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# ENERGY IN THE HEART OF WAVES

## RESOURCES AND ENVIRONMENT: SMART MANAGEMENT

### First Harmonic Approximation of a Full Bridge Rectifier with a Weak Capacitive Filter

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*Keywords: First Harmonic Approximation, Full Bridge Rectifier, Weak Capacitive Filter, DIPT*

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#### Abstract/Résumé

This paper provides an exhaustive model of a full bridge rectifier connected to a weak capacitive filter using the first harmonic approximation. The developed model aims to better analyze the behavior of dynamic inductive power transfer systems. It is represented as an equivalent resistance with its value given as a function of the circuit's parameters. The result is then confirmed by performing simulations using Matlab Simulink.

L'objectif de cet article est de fournir un modèle exhaustif d'un pont redresseur complet connecté à un filtre capacitif faible en utilisant l'approximation du premier harmonique. Ce modèle est développé pour mieux analyser le comportement des systèmes de transfert de puissance inductifs dynamiques. Le modèle équivalent résultant est représenté par une résistance équivalente dont la valeur est donnée en fonction des paramètres du circuit. Le résultat est ensuite confirmé en effectuant des simulations à l'aide de Matlab Simulink.

#### 1 Introduction

High-frequency rectifiers with capacitive filtering play an important role in Dynamic Inductive Power Transfer (DIPT) systems [1]. They are used on the receiver's side to provide DC power from high-frequency AC. A comprehensive model is needed to better analyze DIPT system's behavior [2]. The first harmonic approximation is widely used to simplify the study of nonlinear power converters, especially in the case of resonant converters [3]. It is a standard practice to replace a full-bridge rectifier with an equivalent resistance proportional to the resistive load ( $\cong R_{load} \frac{8}{\pi^2}$ ) [4]. However, such an approximation is only valid when a strong (ripple-free) capacitive filtering is provided at the rectifier's output. Therefore, with weak<sup>1</sup> output capacitance a new precise model is required. As a result, a new precise model is required and has been proposed in this paper.

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<sup>1</sup> DC voltage bus has excessive variation. Therefore, could not be considered as a constant (ripple-free).

## 2 Problem Formulation

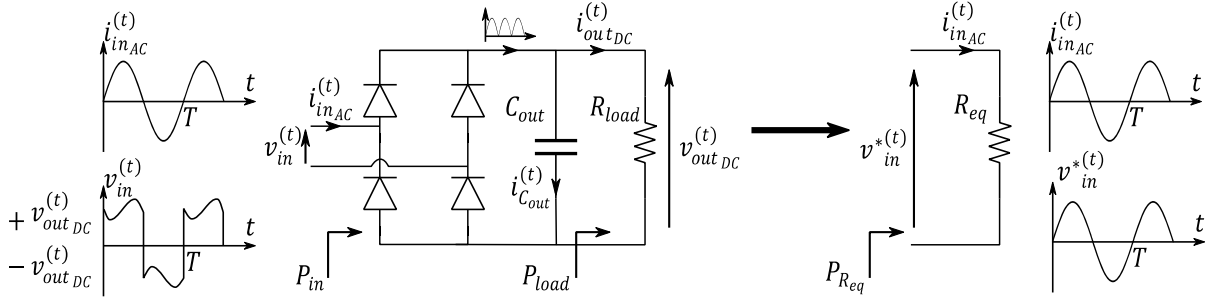


Figure 1: First harmonics approximation of a full bridge with a weak capacitive filter and a resistive Load

In the following, we consider a sinusoidal current source at the rectifier's input delivering  $I_{inAC}$  (RMS value). Figure 1 summarizes the basics of the first harmonics approximation in case of an under-sized  $C_{out}$  with excessive voltage variations noticed in a single period. According to this method, nonlinear loads causing non-sinusoidal voltages ( $v_{in}^{(t)}$ ) can be approximated by an equivalent resistance ( $R_{eq}$ ) with a sinusoidal voltage drop across ( $v_{in}^{*(t)}$ ) where  $R_{eq}$  is based on the power identity<sup>2</sup> ( $P_{in} = P_{Req}$ ). Before proceeding in the calculation of  $P_{in}$  we should solve for  $v_{outDC}^{(t)}$ . Since the frequency of the signals after the rectifier is doubled, calculations are solely performed over  $\frac{T}{2}$ . Based on Figure 1 the following basic equations can be established (Eq.(1)) :

$$\begin{cases} i_{inAC}^{(t)} = \sqrt{2}I_{inAC} \sin(\omega t) \\ |i_{inAC}^{(t)}| = i_{Cout}^{(t)} + i_{outDC}^{(t)} \\ i_{Cout}^{(t)} = C_{out} \frac{dv_{outDC}^{(t)}}{dt} \\ v_{outDC}^{(t)} = R_{load} i_{outDC}^{(t)} \end{cases} \quad (1)$$

By rearranging Eq.(1), the following linear 1<sup>st</sup> order equation can be defined (Eq.(2)):

$$\frac{dv_{outDC}^{(t)}}{dt} + \frac{1}{C_{out}R_{load}} v_{outDC}^{(t)} = \frac{\sqrt{2}I_{inAC}}{C_{out}} |\sin(\omega t)| \quad (2)$$

## 3 Solving for the equivalent model

Using the Bernoulli method for solving differential equations, Eq.(2) can be solved to find  $v_{outDC}^{(t)}$  (Eq.(3))

$$\begin{cases} v_{outDC}^{(t)} = \text{sign}(\sin(\omega t)) \frac{\sqrt{2}I_{inAC}R_{load}(\sin(\omega t) - C_{out}R_{load}\omega \cos(\omega t))}{1 + C_{out}^2R_{load}^2\omega^2} + Q_n e^{\frac{-t}{C_{out}R_{load}}} \\ Q_n = \frac{2\sqrt{2}I_{inAC}C_{out}R_{load}\omega}{1 + C_{out}^2R_{load}^2\omega^2} e^{\frac{nT}{C_{out}R_{load}}} \left( \frac{1}{2} e^{\frac{-n\pi}{\omega C_{out}R_{load}}} + \sum_{a=0}^{n-1} e^{\frac{-a\pi}{\omega C_{out}R_{load}}} \right) \\ \text{valid for: } n\frac{T}{2} < t < (n+1)\frac{T}{2}; \forall n \in \mathbb{N}_0 \\ Q \text{ is a constant that should be updated in each interval to insure continuity} \end{cases} \quad (3)$$

<sup>2</sup> All semiconductors and passive components are supposed to be ideal and the ESR of the capacitor is not taken in to account in this study.

Using the founded expression of  $v_{outDC}^{(t)}$ ,  $P_{in}$  can be calculated in the steady state as the following:

$$\begin{aligned}
P_{in} &= \frac{1}{T} \int_t^{t+T} v_{in}^{(t)} i_{inAC}^{(t)} dt = \frac{2}{T} \int_{n\frac{T}{2}}^{(n+1)\frac{T}{2}} v_{outDC}^{(t)} i_{inAC}^{(t)} dt = \frac{2}{T} \int_{n\frac{T}{2}}^{(n+1)\frac{T}{2}} v_{outDC}^{(t)} (i_{C_{out}}^{(t)} + i_{outDC}^{(t)}) dt \\
&= \frac{2}{T} \int_{n\frac{T}{2}}^{(n+1)\frac{T}{2}} v_{outDC}^{(t)} \left( C_{out} \frac{dv_{outDC}^{(t)}}{dt} + \frac{v_{outDC}^{(t)}}{R_{load}} \right) dt \\
&= C_{out} \frac{2}{T} \int_{n\frac{T}{2}}^{(n+1)\frac{T}{2}} v_{outDC}^{(t)} dv_{outDC}^{(t)} + \frac{1}{R_{load}} \frac{2}{T} \int_{n\frac{T}{2}}^{(n+1)\frac{T}{2}} (v_{outDC}^{(t)})^2 dt \\
&= C_{out} \frac{1}{T} [v_{outDC}^2]_{n\frac{T}{2}}^{(n+1)\frac{T}{2}} + \frac{1}{R_{load}} \frac{2}{T} \int_{n\frac{T}{2}}^{(n+1)\frac{T}{2}} (v_{outDC}^{(t)})^2 dt \\
&\cong 0 + \frac{1}{R_{load}} \frac{2}{T} \int_{n\frac{T}{2}}^{(n+1)\frac{T}{2}} (v_{outDC}^{(t)})^2 dt \\
P_{in} &\cong \frac{1}{R_{load}} \frac{2}{T} \int_{n\frac{T}{2}}^{(n+1)\frac{T}{2}} (v_{outDC}^{(t)})^2 dt \\
&\text{steady state: } \left( v_{outDC}(t) \cong v_{outDC} \left( t + \frac{T}{2} \right) \right)
\end{aligned} \tag{4}$$

It can be noted that reaching the steady state is faster when the product value  $C_{out}R_{load}$  is smaller with respect to the period of  $i_{inAC}^{(t)}$ . In order to find the expression of  $P_{in}$ , Eq.(3) and Eq.(4) are combined. The result is given in Eq.(5).

$$P_{in} \cong \frac{2\pi I_{inAC}^2 R_{load}}{2\pi(C_{out}^2 R_{load}^2 \omega^2 + 1)} + \frac{C_{out} \omega Q_n^2}{2\pi} \left( \frac{e^{\frac{2\pi(n+1)}{\omega C_{out} R_{load}}} - e^{\frac{2\pi n}{\omega C_{out} R_{load}}}}{e^{\frac{2\pi(2n+1)}{\omega C_{out} R_{load}}}} \right) \tag{5}$$

By replacing the value of  $Q_n$  calculated in Eq. (3), Eq.(5) can be simplified to the following:

$$\left\{ \begin{aligned} P_{in} &\cong \frac{2\pi I_{inAC}^2 R_{load}}{2\pi(C_{out}^2 R_{load}^2 \omega^2 + 1)} + \frac{8I_{inAC}^2 C_{out}^3 R_{load}^4 \omega^3}{2\pi(C_{out}^2 R_{load}^2 \omega^2 + 1)^2} \left( 1 - e^{\frac{-2\pi}{\omega C_{out} R_{load}}} \right) A_n^2 \\ A_n &= \frac{1}{2} e^{\frac{-n\pi}{\omega C_{out} R_{load}}} + \sum_{a=0}^{a=n-1} e^{\frac{-a\pi}{\omega C_{out} R_{load}}} \end{aligned} \right. \tag{6}$$

$A_n$  could be simplified using the geometrical sum identity presented in Eq.(7).

$$\sum_{a=0}^{a=m} h^a = \frac{1 - h^{m+1}}{1 - h} \tag{7}$$

Therefore  $A_n$  could be rewritten as:

$$A_n = \frac{1}{2} e^{\frac{-n\pi}{\omega C_{out} R_{load}}} + \frac{1 - e^{\frac{-\pi n}{\omega C_{out} R_{load}}}}{1 - e^{\frac{-\pi}{\omega C_{out} R_{load}}}} \tag{8}$$

In order to estimate calculate  $P_{in}$  at the steady state the limit of  $A_n$  is computed as  $n \rightarrow +\infty$ .

$$\lim_{n \rightarrow +\infty} A_n^2 = \frac{1}{\left( 1 - e^{\frac{-\pi}{\omega C_{out} R_{load}}} \right)^2} \tag{9}$$

Combining Eq.(6) and Eq.(9) an estimation of  $P_{in}$  at the steady state could be written as the following:

$$\lim_{n \rightarrow \infty} P_{in} = \frac{2\pi I_{inAC}^2 R_{load}}{2\pi(C_{out}^2 R_{load}^2 \omega^2 + 1)} + \frac{8I_{inAC}^2 C_{out}^3 R_{load}^4 \omega^3}{2\pi(C_{out}^2 R_{load}^2 \omega^2 + 1)^2} \left( \frac{1 - e^{\frac{-2\pi}{\omega C_{out} R_{load}}}}{\left(1 - e^{\frac{-\pi}{\omega C_{out} R_{load}}}\right)^2} \right) \quad (10)$$

Now the expression of  $R_{eq}$ , at steady state, that is based on the power identity ( $P_{in} = P_{Req}$ ) is provided in Eq.(11):

$$R_{eq} \cong \frac{2\pi R_{load}}{2\pi(C_{out}^2 R_{load}^2 \omega^2 + 1)} + \frac{8C_{out}^3 R_{load}^4 \omega^3}{2\pi(C_{out}^2 R_{load}^2 \omega^2 + 1)^2} \left( \frac{1 - e^{\frac{-2\pi}{\omega C_{out} R_{load}}}}{\left(1 - e^{\frac{-\pi}{\omega C_{out} R_{load}}}\right)^2} \right) \quad (11)$$

#### 4 Simulations

Figure 2, compares the results of the first harmonics approximation based on a weak capacitive filtering ( $C_{out}R_{load} \cong \frac{T}{10}$ ) at steady state (where T is the period of the sinusoidal input current). Results show a great precision of the developed expression in Eq.(11) (named method 2). On the other hand, results presented in Figure 3 shows that strong capacitive filtering could be considered starting from ( $C_{out}R_{load} \cong T$ ) where the known equation  $R_{load} \frac{8}{\pi^2}$  (named method 1) could be used with negligible error. It is important to keep in mind that the developed first harmonics approximation is only valid at steady state. If either the magnitude, phase or frequency of  $i_{inAC}^{(t)}$  changes faster than the time needed to reach the steady state, then static values of  $R_{eq}$  could not be used. In that case, first harmonic method would not be a suitable approach.

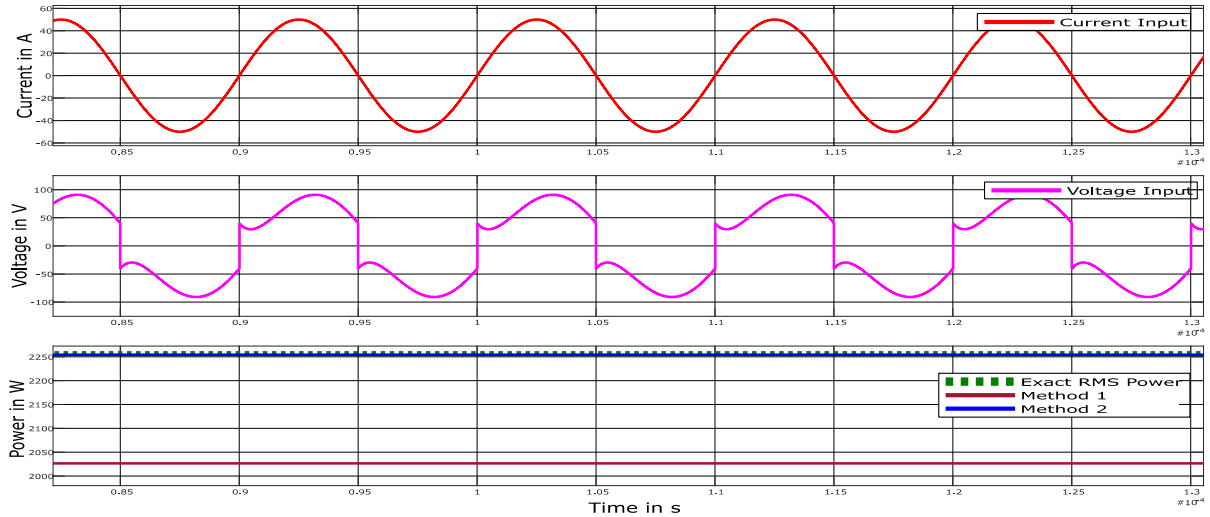


Figure 2: Comparison of first harmonic approximation based on method 1 ( $R_{load} \frac{8}{\pi^2}$ ) and method 2 (Eq.(11)) in a **weak** capacitive filtering ( $C_{out}R_{load} \cong \frac{T}{10}$ )

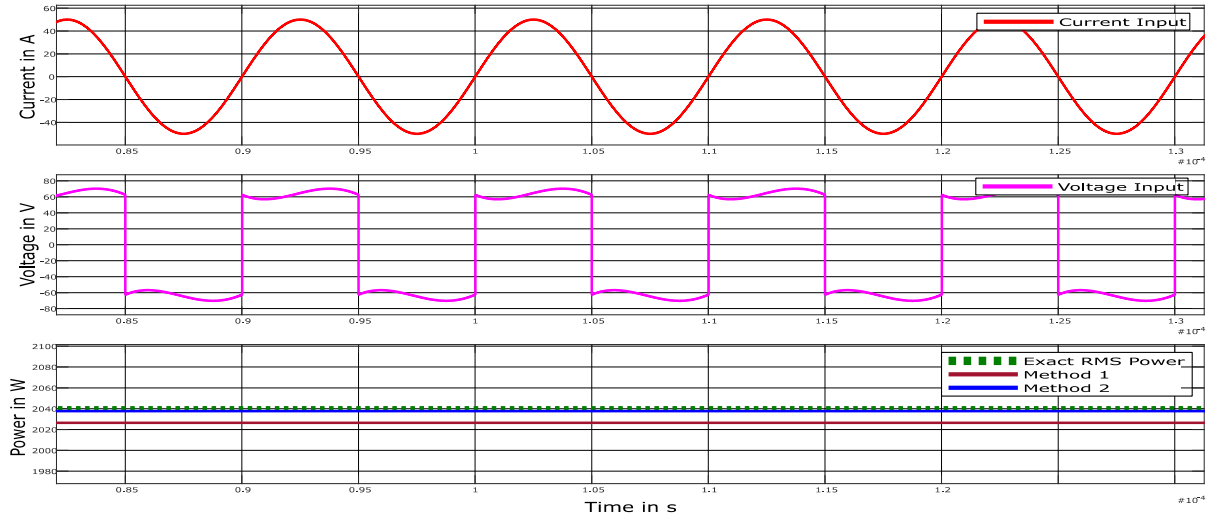


Figure 3: Comparison of first harmonic approximation based on method 1 ( $\cong R_{load} \frac{8}{\pi^2}$ ) and method 2 (Eq.(11)) in a **strong** capacitive filtering ( $C_{out}R_{load} \cong T$ )

## 5 Conclusion

This paper's contribution lies in providing an exhaustive model for a full bridge rectifier connected to a weak capacitive filter using the first harmonic approximation. The developed model permits to better analyze the DIPT systems where a full bridge rectifier plays an important role. The resulting equivalent model was formalised as an equivalent resistance with its value given in function of the circuit's parameters. Details of the calculations were provided. Moreover, the model was validated using Matlab Simulink.

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