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# Bayesian interface based calibration of a novel rockfall protection structure modelled in the non-smooth contact dynamics framework

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## Abstract

This article presents the development and calibration of a numerical model simulating the response of a novel rockfall protection structure subjected to localised dynamic loading. This structure is made of piled-up concrete blocks interconnected via metallic components whose dynamics response under projectile impact is examined via real-scale experiments. The corresponding numerical model is developed in a python based open source software SICONOS which implements the Non-Smooth Contact Dynamics (NSCD) method. The geometrical features and mechanical properties are incorporated in the model via specific developments pertinent to the modelling requirements in SICONOS. Some parameters peculiar to the numerical model are calibrated against the spatial-temporal measurements from two full-scale impact experiments. The Bayesian interface statistical learning method aided by the polynomial chaos expansion based meta-model of the NSCD model is deployed for the calibration. The additional understanding of the model dynamics through the byproducts of the meta-model is highlighted. In the end, the NSCD model is successfully calibrated against the spatial-temporal response of the experimental structure with more than 90% accuracy for impact energies up to 1 MJ.

*Keywords:* rockfall, impact, meta-model, statistical learning, NSCD modelling, calibration, Bayesian interface

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## 1. Introduction

The Non-Smooth Contact Dynamics (NSCD) method developed by M. Jean and J.J. Moreau [1, 2] is a modelling and numerical framework for the dynamics of multi-body systems in interaction through frictional contact interfaces. The present work extends the use of NSCD modelling [3, 4] to study the response to impacts of a complex structure made of individual concrete blocks interconnected via metallic

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6 components, thus forming an articulated structure. The structure is proposed by the company Géolithe  
7 and is intended to serve as passive protection against gravity-driven natural hazards. It is in particular  
8 designed to intercept rockfall, similarly to other massive structure types including embankments [5],  
9 concrete walls [6], concrete blocks cushioned with gabions [7], gabion structures [8] and sea containers [9].  
10 This innovative technology offers the possibility to build massive vertical walls, with reduced footprint,  
11 high deformability, and versatility.

12 The development of this technology is based on small-scale experiments, to address the structure re-  
13 sponse considering different design options, and real-scale impact experiments to demonstrate the struc-  
14 ture capacity in arresting rock blocks with kinetic energy exceeding 1 MJ [10]. These experiments revealed  
15 the intricacy of the impact response of structures made from interconnected concrete blocks. The impact  
16 induces sliding at the base and tilting of the structure, where the amplitude depends on the impact energy  
17 and distance to the impact point. In addition, the interconnection between concrete blocks improves the  
18 structure stability preventing excessive concrete block displacements in the impact vicinity.

19 The impact response of small- and real-scale structures has been previously investigated in the nu-  
20 merical modelling framework of the finite volume formulation code FLAC3D [10]. These models provided  
21 detailed insights into the energy dissipation capacities of the protection walls. However, the computation  
22 time for the real-scale structures ranging from 10 to 20 hours hinders the exhaustive investigations of  
23 the structure's mechanical and dynamic response. In particular, improving the design of such structures  
24 requires better quantifying their efficiency when exposed to rockfall. In line with some previous research  
25 on flexible barriers (*e.g.*, [11, 12, 13]), this implies taking into account the variety of loading cases ob-  
26 served during real rockfalls and therefore performing a very large number of numerical simulations of the  
27 structure's response to the impact.

28 Given the above, we developed an NSCD-based model of the structure, using the SICONOS soft-  
29 ware [14]. SICONOS is an open-source scientific software primarily targeted at modelling and simulating  
30 non-smooth dynamical systems in C++ and in Python, including mechanical systems, switched electri-  
31 cal circuits, sliding mode control systems, and biology. The NSCD method guarantees that Signorini's  
32 condition at the velocity level is satisfied without introducing contact stiffnesses and that the coulomb  
33 friction, especially in the sticking mode is satisfied without resorting to viscous friction. This peculiarity  
34 of NSCD is enforced with the use of a specific implicit scheme (Moreau–Jean scheme) and yields a robust

35 numerical method, which consistently models the threshold phenomena (friction, contact) and the dis-  
36 sipation properties of the model in discrete time, in particular impact dissipation and energy properties  
37 [15].

38 Considering its intended use, the developed model is kept as simple as possible while allowing satisfac-  
39 tory mimicking of the whole structure’s response to impact, as observed during the real-scale experiments.  
40 Such simplifications drastically reduce the computational time as each component of the structure is mod-  
41 elled as a rigid body instead of a deformable mesh resulting in a huge reduction in the degrees of freedom  
42 of the model. The energy dissipation due to plastic strains and breakages of the blocks during impact is  
43 modelled in a simple manner as energy dissipation at the contact points. Escallón et al. [16] reported a  
44 conceptually similar approach for the FEM model of flexible rockfall protection barriers and observed a  
45 significant reduction in the model computational time.

46 Further, the calibration of model parameters becomes necessary to validate the model performance.  
47 For this, an existing experimental database [10] is used as a reference and some identified model param-  
48 eters are calibrated using the Bayesian interface statistical learning method.

49 A common requirement of any stochastic method is a large number of model computations which  
50 are impractical to implement due to a large computation cost. This gap is bridged by the use of a  
51 meta-model, representative of the actual model having a negligible computational cost and possessing  
52 high accuracy and reliability. Such meta-models have been used by many researchers, *e.g.*, for dam  
53 engineering problems [17, 18] and for nuclear containment structures [19].

54 In the present work, the NSCD-based model is first described in detail, including specific developments  
55 to cope with the particular features of this structure type. A polynomial chaos expansion (PCE) based  
56 meta-model of the NSCD model is created for the stochastic analysis. The additional byproduct of PCE-  
57 based meta-models towards the relative influence of model parameters is discussed. The model parameters  
58 are calibrated based on the Bayesian interface approach considering the measurements obtained from  
59 two impact experiments. A discussion on model features in line with the perspectives for future research  
60 concludes the work.

## 61 **2. Experimental structure and its impact response**

62 Referring to [10] for further information, the experimental details use for the calibration of the NSCD

63 model are summarised hereinafter. The considered articulated structures consist of piled-up blocks, made  
 64 from concrete, and are reinforced with steel rebars, presented in Figure 1. The block extremities in the  
 65 horizontal plane are rounded and their upper and lower faces present empty spaces. The wall is made  
 66 up of 38 blocks and four half-blocks stacked in four rows. The blocks are arranged following a zig-zag  
 67 pattern (at  $45^\circ$  angle) to improve the structure stability against titling. The wall is 3.2 m in height and  
 68 about 14 m in length.

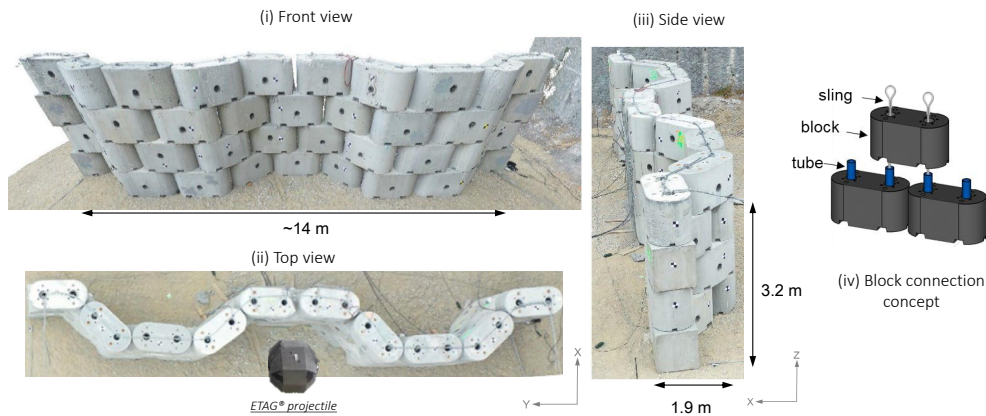


Figure 1: Experimental full-scale structure, recreated after [10]

69 Each block is traversed by two cylindrical holes of diameter 154mm along the vertical axis to receive  
 70 tubes and slings (see Figure 1(iv)). These latter metallic components, hereafter referred to as connectors,  
 71 link blocks together. More precisely, a hollow steel tube of diameter 139.7mm connects each couple  
 72 of adjacent layer blocks, preventing their relative lateral displacement. In addition, a cable runs from  
 73 the structure base to its top, in the succession of tubes, and mainly prevents relative displacements  
 74 along the vertical direction. From a conceptual point of view, these connectors provide the structure  
 75 with mechanical continuity with the aim of increasing the number of blocks involved in the structure's  
 76 response proportional to the impact loading. Another key feature in the design is the presence of various  
 77 free spaces (hereby referred to as 'mechanical plays' or simply 'plays'): a 40 mm side-to-side distance  
 78 between the same layer concrete blocks, about 14 mm difference between the external diameter of the  
 79 metallic tubes and the inner diameter of the cylindrical holes and, a precisely non-quantifiable slack in  
 80 the vertical cables. These plays give the structure a certain deformation capacity. All geometrical and  
 81 mechanical characteristics of this structure are presented in Table 1 along with the ones of the impacting  
 82 projectile.

Table 1: Geometrical and mechanical characteristics of the articulated concrete blocks structure

Component	Parameter	Value	Unit
Concrete Block	Length	1.56	m
	Width	0.76	m
Concrete Block	Height	0.80	m
	Mass ( $m_{block}$ )	1800	kg
	Hole diameter	154	mm
Cylindrical bars	Diameter (ext.)	139.7	mm
	Diameter (int.)	123.7	mm
Structure	Length	0.8 (3 pcs.)	m
		0.4 (2 pcs.)	m
Structure	Length (edge-to-edge)	$\approx 14$	m
	Height	3.2	m
Projectile	Pattern angle	45 (from y-axis)	$^{\circ}$
	Side length ( $l_{proj}$ )	1.1	m
	Mass ( $m_{proj}$ )	2600	kg
	Impact angle	0	$^{\circ}$
Projectile	Impact point location	$y \approx 7.0, z = 1.7$	m

Two identical real-scale walls were built and subjected to one impact each at different energy levels. The impact experiments were performed with the pendulum testing facility of the Université Gustave Eiffel test site (Montagnole, France). It involved a 2600 kg reinforced concrete projectile whose shape is in accordance with the requirements for flexible barriers testing [20]. Its size is about one-third the structure's height (1.1 m). The velocity at impact is 20m/s and 28m/s during the first and second impacts, corresponding to kinetic energies of 520 kJ and 1020 kJ respectively. The angle of incidence and rotation velocity at impact were zero. The projectile impacted the structure at its mid-length and at about 1.7m from the ground. These impacts in particular induced wall sliding at its base, tilting, change

91 in conformation as well as concrete block damage.

92 The structure impact response with time is described based on measurements of displacement and  
93 acceleration along two vertical lines, as presented in Figure 2. They are termed ‘impact’ and ‘distant’  
94 axis which are representative of the impact point and 3.5m to its left respectively. The experimental data  
95 used in this study concerned the displacement of concrete blocks at the *Top* and *Base* in the impact axis  
96 and points *C* and *D* in the distant distant axis. Besides, the data from two points *A* and *B* in the impact  
97 axis is also available which is used for the cross-reference purpose only.

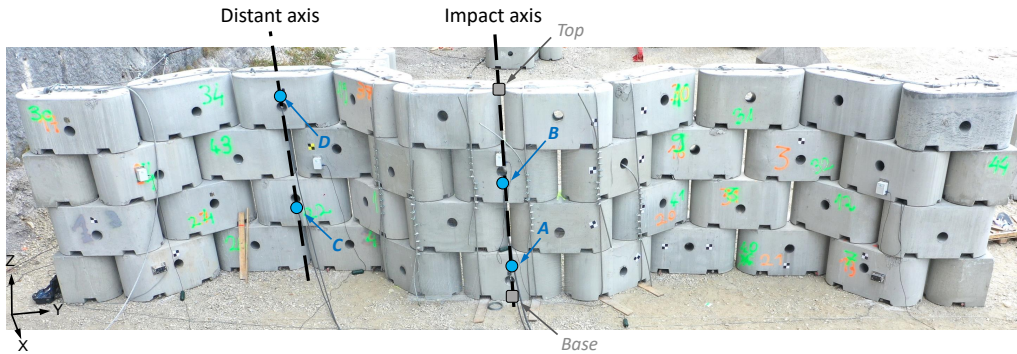


Figure 2: Data acquisition locations in the experimented full-scale structure, recreated after [10]

98 The displacement governs the structure sliding and tilting, both leading to structure failure if in  
99 excess. In this study, the database of displacement evolution with time at the impact vicinity and at  
100 distance is considered. The displacement data used in this study is plotted in Figure 3 for both impact  
101 and distant axes. A secondary y-axis is added for the distant axis albeit to the relatively low displacement  
102 amplitude compared to the impact axis.

103 Measures along the impact axis are derived from video records while measures from cable extensome-  
104 ters are considered along the distant axis. Due to the absence of this later data for the 520 kJ impact  
105 test at point *C*, the evolution obtained from simulations presented in [10] are considered. This exception  
106 is thought to have a minor influence due to the very small amplitude in displacement observed in this  
107 specific case. The dynamic response of the structure is clearly visible through the relative initiation time  
108 lag between observations corresponding to impact and distant axes. Besides, for the 520kJ test, nearly  
109 equal displacement magnitudes are observed at the end of the test for the points in the same axis, indi-  
110 rectly presenting nearly no-permanent post-impact tilt in the structure and small relative displacement  
111 between adjacent layer blocks. However, for the 1020kJ test, a significant difference is observed for both  
112 axes, inferring a combination of permanent tilt and relative displacement between the same axis blocks.

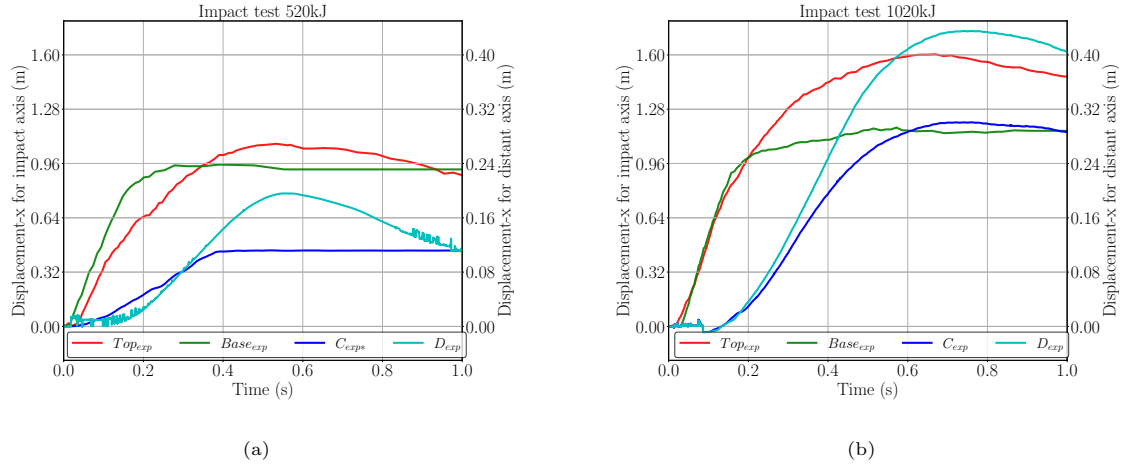


Figure 3: Post-processed experimental database for the time evolution of displacement for (a) 520 kJ and (b) 1020kJ impact test

113 The observed response at the point  $Top$  for the 1020kJ test is an exception due to the severe damage in  
 114 concrete blocks in the impacted vicinity, up to fracture. This resulted in the loss of mechanical continuity  
 115 within the structure, which was thought to cause a relatively larger block displacement at the wall top  
 116 than only due to the combination of a permanent tilt and relative block movements in the structure.

### 117 3. Numerical model

118 The rockfall protection structure is modelled in a python-based software package named SICONOS,  
 119 which implements the NSCD method with complex geometries [14]. In this section, the NSCD framework  
 120 is first introduced, followed by the numerical model development and finally the model response evaluation  
 121 aiming to reproduce the whole structural displacement response upon impact by a projectile.

#### 122 3.1. NSCD main principles and relevance

123 The NSCD method was developed to solve multi-body multi-contact problems with rigid and/or  
 124 deformable bodies [1, 2, 3] with rigid contact laws such as the Signorini's model of unilateral contact and  
 125 the Coulomb's dry friction without any kind of regularization (viscous friction) or compliance. It finds  
 126 many modelling applications in the field of civil engineering and geomechanics such as; masonry and  
 127 stone structures exposed to static, cyclic, and dynamic loading [21]; cohesive and non-cohesive granular  
 128 materials [22]; standard Finite Element Method (FEM) from the early stage of its development [23];  
 129 rockfall propagation on slopes [24] in particular and mechanisms and linkages [25].



130 In this study, the system is modelled as a collection of rigid blocks connected by unilateral constraints  
 131 with Coulomb friction. The finite-freedom dynamics of rigid bodies with unilateral constraints is known  
 132 to be non-smooth, in the sense that the velocities of the system possess jumps when a contact is closing  
 133 with a positive relative velocity. These velocity jumps are described through the introduction of an impact  
 134 law. Thereby, the NSCD method is able to perform the numerical time integration of the multi-body  
 135 system under impacts (of a projectile, in our case) in a dynamic condition. Some details and notation on  
 136 the NSCD method are given in [Appendix A](#) to support the following discussion.

137 Thus, with the NSCD method, robust and efficient simulations of the dynamics and the quasi-statics  
 138 of multi-body systems with contact, impact, and Coulomb friction are possible. For each body, the  
 139 Newton-Euler equations are discretized in time using an implicit scheme:

$$M(v_{k+1} - v_k) - hF_{k+\theta} = G^\top(q_{k+1})P_{k+1} \quad (1)$$

140 here,  $M$  is the inertia matrix,  $v$  is the body twist (translational and rotational velocity) vector,  $q$  is  
 141 representative of body position and orientation and  $G$  is the Jacobian of the gap function (see [Appendix](#)  
 142 [A.2](#) for details),  $h$  is the timestep.  $F_{k+\theta}$  refers to the external forces applied to the bodies (limited to  
 143 gravity forces, in this context) except contact interactions.  $P_{k+1}$  refers to all contact interactions that  
 144 are modelled as percussions.

145 The percussions  $P_{k+1}$  allow modelling impact involving Coulomb friction in the context of the study.  
 146 The percussion calculations are governed by the user-defined friction ( $\mu$ ) and restitution ( $e$ ) coefficients.  
 147 The tangential component of percussion calculations reflects the energy dissipation due to friction and  
 148 the normal component accounts for energy dissipation due to inelastic collision which, in other words,  
 149 represents the damage in the model body.

150 Notably, in this work, the restitution coefficient of magnitude less than one (*i.e.*, inelastic collision)  
 151 globally accounts for the energy dissipation due to plasticity and fracture in the structure. Nonetheless, a  
 152 complex representation of plasticity, damage, and fracture processes is possible in the NSCD framework.  
 153 For example, modelling blocks as deformable bodies using Finite Element based approaches involving  
 154 block plasticity models (*e.g.*, [26]) and cohesive zone models in-between the finite elements (*e.g.*, [27])  
 155 where fracture could occur. However, such a complex block model shall substantially increase the com-  
 156 putational duration and hence it is not adapted in the present work.

157 The non-impulse terms of the dynamics are approximated with a  $\theta$ -method. For  $\theta = 1$ , the scheme  
158 is fully implicit and maximizes the numerical dissipation. It is therefore well adapted to quasi-static  
159 evolutions, or for efficiently reaching a static equilibrium with large time-steps. For  $\theta = 1/2$ , the me-  
160 chanical energy (the sum of the kinetic and potential energy) is conserved. Furthermore, the dissipation  
161 is always positive and is given by a second order approximation of the actual dissipation. These energy  
162 and dissipation properties detailed in [15] render the scheme robust and stable with quite large time-  
163 steps, and are of particular interest for the analysis of the dissipative processes in protection structures.  
164 These are the main advantages with respect to classical smooth DEM approaches [28, 29] where the use  
165 of explicit scheme renders difficult the energy analysis and the contact compliance generates spurious  
166 oscillations that prevent to obtain quickly static equilibrium without artificial damping. With a non-  
167 smooth dynamics, direct higher order approximations are possible, but some care has to be taken. The  
168 use of standard higher scheme — Newmark, HHT, generalised- $\alpha$ , implicit Runge-Kutta to name a few  
169 — is not possible and we need to rely on specific schemes such as non-smooth Newmark or generalised- $\alpha$   
170 schemes [30, 31, 32], or time discontinuous Galerkin methods [33, 34].

### 171 3.2. NSCD structure model development in SICONOS

172 By contrast with common applications of NSCD, new challenges arise when modelling such articulated-  
173 concrete-blocks structures. These relate to the shape of the concrete blocks, the presence of structural  
174 components passing through these blocks, the interaction between these later components, and the mod-  
175 elling of the connectors.

176 In SICONOS, each object (*i.e.*, block and connector) of the wall is modelled as an assembly of many rigid  
177 body components with simple geometrical shapes (referred to as ‘primitive shapes’) for contact detection.  
178 These shapes present computational time efficiency and accuracy in contact detection in comparison to  
179 a typical numerical mesh. The interaction between different objects is user-controlled through so-called  
180 ‘contactors’, inferring that in its absence, two physical objects placed adjacently, would be invisible to  
181 each other. The model configuration and its components are shown in Figure 4. The modelling strategy  
182 that is developed to cope with these specific features is presented hereafter.

#### 183 3.2.1. Blocks modelling

184 The concrete blocks geometry feature from the experimental structure is presented in Figure 5, in-  
185 clusive of the effective contact area between two adjacent layer blocks in the wall. The block’s surface

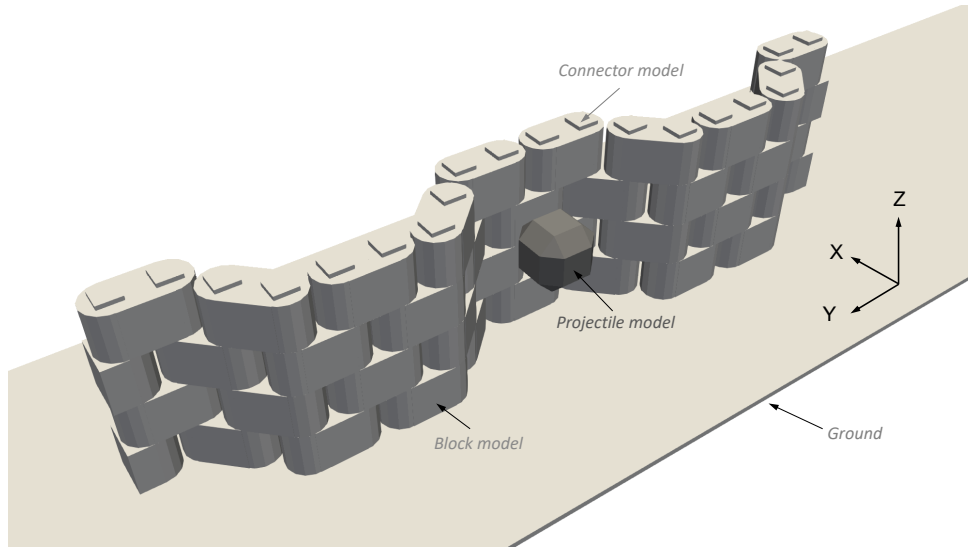


Figure 4: The NSCD model of the articulated concrete block structure and its components

186 comprising of the upper, lower and lateral side faces, is modelled as a combination of ‘primitive’ shapes.

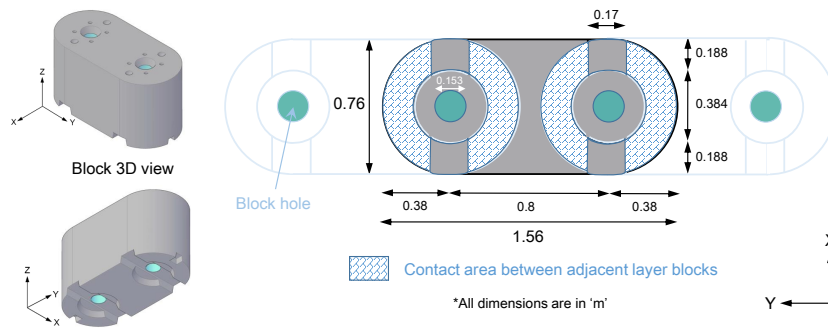


Figure 5: Experimental concrete block design and effective contact surfaces between adjacent layer blocks

187 Different combinations of primitive shapes are considered for geometry and contact area relevance  
 188 with the experimental block, and for numerical efficiency and computational accuracy in the NSCD  
 189 framework. This resulted in the best block model option, presented in Figure 6.

190 The user-controlled contactor definition feature enables to place cylinder and box superimposed to  
 191 collectively constitute the block model. Subsequently, the interaction between layered concrete blocks  
 192 in the wall is enabled through cylinder primitives and interaction between the projectile and the wall  
 193 through both cylinder and cuboidal box primitives, detailed later in the section 3.2.4.

194 Several contact points are set along the contact surface to model the interaction between adjacent  
 195 layer blocks. The location of the contact point is defined in order to maximize the contact surface between  
 196 the interacting bodies. Notably, as the friction and restitution coefficients control the interaction between  
 197 bodies, the difference in the physical contact area between adjacent layer numerical blocks being different

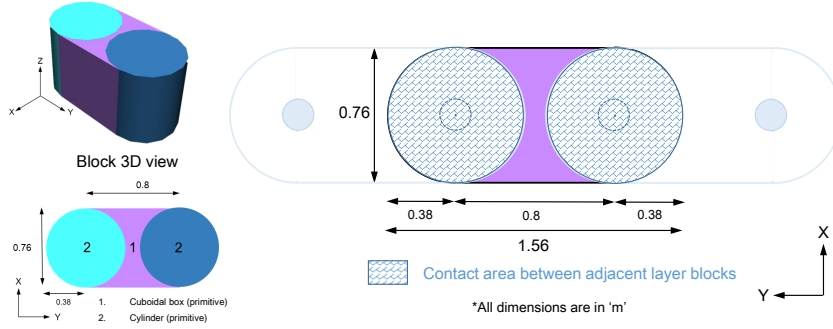


Figure 6: Block design in the NSCD model

198 from the experimental counterpart becomes irrelevant.

199 The block model is assigned with the same mass as of the real block and the inertia of the model  
 200 blocks is obtained through a 2D surface mesh using the Convex Hull [35] feature in SICONOS. Notably,  
 201 modelling the block cylindrical holes is ignored and a specific strategy is developed for accounting for the  
 202 interaction between the block and the connectors, detailed in the next section.

### 203 3.2.2. Connectors modelling

204 The connector between blocks (see Figure 1.iv), are modelled as a single component accounting for the  
 205 influence on the structure response of both the sling and tubes. This required defining a mechanical model  
 206 governing the connector-block interaction (which is insured by the tubes) and the interaction between  
 207 connectors (which is in particular controlled by the slings). The collective connector model illustrative  
 208 profile is presented in Figure 7 for before and after impact situations.

209 It consists of four cylindrical primitive shape components, named as ‘bars’, at locations corresponding  
 210 to the real-structure tube positions and possessing the same length (3 bars with 0.8m and 1 bar with  
 211 0.4m), diameter, inertia and mass as the tube. Notably, a fifth bar is added at the top block having  
 212 similar geometrical characteristics as the short-length bar, however with a negligible mass. This is done  
 213 to complete the configuration and the bounding limits of the sling in the real structure.

214 The mechanical plays existing in the structure, in both the horizontal and vertical directions, are  
 215 accounted for in the model. The horizontal play in the real structure results from the difference between  
 216 the external diameter of the tube (139.7mm) and the diameter of the hollow cylinder in the block (154mm).  
 217 The vertical play in the real structure results from the relative looseness in the sling. The detailed  
 218 connector modelling strategy to accommodate horizontal and vertical plays is presented hereafter.

219 The user control for assigning the contact detection enables, the connector bars passing through the

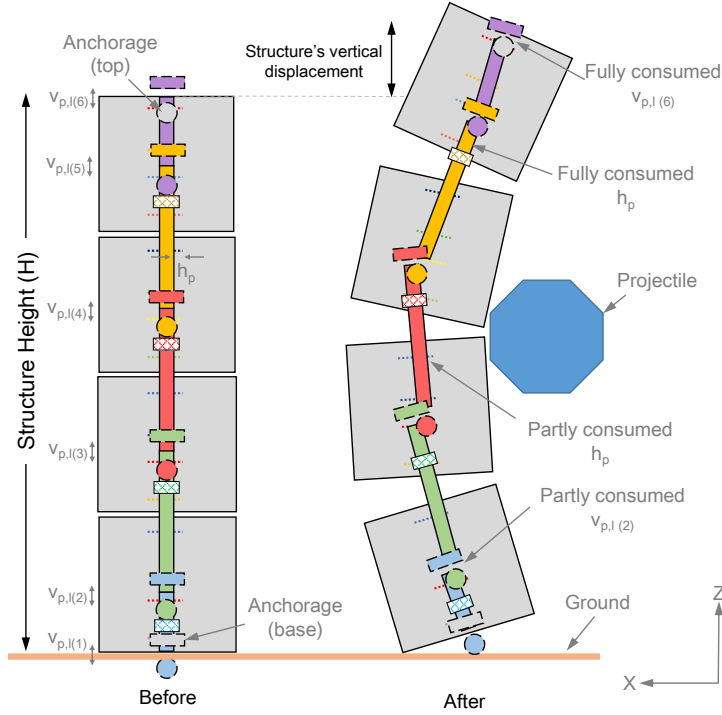


Figure 7: Schematic representation of the deformation of the structure upon projectile impact showing plays in action

220 concrete blocks without a direct interaction between them. However, in reality, the steel bars inserted  
 221 in cylindrical block holes, interact with the concrete blocks. Hence, this interaction is assigned through  
 222 additional components named as hollow circular disks added to the block model object, as illustrated in  
 223 Figure 8. These disks are modelled as 2D mesh and have the same internal diameter as that of the hole  
 224 in the real block, hence ensuring the presence of the requisite horizontal play of about 7mm. Each block  
 225 is equipped with four hollow disks meaning that each of the two bars passing through a block interacts  
 226 with this block via two disks. The position of these disks with respect to the lower/upper block faces  
 227 and to the extremities of the bar is characterized by parameter  $d_z$ , which is kept alike for simplicity. The  
 228 positioning of these disks can not be explicitly defined and hence it is characterized as a model parameter.

229 The vertical play is considered equally distributed along the structure's height as six local vertical  
 230 plays ( $v_{p,l} = v_p/6$ ) whose development is described as follows. Each bar object is equipped with three  
 231 contacting components to represent the influence of the sling, presented in Figure 9.

232 These components are comprised of a cuboid box of length equal to 2.5 times the bar diameter (placed  
 233 above the bar), a sphere of diameter equal to the bar diameter (placed below the bar) and another cuboid  
 234 box of length equal to 1.5 times the bar diameter (placed within the bar) named as 'bar-bar penetration  
 235 stopper'. The vertical position of these components is assigned such that the sphere from the adjacent

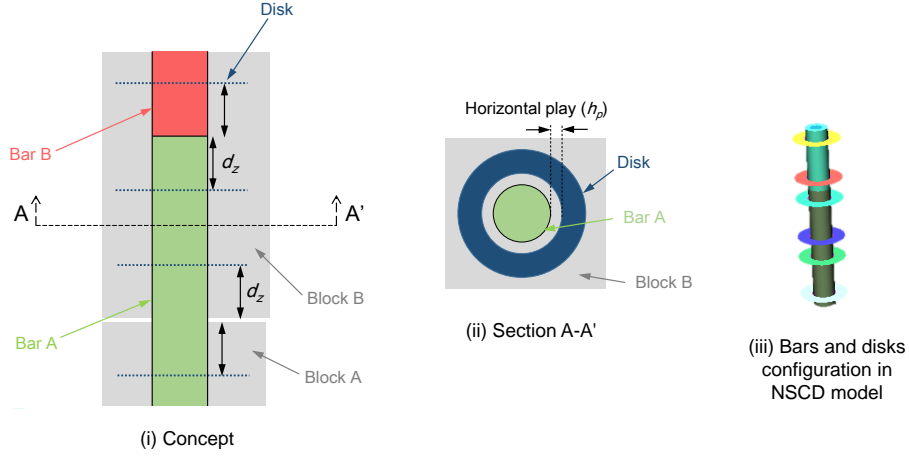


Figure 8: Concept of the connector modelling incorporating horizontal play

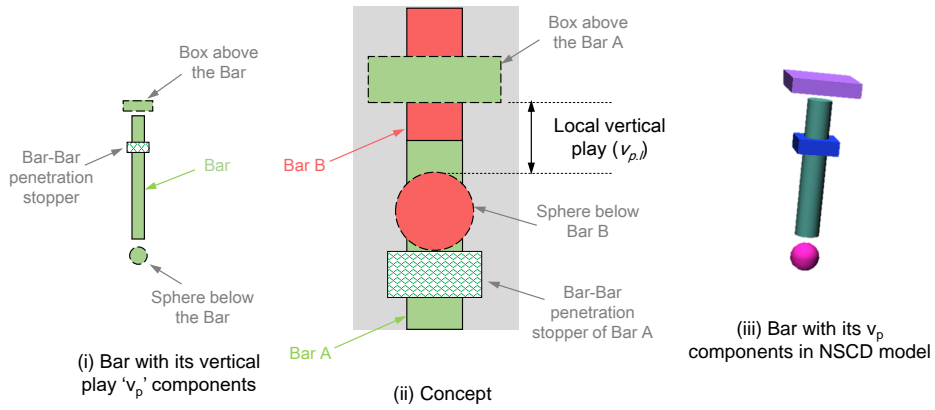


Figure 9: Concept of the connector modelling incorporating vertical play

236 bar detects contact with the boxes of the current bar. The distance between the box placed above the  
 237 bar and the bar surface, as well as the distance between the sphere and the bar surface, is half the local  
 238 vertical play. This way, the box-sphere combination from two adjacent bars collectively accounts for a  
 239 local vertical play. The adjacent bars are assigned invisible to each other and their interpenetration is  
 240 avoided via contact detection between this bar-bar penetration stopper and the sphere from the adjacent  
 241 bar.

242 In the NSCD model, all components of each model object has no relative movement between them  
 243 meaning that the circular disks in the block model object move according to the corresponding block.  
 244 Similarly, all three components of the bar object move according to the bar movement. This feature  
 245 enables to indirectly implement the block-bar interaction and sling functionality representative of the  
 246 real structure. An illustrative representation of the connector model (inclusive of the plays) in action

247 is added in Figure 7 for understanding. Here, the interaction of the vertical play constituents (box and  
248 sphere) can be clearly seen. The distribution of vertical play (as six local vertical plays) enables mimicking  
249 the looseness in the sling as an indirect chain made from multiple linkages. Once the complete vertical  
250 play is consumed, the setup of bars mimics the tension in the sling. Such a state is reached depending  
251 (mostly) on the assigned looseness in the vertical play and the energy of the impacting projectile. This  
252 makes the vertical play ( $v_p$ ) a model parameter.

253 This intricate setup allows us to model the large displacements in the model structure courtesy to  
254 the accompanying horizontal play ( $h_p$ ). Its presence controls the maximum relative lateral displacement  
255 between the box and sphere components of the vertical play. Thereby, a possible contact between the  
256 vertical play components from adjacent bars is always ensured, which makes the connector model's  
257 functionality robust. An additional box and a sphere are added at the bottom and topmost layer of block  
258 objects respectively. This is to respectively mimic the anchorage of the sling with the structure's bottom  
259 and to make sure the bars do not fly out of the structure's top surface upon impact.

### 260 3.2.3. Projectile modelling

261 The experimental equivalent projectile is modelled as a convex 2D mesh of manually assigned vertices.  
262 The projectile shape and size ( $1/3^{rd}$  of the wall height) features are in accordance with the requirements  
263 for flexible barriers testing [20]). The mass of the projectile is assigned 2600 kg, and the inertia is assigned  
264 through the convex hull (as used for the block model). The input kinetic energy corresponding to the  
265 two impact tests (*i.e.*, 520kJ and 1020kJ) is derived to assign the input velocity to the projectile model.

### 266 3.2.4. Interaction laws

267 The model objects (*i.e.*, blocks, connectors, projectile, and ground) are defined as a combination of  
268 different contactors, collectively making the model equivalent to the real structure. These contactors are  
269 defined with a uniquely assigned collision group to identify them and differentiate one from another. The  
270 collision groups of the contactors in the model bodies are listed in Table 2.

271 The interaction between model bodies is assigned through Newton impact friction non-smooth law [3].  
272 This law governs the interaction between a pair of contactors (identified through their respective collision  
273 groups) via a user-defined coefficient of friction ( $\mu$ ) and coefficient of restitution ( $e$ ). The assigned  
274 interaction laws in our NSCD model are listed in Table 3. Here, the various interactions in the numerical  
275 model bodies are chronologically listed, analogous to the construction of the real structure.

Table 2: Model bodies constitution for interaction

Collision group	Contact	Collision group	Contact
0	Block (cuboid portion)	5	Hollow disk
1	Block (curvature portion)	6	Box (vertical play)
2	Bar (odd)	7	Sphere (vertical play)
3	Bar (even)	8	Projectile
4	Bar-Bar penetration stopper	9	Ground

The frictional interaction between the projectile and the blocks and that between blocks are governed by the concrete-concrete friction coefficient ( $\mu_{cc}$ ). Similarly, the interaction of each block of the wall with the ground is governed by the concrete-soil friction coefficient ( $\mu_{cs}$ ). The restitution coefficient ( $e$ ) is kept the same for both of these interaction types.

Besides, all interactions between steel material and blocks and ground are assigned a constant friction coefficient of a relatively low magnitude of 0.2 for a minimum energy dissipation through these frictional interactions in the overall model system. Also, the restitution coefficient with zero magnitude is assigned presuming all such contacts being perfectly inelastic (*i.e.*, no rebound after contact). Moreover, as the sling is a continuous element and bars do not interact with each other, the friction coefficient between the boxes and sphere of connector elements (vertical play and penetration stopper) is assigned zero. This is to indirectly represent consistent stretching in the sling as local vertical plays start to be fully consumed and subsequently tension in the sling in the event of complete consumption of the available vertical play. Also,  $e = 0$  in the bar-to-disk contact assures no relative rebound once the horizontal play is consumed and the bar comes in contact with the disk (analogues to the tube and concrete block hole interaction in the real structure.)

### 3.2.5. Simulation database

The structure model is impacted at the wall center height and in the middle of its footprint length. The model wall response is computed for a one-second duration with a 0.25 ms time step for which the simulation duration of about 20 minutes is reported. A demonstrative post-impact deformed structural



Table 3: Assigning the interaction between model bodies contactors

Construction	Interacting pair (Colliding groups)	Friction	Restitution
Foundation	Block (1) - Ground (9)	$\mu_{cs}$	$e$
	Odd bar (2) - Ground (9)	0.2	0
Stacking blocks	Block (1) - Block (1)	$\mu_{cc}$	$e$
	Odd bar (2) - Disk (5)	0.2	0
Inserting tubes	Even bar (3) - Disk (5)	0.2	0
	Box (6) - Sphere (7)	0	0
Inserting sling	Penetration stopper (4) - Sphere (7)	0	0
	Block (0,1) - Projectile (8)	$\mu_{cc}$	$e$

295 conformation is presented in Figure 10. Here, the initial (pre-impact) configuration is also presented  
 296 (similar to Figure 4) where transparent blocks reveal the underlying connector model characteristics.

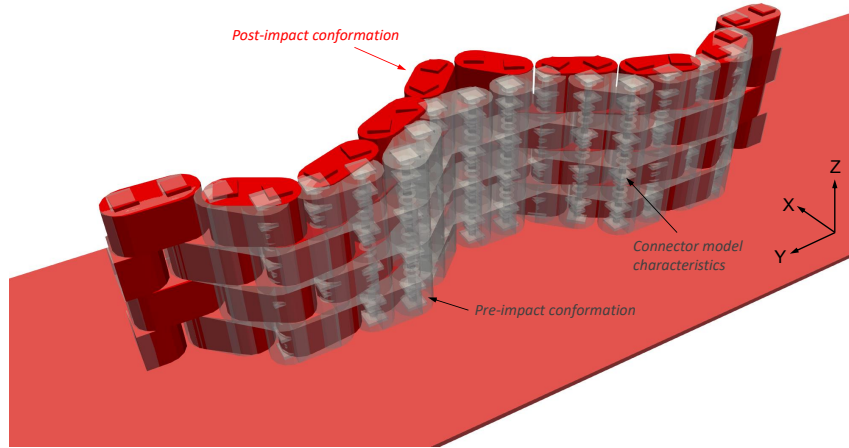


Figure 10: Qualitative representation of initial and post-impact model conformations and highlighted connector model characteristics

297 The output files of the model impact tests provided detailed information on the displacement, rotation,  
 298 and velocities of each block and thereby enabled direct comparison with the experimentally obtained  
 299 results at the corresponding locations. Besides, a detailed output database recorded for the contact  
 300 forces also enabled the additional post-processing of the model structure behavior extending the limits

301 of the experimentally acquired database to a computationally advanced level.

### 302 3.3. Evaluation of the NSCD model response

303 The presented NSCD model comprises of many input parameters, most of which are directly taken  
 304 from the real structure description (Table 1). By contrast, five model parameters couldn't be given a  
 305 precise value from this description. The first two take into account the imposed constraints for the block-  
 306 connector ( $d_z$ ) and connector-connector ( $v_p$ ) interaction while the other three concern the interaction  
 307 laws between contacting bodies ( $\mu_{cc}$ ,  $\mu_{cs}$  and  $e$ ).

308 The relative disk position ( $d_z$ ) could not be precisely derived from the block and tube geometry. The  
 309 vertical play ( $v_p$ ) magnitude is variable from place to place in the structure and it is difficult to measure  
 310 precisely. The coefficients governing friction between blocks ( $\mu_{cc}$ ) and that between the blocks and the  
 311 soil ( $\mu_{cs}$ ) are difficult to measure precisely as dealing with the dynamic loading of an articulated structure  
 312 where relative movement between bodies include translation and rotation. Last, the restitution coefficient  
 313 ( $e$ ) is a model parameter whose precise value couldn't be derived from some mechanical or geometrical  
 314 features of the real structure.

Table 4: Model parameters with their considered range and mean value

Parameter	Possible range	Mean value	Unit
Bar-block contactor disk position ( $d_z$ )	5 - 10	7.5	cm
Vertical play ( $v_p$ )	1 - 10	5.5	cm
Friction coefficient concrete-concrete ( $\mu_{cc}$ )	0.25 - 0.55	0.4	(-)
Friction coefficient concrete-soil ( $\mu_{cs}$ )	0.3 - 0.6	0.45	(-)
Restitution coefficient ( $e$ )	0 - 0.3	0.15	(-)

315 These parameters are listed in Table 4 with their range of possible values. The range for the vertical  
 316 play is derived from observations of the real structure. The range for the disk position is determined as a  
 317 fraction of the block height. The concrete-concrete friction angle is considered with the applicable range  
 318 information retrieved from the literature [36]. The range for the friction angle between concrete and  
 319 soil is established assuming slightly higher values than between concrete and concrete. The restitution

320 coefficient magnitude is considered ranging from perfectly inelastic collision (*i.e.*,  $e = 0$ ) upto a magnitude  
 321 towards perfectly elastic collision. This upper limit is selected based on multiple simulations such that  
 322 the rebound effect does not cause excessive inter-blocks displacement.

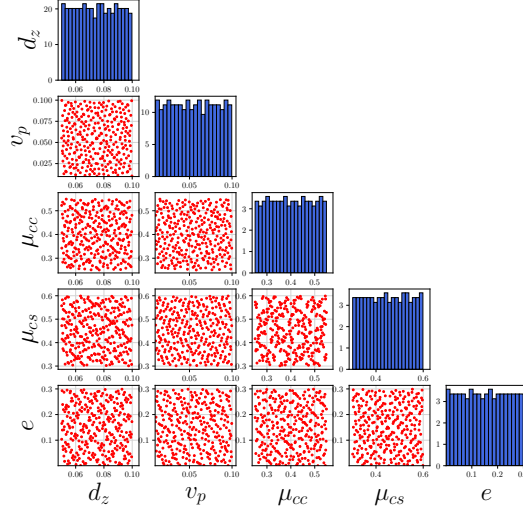
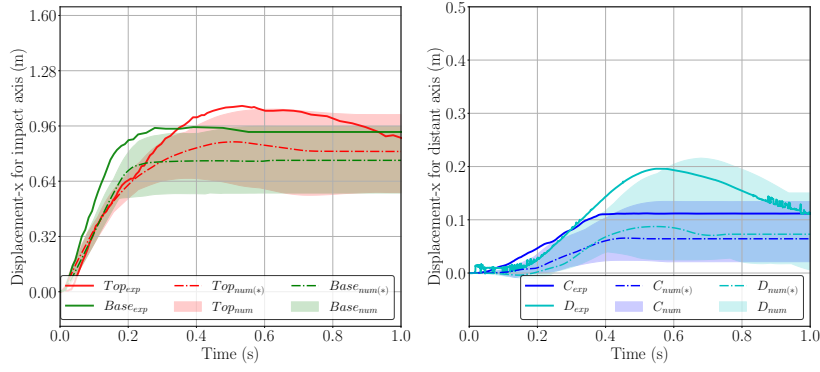


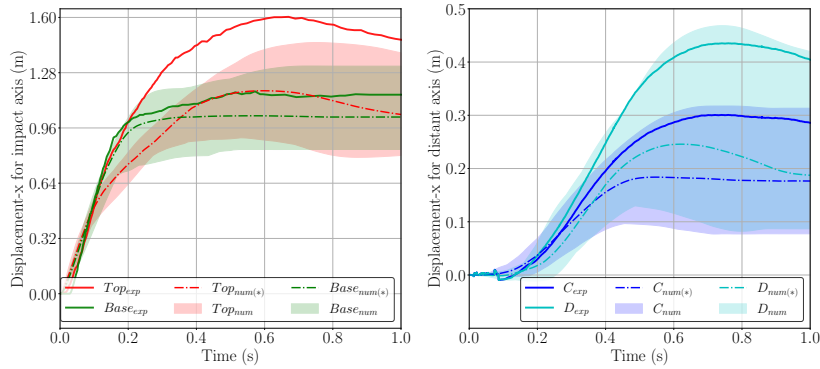
Figure 11: Input sample of size 300 (based on Sobol' sampling method) comprising of five parameters following uniform distribution in their respective ranges of variability

323 As a first step, the overall response of the NSCD model is assessed through a sample set of different  
 324 combinations of the five input parameters. This sample set is statistically generated using sampling  
 325 methods based on Monte-Carlo (MC) and quasi MC (such as Sobol', Halton) [37] implemented in the  
 326 UQlab input module [38]. The Sobol' sampling method is used to define a set of 300 combinations of  
 327 these five input parameters, as presented in Figure 11. This sampling method provides a random selection  
 328 of the input parameters within their predefined ranges such that the input space is well-filled. Besides,  
 329 without any prior bias to the magnitude of the five model parameters, uniform distributions are assigned  
 330 for all.

331 The global minimum and maximum range of outcomes from these 300 different simulations are pre-  
 332 sented in Figures 12a and 12b together with the NSCD model outcome from the mean value set of the  
 333 input parameters. Globally, the range of numerical outcomes envelops the experimental observations  
 334 at all measurement points. However, the range of model outcomes for the point *Top* remains below  
 335 the experimental observations for the 1020 kJ impact. The loss of mechanical continuity (as reported  
 336 in section 2) introduces a difference between the real structure and its numerical counterpart where no



(a)



(b)

Figure 12: Comparison of the experimental observations against the range of numerical computation response for displacement for (a) 520kJ and (b) 1020 kJ impact energy tests. The response obtained from the deterministic set of mean values (\*) of model parameters is also highlighted.

337 concrete block fracture is possible.

338 In spite of the difference at the point *Top* for 1020kJ test, the overall model-to-experiment correspon-  
 339 dence is good considering all the data describing the structure response. This suggests that the NSCD  
 340 model has the capacity to model the structure global response, assuming no block fracture. Also, the  
 341 envelope infers that there could exist at least one set of five parameters that allow the NSCD model to  
 342 possibly replicate the experimental response. This demands an extensive model calibration procedure  
 343 presented hereinafter.

#### 344 4. Calibration of the NSCD model

345 The aim of the calibration is to find the best set of the five model parameters (Table 4) such that  
 346 the simulated spatio-temporal structure impact response mimics that of the real structure considering

347 four observation points ( *i.e.*, points *Top*, *Base*, *C* and *D* as shown in Figure 2). The time evolution  
 348 of the displacement along the x-axis is accounted for considering three representative time instants: the  
 349 initial stage, the maximum, and the rest, referred to as *init*, *max* and *rest* respectively. The displace-  
 350 ment measured at these three-time instants is considered to present a global narrative of the structure's  
 351 displacement, and thus performance from the point of view of its practical application. Notably, the *init*  
 352 time instant, where the displacement evolves rather linearly, is aimed at reflecting the dynamics of the  
 353 structure, is arbitrarily fixed to 0.1 s after the impact beginning in the impact axis (*i.e.*, for points *Top*  
 354 and *Base*) and 0.25 s in the distant plane (*i.e.*, for points *C* and *D*).

#### 355 4.1. Calibration strategy

356 The five model parameters are calibrated considering the displacement at the four measuring points,  
 357 at the three different times and for the two impact experiments. The calibration is performed using the  
 358 Bayesian interface statistical learning procedure on the meta-model of the articulated model structure,  
 359 following the conceptual pathway presented in Figure 13.

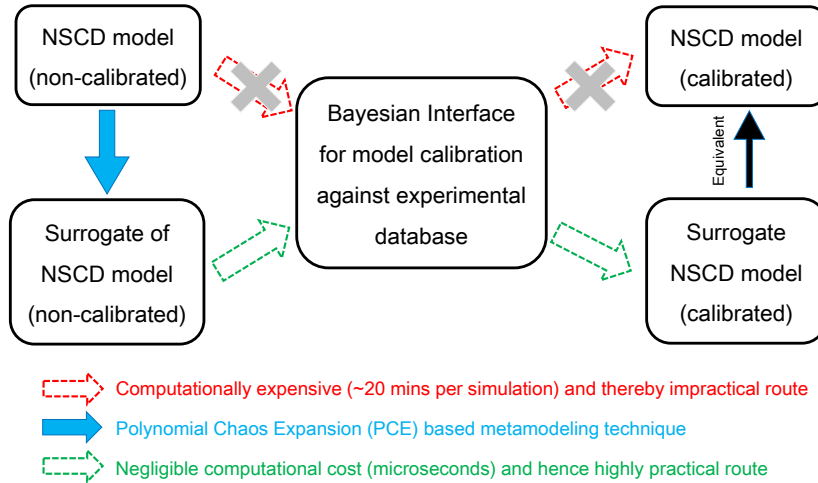


Figure 13: Conceptual workflow of the NSCD model calibration using a combination of stochastic methods

360 This complex pathway is motivated by the fact that reaching this ambitious calibration objective  
 361 demands a very large number of model computations. This is practically impossible with each model  
 362 computation taking about 20 minutes. This limitation is averted using surrogate of NSCD model, rep-  
 363 resentative of the structure response for different points at various time instances, through a collection  
 364 of meta-models. Here, we use polynomial chaos expansion (PCE) based meta-modelling technique for  
 365 surrogate model development, presented hereinafter.

#### 366 4.2. Meta-model creation and accuracy assessment

367 A meta-model is a mathematical modelling tool that bridges the gap between the current version of  
368 the numerical model and the requisite large number of computations for stochastic analysis. Here, the  
369 small statistically created input sample of the model parameters (presented in Figure 11) is used to define  
370 the mathematical expression incorporating the model behaviour with the desired accuracy.

371 A polynomial chaos expansion (PCE) based meta-modelling technique implemented in the framework  
372 of UQ[py]Lab (a python library of UQLab [39]) is used in this work (see Appendix B for details). A total  
373 of 24 meta-models are created from the displacement response at three time instances - for four different  
374 points - for two different energies (*i.e.*,  $3 \times 4 \times 2 = 24$ ). For each meta-model creation, a mathematical  
375 relation (as presented in Equation B.2), is established between the 300 distinct sets of input parameters  
376 and the corresponding displacement output set. The mathematical relation here can be analogically  
377 referred to as obtaining a regression for a 2D database.

378 The accuracy of a meta-model is accessed through the leave-one-out (LOO) error (Equation B.5)  
379 in the order of  $10^{-2}$  to  $10^{-1}$  for all the computational cases. Thereby, as the NSCD model is highly  
380 nonlinear and non-smooth, we accept the meta-model accuracy and the underlying uncertainty. Such a  
381 level of accuracy with the added advantage of a single model computation within micro-seconds is deemed  
382 sufficient for the upcoming Bayesian interface based calibration method.

383 Further, the accuracy of the meta-models is assessed by comparing the meta-models predictions with  
384 the NSCD model computations for the 520 kJ and 1020 kJ impact tests (Figures 14a and 14b). Notably,  
385 the set of 300 NSCD model simulations that was used to create the meta-model is reused as a validation.  
386 The choice of not using an independent set is made based on the order of magnitude of LOO and the  
387 relative difference between NSCD and meta-model outputs for the same set of input parameters.

388 The cloud of points is rather well aligned with the diagonals in these plots, indicating that the predicted  
389 response by the meta-models fits with the corresponding NSCD simulations results. The reliability of the  
390 predictions along the impact axis is relatively better than in the distant one for both impacts. The distant  
391 axis points move relatively less and present more divergence from the diagonal line which highlights the  
392 significance of the variability in model dynamics farther from the impact axis as the model interaction  
393 parameters vary.

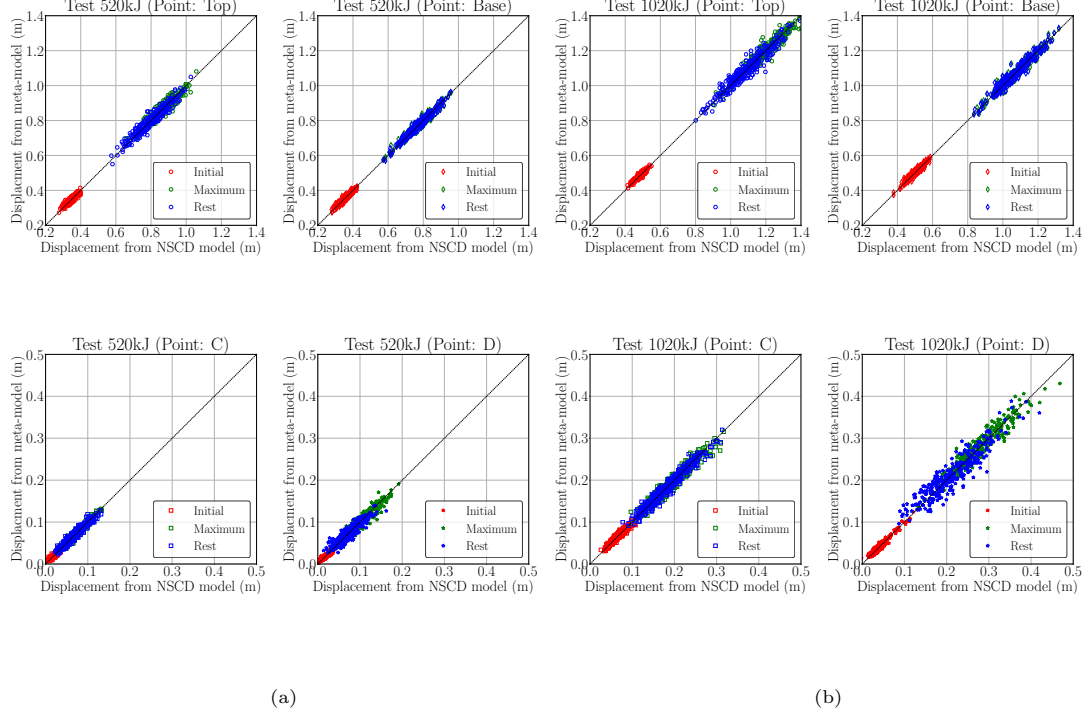


Figure 14: Comparison of displacements evaluated at four points (*Top*, *Base*, *C* and *D*) each at three time instances (initial, maximum and rest) from 300 NSCD model runs, and the corresponding evaluation by the PCE based meta-models with LOO error of order  $10^{-2}$  to  $10^{-1}$  for (a) 520kJ and (b) 1020 kJ impact tests

### 394 4.3. Relative influence of the model parameters

395 Before proceeding with the calibration process, the influence of each parameter on the spatio-temporal  
 396 displacement response of the structure is investigated through the Sobol sensitivity method also known  
 397 as analysis of variance [40]. This method decomposes the variance of the output parameters as the sum  
 398 of the contributions of the different input parameters including the possible interaction between input  
 399 parameters. Each contribution is characterized by the ratios of the partial variance to the total variance,  
 400 called Sobol sensitivity indices.

401 The accurate Sobol indices computation demands a large number of model computations (of order  
 402  $10^6$ ). This is highly impractical in the present study if the NSCD model is to be used directly. Nonetheless,  
 403 the meta-model of the NSCD model can be directly used to compute Sobol indices at zero cost [41, 42].  
 404 The UQlab sensitivity analysis module [43] is used for such computations. The total Sobol indices for  
 405 the three time instances, at all considered points and for both impact tests are presented in Figure 15.

406 The Sobol indices are observed to significantly differ from one parameter to the other. The higher  
 407 values are observed for the vertical play ( $v_p$ ) and concrete-to-soil friction coefficient ( $\mu_{cs}$ ) *e.g.*, at points

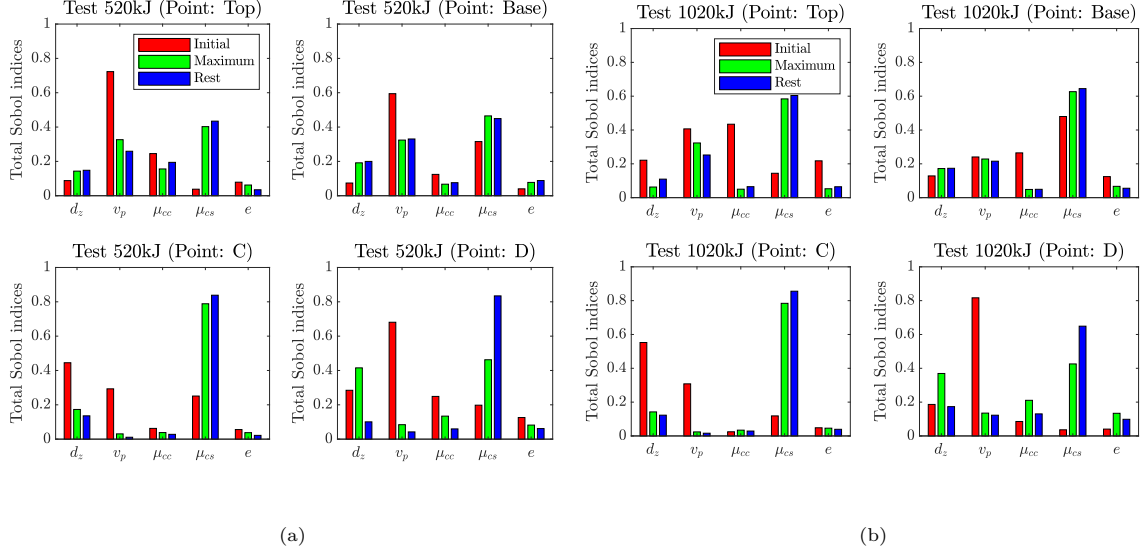


Figure 15: Total Sobol indices of the spatio-temporal displacement response for the sensitivity analysis of the NSCD model parameters for (a) 520kJ and (b) 1020kJ impact test

408 D and C respectively. Even though other parameters globally exhibit lower Sobol indices, their overall  
 409 influence cannot be neglected. Besides, we observe that the Sobol index for a given parameter varies over  
 410 time and is different from one impact energy to the other, *e.g.*, ‘ $v_p$ ’ ranges from 0.01 to 0.7, and ‘ $e$ ’ ranges  
 411 from 0.04 to 0.22.

412 Overall, these results aid in understanding the structure response highlighting the mechanisms at work  
 413 with time, complementing the insights to the experimental data discussed in Section 2. The influence of  
 414 the vertical play ( $v_p$ ) on the displacement of the upper blocks during both impact tests decreased with  
 415 time. This highlighted the significant upward displacement (*i.e.*, positive z-axis) of the blocks initiated  
 416 in the early stage after the impact happened. By contrast, the influence of the concrete-to-soil friction  
 417 coefficient ( $\mu_{cs}$ ) showed a global increasing trend with time, which is attributed to the progressive base  
 418 sliding of the wall. Furthermore, the Sobol index of the concrete-to-concrete friction coefficient,  $\mu_{cc}$   
 419 suggested that the relative displacement between blocks is much higher in the upper part of the structure  
 420 (points *Top* and *D*).

421 Besides, the sum of indices for a given situation exceeds one, expressing the interaction between input  
 422 parameters. In other words, changing the value of a given parameter modifies the structure dynamic  
 423 response which consequently alters the influence of other parameters. Overall, non-zero magnitude of all  
 424 parameters affirms to consider their variability for calibrating the structure response. It also validates  
 425 the identifiability of all parameters [44] for the Bayesian interface based studies presented hereinafter.



#### 426 4.4. Bayesian interface for model calibration

427 The Bayesian interface is inspired by the Bayes' theorem [45] - a representation of the changing  
428 beliefs - simply demonstrating that probability of a 'hypothesis' being correct becomes more reliable with  
429 supporting 'evidence'. In our work, the 'hypothesis' states that the 300 distinct sets of input parameters  
430 give a correct range of displacement output. The idea of reliability increase in on our hypothesis, given  
431 the experimental database as 'evidence', means that there exist at least one set of input parameters such  
432 that the recorded evidence is reproduced, in other words, the model is calibrated. Hence, our goal is to  
433 find the optimal values of the input parameters that allow one to best fit the model predictions to the  
434 observations. A brief description of the Bayesian interface is presented in [Appendix C](#).

##### 435 4.4.1. Representation

436 The available displacement experimental data at the points of interest and at three time instances  
437 are taken as a benchmark to calibrate the NSCD model parameters to obtain the requisite set of param-  
438 eters. Here, we use the PCE-based meta-models of NSCD model as the forward model or '*prior*' (see  
439 Equation C.2) to accelerate Bayesian computations, an approach extensively used by many researchers  
440 [46, 47, 48]. The meta-model based calibrated set of input parameters *i.e.*, '*posterior*' (see Equation C.3)  
441 is the representative of the NSCD model calibration.

442 In the UQlab Bayesian interface framework, the uncertainty in the model prediction is assigned via  
443 added Gaussian discrepancy (see Equation C.5) when correlated with the recorded experimental data. In  
444 the present work, as the model calibration relies on a single corresponding experimental measurement,  
445 deemed precise, the discrepancy with a known residual variance of order  $10^{-20}$  is manually assigned.

##### 446 4.4.2. Implementation

447 The Bayesian interface module in UQLab [49] is implemented to obtain a set of input parameters  
448 considering three approaches referred to as 'point', 'energy', and 'all'. These three approaches are inspired  
449 by the scope of the user's interest to reproduce the experimental outcome which ranges from a particular  
450 time instance calibration to overall model behavior calibration in both space and time.

451 The first approach amounts to giving priority to a given point, meaning that the model is calibrated to  
452 provide precise predictions over time for this specific point courtesy of the multiple model output feature  
453 (demonstrated in Equation C.8). The second approach, 'energy' is in line with strategies consisting

454 in calibrating a model at a given impact energy before using it for another impact energy. The ‘all’  
 455 approach aims at ensuring that the model provides reliable predictions at any point and whatever the  
 456 impact energy. The second and third approaches are implemented courtesy of the multiple forward model  
 457 feature (demonstrated in Equation C.10).

Table 5: Calibrated sets of model parameters following three different approaches

Parameter	Approaches											Units
	Point								Energy		All	
	520kJ				1020kJ				520kJ	1020kJ		
	Top	Base	C	D	Top	Base	C	D				
$d_z$	5.7	9.6	5.3	5.4	9.5	6.2	6.8	5.0	7.2	6.8	6.8	cm
$v_p$	5.2	3.8	6.7	6.5	9.0	8.0	7.6	7.0	6.5	7.1	7.1	cm
$\mu_{cc}$	0.251	0.419	0.474	0.265	0.378	0.512	0.510	0.342	0.351	0.323	0.316	(-)
$\mu_{cs}$	0.311	0.304	0.340	0.346	0.306	0.351	0.301	0.301	0.320	0.308	0.307	(-)
$e$	0.190	0.194	0.138	0.033	0.152	0.290	0.283	0.200	0.198	0.220	0.222	(-)

458 The calibrated sets of input parameters are reported in Table 5 obtained as a ‘*maximum a posteriori*’  
 459 and ‘*mean*’ point estimates (see Equation C.15) for ‘point’ approach and for both other approaches  
 460 respectively. Depending on the calibration approach, significant differences in parameter values are  
 461 observed. For example,  $v_p$  varied from 3.8 to 9 cm depending on the calibration strategy. A ratio of  
 462 about two between the minimum and the maximum values attributed to a parameter is also observed  
 463 for  $d_z$  and  $e$  in particular. Interestingly, the set of values calibrated following the ‘all’ approach is very  
 464 close to that obtained following the ‘energy’ approach while considering the 1020kJ impact. Notably,  
 465 the results obtained from the basic single-time instance approach are not presented in this paper due to  
 466 limited interest.

#### 467 4.4.3. Interpretation

468 The displacement response comparison between experimental results and NSCD model results ob-  
 469 tained considering the magnitude of these input parameters is presented in Figure 16. The interpretation  
 470 of these observations is presented hereinafter.

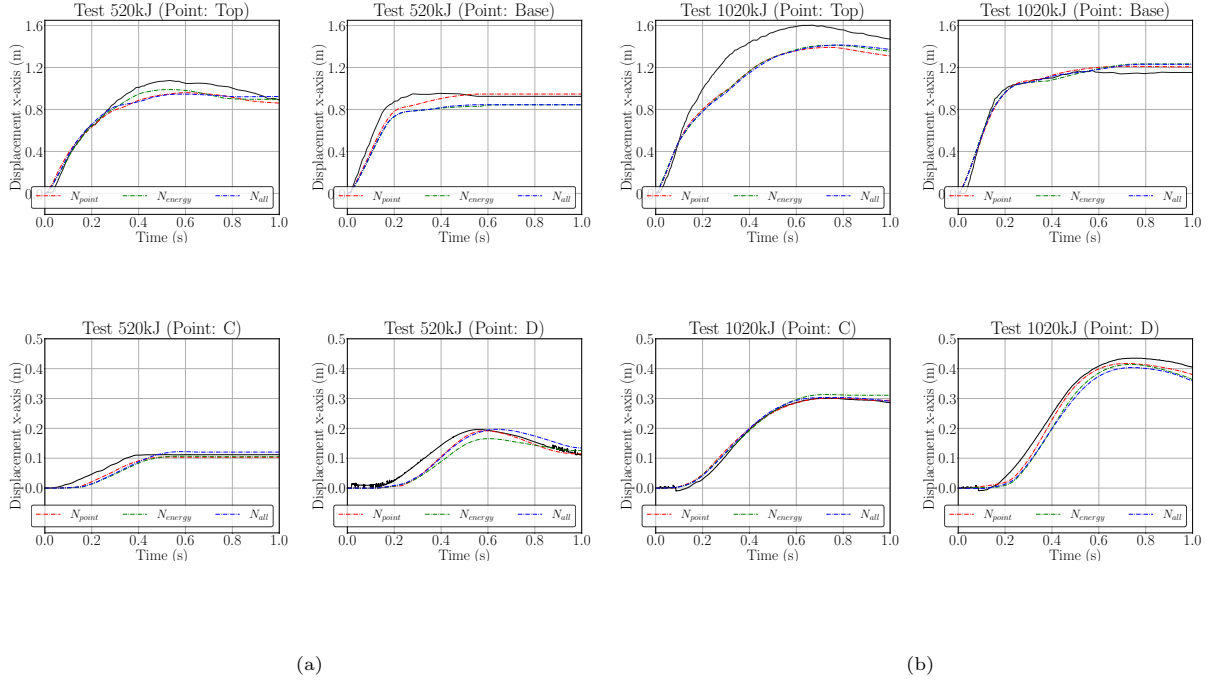


Figure 16: Bayesian inversion based calibration of (a) 520kJ and (b) 1020kJ impact test. Curves in *black* show the experimental data. Curves in *red*, *green* and *blue* show the NSCD simulation results corresponding to approaches ‘point’, ‘energy’ and ‘all’, respectively.

471 The first approach (*i.e.*, ‘point’ approach) results in a total of four calibrated sets of input parameters  
 472 for each impact test. This approach allows us to represent the influence of model dynamics at different  
 473 points as the best set of input parameters is reported distinct for each point and for both energies. The  
 474 experimental and numerical response, respectively in *black* and *red* in Figure 16, are in a very good  
 475 quantitative and qualitative agreement with each other justifying the usage of stochastic methods for  
 476 model calibration. This very good agreement emphasizes the success of choosing only three representative  
 477 time instances over the whole impact duration. The significant difference observed at the point *Top* for  
 478 the 1020kJ test is attributed to the loss of mechanical continuity as discussed in section 2. The obtained  
 479 eight distinct sets of parameters (Table 5) highlight the significance of local dependency of the model  
 480 constitution to accurately replicate the experimental response.

481 The second approach (*i.e.*, ‘energy’) collectively takes into account the model outputs at three time  
 482 instances for all four measurement points for a given impact test. This way, two calibrated sets of input  
 483 parameters are obtained from the Bayesian interface one each for the 520kJ and 1020kJ impact test. This  
 484 approach presents the influence of the impact energy on the global model response. The corresponding  
 485 response obtained from the NSCD model run (*in green*) is compared against the experimental observation.

486 Similar to the ‘point’ approach, a good qualitative agreement is observed between the experimental and  
487 numerical responses. Interestingly, the recorded ‘energy’ response using a single set of input parameters  
488 presented a similar response to the four distinct sets of these parameters in the ‘point’ approach for each  
489 energy level, highlighting the successful implementation of the multiple forward model feature of UQLab.

490 The third approach (*i.e.*, ‘all’) collectively takes into account the model outputs at three time instances  
491 for all four measurement points for both impact tests. Hence, a single set of calibrated input parameters  
492 is obtained representing the global response of the NSCD model, with time and space. Here again, the  
493 comparison between the corresponding response obtained from the NSCD model (*in blue*) is in good  
494 qualitative and quantitative agreement with the experimental observation.

#### 495 4.4.4. Application

496 Finally, the counterpart of the input sample distribution before the calibration (see Figure 11), the  
497 posterior input sample distribution obtained from the ‘all’ approach is presented in Figure 17. The data  
498 point corresponding to the calibrated set is also highlighted. Notably, from the histogram of the outputs,  
499 all input parameters present a locally emphasised magnitudes corresponding to the good fits with the  
500 desired output. However, the final set of parameters deviates slightly from the most probable magnitude  
501 of a particular input parameter. This difference is negligible for all parameters except  $\mu_{cc}$  where the  
502 final selected parameter magnitude is slightly larger than the most probable one. Besides, a very narrow  
503 range of  $\mu_{cs}$  parameter indirectly highlights its relatively high sensitivity to the model response (as also  
504 highlighted in Figure 15).

505 Further, as the counterpart of the displacement response before the calibration process (see Figure 12),  
506 the post-calibration response (from the ‘all’ approach) is presented in Figure 18. Overall, a good quali-  
507 tative agreement between experimental and numerical observations is reported favouring the calibrated  
508 set of input parameters. From the quantitative comparison of the collective experimental and numerical  
509 (posterior mean) displacement measurements at three time representatives, a root mean square error  
510 (RMSE) of 9.9cm and 2.3cm is reported for impact and distant axis respectively which account for about  
511 8.6% and 5.6% of representative displacements from the respective axis. A relatively better accuracy  
512 to reproduce the distant axis response favours the model’s practical utility. Continuing with the loss  
513 of mechanical continuity argument for the point *Top* during the 1020-kJ test (refer to section 2), the  
514 reported relatively large quantitative and qualitative difference is accepted and also is not considered in

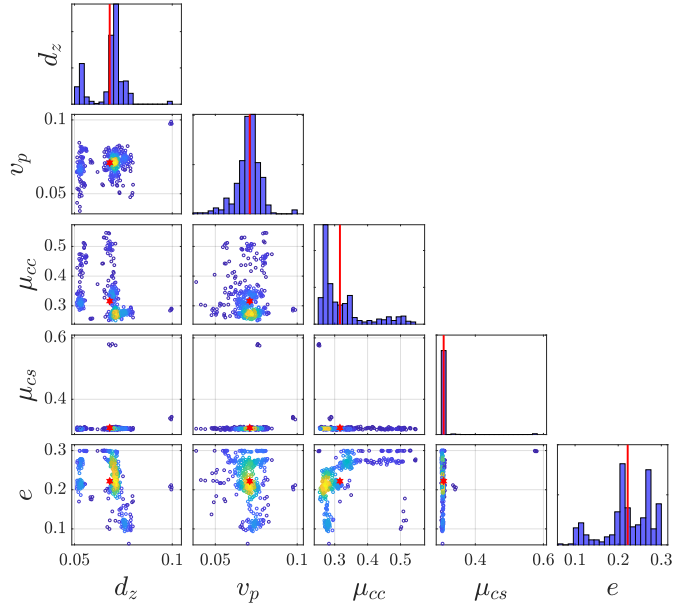


Figure 17: Output sample obtained after Bayesian calibration corresponding to ‘all’ approach. Here, diagonal plots present histograms of the five model parameters and other plots present their distribution, where colours reflect the density of points. The red  $\star$  and line present the calibrated value of the respective model parameter.

515 RMSE estimation.

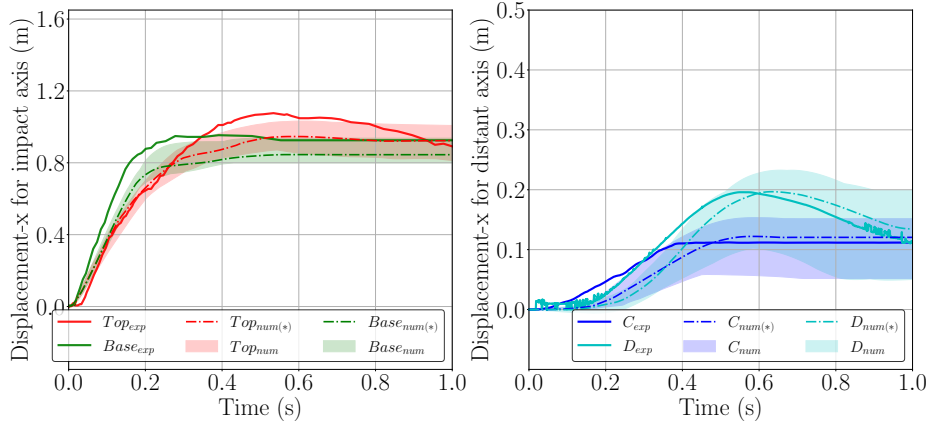
516 Moreover, the range of possible model outputs is also presented using all sets of input parameters  
 517 from the posterior distribution. The relatively narrow width of the band in comparison to the prior  
 518 sample validates the computational efficiency of the Bayesian interface. Besides, in co-relation to the  
 519 input sample distribution (see Figure 18), most of the output trends in the band are likely to be closer to  
 520 the posterior-mean response. Nonetheless, such co-relation is not made as the eventual goal is to obtain  
 521 a deterministic set of input parameters for the further utility of the NSCD model.

522 Globally, the NSCD model is well-calibrated as a positive consequence of the Bayesian interface  
 523 approach where a large number of model computations are favoured by the usage of the PCE-based  
 524 meta-models. As an extension, for better and more reliable model calibration, the diversity in the  
 525 experimental data both in time and space is favourable.

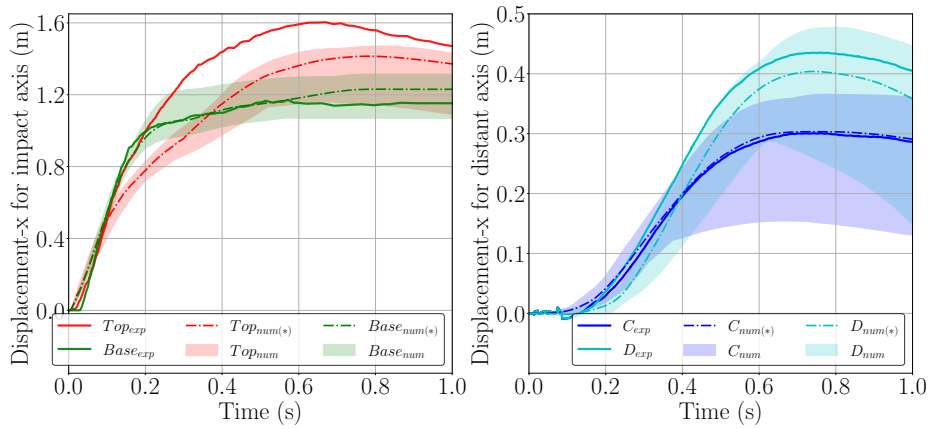
## 526 5. Discussion

### 527 5.1. Benefits of the calibration approach

528 This research proposed a complex approach making use of Bayesian interface accelerated with PCE-  
 529 based meta-models for calibrating the developed NSCD model.



(a) posterior for 520kJ



(b) posterior for 1020kJ

Figure 18: Comparison of the experimental displacement response against the NSCD model response obtained from the best calibrated set of input parameters along with the band of possible posterior sample based outcomes for (a) 520kJ and (b) 1020kJ impact tests respectively

530 The model calibration concerned five input parameters whose values could not be precisely measured  
 531 or determined a priori. The proposed stochastic analysis required an investment of 300 NSCD model runs  
 532 to retrieve the essential database. By comparison, a simple calibration method, for example based on  
 533 Monte Carlo simulations, would require a much larger number of simulations for obtaining a reliable set of  
 534 model parameters. The combination of the meta-model and Bayesian interface thus presented a practical  
 535 route to surpass the induced computational time constraint. On that subject, it is worth highlighting  
 536 that the average computational time for the simulation of one structure impact response is of about 20  
 537 minutes with the developed NSCD model. By comparison, a typical duration of 10 hours was necessary  
 538 with the finite difference model proposed by Furet et al. [10]. This highly significant difference further

539 highlights the practical implementation potential of the NSCD model given the output of interest.

540 The five model parameters were calibrated considering 24 outputs describing the structure displace-  
541 ment at four points and three-time instances and for two impact energies. The best set of model param-  
542 eters was determined minimizing the deviation with the experimental data for this set of 24 outputs. A  
543 simple calibration method would place an emphasis on a limited number of outputs (thus considering a  
544 smaller number of points and time instances) or would rely on a qualitative evaluation of the simulated  
545 response with time, without any quantification of the deviation. For these reasons, a simple calibration  
546 method would fail in satisfactorily describing the complex spatio-temporal response of the structure.

547 The proposed calibration approach thus appears to result in a much more reliable set of model  
548 parameters. It could advantageously be used for calibrating any model where many parameters cannot  
549 be precisely determined a priori.

## 550 *5.2. On the optimum calibration strategy*

551 The results presented in section 4.4 revealed that the calibrated value of a particular model parameter  
552 significantly depends on the chosen calibration strategy. Ratios of about two are observed between  
553 extreme values of a given model parameter from one strategy to the other. This in particular indicates  
554 that a calibration based on the recorded structural response at a single point only may result in a set of  
555 parameters that may not be reliable for estimating the structure response at another location. This is  
556 demonstrated in Figure 19, where the focus is placed on the model calibrated against the displacement  
557 of the point *Base* during the 520kJ energy impact test. It is clearly seen that, despite a very good  
558 qualitative and quantitative agreement of the predictions by the model with the experimental response  
559 at this specific point, the model fails to correctly reproduce the responses at nearly all remaining locations  
560 for the 520kJ-energy test and at all four locations for the 1020 kJ-energy test. This is in line with the  
561 basic idea that the intended use of the model should be taken into account when defining the model  
562 parameters calibration strategy.

563 By contrast, the ‘all’ calibration approach appeared to result in displacement predictions at a specific  
564 point as good as the prediction by the model calibrated for this specific point according to the ‘point’  
565 approach. This demonstrates that calibrating the model considering a large number of data is not at  
566 the expense of a reduction in prediction capacity as compared to models calibrated for a more specific  
567 purpose. In the end, the significance of using all spatio-temporal database for calibrating the model is

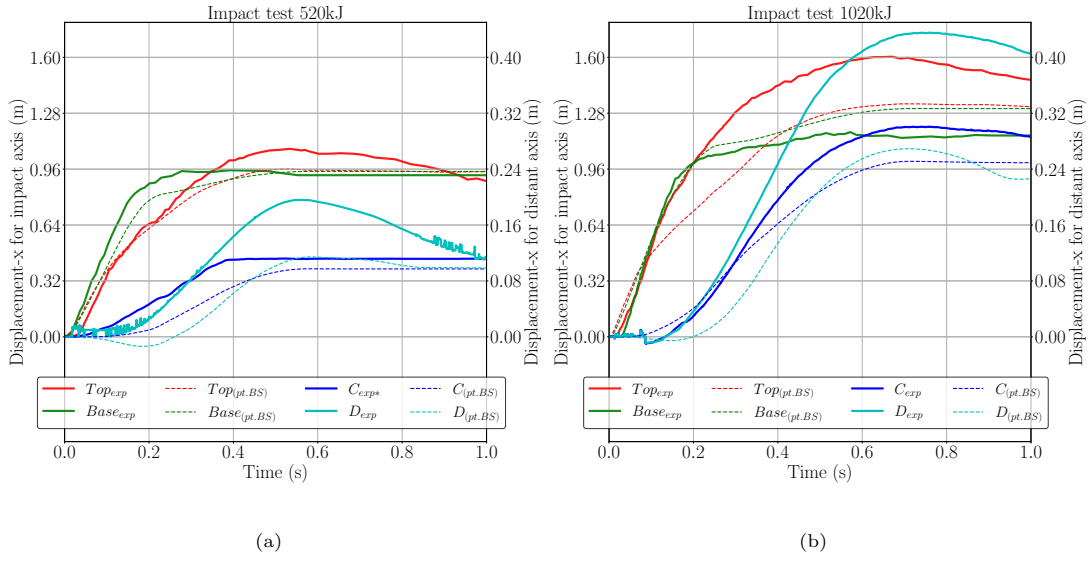


Figure 19: Comparison of experimental displacement response with the NSCD model response obtained from the point *Base* calibration set from 520kJ for (a) 520 kJ and (b) 1020kJ impact test

568 fully justified.

### 569 5.3. Energy dissipation

570 Estimating the energy dissipative capacities of the protective structures exposed to a localized dy-  
 571 namic loading is of paramount importance for understanding their mechanical response and improving  
 572 their design. Numerous research related to rockfall protection has considered the energy issue based on  
 573 simulation results, with application to flexible barriers [50, 51, 52], embankments and walls [5, 10] and  
 574 dissipative materials or components [53, 54]. The provided data mainly aimed at explaining how the  
 575 incident block kinetic energy is transferred and dissipated in the impacted structure and its foundation.

576 As for articulated concrete block structures, finite difference numerical simulations presented in [10]  
 577 suggested that 50% of the projectile kinetic energy is dissipated by concrete plasticization due to shear  
 578 and tension. The NSCD model proposed in the present study accounts for energy dissipation via  $e$ ,  $\mu_{cc}$   
 579 and  $\mu_{cs}$ . The former indirectly accounted for damage to concrete blocks, while the two latter accounted  
 580 for dissipation by friction between structure components.

581 Nevertheless, the significant difference in model parameters from one calibration strategy to the other,  
 582 including the parameters governing energy dissipation ( $e$ ,  $\mu_{cc}$  and  $\mu_{cs}$ ), suggests that energy dissipation  
 583 estimated from the corresponding simulations may lead to different conclusions, in particular in terms of  
 584 the respective contribution of each dissipative mechanism.



585 In the end, this highlights that simulation results interpreted in terms of energy dissipation should  
586 be considered with caution. The contribution by the various dissipative mechanisms (plasticization, fric-  
587 tion...) strongly depends on the modelling strategy, on the constitutive laws and on the value attributed  
588 to the model parameters. This comes in addition to the classical energy conservation issue associated with  
589 some numerical schemes. These general but critical comments undoubtedly hold for any numerical model  
590 of structures exposed to dynamic loading, such as rockfall protective structures (e.g. [5, 10, 50, 52]). Fur-  
591 ther investigation is necessary for evaluating the influence of the model parameters on the contribution  
592 of each mechanism to energy dissipation.

## 593 6. Conclusions and perspectives

594 In this work, a numerical model of a structure consisting of concrete blocks connected one to the others  
595 via metallic components and designed to intercept rockfall is presented and successfully calibrated. The  
596 model is kept simple and developed in SICONOS software package based on NSCD approach mainly for the  
597 computation time-saving purpose. NSCD revealed efficiency for modelling such a complex structure where  
598 many components interact one with each other and some are invisible to each other. Each component in  
599 the real structure is directly and indirectly reproduced in the NSCD model considering the interaction  
600 mechanisms and the design framework of objects with contactors in SICONOS. An average computation  
601 time of 20 mins is reported for the NSCD model in comparison to highly expensive FEM computations  
602 lasting for about 10 hours for the same computation.

603 Five model parameters required calibration. Three parameters governed the mechanical interaction  
604 between the various model components and two concerned the structure description. The calibration of  
605 these five model parameters is conducted based on the displacement with time at four different locations  
606 in the structure measured during real-scale experiments considering two projectile kinetic energies. The  
607 calibration is conducted considering the Bayesian interface statistical learning method, accompanied by  
608 the meta-modelling techniques. This is done in view of accounting for the spatial and temporal displace-  
609 ment response of the structure upon a projectile impact. The meta-modelling techniques presented a  
610 surrogate of the NSCD model which represented a similar response albeit negligible computation time  
611 (microseconds) in comparison to the approximated 20 minutes time for one NSCD model computation.  
612 Subsequently, a large number of model computations are made possible by meta-models for the efficient

613 usage of the Bayesian interface for model calibration. Besides, the Sobol sensitivity analysis is made pos-  
614 sible through the surrogate model which presented the relative influence of one parameter to the other  
615 both in space and time.

616 In the calibration process different calibration approaches, ranging from the local point to the global  
617 structure's response are presented. It is evidenced that the model parameters value significantly depended  
618 on the calibration strategy, meaning on the number and variety of data used in this purpose. The final  
619 set of parameters value is reported to nearly mimic the spatial-temporal response of the real structure for  
620 both impact tests. An overall quantitative deviation of numerical results with the experimental evolution  
621 is reported to be 8.6% for the impact axis and 5.6% for the distant axis.

622 As a perspective, the developed NSCD model will be used to investigate the response of articulated  
623 concrete blocks structures when exposed to impacts under different conditions, in view of quantifying  
624 their real efficiency, in a similar approach as in Mentani et al. [11], Toe et al. [12] and Lambert et al. [13].

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771 **Appendix A. NSCD method**

772 The constitutive framework of the NSCD method is described hereinafter.

773 *Appendix A.1. Newton-Euler equations*

774 The configuration of each rigid body in three dimensions is described by the position of its center  
 775 of mass  $x_{\mathbf{g}} \in \mathbb{R}^3$  and the orientation of the body-fixed frame with respect to a given inertial frame  
 776  $R \in SO^+(3)$ . The velocity of the body is given by the velocity of the center of mass  $v_{\mathbf{g}} = \dot{x}_{\mathbf{g}} \in \mathbb{R}^3$  and  
 777 the angular velocity of the body expressed in the body-fixed frame  $\Omega \in \mathbb{R}^3$ . A possible formulation of  
 778 the Newton-Euler equations of motion for each body is

$$\left\{ \begin{array}{l} m \dot{v}_{\mathbf{g}} = f(t, x_{\mathbf{g}}, v_{\mathbf{g}}, R, \Omega) \\ I \dot{\Omega} + \Omega \times I \Omega = M(t, x_{\mathbf{g}}, v_{\mathbf{g}}, R, \Omega) \\ \dot{x}_{\mathbf{g}} = v_{\mathbf{g}} \\ \dot{R} = R \tilde{\Omega} \end{array} \right. \quad (\text{A.1})$$

779 where  $m > 0$  is the mass,  $I \in \mathbb{R}^{3 \times 3}$  is the matrix of moments of inertia around the center of mass  
 780 and the axis of the body-fixed frame, and  $f(\cdot) \in \mathbb{R}^3$ , respectively  $M(\cdot) \in \mathbb{R}^3$ , are the total forces and  
 781 respectively torques with respect to the center of gravity applied to the body. The matrix  $\tilde{\Omega} \in \mathbb{R}^{3 \times 3}$  is  
 782 given by  $\tilde{\Omega}x = \Omega \times x$  for all  $x \in \mathbb{R}^3$ . In the implementation of SICONOS, the orientation matrix  $R$  is  
 783 parameterized by a unit quaternion  $p$  such that  $R = \Phi(p)$ . In the equations of motion (Equation A.1),  
 784 the occurrences of  $R$  are substituted by their corresponding expressions in terms of  $p$ . The differential  
 785 equation of Lie type  $\dot{R} = R \tilde{\Omega}$  is replaced by  $\dot{p} = \Psi(p)\Omega$  in the unit quaternion space. The closed formulae  
 786 for  $\Phi$  and  $\Psi$  can be found in any textbook on rigid body dynamics. We denote by  $q$  the vector of  
 787 coordinates of the position and the orientation of the body, and by  $v$  the body twist. In matrix notation,  
 788 the relation between the body twist  $v$  and the time derivative of  $q$  is

$$q := \begin{bmatrix} x_{\mathbf{g}} \\ p \end{bmatrix}, \quad v := \begin{bmatrix} v_{\mathbf{g}} \\ \Omega \end{bmatrix}, \quad \dot{q} = \begin{bmatrix} \dot{x}_{\mathbf{g}} \\ \Psi(p)\Omega \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0 & \Psi(p) \end{bmatrix} v := T(q)v \quad (\text{A.2})$$

789 with  $T(q) \in \mathbb{R}^{7 \times 6}$ . A compact matrix form of the Newton-Euler equations is

$$\left\{ \begin{array}{l} \dot{q} = T(q)v, \\ M\dot{v} = F(t, q, v), \end{array} \right. \quad (\text{A.3})$$



790 where  $M \in \mathbb{R}^{6 \times 6}$  is the total inertia matrix and  $F(t, q, v) \in \mathbb{R}^6$  collects all the forces and torques applied  
 791 to the body given by

$$M := \begin{pmatrix} mI_{3 \times 3} & 0 \\ 0 & I \end{pmatrix}, \quad F(t, q, v) := \begin{pmatrix} f(t, x_{\mathbf{g}}, v_{\mathbf{g}}, R, \Omega) \\ I\Omega \times \Omega + M(t, x_{\mathbf{g}}, v_{\mathbf{g}}, R, \Omega) \end{pmatrix}. \quad (\text{A.4})$$

792 In the sequel, we assume that the vectors  $q \in \mathbb{R}^{7n}$  and  $v \in \mathbb{R}^{6n}$  collect the configuration parameters  
 793 and the velocities of the  $n$  bodies in the model. The equation of motion (Equation A.3) is rearranged  
 794 accordingly.

### 795 *Appendix A.2. Contact modelling as unilateral constraints and Coulomb friction*

796 For two contacting bodies  $A$  and  $B$ , we assume that we are able to define one or more contact pair  
 797 composed of two candidate contact points  $C_A$  and  $C_B$  and a local frame at contact  $(C_A, \mathbf{N}, \mathbf{T}_1, \mathbf{T}_2)$  (see  
 798 Figure A.20 for details). In this local frame, the normal gap function  $g_N$  is defined as the signed distance  
 799 between the point  $C_A$  and  $C_B$ , that is  $g_N = (C_B - C_A) \cdot \mathbf{N}$ . The unilateral contact is given by Signorini's  
 800 condition as

$$0 \leq g_N \perp r_N \geq 0, \quad (\text{A.5})$$

801 where  $r_N$  is the local contact normal force. The symbol  $g_N \perp r_N$  means that  $g_N r_N = 0$ . If the contact  
 802 is open  $g_N > 0$ , the contact force  $r_N$  has to be equal to zero. If the contact is closed  $g_N = 0$ , then the  
 803 contact force is nonnegative  $r_N \geq 0$ . As we said before, the dynamics of finite dimensional system requires  
 804 the definition of an impact law. Let's consider the relative velocity  $u_N$  defined by  $u_N = \dot{g}_N$ . One of the  
 805 simplest impact is the Newton impact law given by

$$u_N^+ = -e u_N^-, \text{ if } g_N = 0 \text{ and } u_N^- \leq 0, \quad (\text{A.6})$$

806 where  $e \in [0, 1]$  is the coefficient of restitution,  $u_N^+$  is the post-impact velocity and  $u_N^-$  the pre-impact  
 807 velocity.

808 For the tangential behaviour of the interface, we consider the Coulomb friction that can be formulated  
 809 with the Signorini's condition at the velocity level and in terms of the complete reaction forces  $r = [r_N, r_T]^\top$

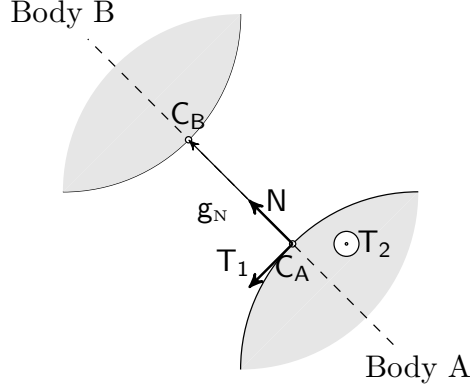


Figure A.20: A contact pair and a local contact frame

810 and the relative velocity  $u = [u_N, u_T]^\top$  as

$$\left\{ \begin{array}{ll} r = 0 & \text{if } g_N > 0 \quad (\text{no contact}) \\ r = 0, u_N \geq 0 & \text{if } g_N = 0 \quad (\text{take-off}) \\ r \in K, u = 0 & \text{if } g_N = 0 \quad (\text{sticking}) \\ r \in \partial K, u_N = 0, \|u_T\| r_T = \|r_T\| u_T & \text{if } g_N = 0 \quad (\text{sliding}) \end{array} \right. \quad (\text{A.7})$$

811 where  $K = \{r \in \mathbb{R}^3, \|r_T\| \leq \mu r_N\}$  is the usual Coulomb friction cone. Using the modified local relative  
 812 velocity introduced by De Saxcé and Feng [55, 56],  $\hat{u} := u + \mu \|u_T\| \mathbf{N}$  and the dual cone of  $K$ , i.e.,  
 813  $K^* = \{z \in \mathbb{R}^3 \mid z^T x \geq 0 \text{ for all } x \in K\}$ , the contact model is equivalent to

$$K^* \ni \hat{u} \perp r \in K. \quad (\text{A.8})$$

### 814 Appendix A.3. Equations of motion with contact and friction

815 The gap function  $g_N$  for a contact is generally a function of the configuration of the bodies  $q$ . Let us  
 816 consider that we have a set of  $m$  contact pairs given by the unilateral constraints:

$$g_N^\alpha(q) \geq 0, \alpha \in \mathcal{I} \subset \mathcal{N}, |\mathcal{I}| = m. \quad (\text{A.9})$$

817 For a contact  $\alpha \in \mathcal{I}$ , the relative normal velocity is related to  $v$  through the relation

$$u_N^\alpha = \dot{g}_N^\alpha(q) = J_{g_N^\alpha}^\alpha(q) \dot{q} = J_{g_N^\alpha}^\alpha(q) T(q) v := G_N^\alpha(q) v, \quad (\text{A.10})$$

818 where  $J_{g_N^\alpha}^\alpha(q)$  is the Jacobian of  $g_N^\alpha$  with respect to  $q$ . The same type of relation can be written for the  
 819 tangential relative velocity  $u_T$  as  $u_T = G_T^\alpha(q) v$  and we get for  $u^\alpha$

$$u^\alpha := G^\alpha(q) v. \quad (\text{A.11})$$

820 By duality, the total force generated by the contact  $\alpha$  is

$$G^{\alpha, \top}(q)r^\alpha := G_N^{\alpha, \top}(q)r_N^\alpha + G_T^{\alpha, \top}(q)r_T^\alpha. \quad (\text{A.12})$$

821 Altogether the equation of motion with contact and Coulomb friction is given by

$$\left\{ \begin{array}{l} \dot{q} = T(q)v, \\ M\dot{v} = F(t, q, v) + G^\top(q)r, \\ r^\alpha = 0, \quad \text{if } g_N^\alpha(q) > 0, \\ K^{\alpha, *} \ni \widehat{u}^\alpha \perp r^\alpha \in K^\alpha, \quad \text{if } g_N^\alpha(q) = 0, \\ u_N^{\alpha, +} = -e^\alpha u_N^{\alpha, -}, \quad \text{if } g_N^\alpha(q) = 0 \text{ and } u_N^{\alpha, -} \leq 0 \end{array} \right\} \alpha \in \mathcal{I},$$

822 *Appendix A.4. Time-discretization*

823 Let us consider a time discretization  $t_0 < \dots < t_{k-1} < t_k < \dots < T$  with a constant time step

824  $h = t_{k+1} - t_k$ . The Moreau–Jean scheme for the system is

$$\left\{ \begin{array}{l} q_{k+1} = q_k + hT(q_{k+\theta})v_{k+\theta} \\ M(v_{k+1} - v_k) - hF_{k+\theta} = G^\top(q_{k+1})P_{k+1}, \\ P_{k+1}^\alpha = 0, \\ K^{\alpha, *} \ni \widehat{u}_{k+1}^\alpha + e^\alpha u_{N,k}^\alpha \mathbf{N} \perp P_{k+1}^\alpha \in K^\alpha \end{array} \right\} \begin{array}{l} \alpha \notin \mathcal{I}_k \\ \alpha \in \mathcal{I}_k. \end{array} \quad (\text{A.13})$$

825 where the notation  $x_{k+\theta} = (1 - \theta)x_k + \theta x_{k+1}$  is used for  $\theta \in [0, 1]$  and the set  $\mathcal{I}_k$  is the set of contact

826 activated at the velocity level

$$\mathcal{I}_k = \{\alpha \in I \mid g_{N,k}^\alpha + \gamma u_{N,k}^\alpha \leq 0\} \text{ with } \gamma \in [0, \frac{1}{2}]. \quad (\text{A.14})$$

827 In the time-stepping method, the unknown  $P_{k+1}$  is an approximation of the impulses of the contact

828 reaction measure  $di$  over the time interval, that is

$$\int_{(t_k, t_{k+1}]} di \approx P_{k+1}. \quad (\text{A.15})$$

829 The contact reaction measure is related to the contact force when the motion is smooth enough by

830  $di = r(t)dt$ . The system (Equation A.13) is a second-order cone complementarity problem that is solved

831 by SICONOS using a Gauss-Seidel method with projection [56].

832 **Appendix B. PCE based meta-model**

833 Consider the articulated structure model represented by a  $\mathcal{M}(\mathbf{X})$  as an equivalent mathematical  
 834 model. Here,  $\mathbf{X} \in \mathbb{R}^M$  is a random vector with independent components described by the joint probability  
 835 density function (PDF)  $f_{\mathbf{X}}$ . Consider also a finite variance computational model as a map  $Y = \mathcal{M}(\mathbf{X})$ ,  
 836 with  $Y \in \mathbb{R}$  such that:

$$\mathbb{E} [Y^2] = \int_{\mathcal{D}_{\mathbf{X}}} \mathcal{M}(\mathbf{x})^2 f_{\mathbf{X}}(\mathbf{x}) d\mathbf{x} < \infty \quad (\text{B.1})$$

837 Then, under the assumption of Equation B.1, the PCE of  $\mathcal{M}(\mathbf{X})$  is defined as:

$$Y = \mathcal{M}(\mathbf{X}) = \sum_{\alpha \in \mathbb{N}^M} y_{\alpha} \Psi_{\alpha}(\mathbf{X}) \quad (\text{B.2})$$

838 where, the  $\Psi_{\alpha}(\mathbf{X})$  are multivariate polynomials orthonormal with respect to  $f_{\mathbf{X}}$ ,  $\alpha \in \mathbb{N}^M$  is a multi-  
 839 index that identifies the components of the multivariate polynomials  $\Psi_{\alpha}$  and the  $y_{\alpha} \in \mathbb{R}$  are the corre-  
 840 sponding coefficients. In practical applications, the sum in Equation B.2 needs to be truncated to a finite  
 841 sum by introducing the truncated polynomial chaos expansion:

$$\mathcal{M}(\mathbf{X}) \approx \mathcal{M}^{PC}(\mathbf{X}) = \sum_{\alpha \in \mathcal{A}} y_{\alpha} \Psi_{\alpha}(\mathbf{X}) \quad (\text{B.3})$$

842 where,  $\mathcal{A} \subset \mathbb{N}^M$  is the set of selected multi-indices of multivariate polynomials.

843 In this work, the least-angle regression (LARS) method is used to create the PCE meta-model trun-  
 844 cated to the maximum polynomial degree ( $p$ ) ranging from 1 to 20, and using hyperbolic truncation  
 845 scheme ( $q$ ) ranging from 0.5 to 1.

$$\mathcal{A}^{M,p,q} = \{\alpha \in \mathcal{A}^{M,p} : \|\alpha\|_q \leq p\}, \text{ where } \|\alpha\|_q = \left( \sum_{i=1}^M \alpha_i^q \right)^{1/q} \quad (\text{B.4})$$

846 The accuracy of the constructed PCE is estimated by computing the leave-one-out (LOO) cross-  
 847 validation error ( $\epsilon_{LOO}$ ). It consists in building  $N$  meta-models  $\mathcal{M}^{PC \setminus i}$ , each one created on a reduced  
 848 experimental design  $\mathfrak{X} \setminus \mathbf{x}^{(i)} = \{\mathbf{x}^{(j)}, j = 1, \dots, N, j \neq i\}$  and comparing its prediction on the excluded  
 849 point  $\mathbf{x}^{(i)}$  with the real value  $y^{(i)} = \mathcal{M}(\mathbf{x}^{(i)})$  [57]. The leave-one-cross-validation error can be written as:

$$\epsilon_{LOO} = \frac{\sum_{i=1}^N \left( \mathcal{M}(\mathbf{x}^{(i)}) - \mathcal{M}^{PC \setminus i}(\mathbf{x}^{(i)}) \right)^2}{\sum_{i=1}^N \left( \mathcal{M}(\mathbf{x}^{(i)}) - \hat{\mu}_Y \right)^2} \quad (\text{B.5})$$

850 where,  $\hat{\mu}_Y$  is the mean of the experimental design sample.

### 851 **Appendix C. Bayesian interface for model calibration**

852 Consider the computational model  $\mathcal{M}$  that allows the analyst to predict certain quantities of interest  
 853 gathered in a vector  $\mathbf{y} \in \mathbb{R}^{N_{out}}$  as a function of input parameters  $\mathbf{x}$ :

$$\mathcal{M} : \mathbf{x} \in \mathcal{D}_{\mathbf{X}} \subset \mathbb{R}^M \mapsto \mathbf{y} = \mathcal{M}(\mathbf{x}) \in \mathbb{R}^{N_{out}} \quad (\text{C.1})$$

854 The Bayesian interface for model calibration focuses on identifying the input parameters of a computa-  
 855 tional model to recover the observations in the collected output data set. It comprises of a computational  
 856 forward model  $\mathcal{M}$ , a set of input parameters  $\mathbf{x} \in \mathcal{D}_{\mathbf{X}}$  that need to be inferred, and a set of experimental  
 857 data  $\mathcal{Y}$ . Here,  $\mathcal{Y} \stackrel{\text{def}}{=} \{\mathbf{y}_1, \dots, \mathbf{y}_N\}$  is a global data set of  $N$  independent measured quantities of interest  
 858  $(\mathbf{y}_i)$ .

859 The forward model  $\mathbf{x} \mapsto \mathcal{M}(\mathbf{x})$  is a mathematical representation of the system under consideration.  
 860 The lack of knowledge on the input parameters is modelled by considering them as a random vector,  
 861 denoted by  $\mathbf{X}$  which is assumed to follow a so-called prior distribution (with support  $\mathcal{D}_{\mathbf{X}}$ ), as presented  
 862 in Figure 11 in the present work.

$$\mathbf{X} \sim \pi(\mathbf{x}) \quad (\text{C.2})$$

863 The Bayesian statistics combine this prior knowledge of the parameters with the few observed data  
 864 points to obtain a statistical model called posterior distribution ( $\pi(\mathbf{x} | \mathbf{y})$ ) of the input parameters, using  
 865 Bayes' theorem [45], expressed as:

$$\pi(\mathbf{x} | \mathbf{y}) = \frac{\pi(\mathbf{y} | \mathbf{x})\pi(\mathbf{x})}{\pi(\mathbf{y})} \quad (\text{C.3})$$

866 Now, considering the available data set ( $\mathcal{Y}$ ) as independent realizations of  $\mathbf{Y} | \mathbf{x} \sim \pi(\mathbf{y} | \mathbf{x})$ , the  
 867 collected measurements result in the definition of the likelihood function  $\mathcal{L}(\mathbf{x}; \mathcal{Y})$ , which is a function of  
 868 input parameters  $\mathbf{x}$ :

$$\mathcal{L} : \mathbf{x} \mapsto \mathcal{L}(\mathbf{x}; \mathcal{Y}) \stackrel{\text{def}}{=} \prod_{i=1}^N \pi(\mathbf{y}_i | \mathbf{x}) \quad (\text{C.4})$$

869 This implicitly assumes independence between individual measurements in  $\mathcal{Y}$ . Intuitively the like-  
870 likelihood function for a given  $\mathbf{x}$  returns the relative likelihood of observing the data at hand, under the  
871 assumption that it follows the prescribed parametric distribution  $\pi(\mathbf{y} | \mathbf{x})$ .

872 As all models are simplifications of the real world, a discrepancy term ( $\epsilon$ ) is introduced to connect  
873 real-world observations ( $\mathcal{Y}$ ) to the predictions by the model. In practice, the discrepancy term represents  
874 the effects of the measurement error and model inaccuracy. The discrepancy term introduced here reads:

$$\mathbf{y} = \mathcal{M}(x) + \epsilon \quad (\text{C.5})$$

875 Here, the  $\epsilon$  is assumed as an additive Gaussian discrepancy [49] with a zero mean and given covariance  
876 matrix ( $\Sigma$ ):

$$\epsilon \sim \mathcal{N}(\epsilon | \mathbf{0}, \Sigma) \quad (\text{C.6})$$

877 Taking insights from the discrepancy term definition, a particular measurement point ( $\mathbf{y}_i \in \mathcal{Y}$ , is a  
878 realization of the Gaussian distribution with mean value  $\mathcal{M}(x)$  and covariance matrix  $\Sigma$ . This distribution  
879 is named as discrepancy model and is expressed as:

$$\pi(\mathbf{y} | \mathbf{x}) = \mathcal{N}(\mathbf{y} | \mathcal{M}(\mathbf{x}), \Sigma) \quad (\text{C.7})$$

880 In application, the discrepancy model defines the connection between the supplied data ( $\mathcal{Y}$ ) and the  
881 forward model. In the present work, as the model calibration relies on a single experimental measurement,  
882 the discrepancy model with known residual variance is assigned.

883 Afterwards, the  $N$  independent available measurements gathered in the data-set (*i.e.*,  $\mathcal{Y} = \{\mathbf{y}_1, \dots, \mathbf{y}_N\}$ )  
884 are used to define the likelihood function as:

$$\begin{aligned} \mathcal{L}(\mathbf{x}; \mathcal{Y}) &= \prod_{i=1}^N \mathcal{N}(\mathbf{y}_i | \mathcal{M}(\mathbf{x}), \Sigma) \\ &= \prod_{i=1}^N \frac{1}{\sqrt{(2\pi)^{N_{out}} \det(\Sigma)}} \exp\left(-\frac{1}{2}(\mathbf{y}_i - \mathcal{M}(\mathbf{x}))^\top \Sigma^{-1}(\mathbf{y}_i - \mathcal{M}(\mathbf{x}))\right) \end{aligned} \quad (\text{C.8})$$

885 Moreover, in the present work, the experimental data is retrieved through various points and for  
886 different impact energies. The Bayesian interface allows incorporating all these data points together  
887 for the model calibration by arranging the elements of  $\mathcal{Y}$  in disjoint data groups and defining different  
888 likelihood functions for each data group [49].

889 Denoting the  $g$ -th data group by  $\mathcal{G}^g = \{\mathbf{y}_i\}_{i \in \mathbf{u}}$ , where  $\mathbf{u} \subseteq \{1, \dots, N\}$ , the full data-set can be combined  
 890 by

$$\mathcal{Y} = \bigcup_{g=1}^{N_{gr}} \mathcal{G}^{(g)} \quad (\text{C.9})$$

891 Here, each of the  $N_{gr}$  data groups contains measurements collected at the same measurement point. In  
 892 the context of the present work, these measurements are experimental data recorded at ‘init’, ‘max’ and  
 893 ‘rest’ time instances at a particular point. This makes it evident to have a different likelihood function  
 894  $\mathcal{L}^{(g)}$  describing the experimental conditions that led to measuring  $\mathcal{G}^{(g)}$ . Assuming the Independence  
 895 between  $N_{gr}$  measurement conditions, the full likelihood function can then be written as

$$\mathcal{L}(\mathbf{x}_{\mathcal{M}}, \mathbf{x}_{\epsilon}; \mathcal{Y}) = \prod_{g=1}^{N_{gr}} \mathcal{L}^{(g)}(\mathbf{x}_{\mathcal{M}}, \mathbf{x}_{\epsilon}^{(g)}; \mathcal{G}^{(g)}) \quad (\text{C.10})$$

896 where,  $\mathbf{x}_{\epsilon}^{(g)}$  are the parameters of the  $g$ -th discrepancy group. The different model output groups are  
 897 assigned through a model output map (MOMap) vector [49].

898 Thereby, following Bayes’ theorem, the posterior distribution  $\pi(\mathbf{x} \mid \mathcal{Y})$  of the parameters ( $\mathbf{x}$ ) given  
 899 the observations in  $\mathcal{Y}$  can be written as:

$$\pi(\mathbf{x} \mid \mathcal{Y}) = \frac{\mathcal{L}(\mathbf{x}; \mathcal{Y})\pi(\mathbf{x})}{Z} \quad (\text{C.11})$$

900 Here,  $Z$  is a normalizing factor, known as the marginal likelihood or evidence, is added to ensure that  
 901 this distribution integrates to 1:

$$Z \stackrel{\text{def}}{=} \int_{\mathcal{D}_{\mathbf{x}}} \mathcal{L}(\mathbf{x}; \mathcal{Y})\pi(\mathbf{x})d\mathbf{x} \quad (\text{C.12})$$

902 The closed-form solutions do not exist in practice the posterior distribution is obtained through  
 903 Markov chain Monte Carlo (MCMC) simulations. In the present work, among many proposed algorithms  
 904 in [49], the Adaptive Metropolis (AM) algorithm is used and 100 parallel chains with 1000 steps are  
 905 assigned to the MCMC solver. The start of the covariance adaptation in AM algorithm is assigned at  
 906 the 100th step (see Wagner et al. [49] for more details).

907 The output ( $\mathbf{y}$ ) predictive capabilities of the Bayesian interface is assessed through the comparison of  
 908 prior ( $\pi(\mathbf{y})$ ) and posterior ( $\pi(\mathbf{y} \mid \mathcal{Y})$ ) output distributions as:

$$\pi(\mathbf{y}) = \int_{\mathcal{D}_{\mathbf{x}}} \pi(\mathbf{y} | \mathbf{x})\pi(\mathbf{x})d\mathbf{x} \quad (\text{C.13})$$

$$\pi(\mathbf{y} | \mathcal{Y}) = \int_{\mathcal{D}_{\mathbf{x}}} \pi(\mathbf{y} | \mathbf{x})\pi(\mathbf{x} | \mathcal{Y})d\mathbf{x} \quad (\text{C.14})$$

909        Lastly, in the present studies, the purpose of the Bayesian interface is to obtain the ‘best set of input  
 910 parameters’. Given the posterior distribution ( $\pi(\mathbf{x} | \mathcal{Y})$ ), we are interested in finding a suitable set among  
 911 the posterior computed set *i.e.*,  $\mathbf{X} | \mathcal{Y}$ . This is done through a point estimator ( $\hat{\mathbf{x}}$ ) computed from:

$$\pi(\mathbf{y} | \mathcal{Y}) \stackrel{\text{def}}{=} \pi(\mathbf{y} | \hat{\mathbf{x}}) \quad (\text{C.15})$$

912        This point estimator can be a mean or mode (maximum a posteriori ‘MAP’) [49] of the posterior  
 913 distribution as per the user’s choice.