

CONTROLLABILITY OF SATELLITES ON PERIODIC ORBITS WITH CONE-CONSTRAINTS ON THE THRUST DIRECTION

Alesia Herasimenka, Ariadna Farrés, and Lamberto Dell'Elce

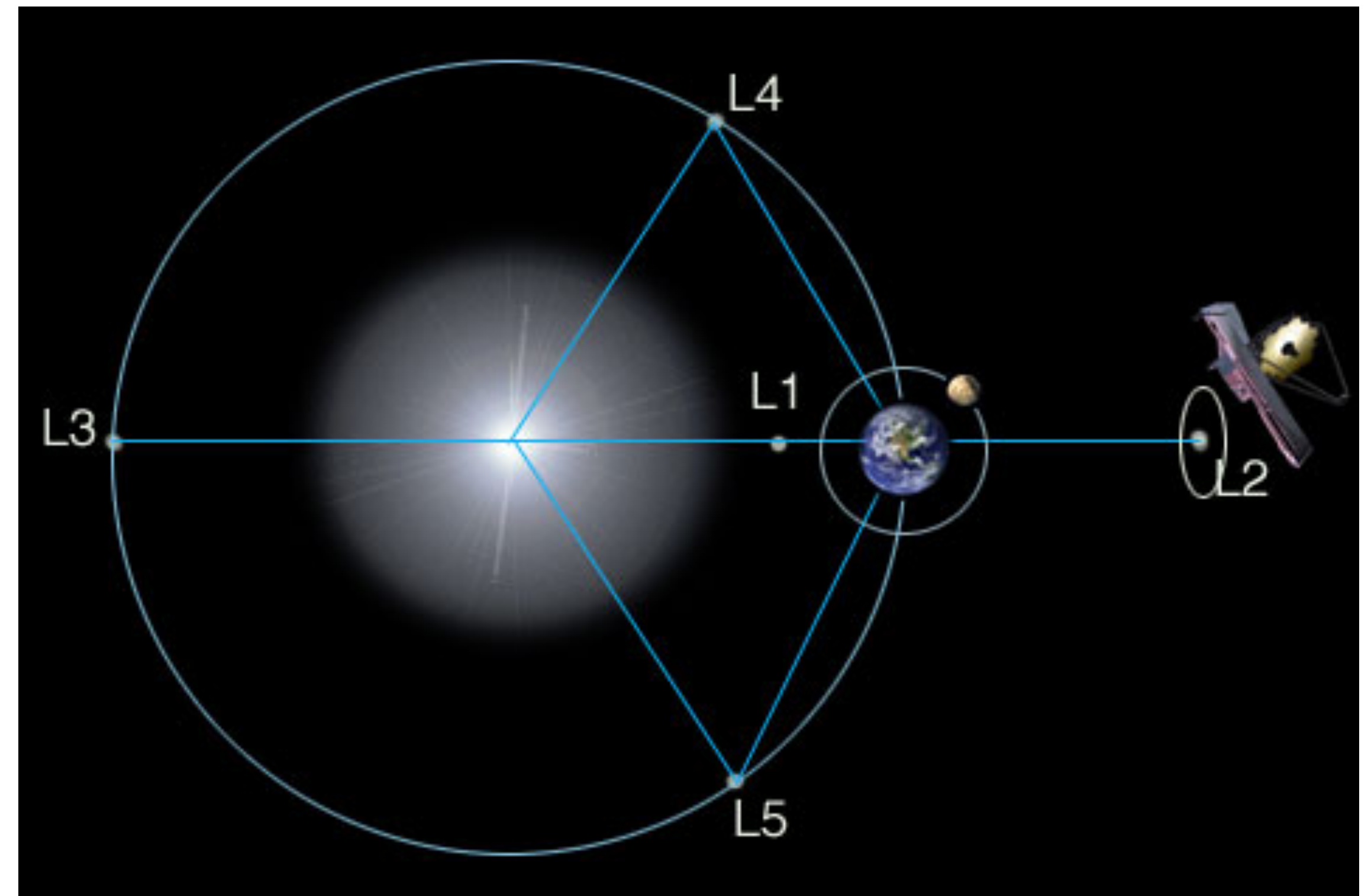
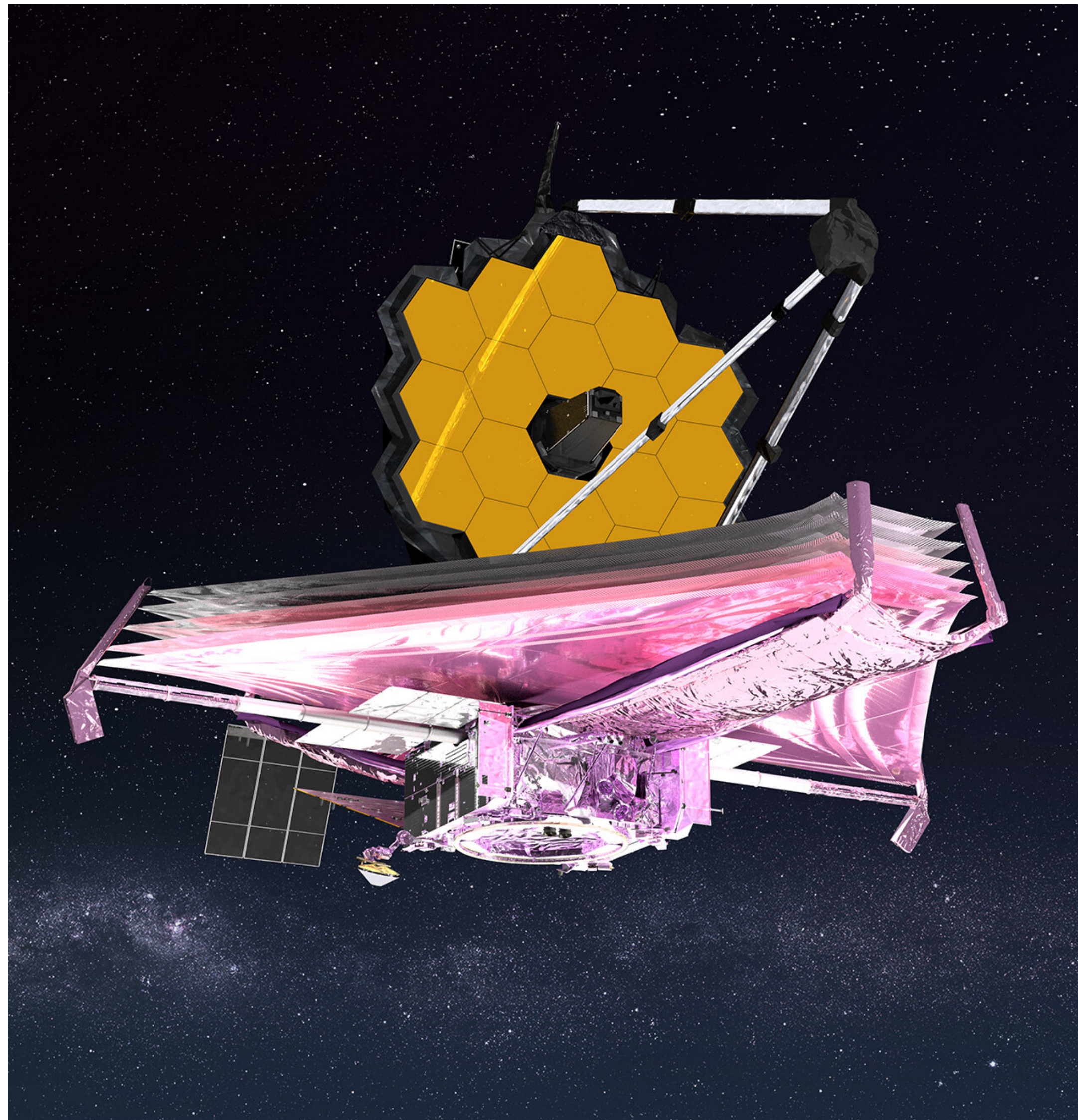
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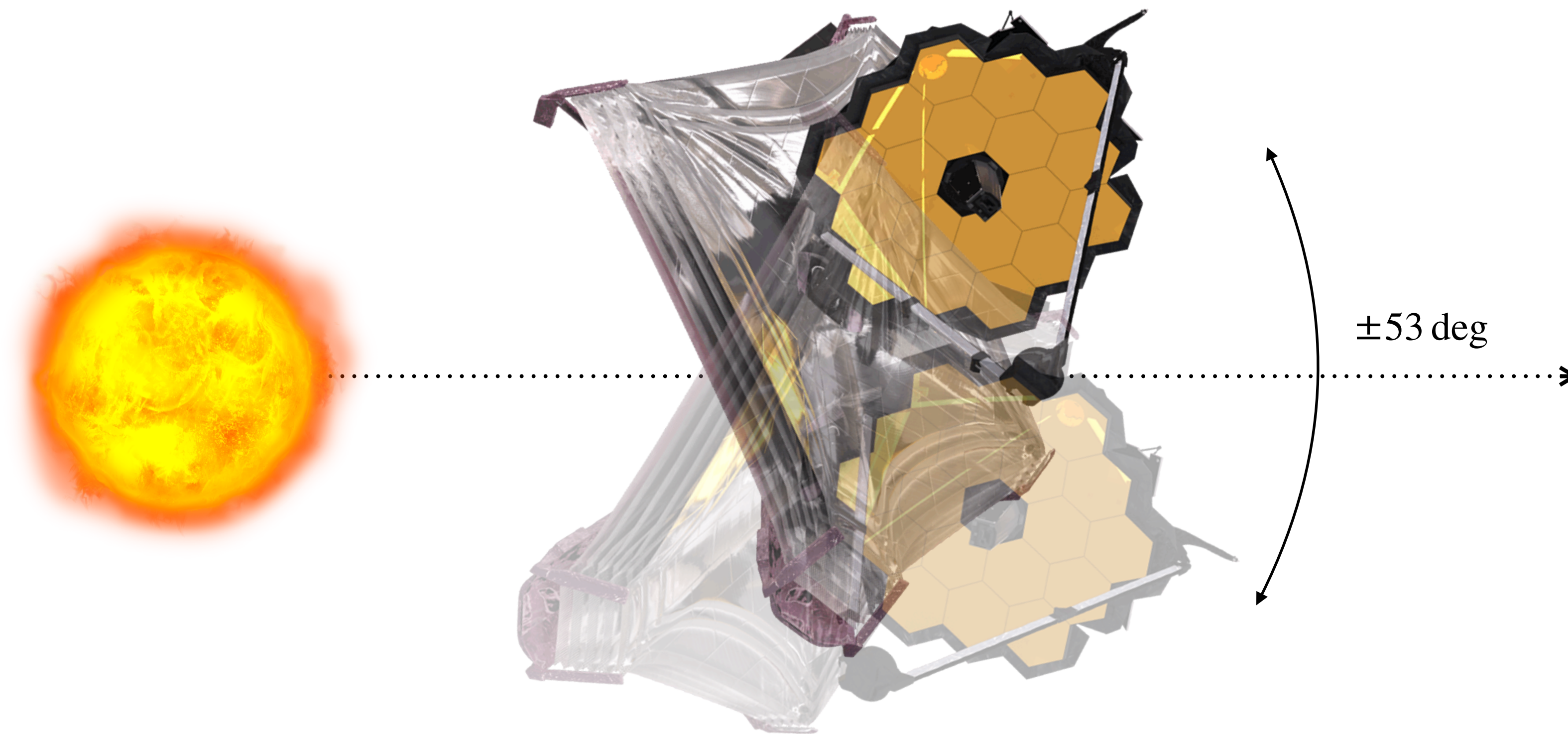
Complex Day, February 2023

Missions with cone-constrained thrust

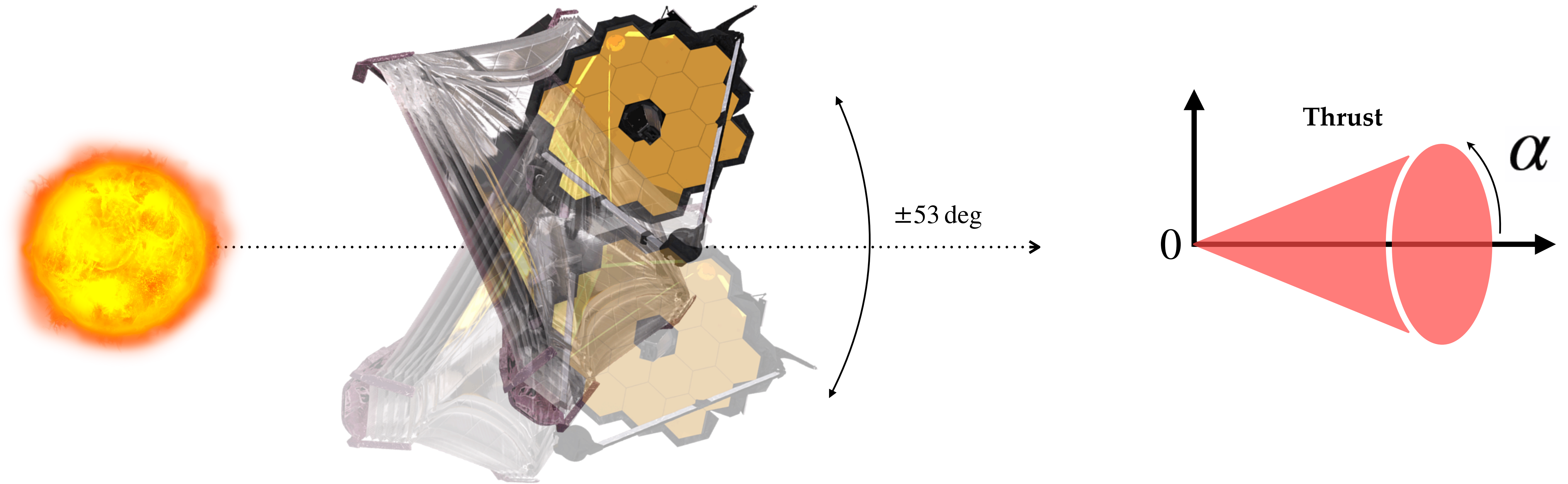


Source: NASA

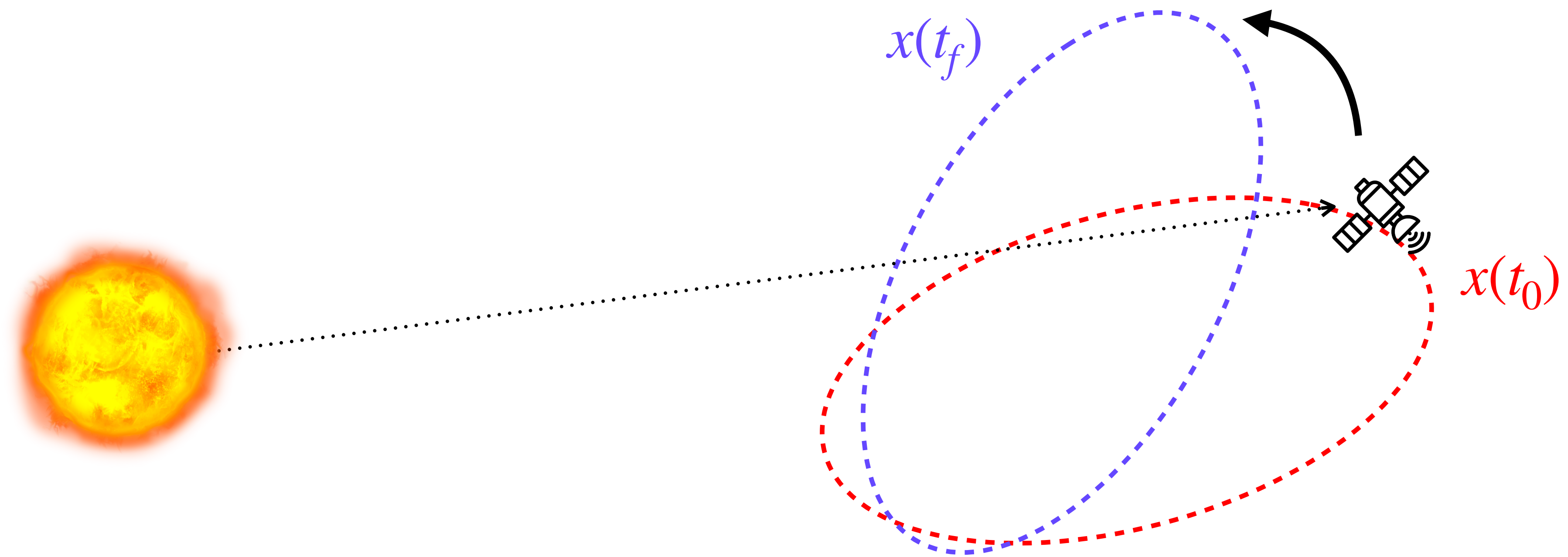
Missions with cone-constrained thrust



Missions with cone-constrained thrust

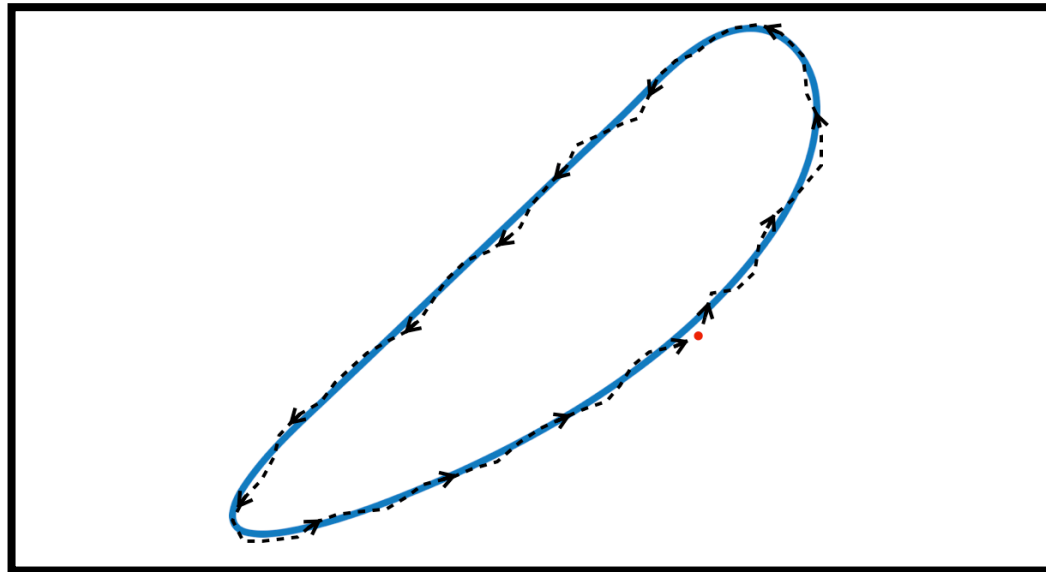


Can these satellites arbitrarily change their orbit?



Is it possible to generate any $x(t_0) \longrightarrow x(t_f)$?

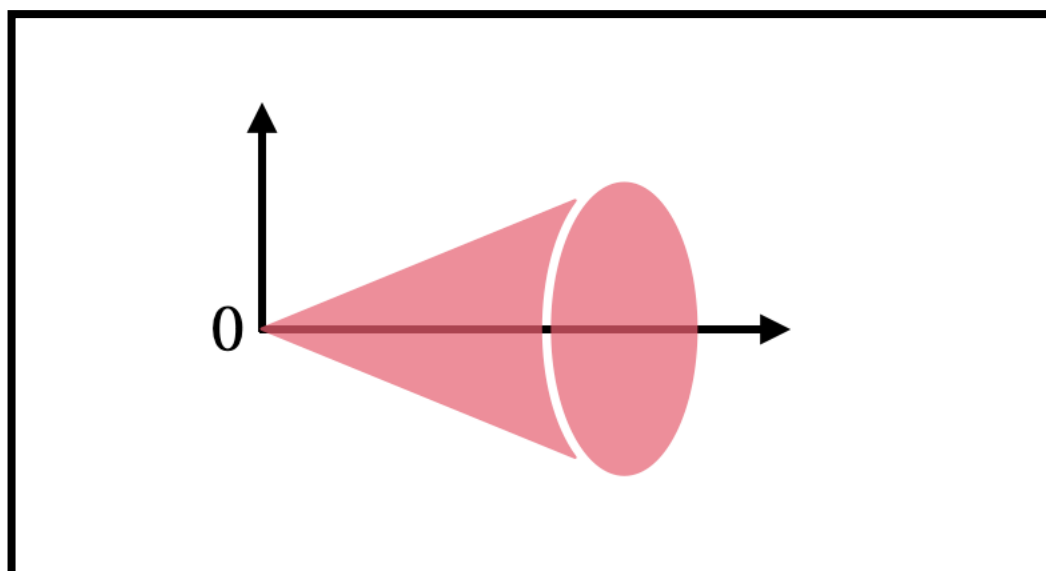
Outline



1. Dynamics of the problem

$$\left\langle p_{\delta x_0}, \frac{d\delta x_0}{dt} \right\rangle \geq 0$$

2. Necessary condition for local controllability

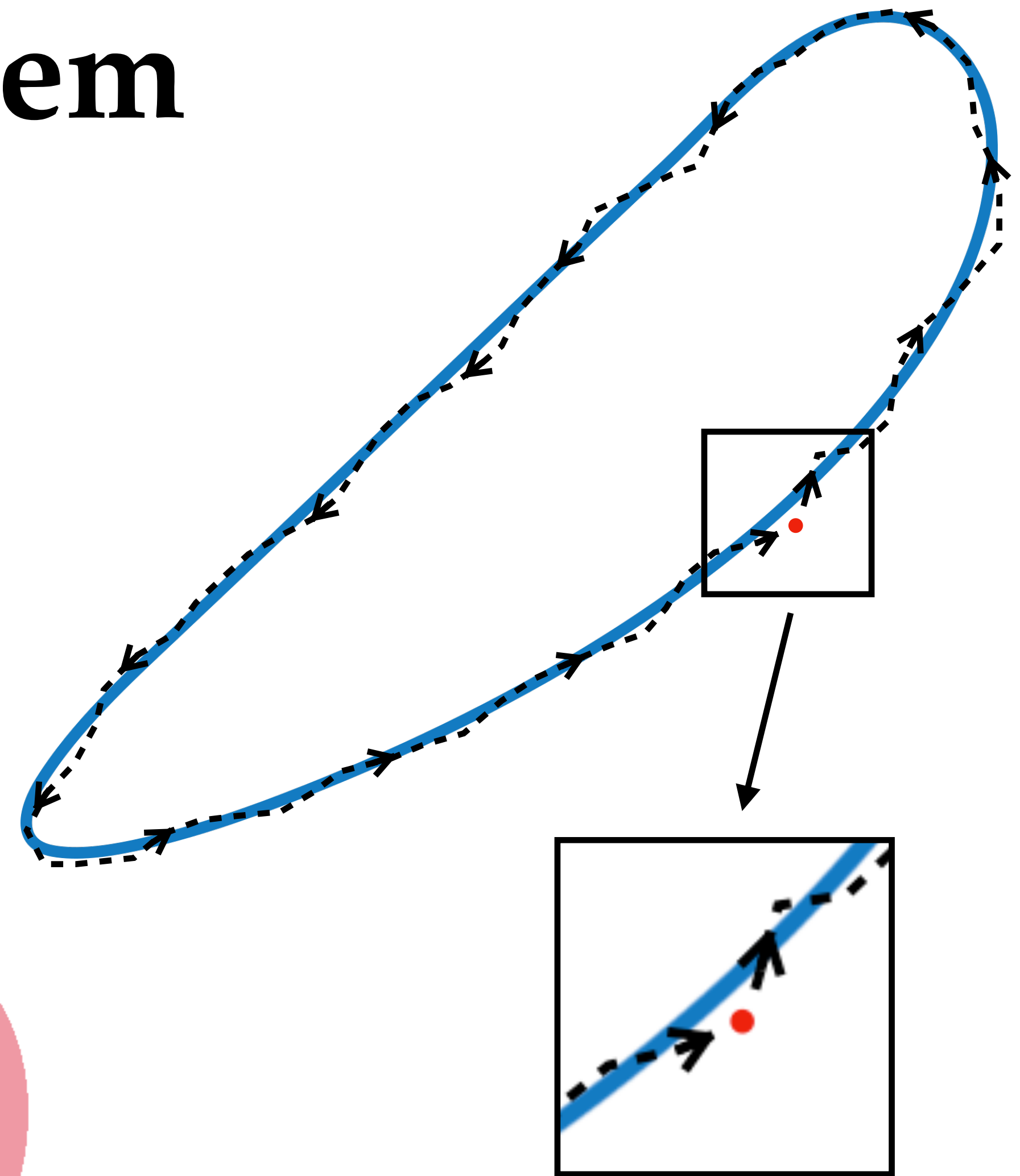
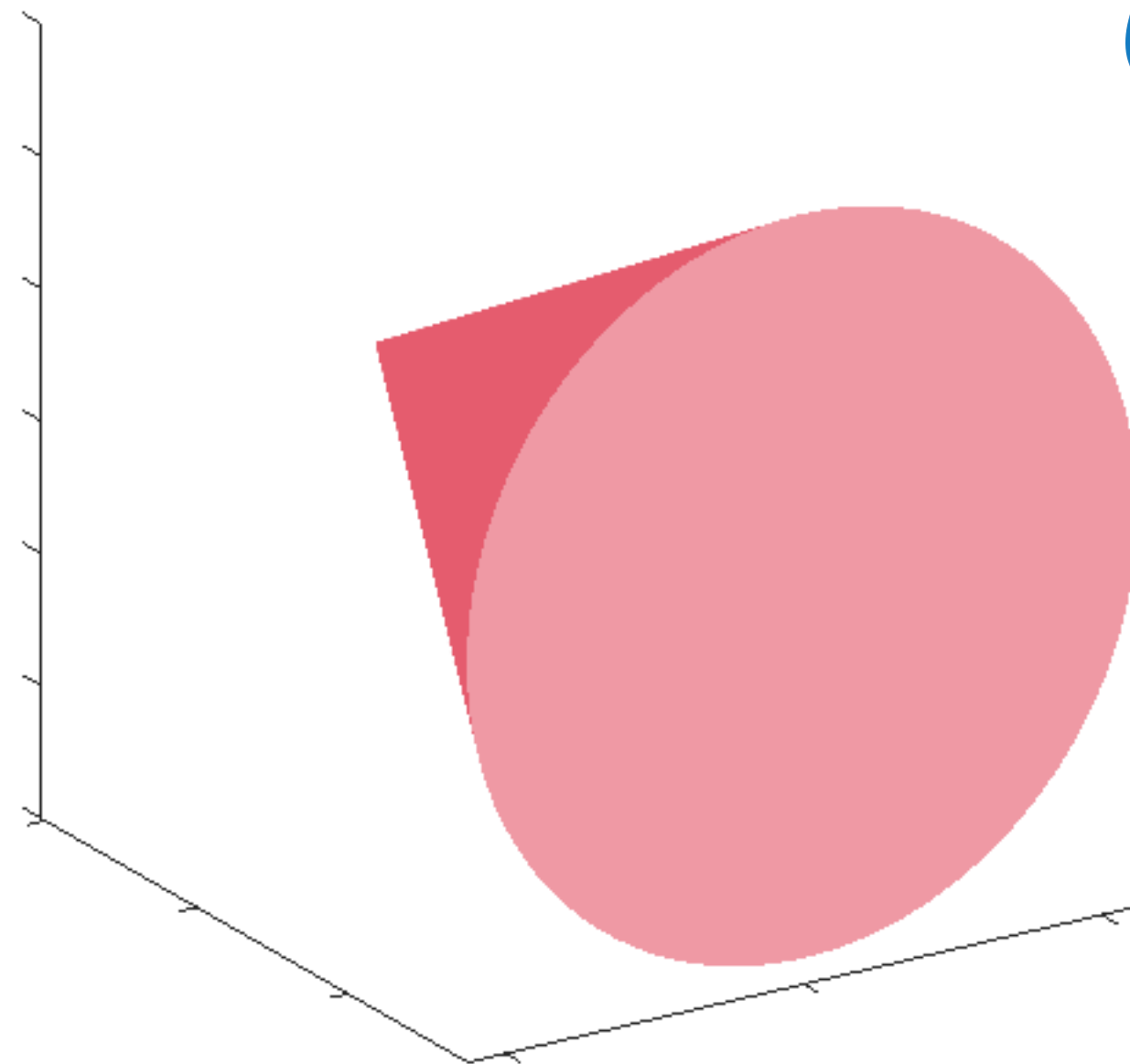


3. Case study

1. Dynamics of the problem

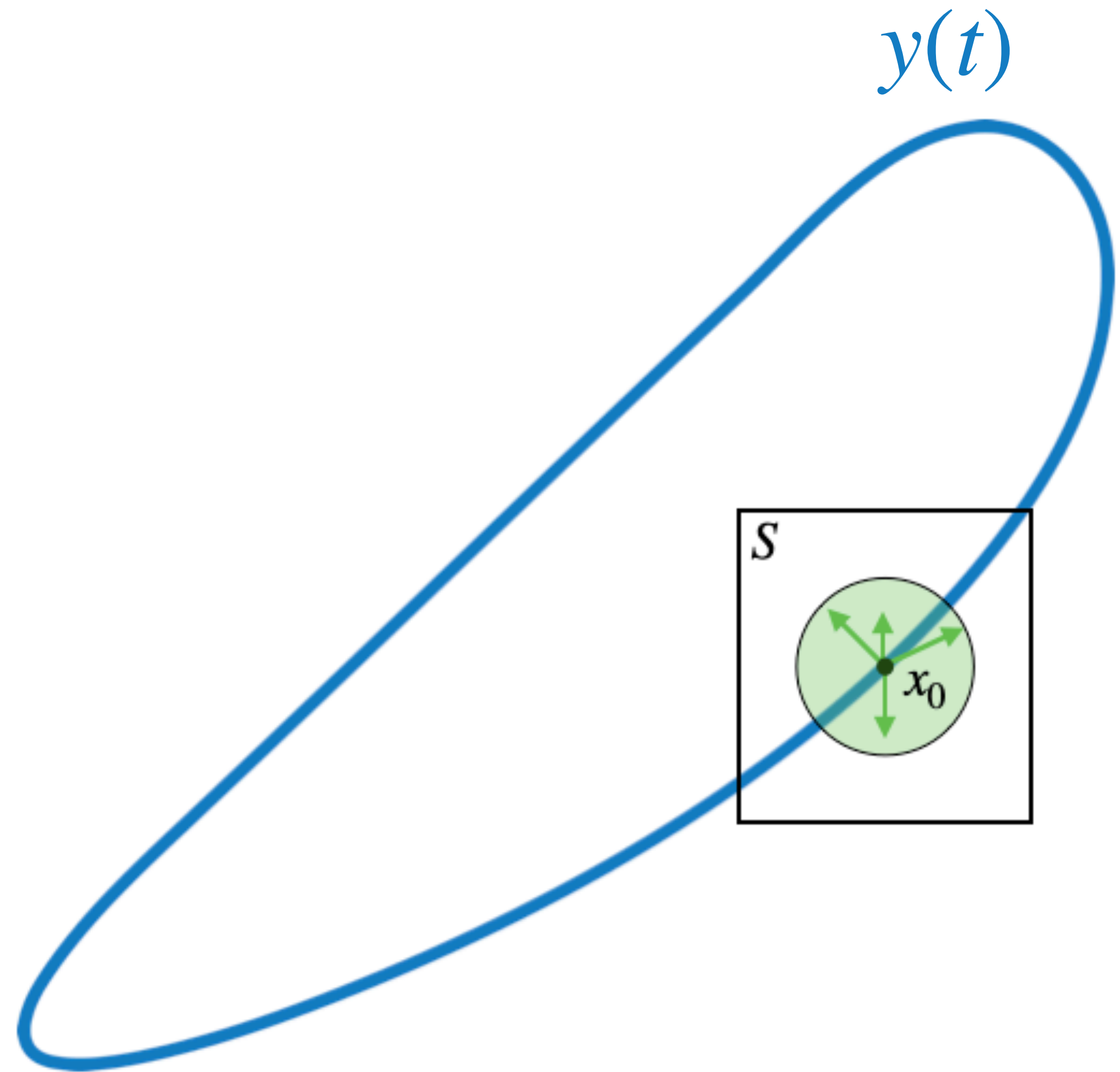
$$\frac{dx}{dt} = f(x) + B(x)u, \quad x \in M \subseteq \mathbb{R}^n$$

$$u \in K_\alpha \subseteq \mathbb{R}^m, \quad \|u\| \leq \varepsilon$$



1. Poincaré map

$$\begin{cases} \frac{dy}{dt} = f(y) \\ y(0) = y(T) = x_0 \\ S(x_0) = 0 \end{cases}$$

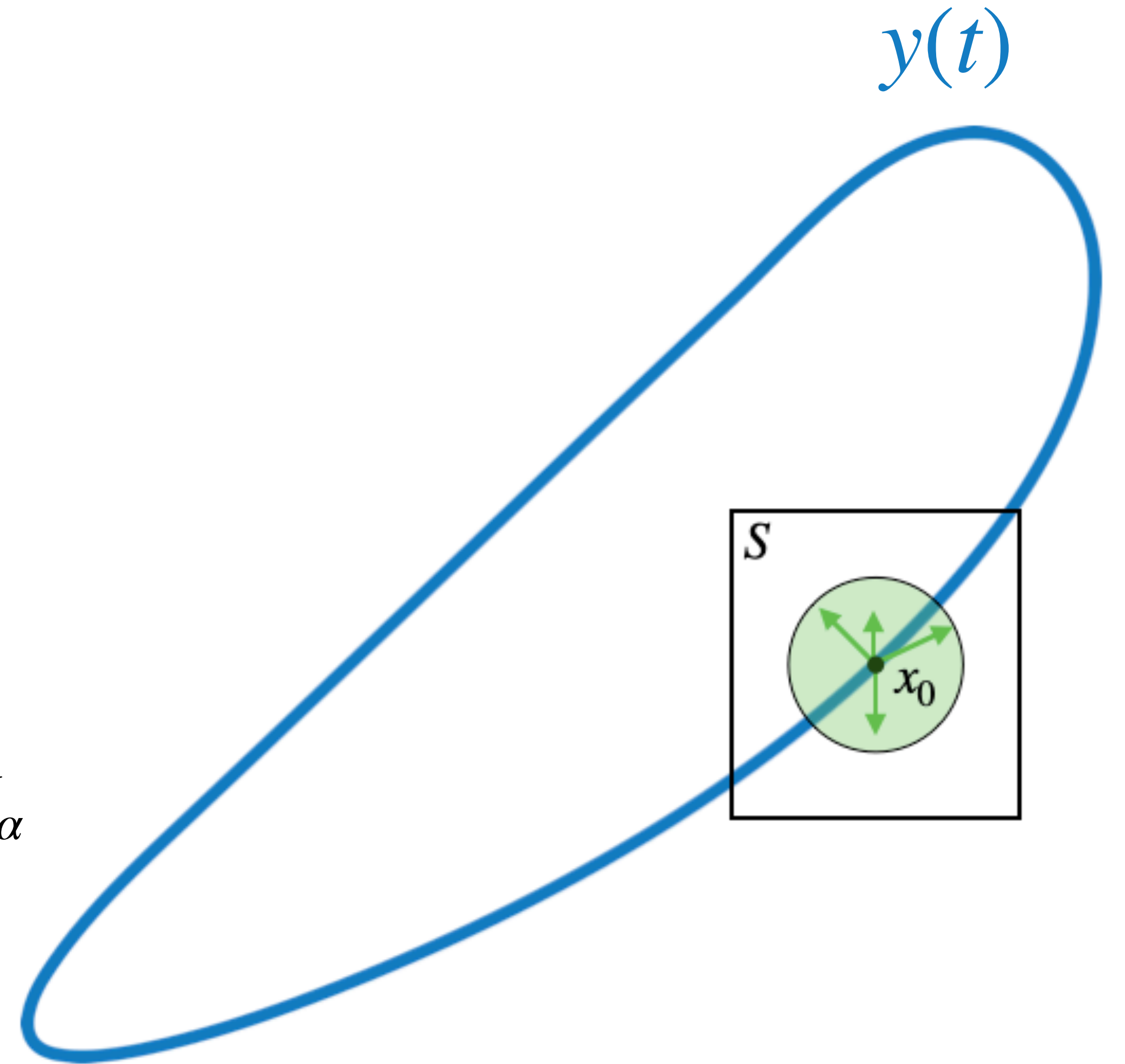


1. Linearization over one orbital period

$$\delta x(t) = \Phi(t, x_0) \delta x_0$$

$$\frac{d\delta x}{dt} = \left. \frac{df}{dx} \right|_y \delta x + B(y) u$$

$$\frac{d\delta x_0}{dt} = \Phi^{-1}(t, x_0) B(y(t)) u, \quad \delta x_0 \in T_{x_0} S, \quad u \in K_\alpha$$



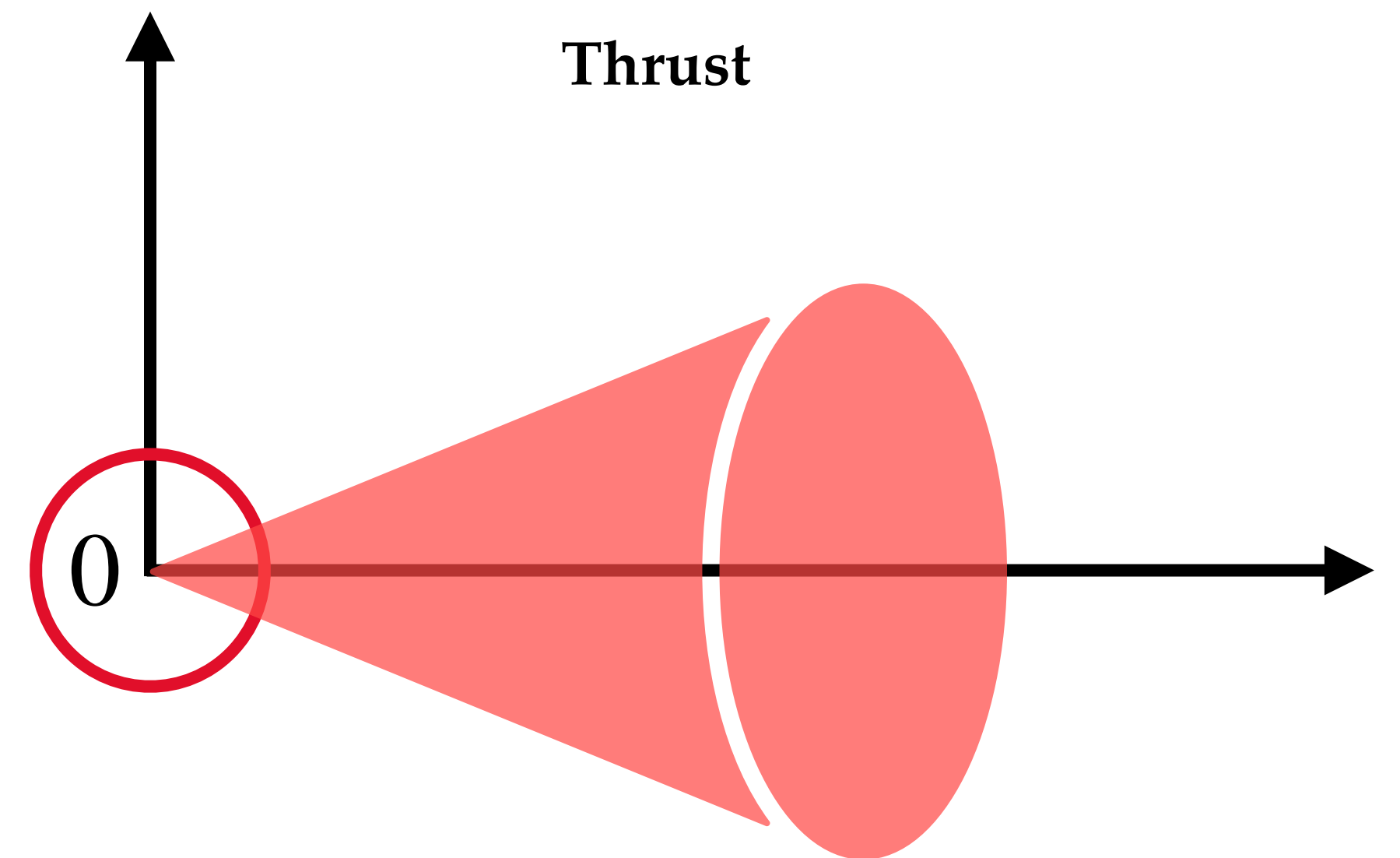
2. Classical approach using Lie brackets

Local controllability if [Jurdjevic, 1996]:

i) Periodic drift \longrightarrow **OK**

ii) Bracket generating \longrightarrow **OK**

iii) $\text{Conv}(\mathcal{U})$ is neighbourhood of the origin \longrightarrow
FAIL

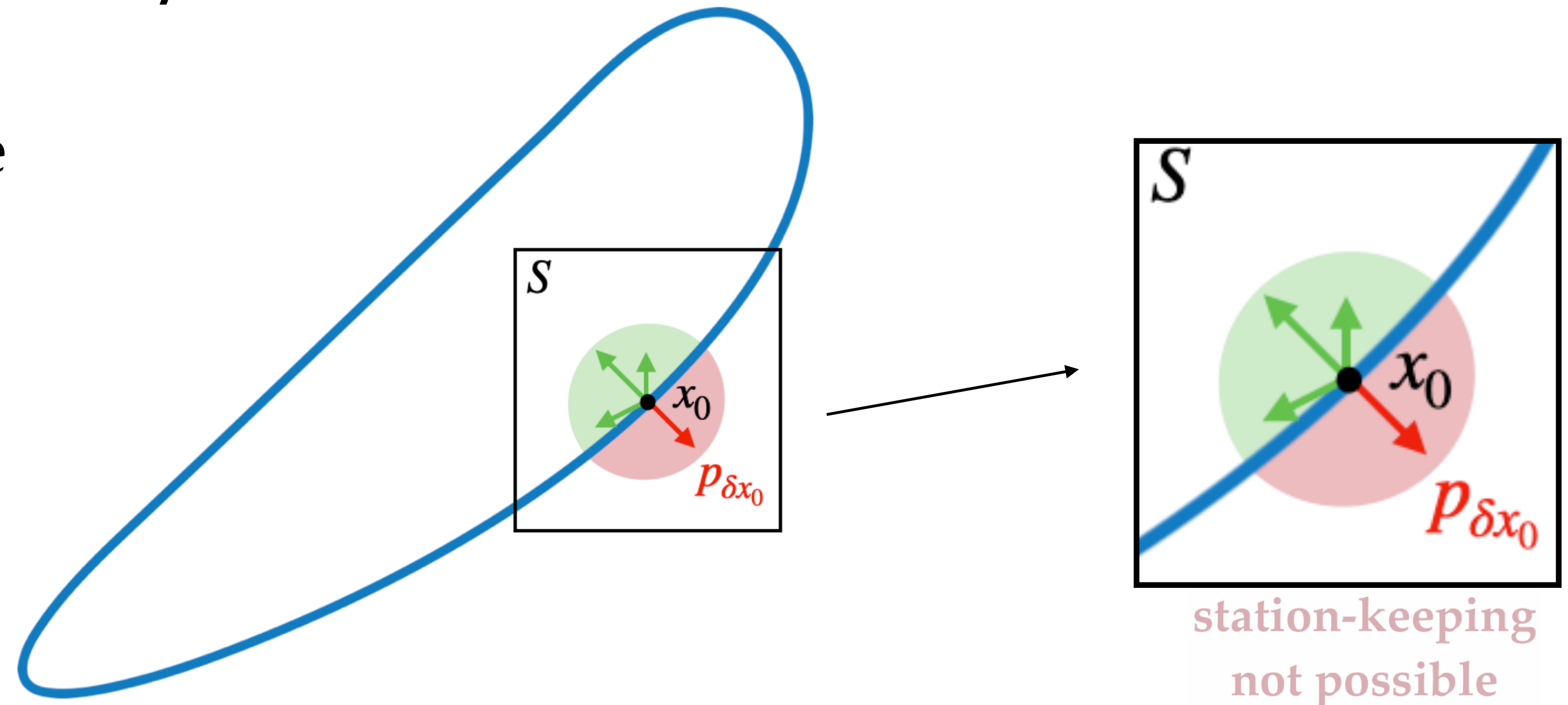


2. Necessary condition for local controllability

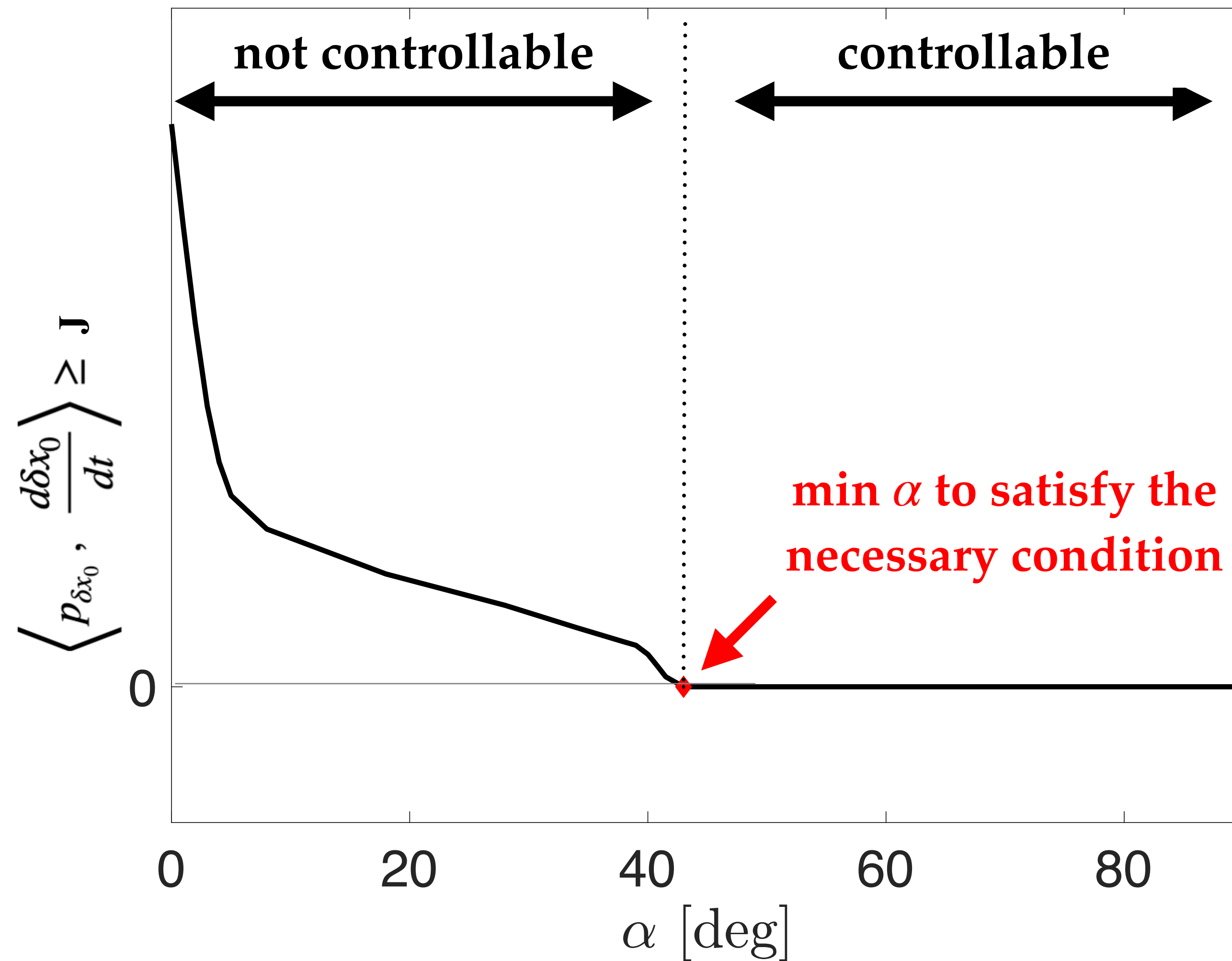
If $\exists p_{\delta x_0} \in T_{x_0}^*S$, $p_{\delta x_0} \neq 0$ such that

$$\left\langle p_{\delta x_0}, \frac{d\delta x_0}{dt} \right\rangle \geq 0, \quad \forall u \in \partial K_\alpha, \|u\| = 1, t \in [0, T)$$

→ not locally controllable

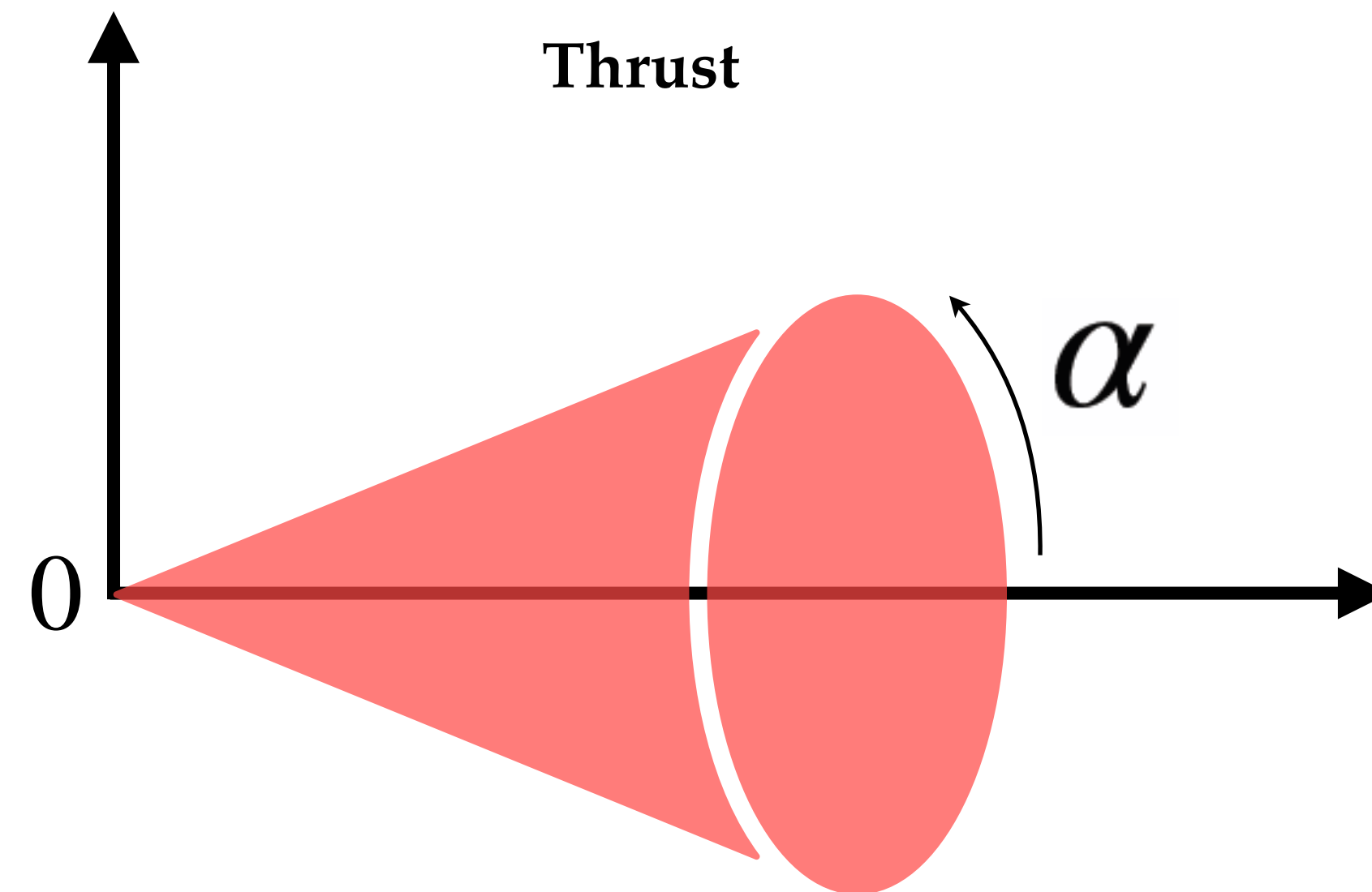


2. Exploitation: minimum requirement on thrust



2. Effective test of the necessary condition

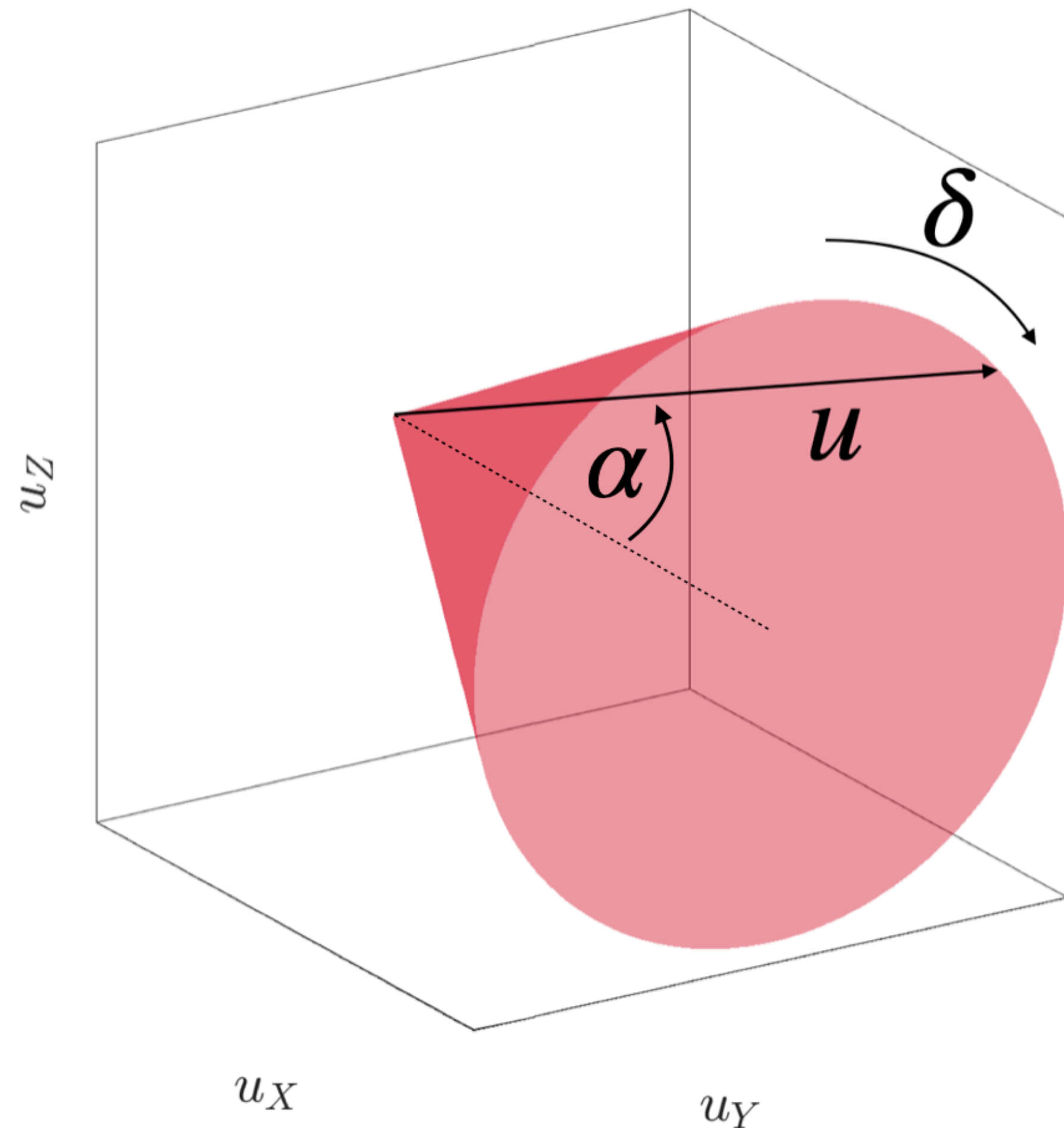
$\min_{u \in K_\alpha} \alpha$ such that the system is controllable



2. Numerical solution of the semi-infinite problem

1. Parametrization of the cone K_α

$$\mathbf{u} = \begin{bmatrix} \cos \alpha \\ \cos \delta \sin \alpha \\ \sin \delta \sin \alpha \end{bmatrix}$$



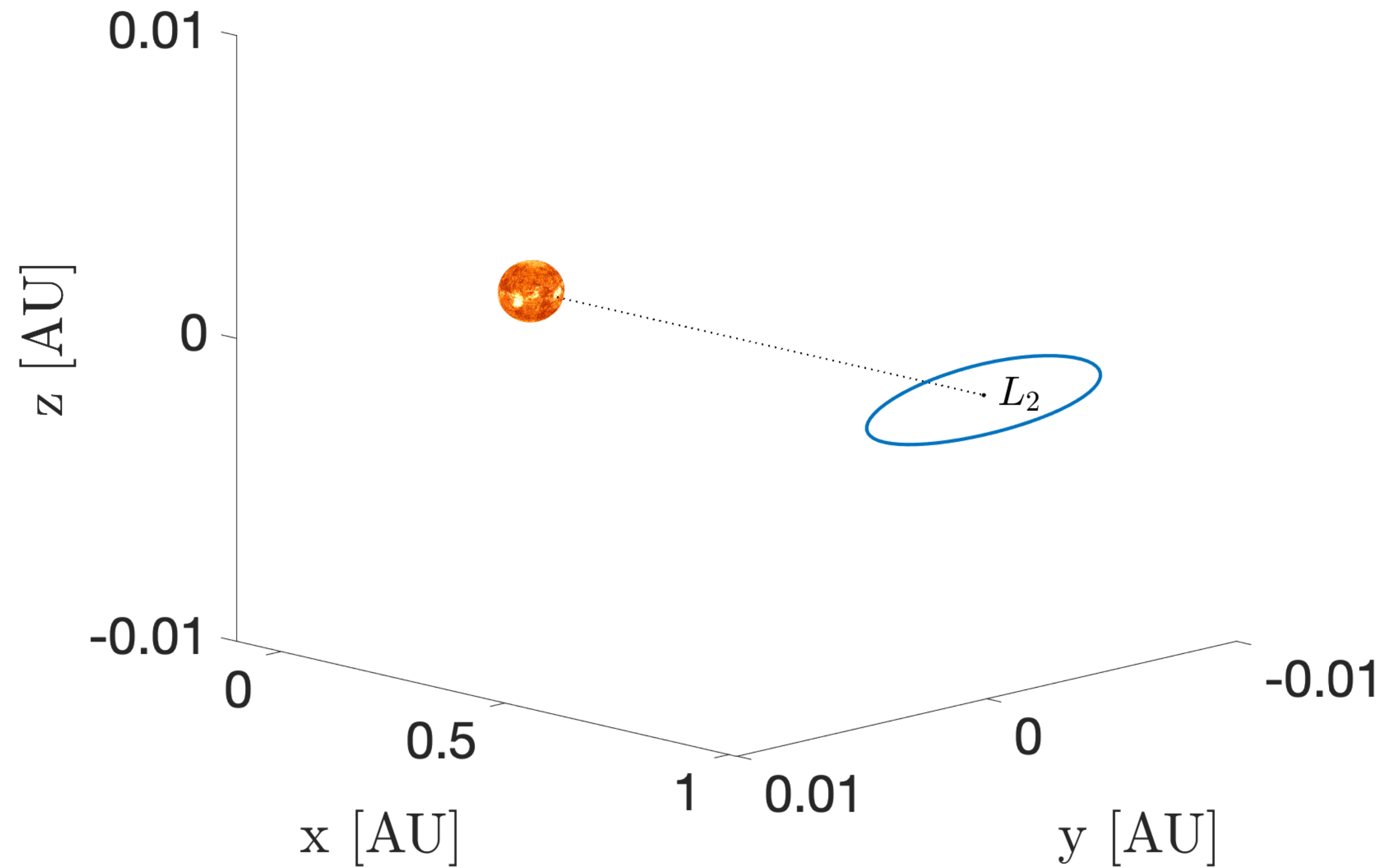
2. Numerical solution of the semi-infinite problem

2. Fourier transform

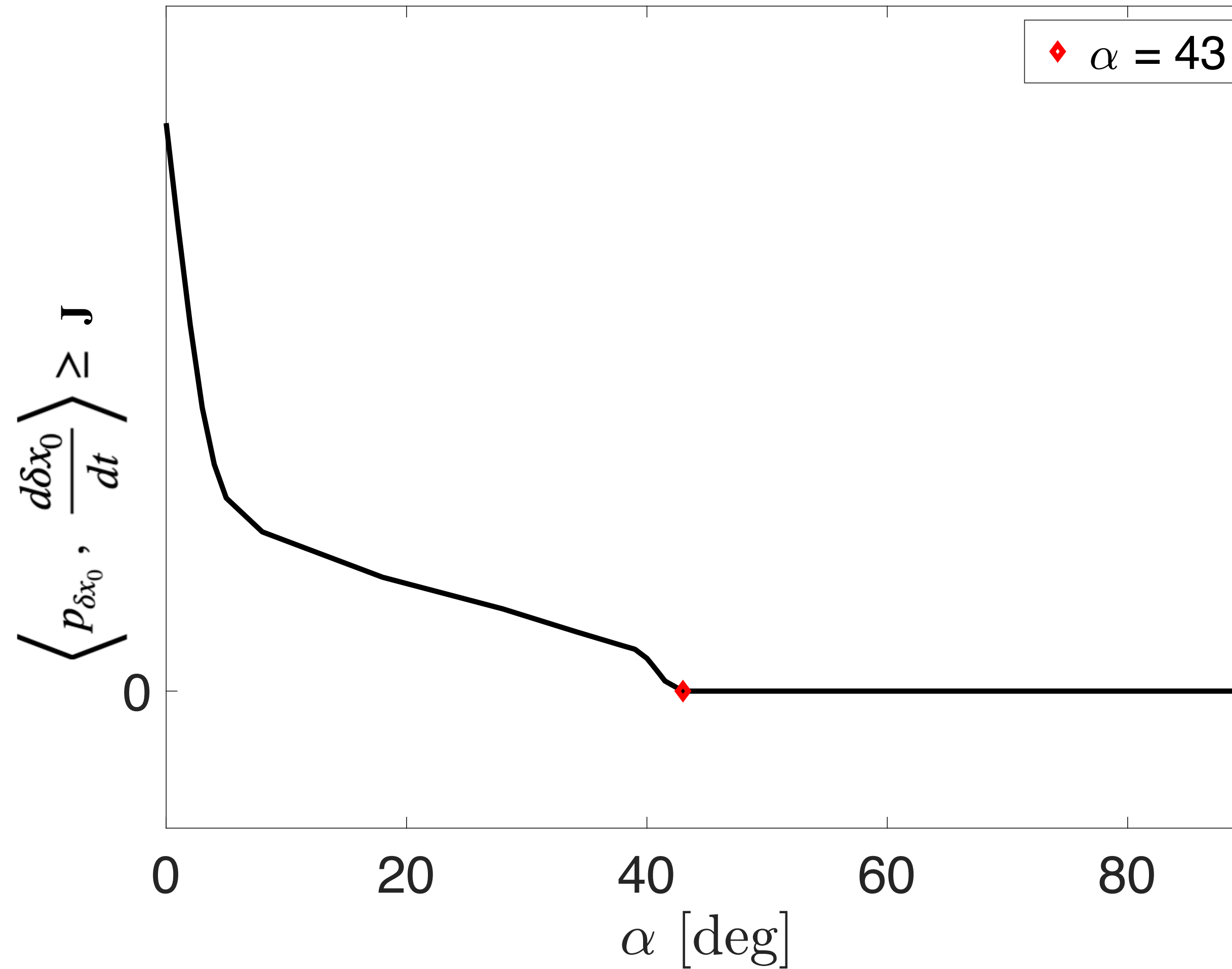
3. Positive polynomials [Nesterov, 2000; Dumitrescu, 2007]

semi-infinite convex problem \iff finite number of LMI

3. Halo orbit around L2



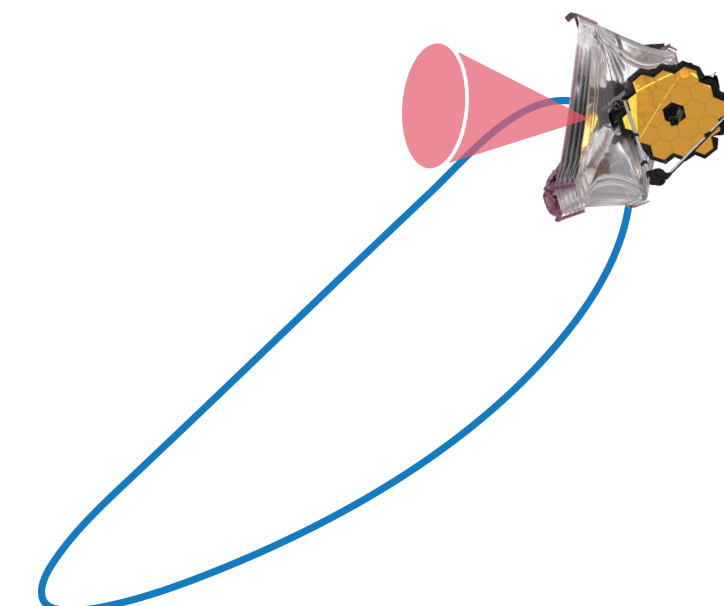
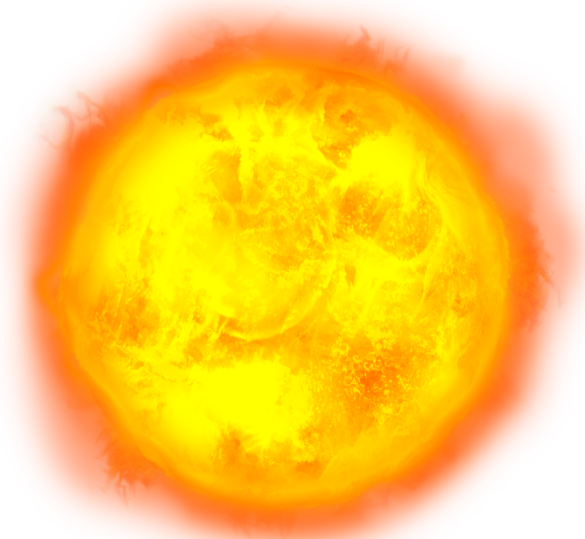
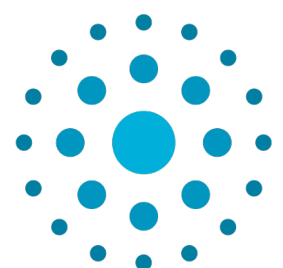
3. Minimum cone angle for thrust directions



Conclusion

We offer a certificate of non-controllability for station-keeping:

- any propulsion (low-thrust, small chemical impulses, solar sails, ...)
- any periodical orbit (around a planet, Halo orbit, ...)



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