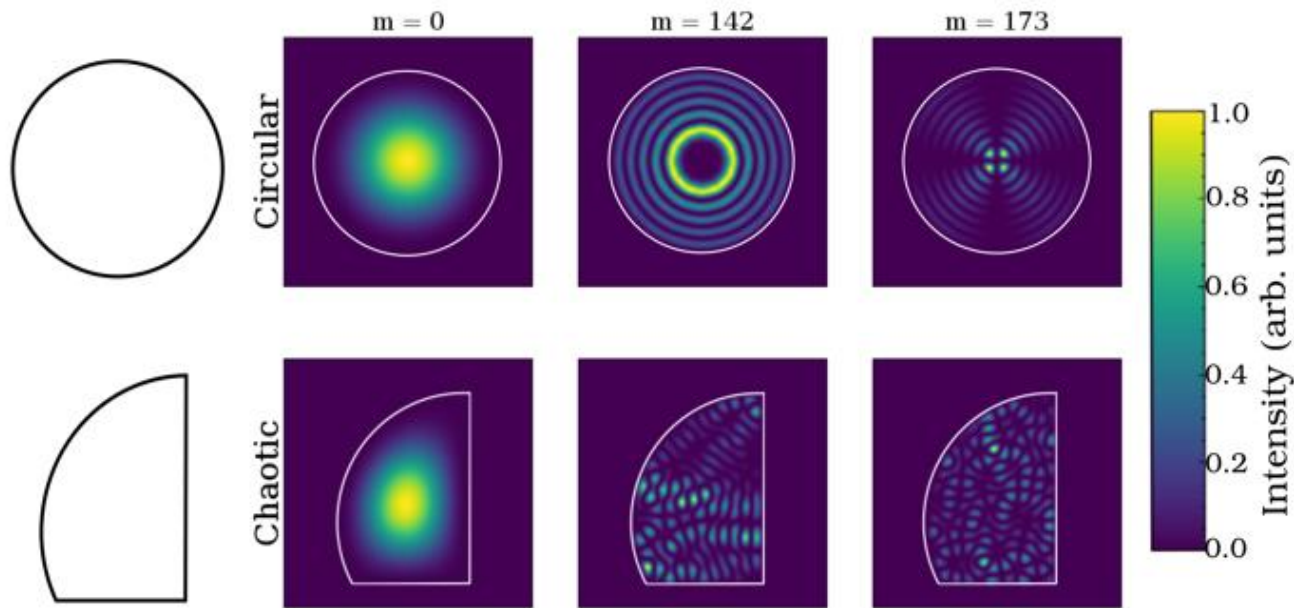
- Influence of chaotic geometry:
Ergodic distribution of modal amplitudes $u_m(\mathbf{r})$
Repulsion between β_m



Samples of spatial distributions for 233 modes chaotic fibre.

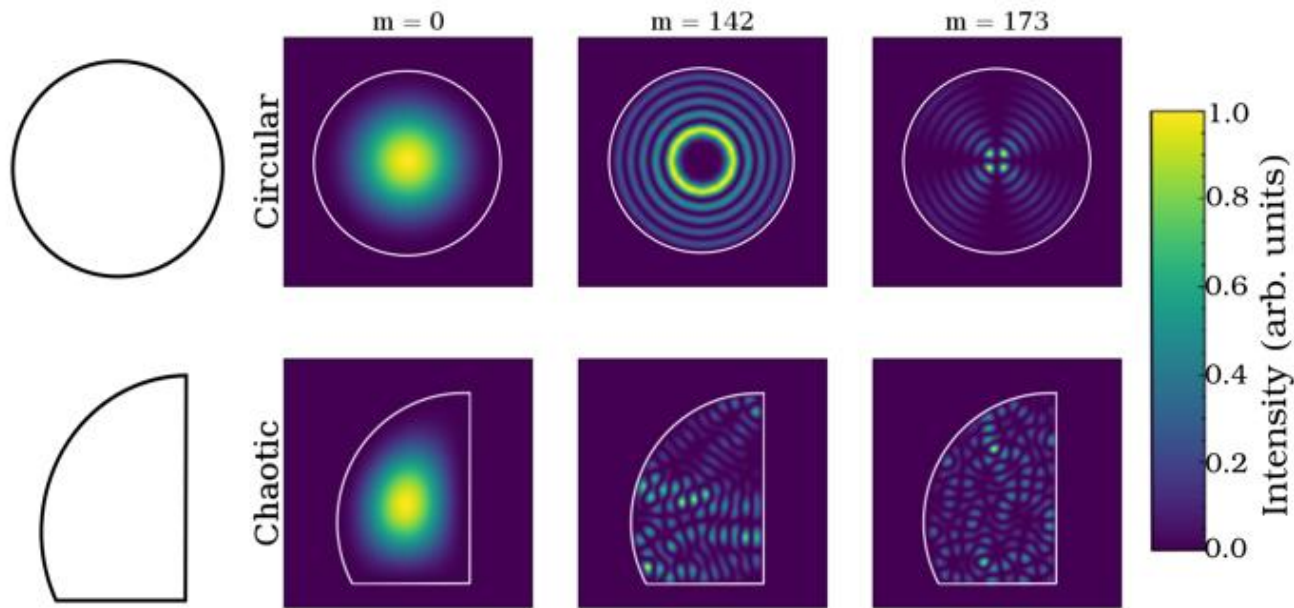
a) 4-modes distributions overlap

Samples of modes intensity distributions for the chaotic (upper row)
and circular fibres (lower row)



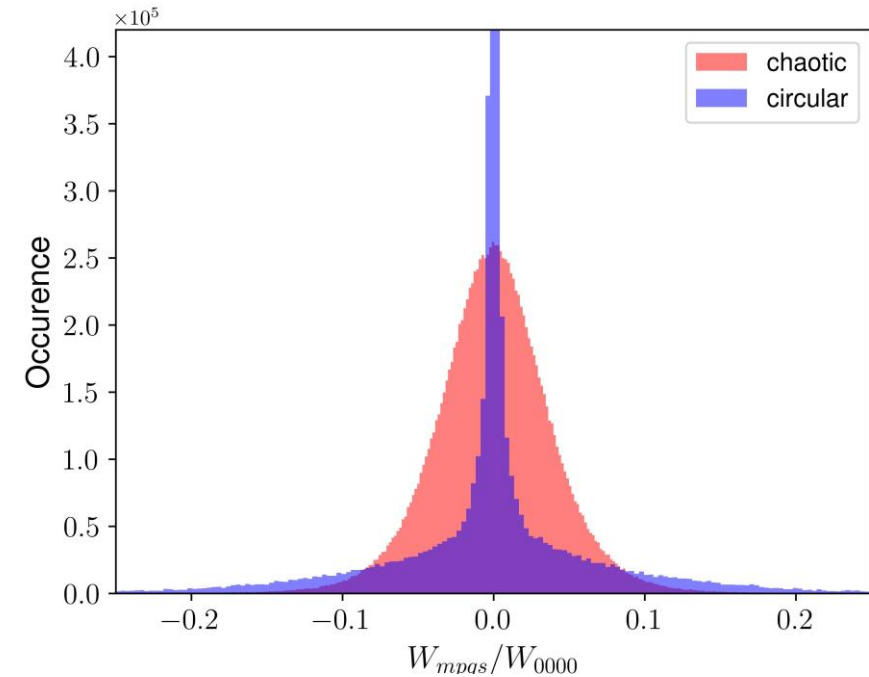
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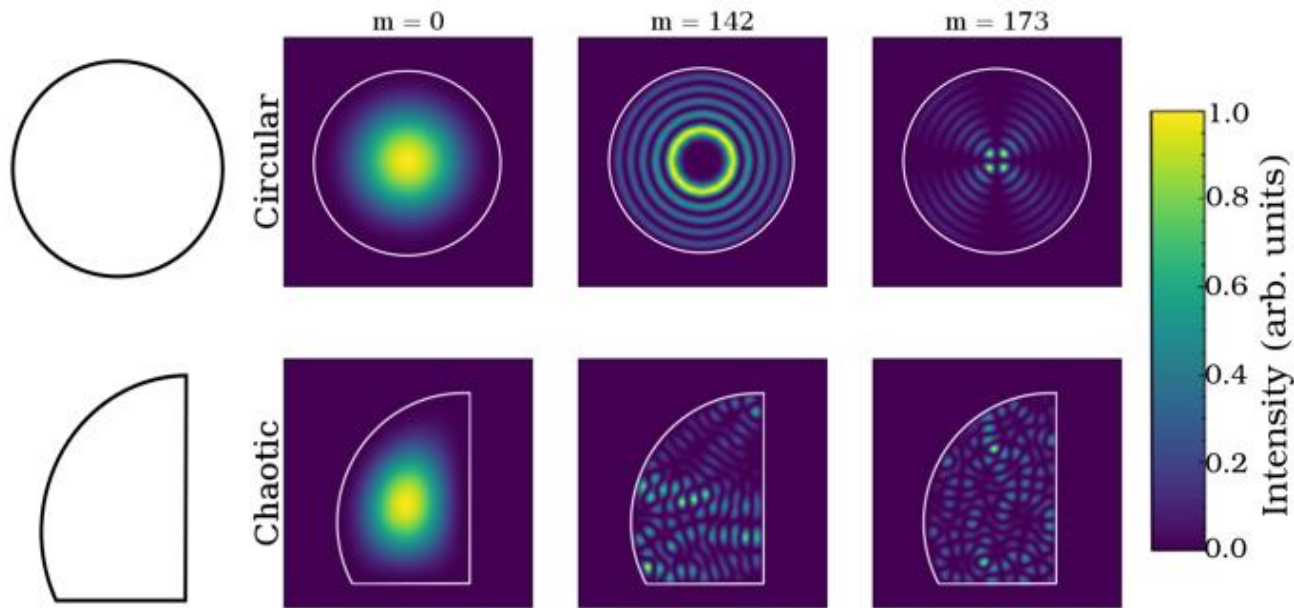
$$W_{mpqs} = \int \mathbf{u}_m(\mathbf{r}) \mathbf{u}_q^*(\mathbf{r}) \mathbf{u}_p(\mathbf{r}) \mathbf{u}_s^*(\mathbf{r}) d\mathbf{r}$$

Histogram of the normalised modal overlap



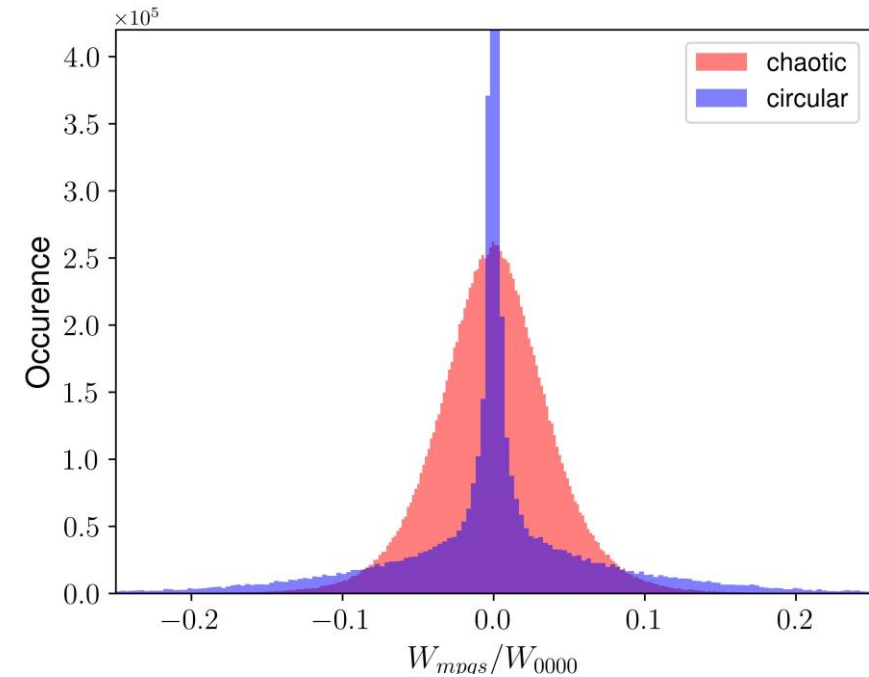
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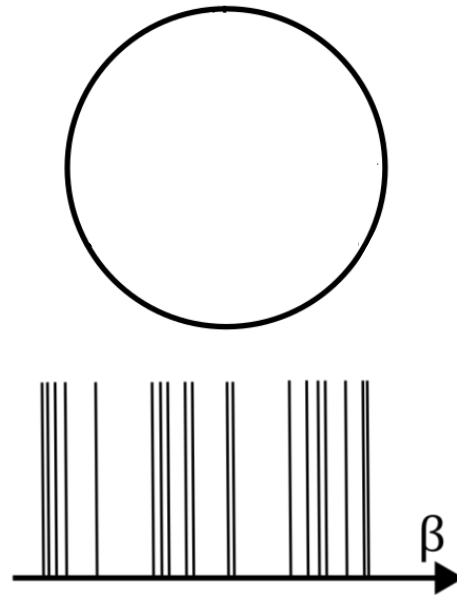
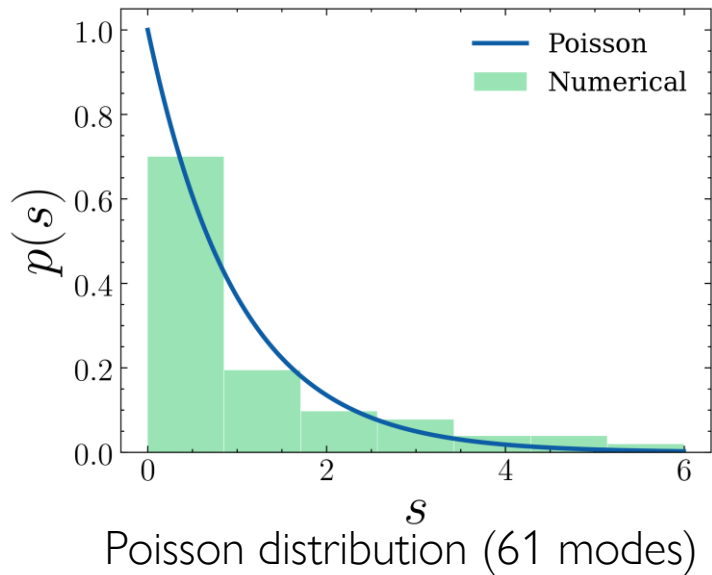


Chaotic fibre : modal ergodicity

More efficient overlap with the chaotic modes

b) Spectral properties

Circular fibre :
Quasi-degeneracy of propagation constants



$$p(s) = e^{-s}$$

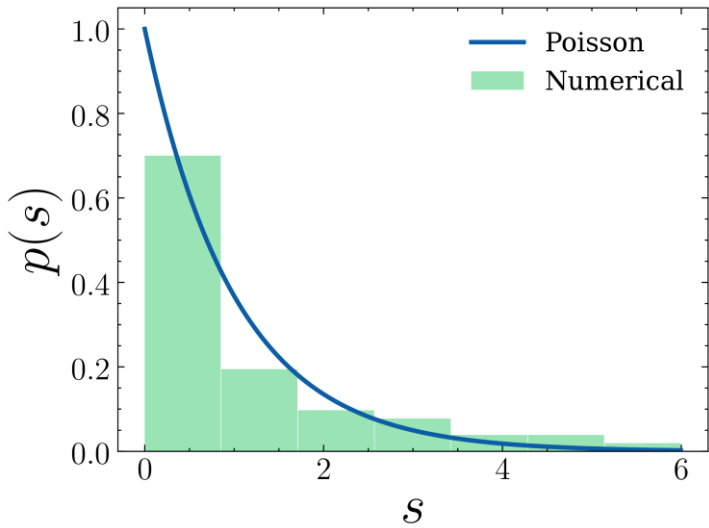
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H.-J. Stöckmann, *Quantum Chaos: an introduction*, Cambridge University Press (1999)

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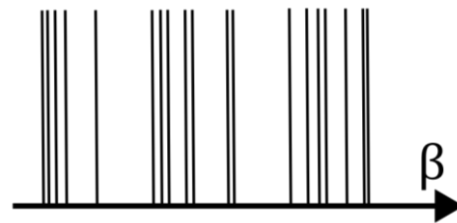
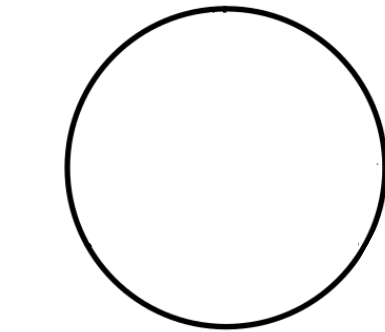
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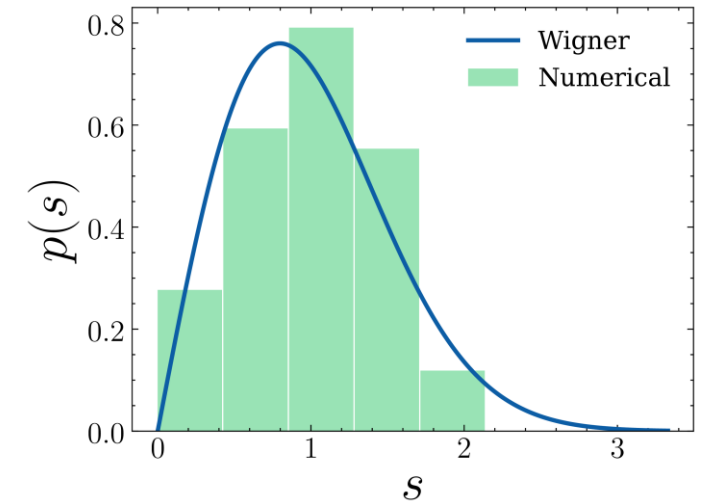
Poisson distribution (61 modes)

$$p(s) = e^{-s}$$



Chaotic fibre :
Spectral rigidity/Repulsion among propagation constants

More quasi-resonances



Wigner distribution (60 modes)

$$p(s) = \frac{\pi S}{2} e^{-\pi s^2/4}$$

$$\Delta\beta_{mpqs} = \beta_m + \beta_q - \beta_p - \beta_s$$

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Results from simulations

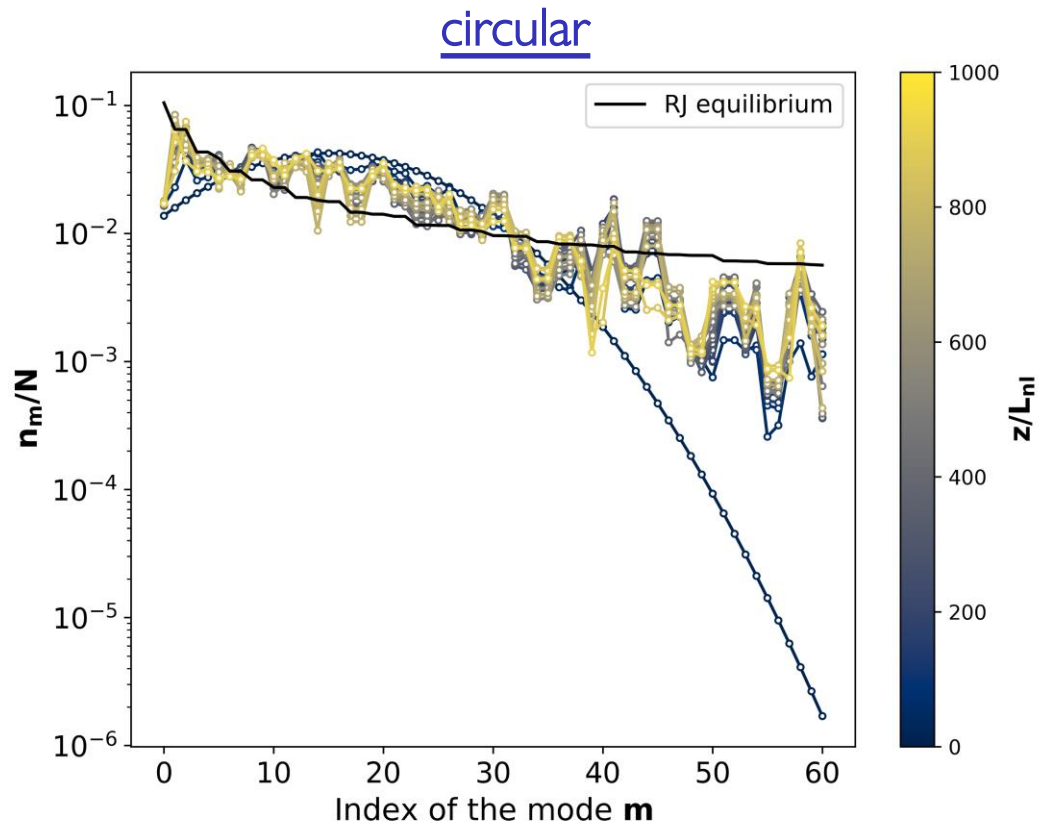
~60 modes fibres :

- Non-linear length : $L_{nl} = 10^{-3}m$
- Propagation over $z = 1m$
- Power and energy conserved
 - Entropy growth along z

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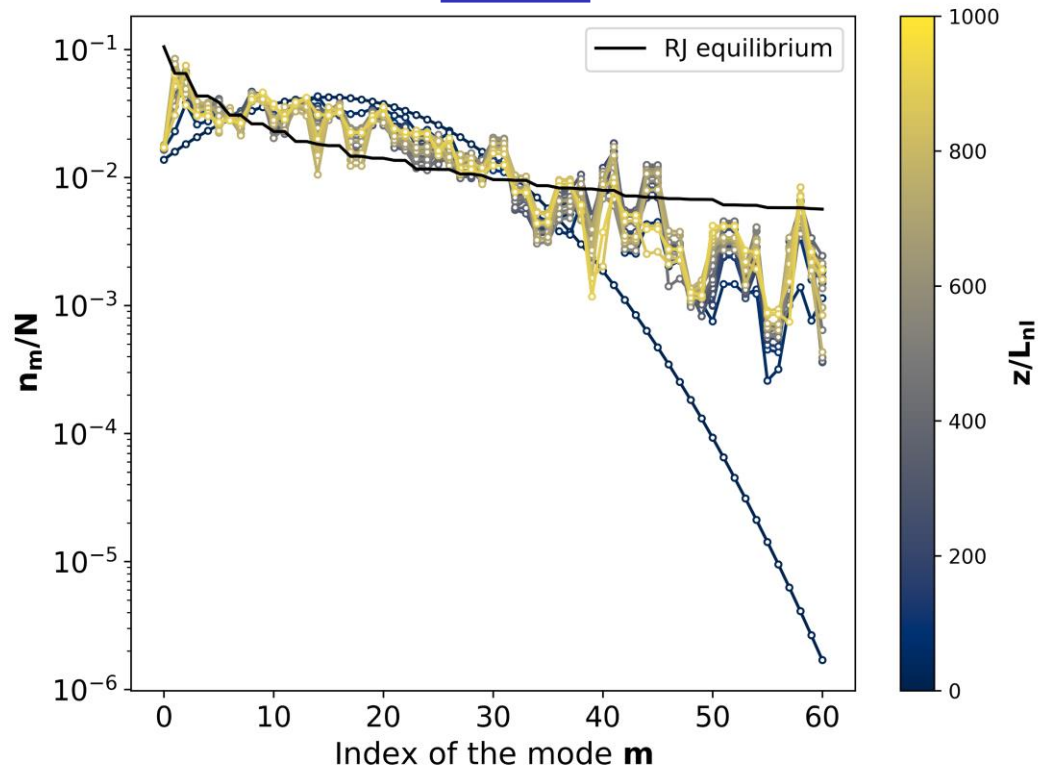


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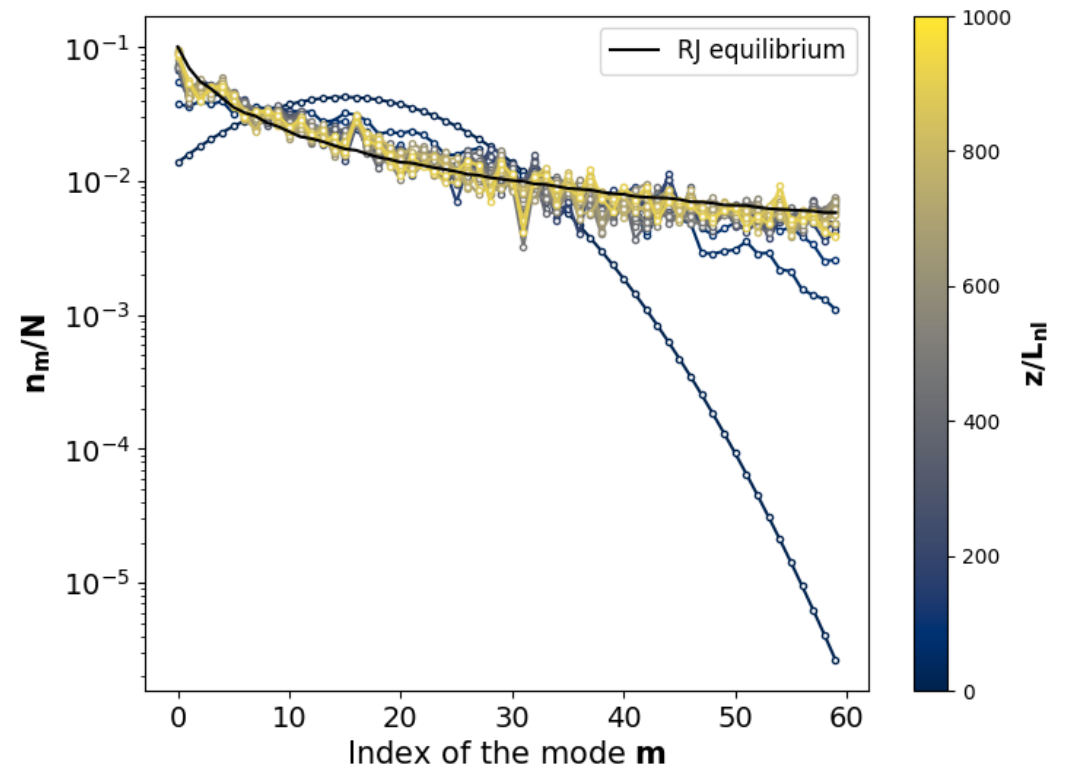
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circular



chaotic



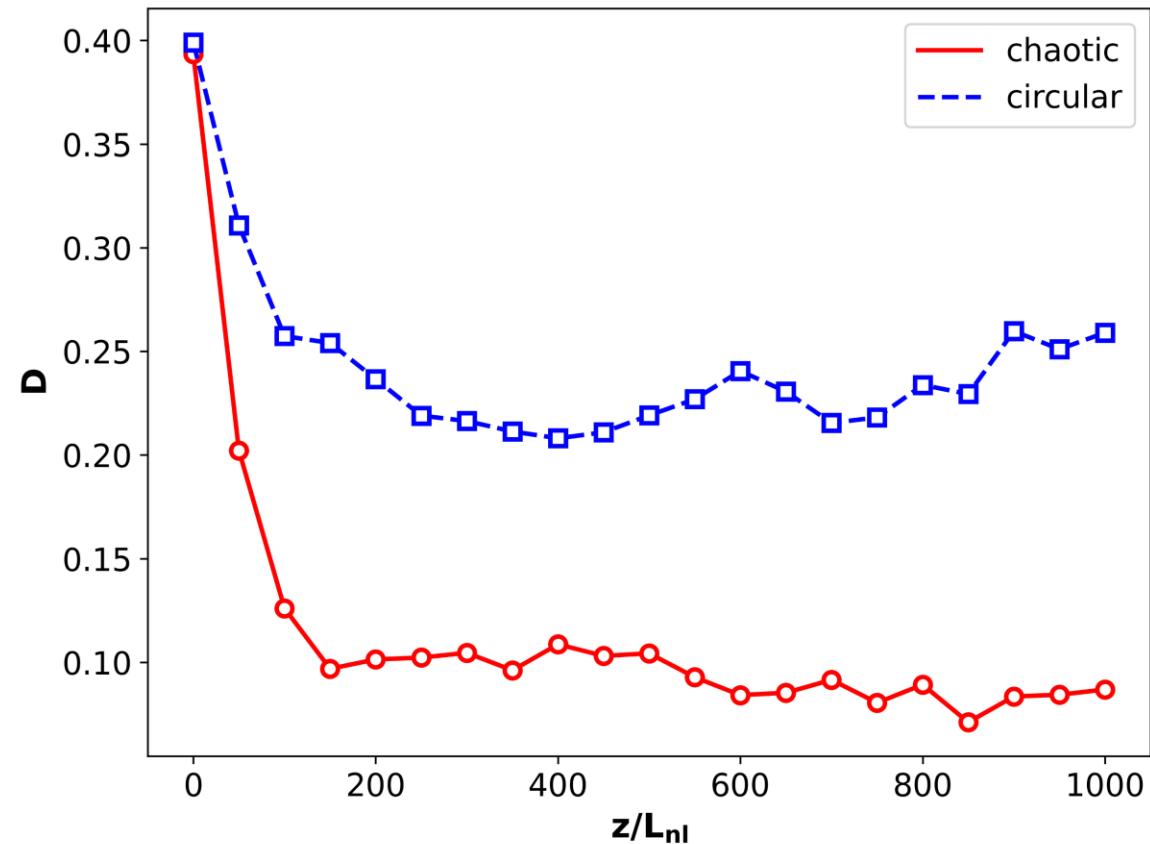
Results from simulations - Comparison

For both fibres :

- Powers and energy conserved
- Entropies growing along propagation

- Distance to R_j :

$$D(z) = \frac{\sum_m |n_m(z) - n_m^{eq}|}{\sum_m (n_m(z) + n_m^{eq})} \in [0,1]$$



Better convergence toward R_j for the chaotic fibre

To sum up

- Thermalisation extended to non-linear optics in MMFs
- Encouraging results for thermalisation for the chaotic fibre
- Current issues and future researches :
 - ➔ Lack of condensation :
 - Agreement to RJ lost at small energies
 - Simulations with NLS directly
 - ➔ Temporal dispersion in GRIN,
Negative temperature thermalisation, ...



C. Michel, INPHYNI



V. Doya, INPHYNI



A. Picozzi, ICB



F. Mortessagne, INPHYNI



J. Garnier, Ecole Polytechnique

Thank you !

Disorder and experimental confirmation

- Simulation without disorder :
FPU recurrences \rightarrow No condensation at small z
- Introduction of disorder : Linear coupling between modes

$$i\partial_z a_m = \beta_m a_m - \gamma F_m(a) + W_m(z) a_m \quad \text{NLSE+ disorder}$$

- Modal power $n_m = \langle a_m(z) a_m^*(z) \rangle$ kinetic equation ?

$$\partial_z n_m = \frac{2\gamma^2}{9\eta} \sum_{p,q,s} \delta(\Delta\beta_{mpqs}) |W_{mpqs}|^2 M_{mpqs}(n(z))$$

$$M_{mpqs}(n(z)) = n_m n_p n_q n_s \times (n_m^{-1} + n_q^{-1} - n_p^{-1} - n_s^{-1})$$

$$\Delta\beta_{mpqs} = \beta_m + \beta_q - \beta_p - \beta_s$$

$$W_{mpqs} = \int u_m(\mathbf{r}) u_q^*(\mathbf{r}) u_p(\mathbf{r}) u_s^*(\mathbf{r}) d\mathbf{r}$$

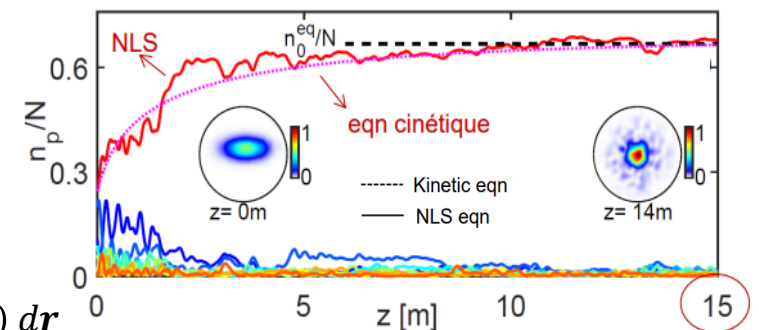
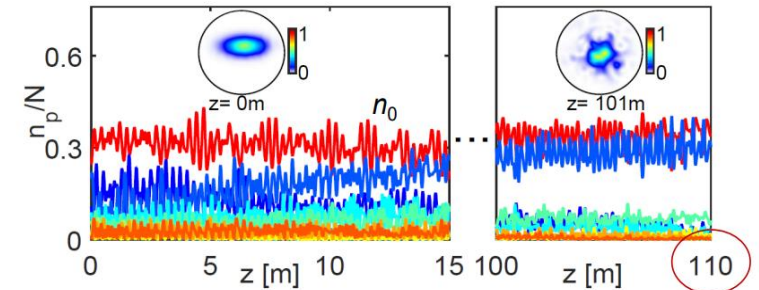
- Conservation : $N = \sum_m n_m$, $E = \sum_m \beta_m n_m$
- H-theorem ($\partial_z S \geq 0$) with $S = \sum_m \ln(n_m)$
- Disorder η accelerates the rate of thermalisation to Rj equilibrium (**several order of magnitude !**)

Maximize S under constraints

$$n_m^{eq} = \frac{T}{\beta_m - \mu}$$

Rayleigh-Jeans

Divergence : $\mu \rightarrow \beta_0$
Condensation

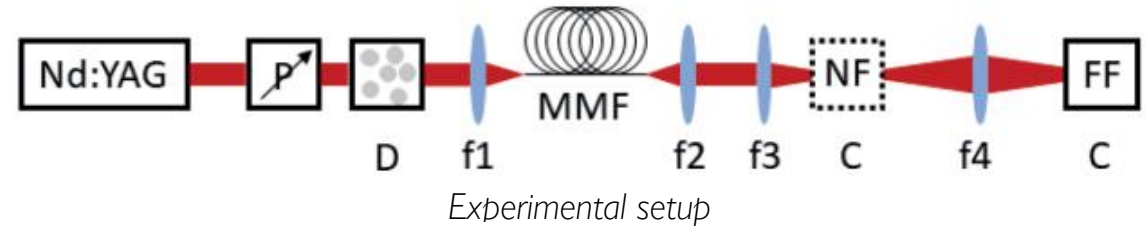


Fusaro et al., PRL 122, 123902 (2019)

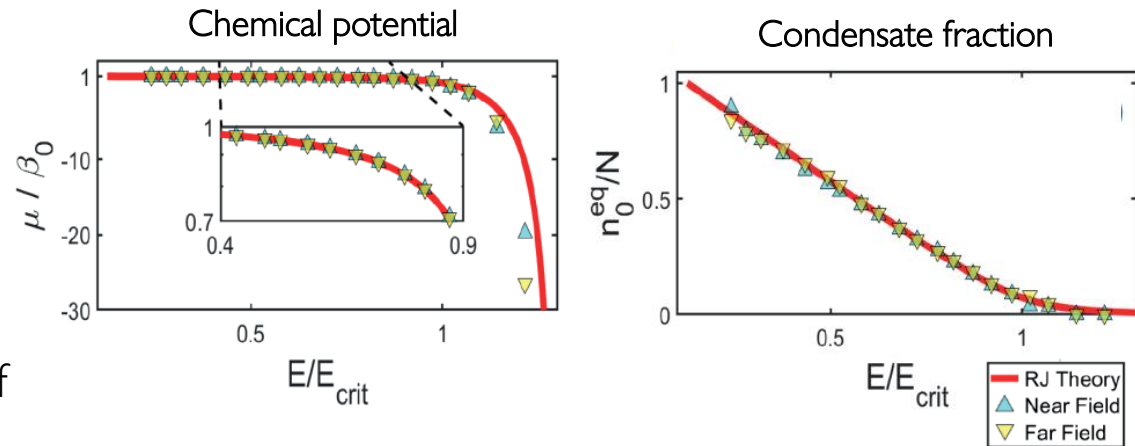
Experimental observation of light condensation

K. Baudin, A. Fusaro, K. Krupa, J. Garnier, G. Millot, A. Picozzi "Rayleigh-Jeans condensation of classical light : Observation and thermodynamic characterization" , PRL, 125, (2020)

- Fibre : 13 m long, radius $R=26 \mu\text{m}$, laser at 7 kW
- Modal power evolution :
 N constant
 E varying parameter (diffuser D)



- Condensation observed by varying E :
 $\mu \rightarrow \beta_0$ by decreasing E
➔ RJ divergence for the fundamental
 Condensate fraction in agreement with RJ theory

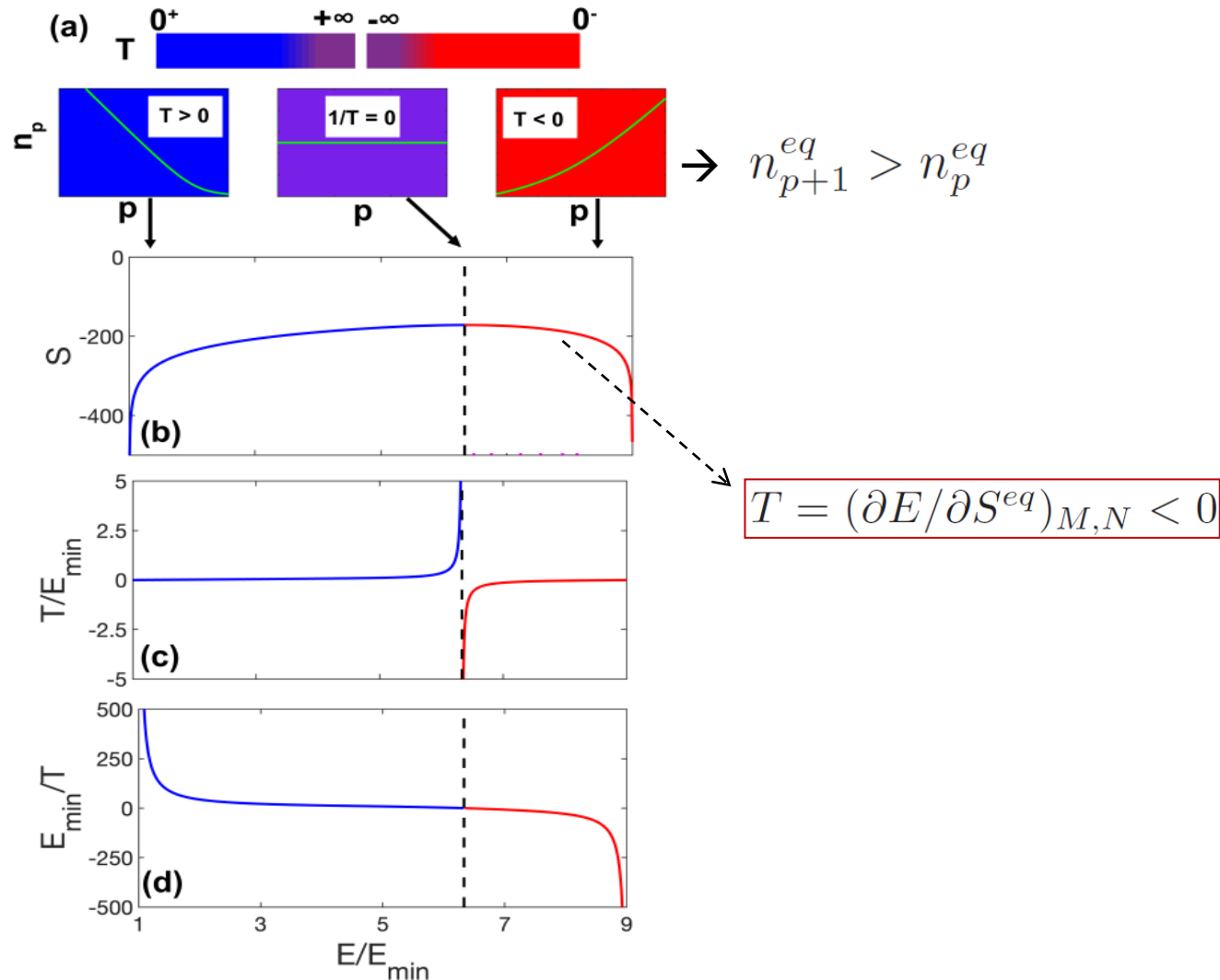


- Recent confirmation of experimental observation of thermalisation in MMFs
Pourbeyram et al., Nature Physics, 18, 685, (2022)

Thermalisation vers des états d'équilibre de Température Négative

Nombre fini de modes \rightarrow borne supérieure pour l'énergie \rightarrow possibilité de $T < 0$

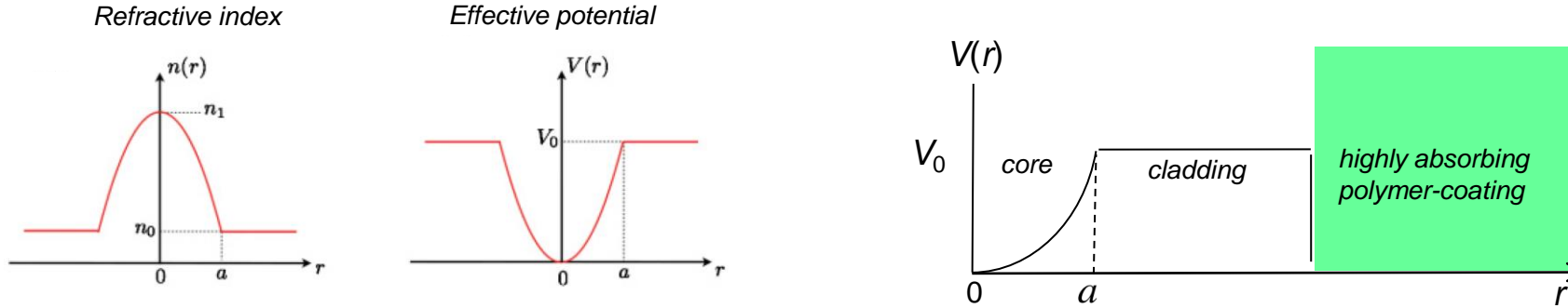
$$E_{\min} = N\beta_0 \leq E \leq E_{\max} = N\beta_{\max} \rightarrow E > E_t = N \langle \beta_p \rangle \rightarrow n_{p+1}^{eq} > n_p^{eq}$$



Etats d'équilibre de $T < 0$ sont plus "chauds" que $T > 0$

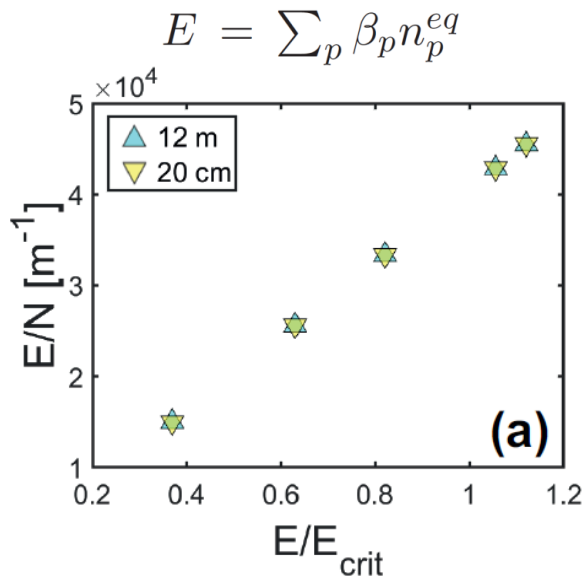
\rightarrow Flux d'énergie va des états $T < 0$ vers les états $T > 0$ quand les systèmes sont mis en contact

Conservation of the energy during propagation in the MMF

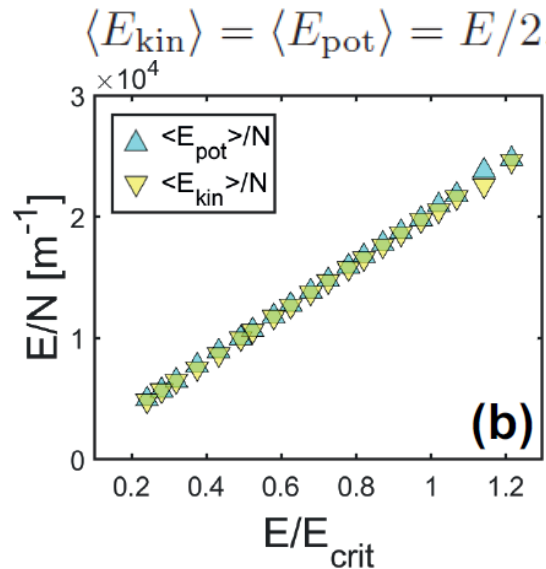


Leaky radiation modes in the cladding are rapidly absorbed during propagation due to their large penetration in the coating.

- Measurement of absorption length of leaky modes $L_{lk} \sim 15\text{cm}$ (linear regime)
- This length L_{lk} is much smaller than that required to generate leaky modes
- The nonlinear excitation of leaky modes can be neglected → effective $k_c = \sqrt{2V_0/\beta_0}/a$
- Confirmed by the conservation of the E & N



No average over realizations



Average over realizations

$$E_{pot} = \int V(\mathbf{r}) |\psi(\mathbf{r})|^2 d\mathbf{r}$$

$$E_{kin} = \int \alpha |\nabla \psi|^2 d\mathbf{r}$$

Classical wave condensation is reminiscent of quantum condensation

(2nd reading)

Quantum kinetic equation (bosons):

$$\begin{cases} \partial_t n_{k_1} = \text{Coll} [n_{k_1}] \\ \text{Coll} [n_{k_1}] = \int d^3k_2 d^3k_3 d^3k_4 \left[n_{k_4} n_{k_3} (1+n_{k_1})(1+n_{k_2}) - n_{k_1} n_{k_2} (1+n_{k_3})(1+n_{k_4}) \right] \delta_k \delta_\varepsilon \end{cases}$$

$$n_k^B = \frac{1}{\exp[\beta(\varepsilon_k - \mu)] - 1}$$

$n_k \gg 1$

WT kinetic eqn

$$n_k = \frac{k_B T}{\varepsilon_k - \mu}$$

WT

NLS eqn

$n_k \ll 1$

Boltzmann kinetic eqn

$$n_k = \exp[-\beta(\varepsilon_k - \mu)]$$

Classical gas

- B. Svistunov, J. Mosc. Phys. Soc. 1, 373 (1991)
- Yu. Kagan, B. Svistunov, PRL 79, 3331 (1997)
- M. J. Davis, R. J. Ballagh, K. Burnett, J. Phys. B 34, 4487 (2001)
- M. J. Davis, S. A. Morgan, K. Burnett, PRL 87, 160402 (2001)