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Bayesian interface based calibration of a novel rockfall protection structure modelled in the Non-smooth contact dynamics framework

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Abstract

This article presents the development and calibration of a numerical model simulating the response of large structure to a localized dynamic loading. A novel rockfall protection structure composed of piled up concrete blocks interconnected via metallic components is studied, where the underlying intricacies are revealed by real-scale impact experiments. The numerical model is developed using the SICONOS software based on the Non-Smooth Contact Dynamics (NSCD) method to further investigate the efficiency and performance of this new structure type. All geometrical features and mechanical properties of the real structure components are incorporated via specific developments pertinent to the modeling requirements in SICONOS. Some parameters peculiar to the numerical model are calibrated against the spatial-temporal measurements from two full-scale impact experiments. The Bayesian interface statistical learning method aided by the polynomial chaos expansion based meta-model of the NSCD model is deployed for the calibration. The additional understanding of the model dynamics through the byproducts of meta-model are highlighted. In the end, the NSCD model is successfully calibrated against the spatial-temporal response of the experimental structure for impact energies up to 1 MJ.

Keywords: rockfall, impact, meta-model, statistical learning, NSCD modeling, calibration, Bayesian interface

1. Introduction

The Non-Smooth Contact Dynamics (NSCD) approach was developed to solve multi-body multi-contact problems with rigid and/or deformable bodies [1, 2, 3, 4]. It finds many modeling application in the field of civil engineering and geomechanics such as; masonry and stone structures exposed to static, cyclic and dynamic loading [5, 6, 7]; cohesive and non-cohesive granular materials [8, 9]; and rockfall

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6 propagation on slopes [10] in particular. All these applications consider bodies interacting with each
7 other via surface contacts. Hence, NSCD approach can also be used for mechanisms and linkages [11]
8 and has also been easily combined with standard Finite Element Method (FEM) from the early stage of
9 its development [12, 13].

10 The present work extends the use of NSCD modeling for the impact study on a complex structure made
11 of individual concrete blocks interconnected one to the other via metallic components, thus forming an
12 articulated structure. These types of structures are proposed by the company Géolithe and are intended
13 to serve as passive protection against gravity driven natural hazards. It is in particular designed to
14 intercept rockfall, similarly as other massive structure types including embankments [14, 15, 16, 17],
15 concrete walls [18, 19, 20], concrete blocks cushioned with gabions [21, 22, 23], gabion structures [24] and
16 sea containers [25]. This innovative technology offers the possibility to build massive vertical walls, with
17 reduced foot print, high deformability and versatility.

18 The development of this technology is based on small-scale experiments, to address the structure
19 response considering different design options [26], and real-scale impact experiments to demonstrate the
20 structure capacity in arresting rock blocks with kinetic energy exceeding 1 MJ [27]. These experiments
21 revealed the intricacy of the impact response of structures made from interconnected concrete blocks.
22 The impact induces sliding at the base and tilting of the structure, where the amplitude depends on
23 the impact energy and distance to the impact point [26]. In addition, interconnection between concrete
24 blocks improve the structure stability preventing from excessive concrete blocks displacements in the
25 impact vicinity.

26 The impact response of small- and real-scale structures has been previously investigated in the numer-
27 ical modeling framework of the finite volume formulation code FLAC3D [26, 27]. These models provided
28 the detailed insights on the energy dissipation capacities of the protection walls. However, the compu-
29 tation time for real-scale structure ranging from 10 to 20 hours hinders the exhaustive investigations of
30 the structure mechanical and dynamic response. In particular, improving the design of such structures
31 requires better quantifying their efficiency when exposed to rockfall. In line with some previous research
32 on flexible barriers [28, 29, 30], this implies taking into account the variety of loading cases observed dur-
33 ing real rockfalls and therefore performing a very large number of simulations of the structure's response
34 to the impact.

35 Given the above, the authors developed a NSCD-based model of the structure, using the SICONOS
36 software [31, 32]. SICONOS is an open-source scientific software primarily targeted at modeling and
37 simulating non-smooth dynamical systems in C++ and in Python, including mechanical systems, switched
38 electrical circuits, sliding mode control systems and biology. The NSCD method guarantees that the
39 Signorini’s condition at the velocity level is satisfied without introducing contact stiffnesses, and that the
40 coulomb friction, especially in the sticking mode is satisfied without resorting to viscous friction. This
41 peculiarity of NSCD is enforced with the use of a specific implicit scheme (Moreau–Jean scheme) and
42 yields a robust numerical method, which consistently models the threshold phenomena (friction, contact)
43 and the dissipation properties of the model in discrete time, in particular impact dissipation and energy
44 properties [33].

45 Considering its intended use, the developed model is kept as simple as possible while allowing satisfac-
46 torily mimicking the whole structure’s response to impact, as observed during the real-scale experiments.
47 Such simplifications drastically reduce the computational time as each component of the structure is
48 modeled as a rigid body instead of a deformable mesh resulting in a huge reduction in the degrees of
49 freedom of the model. The energy dissipation due to plastic strains and breakages of the blocks during
50 impact is modeled in a simple manner as energy dissipation at the contact points. Escallón et al. [34]
51 reported a conceptually similar approach for the FEM model of flexible rockfall protection barriers and
52 observed a significant reduction in the model computational time.

53 Further, the calibration of model parameters becomes necessary to validate the model performance.
54 For this, an existing experimental database [27] is used as a reference and some identified model param-
55 eters are calibrated using Bayesian interface method. This method belongs to the category of parameters
56 identification through inverse formulation [34, 35] and a calibration technique [36, 37] extensively used to
57 reproduce the experimental or practical observations through optimum model parameters identification.
58 It statistically learns the trends of the model output response on the large dataset of the model input
59 parameters as per the recorded experimental observations.

60 A common requirement of any stochastic method is a large number of model computations for reli-
61 ability and abiding with the law of the large numbers and the central limit theorems. Therefore, even
62 after a significant reduction in model computation time (say to within a few minutes), the stochastic
63 methods continue to be impractical to implement due to a large computation cost. This gap is bridged

64 by the use of meta-model, representative of the actual model having a negligible computational cost and
 65 possess high accuracy and reliability. Such meta-models (or surrogate models) have been used by many
 66 researchers, *e.g.*, for dam engineering problems [38, 39] and for nuclear containment structures [40].

67 In the present work, the NSCD-based model is first described in detail, including specific developments
 68 to cope with the particular features of this structure type. A polynomial chaos expansion-based meta-
 69 model of the NSCD model is created for the stochastic analysis. The additional byproduct of PCE based
 70 meta-models towards the relative influence of model parameters is discussed. The model parameters are
 71 calibrated based on the Bayesian interface approach considering the measurements obtained from two
 72 impact experiments. A discussion on model features in line with the perspectives for the future research
 73 concludes the work.

74 2. Experimental structure and its impact response

75 The considered articulated structures consist in piled up blocks, made from concrete and are reinforced
 76 with steel rebars [27], presented in Figure 1. The block extremities in the horizontal plane are rounded
 77 and their upper and lower faces present empty spaces. The wall is made up of 38 blocks and four half-
 78 blocks stacked in four rows. The blocks are arranged following a zig-zag pattern (at 45° angle) to improve
 79 the structure stability vs titling. The wall is 3.2 m in height and about 14 m in length.

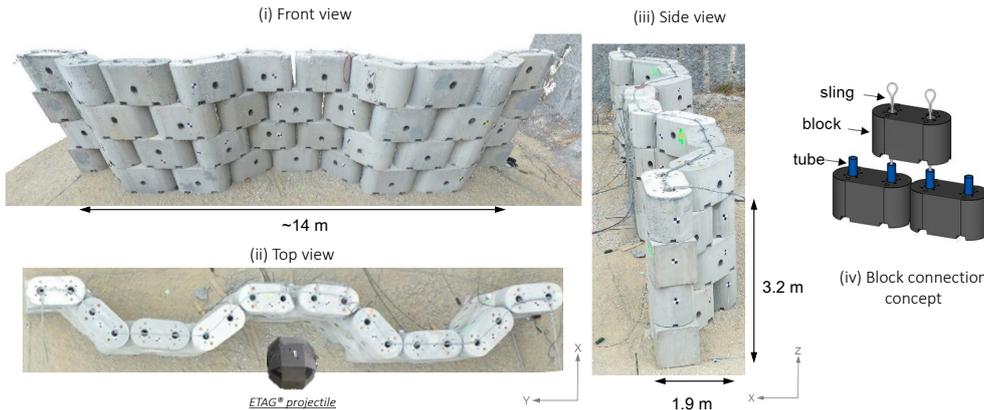


Figure 1: Experimental full-scale structure, modified after [27]

80 Each block is traversed by two cylindrical holes of diameter 154mm along the vertical axis to receive
 81 tubes and slings. These latter metallic components, hereafter referred to as connectors, link blocks
 82 together. More precisely, a hollow steel tube of diameter 139.7mm connects each couple of superimposed

83 blocks, preventing from their relative displacements along the horizontal direction. In addition, a cable
84 runs from the structure base to its top, in the succession of tubes, and mainly prevents from relative
85 displacements along the vertical direction. On a conceptual point of view, these connectors provide
86 the structure with mechanical continuity with the aim of increasing the number of blocks involved in
87 the structure's response proportional to the impact loading. Another key feature in the design is the
88 presence of various plays : a 40 mm distance between adjacent concrete blocks, an approx. 14 mm
89 difference between the external diameter of the metallic tubes and the inner diameter of the cylindrical
90 holes and, a slack in the vertical cables [27]. These plays give the structure a certain deformation capacity.
91 All geometrical and mechanical characteristics of this structure are presented in Table 1 along with the
92 ones of the impacting projectile.

Table 1: Geometrical and mechanical characteristics of the articulated concrete blocks structure

Component	Parameter	Value	Unit
Concrete Block	Length	1.56	m
	Width	0.76	m
	Height	0.80	m
	Mass (m_{block})	1800	kg
	Hole diameter	154	mm
Cylindrical bars	Diameter (ext.)	139.7	mm
	Diameter (int.)	123.7	mm
	Length	0.8 (3 pcs.)	m
		0.4 (2 pcs.)	m
Structure	Length (edge-to-edge)	≈ 14	m
	Height	3.2	m
	Pattern angle	45 (from y-axis)	$^{\circ}$
Projectile	Side length (l_{proj})	1.1	m
	Mass (m_{proj})	2600	kg
	Impact angle	0	$^{\circ}$
	Impact point location	$y \approx 7.0, z = 1.7$	m

93 Two identical real-scale walls were built and submitted to one impact each. The impact experiments
 94 were performed with the pendulum testing facility of the Université Gustave Eiffel test site (Montagnole,
 95 France). It involved a 2600 kg in mass reinforced concrete projectile which shape is in accordance with
 96 the requirements for flexible barriers testing (EOTA 2018). Its size is about one-third the structure height
 97 (1.1 m). The velocity at impact is 20m/s and 28m/s during the first and second impacts, corresponding
 98 to kinetic energies of 520 and 1020 kJ respectively. The angle of incidence and rotation velocity at impact
 99 were zero. The projectile impacted the structure at its mid-length and at about 1.7m from the ground.
 100 These impacts in particular induced wall sliding at its base, tilting, change in conformation as well as
 101 concrete blocks damage.

102 The structure impact response with time is described based on measurements of displacement and
 103 acceleration along two vertical lines, as presented in Figure 2. They are termed as impact and distant
 104 axis which are the representative of the impact point and 3.5m to its left respectively. The experimental
 105 data used in this study concerned the concrete blocks at the top and at the base in the impact axis and
 106 points *C* and *D* in the distant distant axis. Besides, the data from two points *A* and *B* in the impact axis
 107 is also available which is used for the cross-reference purpose only. Further details on the experiments
 108 and data are given in [27].

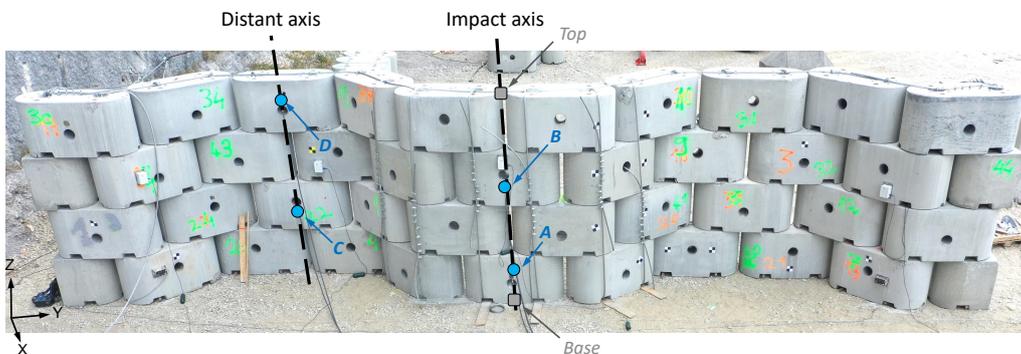


Figure 2: Data acquisition locations in the experimented full-scale structure, recreated after [27]

109 The displacement governs the structure sliding and tilting and thereby, both leading to structure
 110 failure if in excess. Thereby, in this study, only the database of displacement evolution with time in both
 111 the impact vicinity and at distance are considered. The displacement data used in this study are plotted
 112 in Figure 3 for both impact tests and distant axis. A secondary y-axis is added for distant axis albeit to
 113 the relatively low displacement amplitude comparing to the impact axis.

114 Measures along the impact axis are derived from video records while measures from cable extensome-

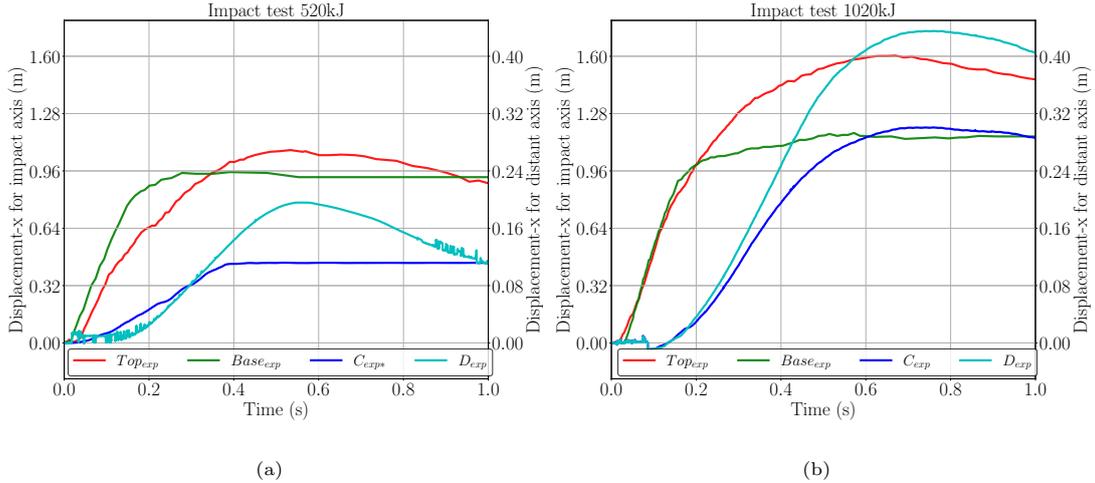


Figure 3: Post-processed experimental database for the time evolution of Displacement for (a) 520 kJ and (b) 1020kJ impact test

115 ters are considered along the distant axis. Due to the absence of this later data for the 520 kJ impact test
 116 at point C , the evolution obtained from simulations presented in [27] are considered. This exception is
 117 thought to have minor influence due to the very small amplitude in displacement observed in this specific
 118 case. The dynamic response of the structure is clearly visible through the relative initiation time lag
 119 between observations corresponding to impact and distant axes.

120 3. Numerical model

121 The rockfall protection structure is modeled in a python based software package named SICONOS,
 122 which implements the NSCD method with complex geometries [31]. The model is developed in the aim
 123 of reproducing the whole structure displacement response upon impact by a projectile. In this section,
 124 first the constitutive framework of NSCD approach is presented followed by the development of the
 125 numerical model in SICONOS.

126 3.1. NSCD main principles and specific features available in SICONOS

127 The Non-Smooth Contact Dynamics (NSCD) method developed by M. Jean and J.J. Moreau [1, 2]
 128 is a modeling and numerical framework for the dynamics of multi-body systems in interaction through
 129 frictional contact interfaces. In our context, the system is modeled as a collection of rigid blocks connected
 130 by unilateral constraints with Coulomb friction. The main feature of the NSCD method is that it can
 131 deal with rigid or flexible bodies with rigid contact laws such as the Signorini's model of unilateral contact

132 and the Coulomb's dry friction without any kind of regularization (viscous friction) or compliance. The
133 finite-freedom dynamics of rigid bodies with unilateral constraints is known to be non-smooth, in the
134 sense that the velocities of the system possess jumps when a contact is closing with a positive relative
135 velocity. These velocity jumps call for the introduction of an impact law. The NSCD method is able to
136 perform the numerical time integration of the multi-body system with impacts in the dynamics. In this
137 section, we give the main ingredients of the NSCD method referring to the following references for more
138 details [3, 4].

139 3.1.1. Newton-Euler equations

140 The configuration of each rigid body in three dimension is described by the position of its center
141 of mass $x_{\mathbf{g}} \in \mathbb{R}^3$ and the orientation of the body-fixed frame with respect to a given inertial frame
142 $R \in SO^+(3)$. The velocity of the body is given by the velocity of the center of mass $v_{\mathbf{g}} = \dot{x}_{\mathbf{g}} \in \mathbb{R}^3$ and
143 the angular velocity of the body expressed in the body-fixed frame $\Omega \in \mathbb{R}^3$. A possible formulation of
144 the Newton-Euler equations of motion for each body is

$$\left\{ \begin{array}{l} m \dot{v}_{\mathbf{g}} = f(t, x_{\mathbf{g}}, v_{\mathbf{g}}, R, \Omega) \\ I \dot{\Omega} + \Omega \times I \Omega = M(t, x_{\mathbf{g}}, v_{\mathbf{g}}, R, \Omega) \\ \dot{x}_{\mathbf{g}} = v_{\mathbf{g}} \\ \dot{R} = R \tilde{\Omega} \end{array} \right. \quad (1)$$

145 where $m > 0$ is the mass, $I \in \mathbb{R}^{3 \times 3}$ is the matrix of moments of inertia around the center of mass
146 and the axis of the body-fixed frame, and $f(\cdot) \in \mathbb{R}^3$, respectively $M(\cdot) \in \mathbb{R}^3$, are the total forces and
147 respectively torques with respect to the center of gravity applied to the body. The matrix $\tilde{\Omega} \in \mathbb{R}^{3 \times 3}$
148 is given by $\tilde{\Omega}x = \Omega \times x$ for all $x \in \mathbb{R}^3$. In the implementation of SICONOS, the orientation matrix R
149 is parameterized by a unit quaternion p such that $R = \Phi(p)$. In the equations of motion (Equation 1),
150 the occurrences of R are substituted by their corresponding expressions in terms of p . The differential
151 equation of Lie type $\dot{R} = R \tilde{\Omega}$ is replaced by $\dot{p} = \Psi(p)\Omega$ in the unit quaternion space. The closed formulae
152 for Φ and Ψ can be found in any textbook on rigid body dynamics. We denote by q the vector of
153 coordinates of the position and the orientation of the body, and by v the body twist. In matrix notation,

154 the relation between the body twist v and the time derivative of q is

$$q := \begin{bmatrix} x_{\mathbf{g}} \\ p \end{bmatrix}, \quad v := \begin{bmatrix} v_{\mathbf{g}} \\ \Omega \end{bmatrix}, \quad \dot{q} = \begin{bmatrix} \dot{x}_{\mathbf{g}} \\ \Psi(p)\Omega \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0 & \Psi(p) \end{bmatrix} v := T(q)v \quad (2)$$

155 with $T(q) \in \mathbb{R}^{7 \times 6}$. A compact matrix form of the Newton Euler equations is

$$\begin{cases} \dot{q} = T(q)v, \\ M\dot{v} = F(t, q, v), \end{cases} \quad (3)$$

156 where $M \in \mathbb{R}^{6 \times 6}$ is the total inertia matrix and $F(t, q, v) \in \mathbb{R}^6$ collects all the forces and torques applied
157 to the body given by

$$M := \begin{pmatrix} mI_{3 \times 3} & 0 \\ 0 & I \end{pmatrix}, \quad F(t, q, v) := \begin{pmatrix} f(t, x_{\mathbf{g}}, v_{\mathbf{g}}, R, \Omega) \\ I\Omega \times \Omega + M(t, x_{\mathbf{g}}, v_{\mathbf{g}}, R, \Omega) \end{pmatrix}. \quad (4)$$

158 In the sequel, we assume that the vectors $q \in \mathbb{R}^{7n}$ and $v \in \mathbb{R}^{6n}$ collect the configuration parameters
159 and the velocities of the n bodies in the model. The equation of motion (Equation 3) is rearranged
160 accordingly.

161 3.1.2. Contact modeling as unilateral constraints and Coulomb friction

162 For two contacting bodies A and B , we assume that we are able to define one or more contact pair
163 composed of two candidate contact points C_A and C_B and a local frame at contact $(C_A, \mathbf{N}, \mathbf{T}_1, \mathbf{T}_2)$ (see
164 Figure 4 for details). In this local frame, the normal gap function g_N is defined as the signed distance
165 between the point C_A and C_B , that is $g_N = (C_B - C_A) \cdot \mathbf{N}$. The unilateral contact is given by the Signorini's
166 condition as

$$0 \leq g_N \perp r_N \geq 0, \quad (5)$$

167 where r_N is the local contact normal force. The symbol $g_N \perp r_N$ means that $g_N r_N = 0$. If the contact is
168 open $g_N > 0$, the contact force r_N has to be equal to zero. If the contact is closed $g_N = 0$, then the contact
169 force is nonnegative $r_N \geq 0$. As we said before, the dynamics of finite dimensional system requires the
170 definition of an impact law. Let consider the relative velocity u_N defined by $u_N = \dot{g}_N$. One of the simplest
171 impact is the Newton impact law given by

$$u_N^+ = -e u_N^-, \text{ if } g_N = 0 \text{ and } u_N^- \leq 0, \quad (6)$$

172 where $e \in [0, 1]$ is the coefficient of restitution, u_N^+ is the post-impact velocity and u_N^- the pre-impact
173 velocity.

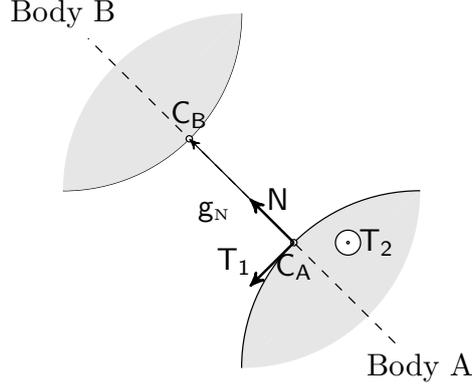


Figure 4: A contact pair and a local contact frame

174 For the tangential behavior of interface, we consider the Coulomb friction that can be formulated with
 175 the Signorini's condition at the velocity level and in terms of the complete reaction forces $r = [r_N, r_T]^\top$
 176 and the relative velocity $u = [u_N, u_T]^\top$ as

$$\left\{ \begin{array}{ll} r = 0 & \text{if } g_N > 0 \quad (\text{no contact}) \\ r = 0, u_N \geq 0 & \text{if } g_N = 0 \quad (\text{take-off}) \\ r \in K, u = 0 & \text{if } g_N = 0 \quad (\text{sticking}) \\ r \in \partial K, u_N = 0, \|u_T\| r_T = \|r_T\| u_T & \text{if } g_N = 0 \quad (\text{sliding}) \end{array} \right. \quad (7)$$

177 where $K = \{r \in \mathbb{R}^3, \|r_T\| \leq \mu r_N\}$ is the usual Coulomb friction cone. Using the modified local relative
 178 velocity introduced by De Saxcé and Feng [41, 42], $\hat{u} := u + \mu \|u_T\| \mathbf{N}$ and the dual cone of K , *i.e.*,
 179 $K^* = \{z \in \mathbb{R}^3 \mid z^T x \geq 0 \text{ for all } x \in K\}$, the contact model is equivalent to

$$K^* \ni \hat{u} \perp r \in K. \quad (8)$$

180 3.1.3. Equations of motion with contact and friction

181 The gap function g_N for a contact is generally a function of the configuration of the bodies q . Let us
 182 consider that we have a set of m contact pairs given by the unilateral constraints:

$$g_N^\alpha(q) \geq 0, \alpha \in \mathcal{I} \subset \mathcal{N}, |\mathcal{I}| = m. \quad (9)$$

183 For a contact $\alpha \in \mathcal{I}$, the relative normal velocity is related to v through the relation

$$u_N^\alpha = \dot{g}_N^\alpha(q) = J_{g_N^\alpha}^\alpha(q) \dot{q} = J_{g_N^\alpha}^\alpha(q) T(q) v := G_N^\alpha(q) v, \quad (10)$$

184 where $J_{g_N^\alpha}^\alpha(q)$ is the Jacobian of g_N^α with respect to q . The same type of relation can be written for the
 185 tangential relative velocity u_T as $u_N = G_T^\alpha(q)v$ and we get for u^α

$$u^\alpha := G^\alpha(q)v. \quad (11)$$

186 By duality, the total force generated by the contact α is

$$G^{\alpha,\top}(q)r^\alpha := G_N^{\alpha,\top}(q)r_N^\alpha + G_T^{\alpha,\top}(q)r_T^\alpha. \quad (12)$$

187 Altogether the equation of motion with contact and Coulomb friction are given by

$$\left\{ \begin{array}{l} \dot{q} = T(q)v, \\ M\dot{v} = F(t, q, v) + G^\top(q)r, \\ r^\alpha = 0, \quad \text{if } g_N^\alpha(q) > 0, \\ K^{\alpha,*} \ni \widehat{u}^\alpha \perp r^\alpha \in K^\alpha, \quad \text{if } g_N^\alpha(q) = 0, \\ u_N^{\alpha,+} = -e^\alpha u_N^{\alpha,-}, \quad \text{if } g_N^\alpha(q) = 0 \text{ and } u_N^{\alpha,-} \leq 0 \end{array} \right\} \alpha \in \mathcal{I},$$

188 3.1.4. Time-discretization

189 Let us consider a time discretization $t_0 < \dots < t_{k-1} < t_k < \dots < T$ with a constant time step
 190 $h = t_{k+1} - t_k$. The Moreau–Jean scheme for the system is

$$\left\{ \begin{array}{l} q_{k+1} = q_k + hT(q_{k+\theta})v_{k+\theta} \\ M(v_{k+1} - v_k) - hF_{k+\theta} = G^\top(q_{k+1})P_{k+1}, \\ P_{k+1}^\alpha = 0, \\ K^{\alpha,*} \ni \widehat{u}_{k+1}^\alpha + e^\alpha u_{N,k}^\alpha \mathbf{N} \perp P_{k+1}^\alpha \in K^\alpha \end{array} \right\} \begin{array}{l} \alpha \notin \mathcal{I}_k \\ \alpha \in \mathcal{I}_k. \end{array} \quad (13)$$

191 where the notation $x_{k+\theta} = (1 - \theta)x_k + \theta x_{k+1}$ is used for $\theta \in [0, 1]$ and the set \mathcal{I}_k is the set of contact
 192 activated at the velocity level

$$\mathcal{I}_k = \{\alpha \in I \mid g_{N,k}^\alpha + \gamma u_{N,k}^\alpha \leq 0\} \text{ with } \gamma \in [0, \frac{1}{2}]. \quad (14)$$

193 In the time-stepping method, the unknown P_{k+1} is an approximation of the impulses of the contact
 194 reaction measure di over the time interval, that is

$$\int_{(t_k, t_{k+1}]} di \approx P_{k+1}. \quad (15)$$

195 The contact reaction measure is related to the contact force when the motion is smooth enough by
 196 $di = r(t)dt$. The system (Equation 13) is a second order cone complementarity problem that is solved
 197 by SICONOS using a Gauss-Seidel method with projection [42].

198 3.1.5. Comments and remarks on the strengths of the NSCD method

199 With the NSCD method, robust and efficient simulations of the dynamics and the quasi-statics of
 200 multi-body systems with contact, impact and Coulomb friction are possible. Among them, the scheme
 201 integrates the impact equation by consistently approximating percussions. To this aim, the scheme is
 202 fully implicit for the unknown P_{k+1} and when the time-step goes to 0, the following impact equations
 203 are retrieved

$$M(v_{k+1} - v_k) = G^\top(q_{k+1})P_{k+1}. \quad (16)$$

204 The non-impulse terms of the dynamics are approximated with a θ -method. For $\theta = 1$, the scheme is fully
 205 implicit and maximizes the numerical dissipation. It is therefore well adapted to quasi-static evolutions,
 206 or for efficiently reaching a static equilibrium with large time-steps. For $\theta = 1/2$, the mechanical energy
 207 (the sum of the kinetic and potential energy) is conserved. Furthermore, the dissipation is always positive
 208 and is given by a second order approximation of the actual dissipation. These energy and dissipation
 209 properties detailed in [33] render the scheme robust and stable with quite large time-steps, and are of
 210 particular interest for the analysis of the dissipative processes in protection structures. These are the main
 211 advantages with respect to classical smooth DEM approaches [43, 44] where the use of explicit scheme
 212 renders difficult the energy analysis and the contact compliance generates spurious oscillations that
 213 prevent to obtain quickly static equilibrium without artificial damping. With a non-smooth dynamics,
 214 direct higher order approximations are possible, but some care has to be taken. The use of standard higher
 215 scheme — Newmark, HHT, generalised- α , implicit Runge-Kutta to name a few — is not possible and we
 216 need to rely on specific schemes such as non-smooth Newmark or generalised- α schemes [45, 46, 47], or
 217 time discontinuous Galerkin methods [48, 49].

218 3.2. NSCD structure model development in SICONOS

219 By contrast with common applications of NSCD, new challenges raise when modeling such articulated-
 220 concrete-blocks structures. These relate to the shape of the concrete blocks, the presence of structural
 221 components passing through these blocks, the interaction between these later components and the mod-

222 eling of the connectors. The model configuration and its components are demonstrated in Figure 5. The
223 modeling strategy that is developed to cope with these specific features is presented hereafter.

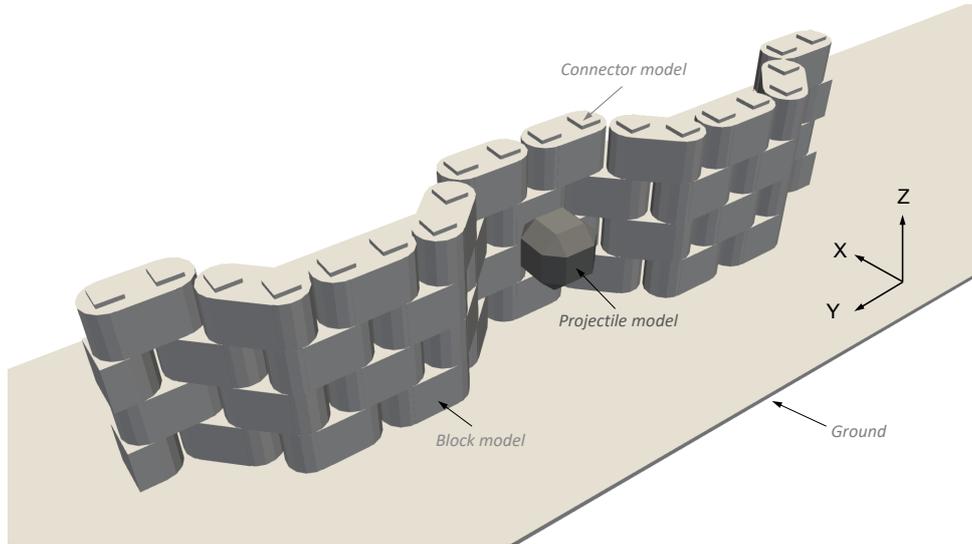


Figure 5: The NSCD model of the articulated concrete block structure and its components

224 3.2.1. Blocks modeling

225 In the development of the concrete block model, a particular attention is paid to the block shape
226 and to the mechanical interaction of the block with the connectors. The former issue results from the
227 rounded extremities of the blocks and from the effective contact surfaces between superimposed blocks,
228 as illustrated in Figure 6. The block shape issue thus concerns the interaction between superimposed
229 concrete blocks and the interaction between the projectile and the wall.

230 The geometry of the blocks was unreasonable to model as a mesh. Replicating the rounded extremities
231 of the block would have induced very large computation times associated with the large number of trian-
232 gular mesh required for describing their upper and lower faces. It would have also induced computational
233 errors associated with the interaction between superimposed blocks and resulting from contact detection
234 problems. The upper, lower and lateral side faces of the block are thus simplified modeling the concrete
235 blocks as a combination of the so-called primitive shapes which are available in SICONOS.

236 Different combinations of primitive shapes are considered for geometry and contact area relevance
237 with the experimental block, and for numerical efficiency and computational accuracy in the NSCD
238 framework. This resulted in the best block model option, presented in Figure 7. Several contact points
239 are set along the contact surface to model the interaction between superimposed blocks. The location

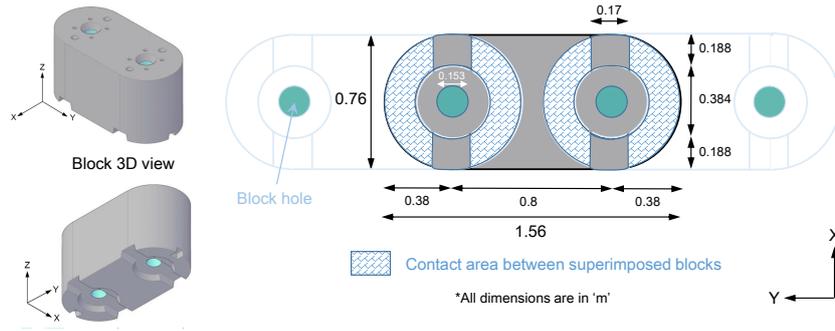


Figure 6: Experimental concrete block design and effective contact surfaces between superimposed blocks

240 of the contact point is defined in order to maximize the contact surface between the interacting bodies.
 241 One can note that, in the case of static equilibrium, the resulting interaction force between the bodies
 242 does not depend on the location of the contact forces.

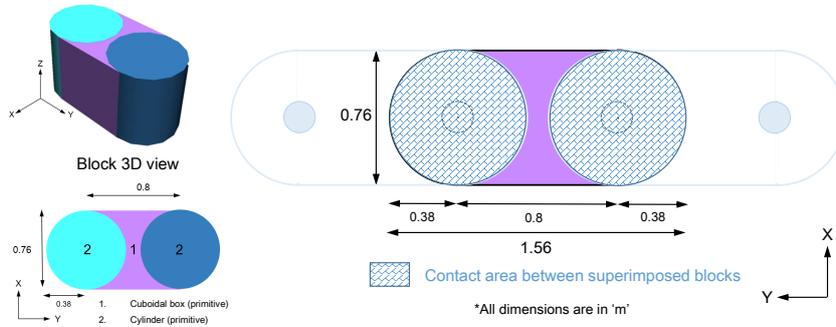


Figure 7: Block design in the NSCD model

243 Modeling the connectors explicitly as hollow cylinders passing through the blocks would also un-
 244 necessarily increase the computation time. These are ignored and a specific strategy is developed for
 245 accounting for the interaction between the block and the connectors, as detailed later in section 3.2.4.
 246 Besides, the block model has the same mass as of the real block. The inertia of the model blocks is
 247 obtained through a 2D surface mesh using the Convex Hull [50] feature in SICONOS.

248 3.2.2. Connectors modeling

249 In the real structure, the stacked concrete blocks are connected together via piled up steel tubes
 250 through which a steel sling runs from the bottom of the wall to its top (see Figure 1.iv). These connectors
 251 were modeled as a single component accounting for the influence on the structure response of both the
 252 sling and tubes. It required defining a mechanical model governing the connector- block interaction
 253 (which is insured by the tubes) and the interaction between connectors (which is in particular controlled
 254 by the slings). The collective connector model illustrative profile is presented in Figure 8 for before and

255 after impact situations.

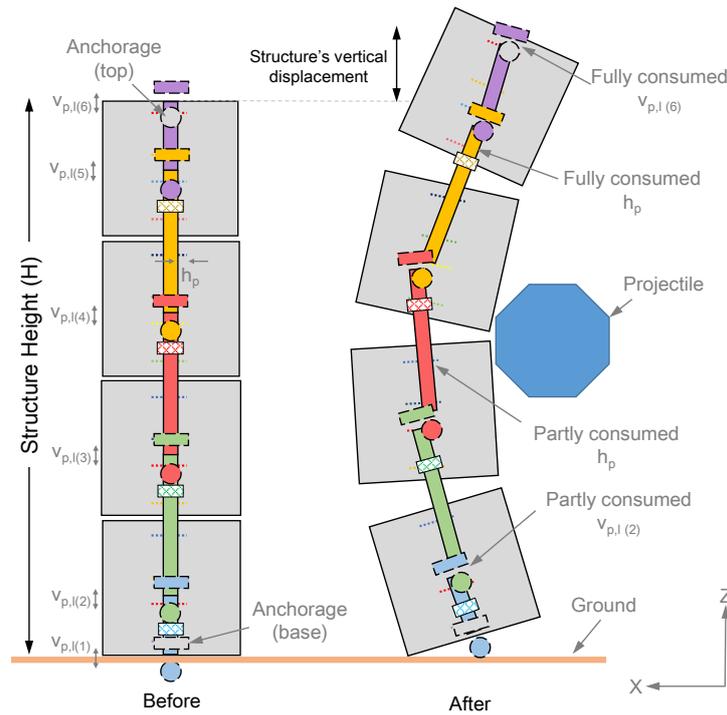


Figure 8: Schematic representation of the deformation of the structure upon projectile impact showing plays in action

256 It consists in five cylindrical primitive shape components, named as bars, at location corresponding
 257 to the real-structure tube positions and possessing same length (3 bars with 0.8m and 1 bar with 0.4m),
 258 diameter, inertia and mass as the tube. Notably, a fifth bar is added at the top block having similar
 259 geometrical characteristics as of the short length bar, however with a negligible mass. This is done to
 260 complete the configuration and the bounding limits of the sling in the real structure.

261 The mechanical plays existing in the structure, in both the horizontal and vertical directions are
 262 accounted for in the model. The horizontal play in the real structure results from the difference between
 263 the external diameter of the tube (139.7mm) and the diameter of the hollow cylinder in the block (154mm).
 264 In the model, the horizontal play (h_p) is rounded to 7mm (Figure 9). The vertical play in the real
 265 structure results from the relative looseness in the sling. The detailed connector modeling strategy to
 266 accommodate horizontal and vertical plays is presented hereafter.

267 In the model, the connectors pass through the concrete blocks but there is no direct interaction
 268 between them. This interaction is assigned through hollow circular disks added to the block model
 269 object, as illustrated in Figure 9. These disks are modeled as 2D mesh and have same internal diameter
 270 as that of the hole in the real block, hence ensuring the presence of the requisite horizontal play. Each

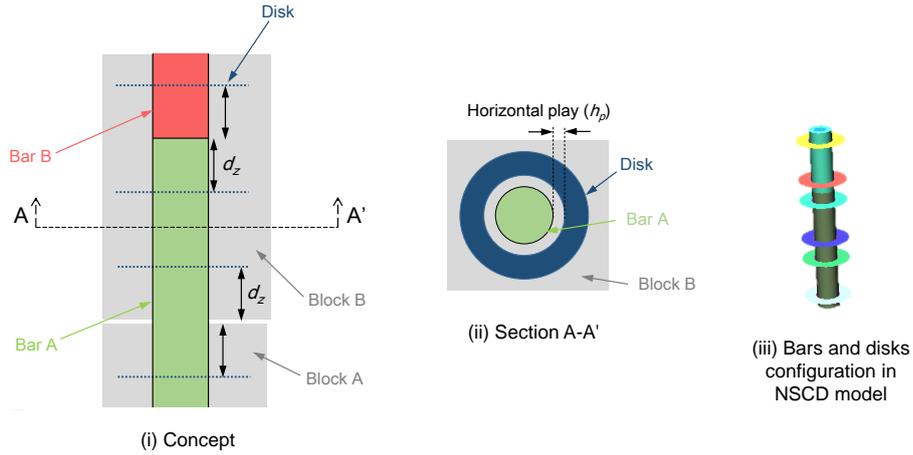


Figure 9: Concept of the connector modeling incorporating horizontal play

271 block is equipped with four hollow disks meaning that each of the two bars passing through a block
 272 interacts with this block via two disks. The position of these disks with respect to the lower/upper block
 273 faces and to the bars extremities is characterized by parameter d_z , which is kept alike for simplicity. The
 274 positioning of these disks can not be explicitly defined and hence it is characterized as a model parameter.

275 The vertical play is considered equally distributed along the structure's height as six local vertical
 276 plays ($v_{p,l} = v_p/6$) whose development is described as follows. Bar objects are assigned as invisible to
 277 each other and are equipped with three additional components accounting for the influence of the sling,
 278 presented in Figure 10.

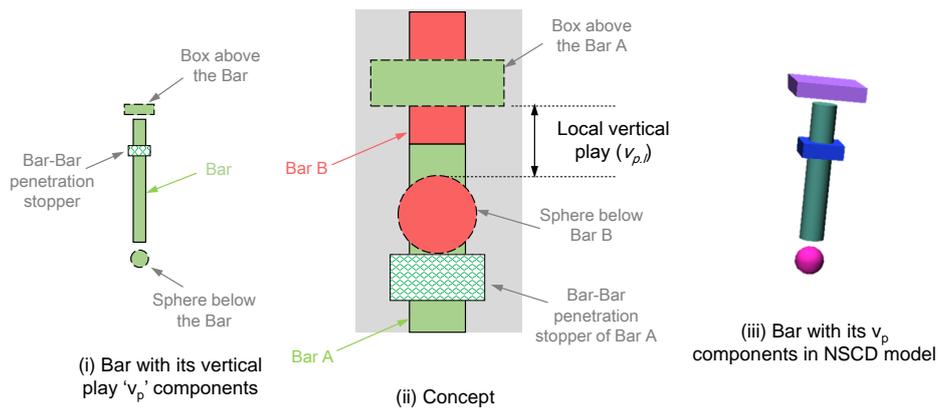


Figure 10: Concept of the connector modeling incorporating vertical play

279 These components are comprised of a cuboid box of length equal to 2.5 times the bar diameter (placed
 280 above the bar), a sphere of diameter equal to the bar diameter (placed below the bar and another cuboid

281 box of length equal to 1.5 times the bar diameter (placed within the bar) named as ‘bar-bar penetration
282 stopper’. The vertical position of these components are assigned such that the sphere from the adjacent
283 bar detects the contact with the boxes of the current bar. The distance between the box placed above
284 the bar and the bar surface, as well as the distance between the sphere and the bar surface, is half the
285 local vertical play. This way, the box-sphere combination from two adjacent bars collectively accounts
286 for the local vertical play. The bars interpenetration is avoided via the contact detection between this
287 bar-bar penetration stopper and the sphere from the adjacent bar.

288 All components of each block (and also bar) model objects have no relative movement between them.
289 This feature enables to indirectly implement the block-bar interaction and sling functionality represen-
290 tative of the real structure. An illustrative representation of the connector model (inclusive of the plays)
291 in action is added in Figure 8 for understanding. Here, the interaction of the vertical play constituents
292 (box and sphere) can be clearly seen. The distribution of vertical play (as six local vertical plays) enables
293 to mimic the looseness in the sling as an indirect chain made from multiple linkages. Once the complete
294 vertical play is consumed, the setup of bars mimic the tension in the sling. Such a state is reached de-
295 pending (mostly) on the assigned looseness in the vertical play and the energy of the impacting projectile.
296 This makes the vertical play (v_p) as a model parameter.

297 This intricate setup allows to model the large displacements in the model structure courtesy to the
298 accompanying horizontal play (h_p). Its presence controls the maximum relative lateral displacement
299 between box and sphere components of the vertical play. Thereby, a possible contact between the vertical
300 play components from adjacent bars is always ensured, which makes the connector model’s functionality
301 robust. An additional box and a sphere are added at the bottom and top most layer of blocks respectively.
302 This is to respectively mimic the anchorage of the sling with the structure’s bottom and to make sure
303 the bars do not fly-out of the structure’s top surface upon impact.

304 3.2.3. Projectile modeling

305 The experimental equivalent projectile is modeled as a convex 2D mesh of manually assigned vertices.
306 The projectile shape and size ($1/3^{rd}$ of the wall height) features are in accordance with the requirements
307 for flexible barriers testing [51]). The mass of the projectile is assigned 2600 kg, and the inertia is assigned
308 through convex hull (as used for block model). The input kinetic energy corresponding to the two impact
309 tests (*i.e.*, 520kJ and 1020kJ) is derived to assign the input velocity to the projectile model.

310 3.2.4. Interaction laws

311 The model objects (*i.e.*, blocks, bars, projectile and ground) are defined as a combination of different
 312 components called contactors, collectively making the model equivalent to the real structure. These
 313 contactors are defined with a uniquely assigned collision group to identify them and differentiate one
 314 from another. The collision groups of the contactors in the model bodies are listed in Table 2.

Table 2: Model bodies constitution for interaction

Collision group	Contactor	Collision group	Contactor
0	Block (cuboid portion)	5	Hollow disk
1	Block (curvature portion)	6	Box (vertical play)
2	Bar (odd)	7	Sphere (vertical play)
3	Bar (even)	8	Projectile
4	Bar-Bar penetration stopper	9	Ground

315 The interaction between model bodies is assigned through Newton impact friction non-smooth law
 316 [3]. This law governs the interaction between a pair of contactors (identified through their respective
 317 collision groups) via user-defined coefficient of friction (μ) and coefficient of restitution (e). The assigned
 318 interaction laws in our NSCD model are listed in Table 3. Here, the various interactions in the numerical
 319 model bodies are chronologically listed, analogous to the construction of real structure.

320 The interaction along the tangential direction between the projectile and the blocks and that between
 321 blocks are governed by the concrete-concrete friction coefficient (μ_{cc}). Similarly, the interaction of each
 322 block of the wall with the ground is governed by the concrete-soil friction coefficient (μ_{cs}). The restitution
 323 coefficient (e) is kept the same for both of these interaction types.

324 Besides, all interactions between steel material and blocks and ground are assigned a constant friction
 325 coefficient of a relatively low magnitude of 0.2 for a minimum energy dissipation through these frictional
 326 interactions in the overall model system. Also, the restitution coefficient with magnitude 0 is assigned
 327 presuming all such contacts being perfectly inelastic (*i.e.*, no rebound after contact). Moreover, as the
 328 sling is a continuous element and bars do not interact with each other, the friction coefficient between the
 329 boxes and sphere of connector elements (vertical play and penetration stopper) is assigned zero. This is

Table 3: Assigning the interaction between model bodies contactors

Construction	Interacting pair (Colliding groups)	Friction	Restitution
Foundation	Block (1) - Ground (9)	μ_{cs}	e
	Cylinder (2) - Ground (9)	0.2	0
Stacking blocks	Block (1) - Block (1)	μ_{cc}	e
Inserting tubes	Odd bar (2) - Disk (5)	0.2	0
	Even bar (3) - Disk (5)	0.2	0
Inserting sling	Box (6) - Sphere (7)	0	0
	Penetration stopper (4) - Sphere (7)	0	0
Impacting the wall	Block (0,1) - Projectile (8)	μ_{cc}	e

330 to indirectly represent consistent stretching in the sling as local vertical plays starts to be fully consumed
331 and subsequently tension in the sling in the event of complete consumption of the available vertical play.
332 Also $e = 0$ in the bar to disk contact assures no relative rebound once the horizontal play is consumed
333 and the bar comes in contact with disk (analogues to tube and concrete block hole interaction in the real
334 structure.)

335 3.3. Simulation database

336 The structure model is impacted at the wall center height and in the middle of its footprint length. A
337 demonstrative post impact deformed structural conformation is presented in Figure 11. Here, the initial
338 (pre-impact) configuration is also presented (similar to Figure 5) where transparent blocks reveal the
339 underlying connector model characteristics.

340 The HDF5 output file of the model impact test provided the detailed information on the displacement,
341 rotation and velocities of each block and thereby enabled a direct comparison with the experimentally
342 obtained results at the corresponding locations. Besides, a detailed output database recorded for the
343 contact forces also enabled the additional post-processing of the model structure behavior extending the
344 limits of the experimentally acquired database to a computationally advanced level.

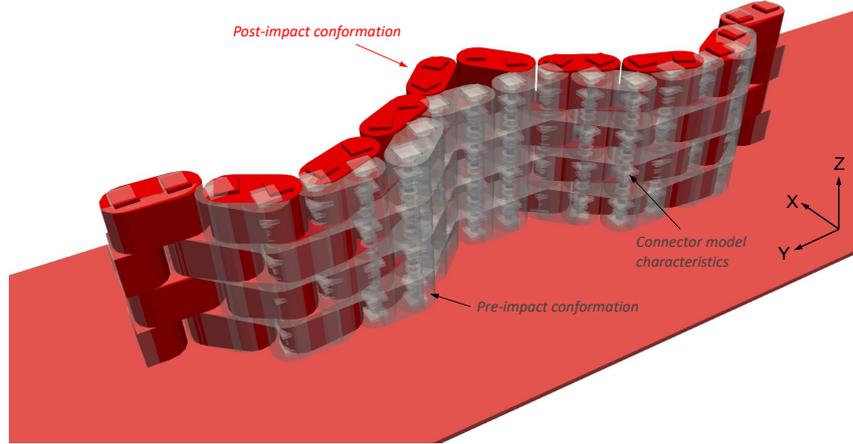


Figure 11: Qualitative representation of initial and post-impact model conformations and highlighted connector model characteristics

3.4. Evaluation of the NSCD model response

The presented NSCD model comprises of many input parameters, most of which are directly taken from the real structure description (Table 1). By contrast, five model parameters couldn't be given a precise value from this description. The first two take into account the imposed constraints for the block-connector (d_z) and connector-connector (v_p) interaction while the other three concern the interaction laws between contacting bodies (μ_{cc} , μ_{cs} and e).

The relative disk position (d_z) could not be precisely derived from the block and tube geometry. The vertical play (v_p) magnitude is variable from place to place in the structure and it is difficult to measure precisely. The coefficients governing friction between blocks (μ_{cc}) and that between the blocks and the soil (μ_{cs}) are difficult to measure precisely as dealing with the dynamic loading of an articulated structure where relative movement between bodies include translation and rotation. Last, the restitution coefficient (e) is a model parameter whose precise value couldn't be derived from some mechanical or geometrical features of the real structure.

These parameters are listed in Table 4 with their range of possible values. The range for the vertical play is derived from observations on the real structure. The range for the disk position is determined as fraction of the block height. The concrete-concrete friction angle is considered with the applicable range information retrieved from the literature [52]. The range for the friction angle between concrete and soil is established assuming slightly higher values than between concrete and concrete. The restitution coefficient magnitude is considered ranging from perfectly inelastic collision (*i.e.*, $e = 0$) up to a magnitude

Table 4: Model parameters with their considered range and mean value

Parameter	Possible range	Mean value	Unit
Bar-block contactor disk position (d_z)	5 - 10	7.5	cm
Vertical play (v_p)	1 - 10	5.5	cm
Friction coefficient concrete-concrete (μ_{cc})	0.25 - 0.55	0.4	(-)
Friction coefficient concrete-soil (μ_{cs})	0.3 - 0.6	0.45	(-)
Restitution coefficient (e)	0 - 0.3	0.15	(-)

364 towards perfectly elastic collision. This upper limit is selected based on multiple simulations such that
 365 the rebound effect does not cause excessive inter-blocks displacement.

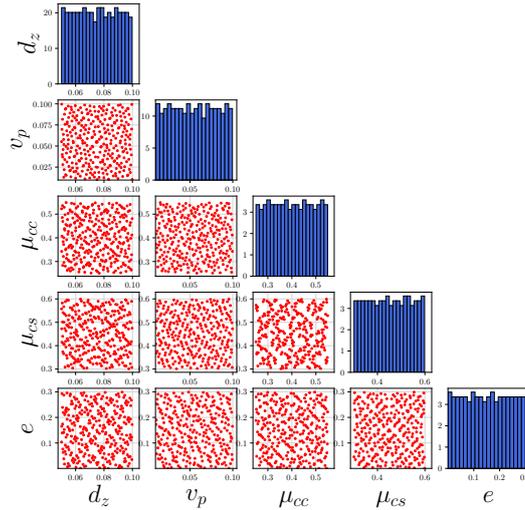


Figure 12: Input sample of size 300 (based on Sobol' sampling method) comprising of five parameters following uniform distribution in their respective ranges of variability

366 As a first step, the overall response of the NSCD model is assessed through a sample set of different
 367 combinations of the five input parameters. This sample set is statistically generated using sampling
 368 methods based on MC and quasi MC (such as Sobol', Halton) [53] implemented in the UQlab input
 369 module [54]. The Sobol' sampling method is used to define a set of 300 combinations of these five input
 370 parameters, as presented in Figure 12. This sampling method provides a random selection of the input
 371 parameters within their predefined ranges such that the input space is well-filled. Besides, without any

372 prior bias to the magnitude of the five model parameters, uniform distribution are assigned for all.

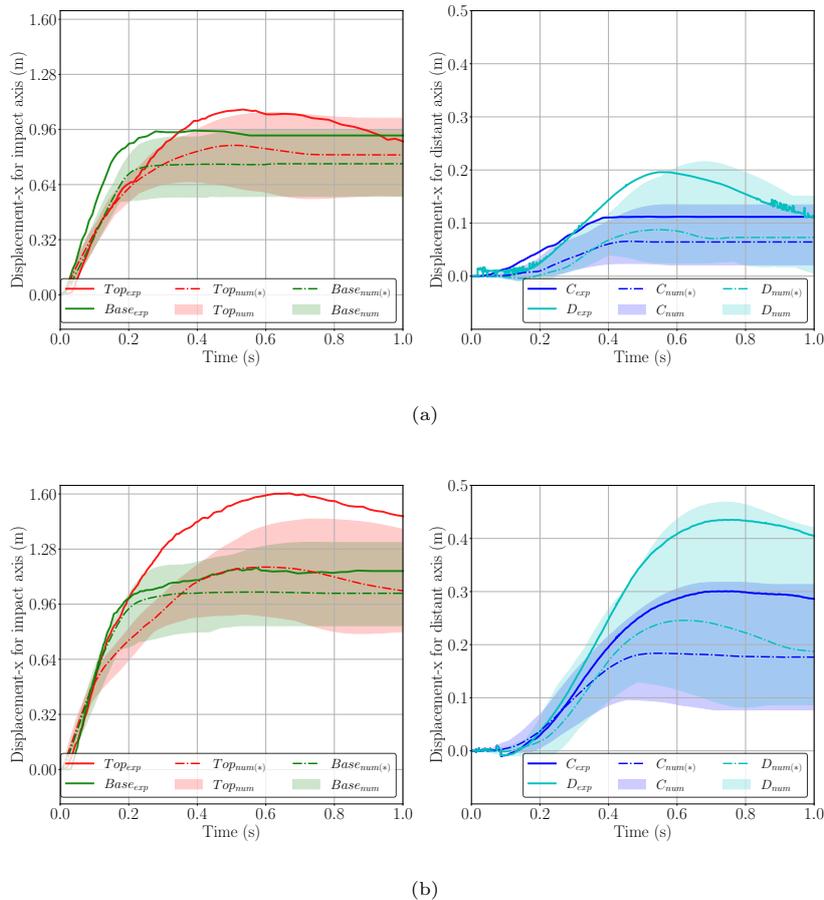


Figure 13: Comparison of the experimental observations against the range of numerical computation response for displacement for (a) 520kJ and (b) 1020 kJ impact energy tests. The response obtained from the deterministic set of mean values of model parameters is also highlighted.

373 The global minimum and maximum range of outcomes from these 300 different simulations is pre-
 374 sented in Figures 13a and 13b together with the NSCD model outcome from the mean value set of the
 375 input parameters. Globally, the range of numerical outcomes envelops the experimental observations at
 376 all measurement points. However, the range of model outcomes for the ‘Top point’ remain below the
 377 experimental observations for the 1020 kJ impact. In this case, concrete blocks in the impact vicinity
 378 experienced severe damage, up to fracture, resulting in a decrease in the mechanical continuity within the
 379 structure, which was thought to result in a higher blocks displacement at the wall top. This introduced
 380 a difference between the real structure and its numerical counterpart where no concrete block fracture is
 381 possible.

382 In spite of this difference, the overall model to experiment correspondence is good considering all the

383 data describing the structure response. This suggests that the NSCD model has the capacity to model
384 the structure global response. Also, the envelop infers that there could exists at least one set of five
385 parameters that allows the NSCD model to possibly replicate the experimental response. This demands
386 an extensive model calibration procedure presented hereafter.

387 4. Calibration of the NSCD model

388 The five parameters presented in Table 4 are calibrated so that the simulated spatio-temporal structure
389 impact response mimics that of the real structure considering four observation points (*i.e.*, points Top,
390 Base, *C* and *D* as shown in Fig. 2). The time evolution of the displacement along the x-axis is accounted
391 for considering three representative time instants. As classically done, the displacement at rest and the
392 maximum displacement are considered. In the following, these are referred to as *rest* and *max* respectively.
393 The third time instant focused on the initial stage where the displacement is fast and varies rather linearly.
394 The corresponding time instant is arbitrarily fixed to 0.1 after the impact beginning in the impact axis
395 (*i.e.*, for points Top and Base) and 0.25 s in the distant plane (*i.e.*, for points *C* and *D*). This third
396 displacement value, hereafter referred to as *init*, aimed at reflecting the dynamics of the structure. The
397 displacement measured at these three time instants are considered to present a global narrative of the
398 structure's displacement, and thus performance in the point of view of its practical application.

399 4.1. Calibration strategy

400 The five model parameters are calibrated considering the displacement at the four measuring points,
401 at the three different times and for the two impact experiments. The calibration is performed using the
402 Bayesian interface statistical learning procedure on the meta-model of the articulated model structure,
403 following the conceptual pathway presented in Figure 14.

404 This complex pathway is motivated by the fact that reaching this ambitious calibration objective
405 demands a very large number of model computations. This is practically impossible with each model
406 computation taking up to 20-30 minutes. This limitation is averted using surrogate models of the structure
407 response for different points at various time instances. The surrogate model development and subsequent
408 model calibration is presented hereafter.

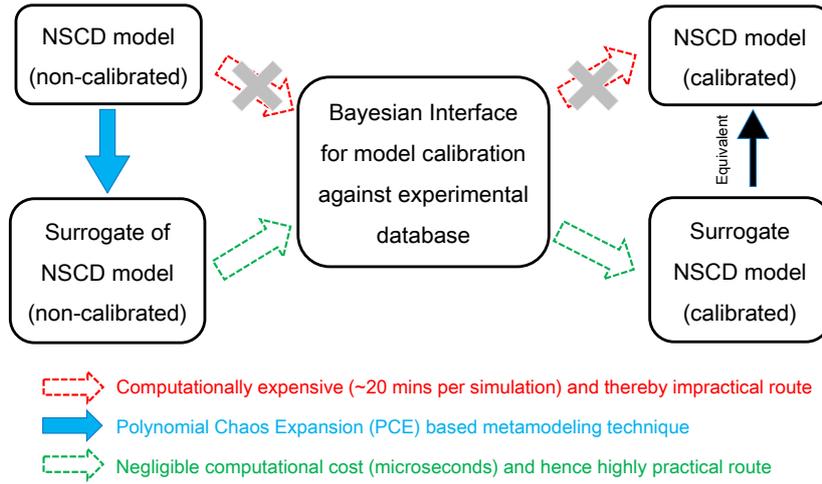


Figure 14: Conceptual work flow of the NSCD model calibration using a combination of stochastic methods

4.2. Meta-model creation and accuracy assessment

A meta-model is a mathematical modeling tool which bridges the gap between the current version of the numerical model and the requisite large number of computations for stochastic analysis. Here, the small statistically created input sample of the model parameters (presented in Figure 12) is used to define the mathematical expression incorporating the model behavior with the desired accuracy. A polynomial chaos expansion (PCE) based meta-modeling technique is used to create the surrogate of the NSCD model in the framework of UQ[py]Lab (a python library of UQLab [55]). Here, the set of results from the 300 NSCD simulations is considered for the meta-model creation following the method described in Appendix A. The resulting mathematical expression (*i.e.*, meta-model) can be analogically referred to as obtaining a regression for a 2D database.

The PCE based meta-models are built upon the displacement response at three time instances-at four different points-for two different energies, thereby, summing to a total of 24 meta models. The accuracy assessment revealed a leave-one-out (LOO) error (Equation A.5) in the order of 10^{-2} to 10^{-1} for all the computational cases. Thereby, as the NSCD model is highly nonlinear and non-smooth, we accept the meta-model accuracy and the underlying uncertainty. Such level of accuracy is deemed sufficient for the upcoming Bayesian interface based calibration method with the advantage of a single model computation within micro-seconds.

Further, the accuracy of the meta-models is assessed by comparing the meta-models predictions with the NSCD model computations for the 520 kJ and 1020 kJ impact tests (Figures 15a and 15b). Notably,

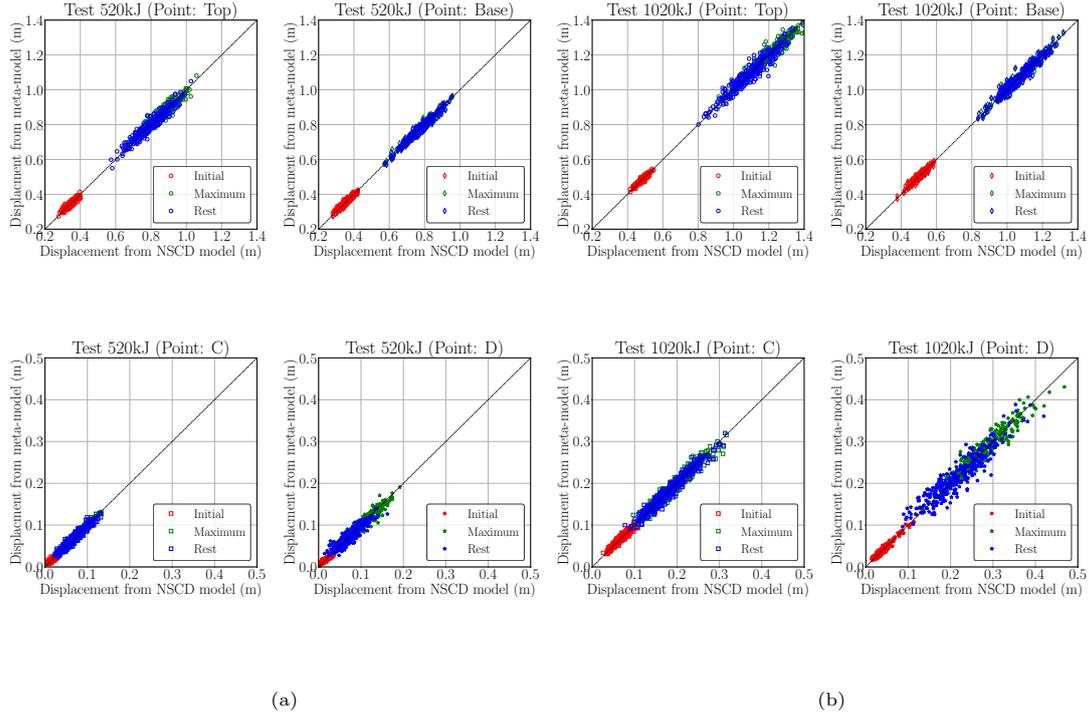


Figure 15: PCE based meta-model validation at three different locations for (a) 520kJ and (b) 1020 kJ impact tests

428 the set of 300 NSCD model simulations that was used to create the meta-model is reused as a validation.
 429 The choice of not using an independent set is made based on the order of magnitude of LOO and the
 430 relative difference between NSCD and meta-model outputs for the same set of input parameters.

431 The cloud of points are rather well aligned with the diagonals in these plots, indicating that the
 432 predicted response by the meta-models fits with the corresponding NSCD simulations results. The
 433 reliability of the predictions along the impact axis is relatively better than in the distant one for both
 434 impacts. The distant axis points move relatively less and presents more divergence from the diagonal
 435 line. It highlights the significance of the variability in model dynamics farther from the impact axis as
 436 the model interaction parameters vary.

437 4.3. Relative influence of the model parameters

438 Before proceeding with the calibration process, the influence of each parameter on the spatio-temporal
 439 displacement response of the structure is investigated through the Sobol sensitivity method also known
 440 as analysis of variance (ANOVA) [56]. This method decomposes the variance of the output parameters as
 441 the sum of the contributions of the different input parameters including the possible interaction between
 442 input parameter. Each contribution is characterized by the ratios of the partial variance to the total

443 variance, called Sobol sensitivity indices. In this study, we used the total Sobol indices to quantify the
 444 total effect of a given input parameter including all possible interactions.

445 The accurate Sobol indices computation demands a large number of model computation (of order 10^6).
 446 This is highly impractical in the present study if the NSCD model is to be used directly. Nonetheless,
 447 the meta-model of the NSCD model can be directly used to compute Sobol indices at zero cost [57, 58].
 448 The UQlab sensitivity analysis module [59] is used for such computations. The total Sobol indices for
 449 the three time instances, at all considered points and for both impact tests are presented in Figure 16.

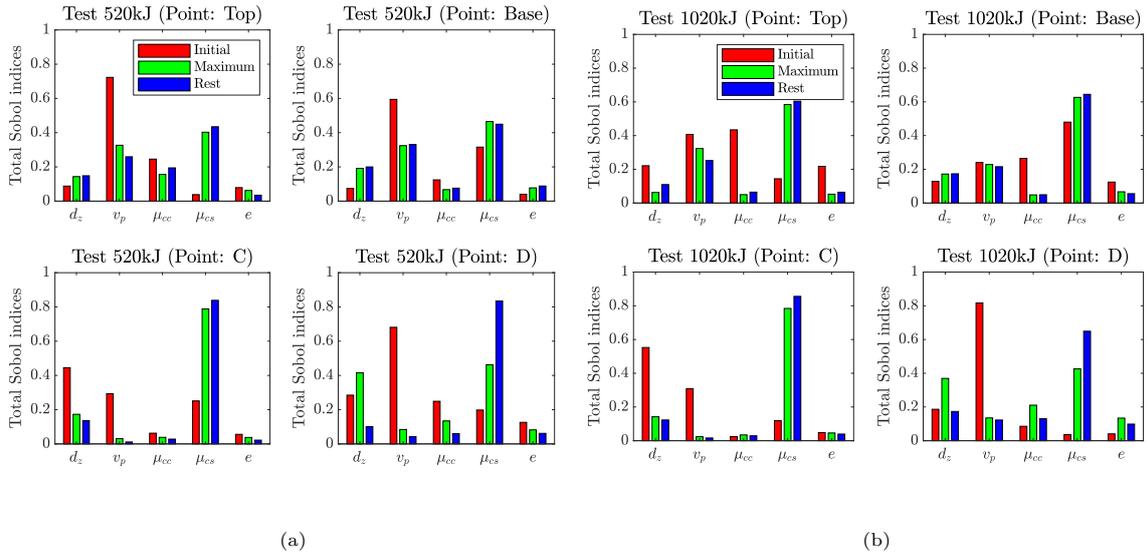


Figure 16: Total Sobol indices of the spatio-temporal displacement response for the sensitivity analysis of the NSCD model parameters for (a) 520kJ and (b) 1020kJ impact test

450 Overall, these results aid in understanding the structure response highlighting the mechanisms at work
 451 with time, complimenting the discussion in [27]. The Sobol indices are observed significantly differ from
 452 one parameter to the other. The higher values are observed for the vertical play (v_p) and concrete-to-soil
 453 friction coefficient (μ_{cs}). For example, the Sobol index of v_p exceeds 0.7 for the initial displacement at
 454 point D , for both impact energies. Also, the Sobol index of μ_{cs} exceeds 0.8 for the displacement at rest
 455 at point C , for both impact energies. Even though other parameters globally exhibit lower Sobol indices,
 456 none is negligible.

457 For example, the restitution coefficient (e) globally exhibit the lower Sobol indices, but a value higher
 458 than 0.2 is obtained for the initial displacement at the top for the 1020kJ impact. Figure 16 also reveals
 459 that the values of Sobol indices for a given parameter vary over time and differ from one impact energy
 460 to the other. The Sobol indices of v_p exceeded 0.7 considering the initial displacement at the top for the

461 the 520kJ impact, while it is close to zero considering the displacement at rest at point C for the 1020-kJ
462 impact. Similar observations are made on the influence of other parameters, as for example comparing
463 the Sobol index of e for the maximal displacement at point C during the 520kJ impact (0.04) to that for
464 the initial displacement at the top during the 1020kJ impact (0.22). The non-zero Sobol indices for each
465 parameter at all computed time instances validates the identifiability of all parameters [60] for upcoming
466 Bayesian interface based studies.

467 Besides, the sum of indices for a given situation exceeds one (in some cases) which expresses the
468 interaction between input parameters, *i.e.*, the influence of some parameters being dependent on the
469 value of some other parameters. The influence of the vertical play, v_p , on the displacement of the upper
470 blocks during both impact tests decreased with time, from the so-called initial stage to the position
471 at rest. This highlighted that the significant upward displacement (*i.e.*, positive z -axis) of the blocks
472 initiated in the early stage after the impact beginning. By contrast, the influence of the concrete-to-soil
473 friction coefficient, μ_{cs} , showed a global increasing trend with time, which is attributed to the progressive
474 base sliding of the wall. Furthermore, the Sobol index of the concrete-to-concrete friction coefficient, μ_{cc}
475 suggested that the relative displacement between blocks, is much higher in the upper part of the structure
476 (Top and Point D). Overall, non-zero magnitude of all parameters affirms to consider their variability
477 for calibrating the structure response presented hereafter.

478 *4.4. Bayesian interface for model calibration*

479 The Bayesian process for model calibration is inspired from the Bayes' theorem [61] which is simply a
480 representation of the changing beliefs. In simple words, it demonstrates that probability of a hypothesis
481 being correct becomes more reliable when supported with the concerning evidences. In the context of the
482 present work, our hypothesis is that the 300 sets of possible input parameters give a range of displacement
483 values output that is correct. In addition, the experimental observations data is the evidence recorded.
484 Now, the idea of increasing the reliability on our hypothesis given the evidence would be to say that there
485 exist at least one set of input parameters such that the recorded evidence is reproduced and hence, in
486 other words, the model is calibrated. In the context of model calibration, the goal is to find the optimal
487 values of the input parameters that allow one to best fit the model predictions to the observations. A
488 brief description of the Bayesian's interface is presented in [Appendix B](#).

489 4.4.1. Representation

490 The available displacement experimental data at the points of interest and at three time instances are
491 taken as a benchmark to calibrate the NSCD model parameters to obtain the requisite set of parameters.
492 The Bayesian interface, similar to a typical stochastic analysis method, demands a large number of
493 model computations for its reliable functionality. In the present work, such essential requirement poses
494 computational time constraint as each NSCD model computational run lasts for about 20 minutes.

495 This point motivated the use of PCE based meta-models of NSCD model, presented above, to accel-
496 erate Bayesian computations, an approach extensively used by many researchers [62, 63, 64]. Here, these
497 meta-models serve as a forward model (or prior) which are created through an extended version (*i.e.*,
498 larger sample size) of the input sample presented in Figure 12, thanks to the surrogate NSCD models.
499 The meta-model based calibrated set of input parameters (*i.e.*, posterior) is thereby a representative of
500 the NSCD model calibration.

501 The discrepancy (see Equation B.5) of order 10^{-20} is manually assigned. This choice is made with an
502 underlying hypothesis that the experimental measurements are very precise and we wish to retrieve the
503 model input parameters which result in the corresponding measurements. Notably, this value influences
504 the calibration process and thereby in further study its magnitude should be carefully chosen so as to
505 represent the true model and the observation error.

506 4.4.2. Implementation

507 The Bayesian interface is implemented to obtain a set of input parameters considering three approaches
508 referred to as ‘point’, ‘energy’ and ‘all’. These three approaches are inspired from the scope of user’s
509 interest to reproduce the experimental outcome which ranges from a particular time instance calibration to
510 overall model behavior calibration in both space and time. The first approach amounts to giving priority
511 for a given point in the calibration process, meaning that the model is calibrated to provide precise
512 predictions over time for this specific point. The second approach, ‘energy’ is in line with strategies
513 consisting in calibrating a model at a given impact energy before using it for an other impact energy. By
514 contrast with the two previous ones, the ‘all’ approach aims at ensuring that the model provides reliable
515 predictions at any point and whatever the impact energy. Bayesian interface module in UQLab [65] is
516 used to implement all three calibration approaches.

517 The calibrated sets of input parameters are reported in Table 5 obtained as a maximum a posteriori

Table 5: Calibrated sets of model parameters following three different approaches

Parameter	Approaches											Units
	Point								Energy		All	
	520kJ				1020kJ				520kJ	1020kJ		
	Top	Base	C	D	Top	Base	C	D				
d_z	5.7	9.6	5.3	5.4	9.5	6.2	6.8	5.0	7.2	6.8	6.8	cm
v_p	5.2	3.8	6.7	6.5	9.0	8.0	7.6	7.0	6.5	7.1	7.1	cm
μ_{cc}	0.251	0.419	0.474	0.265	0.378	0.512	0.510	0.342	0.351	0.323	0.316	(-)
μ_{cs}	0.311	0.304	0.340	0.346	0.306	0.351	0.301	0.301	0.320	0.308	0.307	(-)
e	0.190	0.194	0.138	0.033	0.152	0.290	0.283	0.200	0.198	0.220	0.222	(-)

518 (MAP) and mean point estimates (see Equation B.15) for ‘point’ approach and for both other approaches
519 respectively. Depending on calibration approach, significant differences in parameter values are observed.
520 For example, v_p varied from 3.8 to 9 cm depending on the calibration strategy. A ratio of about two
521 between the minimum and the maximum values attributed to a parameter is also observed for d_z and e
522 in particular. Interestingly, the set of values calibrated following the ‘all’ approach is very close to that
523 obtained following the ‘energy’ approach while considering the 1020kJ impact.

524 4.4.3. Interpretation

525 The displacement response comparison between experimental results and NSCD model results ob-
526 tained considering the magnitude of these input parameters is presented in Figure 17. The interpretation
527 on these observations is presented hereafter.

528 The first approach (*i.e.*, ‘point’ approach) results in a total of four calibrated set of input parameters
529 for each impact test. This approach allows to represent the influence of model dynamics at different
530 points as the best set of input parameters is reported distinct for each point and for both energies. The
531 experimental and numerical response, respectively in *black* and *red* in Figure 17, are in a very good
532 quantitative and qualitative agreement with each other justifying the usage of stochastic methods for
533 model calibration. This very good agreement emphasizes the success of choosing only three representative
534 time instances over the whole impact duration. The significant difference observed at top point for

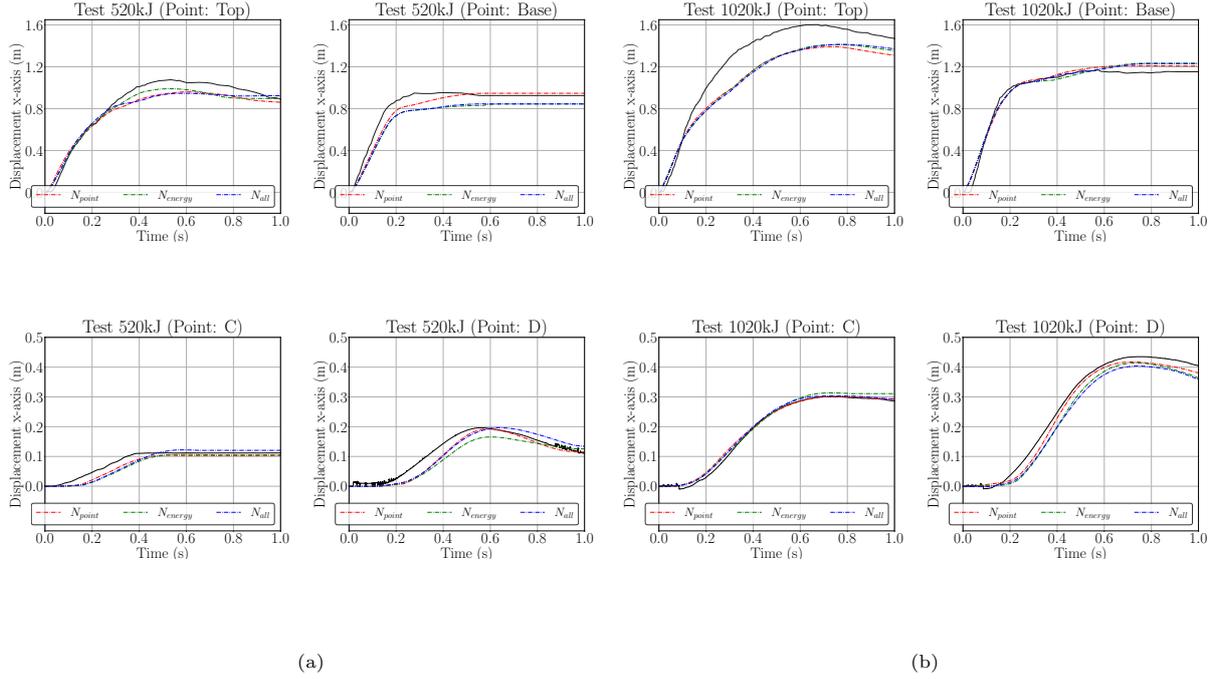


Figure 17: Bayesian inversion based calibration of (a) 520kJ and (b) 1020kJ impact test. Curves in *black* show the experimental data. Curves in *red*, *green* and *blue* show the NSCD simulation results corresponding to approaches ‘point’, ‘energy’ and ‘all’, respectively.

535 1020kJ test is attributed to the loss of mechanical continuity as discussed in section 3.4. The obtained
 536 eight distinct sets of parameters (Table 5) highlight the significance of local dependency of the model
 537 constitution to accurately replicate the experimental response.

538 The second approach (*i.e.*, ‘energy’) collectively takes into account the model outputs at three time
 539 instances for all four measurement points for a given impact test. This way, two calibrated sets of input
 540 parameters are obtained from the Bayesian interface one each for 520kJ and 1020kJ impact test. This
 541 approach presents the influence of the impact energy on the global model response. The corresponding
 542 response obtained from the NSCD model run (*in green*) is compared against the experimental observation.
 543 Similar to the ‘point’ approach, a good qualitative agreement is observed between the experimental and
 544 numerical responses. Courtesy to the multi-model calibration possibility (through group Likelihood
 545 computation, see Equation B.10) in Bayesian interface of UQlab [65], the recorded ‘energy’ response
 546 using a single set of input parameters presented a similar response to the four distinct sets of these
 547 parameters in ‘point’ approach for each energy level.

548 The third approach (*i.e.*, ‘all’) collectively takes into account the model outputs at three time instances
 549 for all four measurement points for both impact tests. Hence, a single set of calibrated input parameters

550 is obtained representing the global response of the NSCD model, with time and space. Here again, the
 551 comparison between the corresponding response obtained from the NSCD model (*in blue*) is in good
 552 qualitative and quantitative agreement with the experimental observation.

553 4.4.4. Application

554 Finally, the counterpart of the input sample distribution before the calibration (see Figure 12, the
 555 posterior input sample distribution obtained from the ‘all’ approach is presented in Figure 18. The data-
 556 point corresponding to the calibrated set - here and in different locations in the text is also highlighted.
 557 Notably, from the histogram of the outputs, all input parameters present a locally emphasized magnitudes
 558 corresponding to the good fits with the desired output. However, the final set of parameters deviates
 559 slightly from the most probable magnitude of a particular input parameter. This difference is negligible
 560 for all parameters except μ_{cc} where the final selected parameters magnitude is slightly larger than the
 561 most probable one. Besides, a very narrow range of μ_{cs} parameter indirectly highlights its relatively high
 562 sensitivity to the model response (as also highlighted in Figure 16).

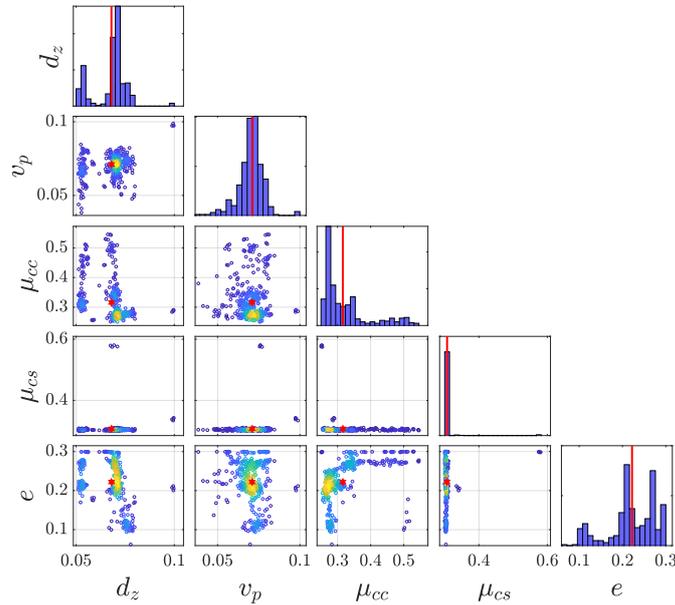
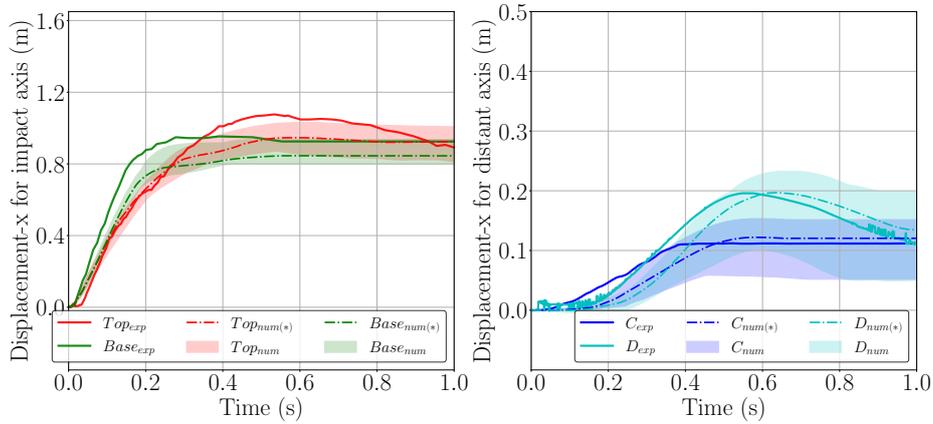


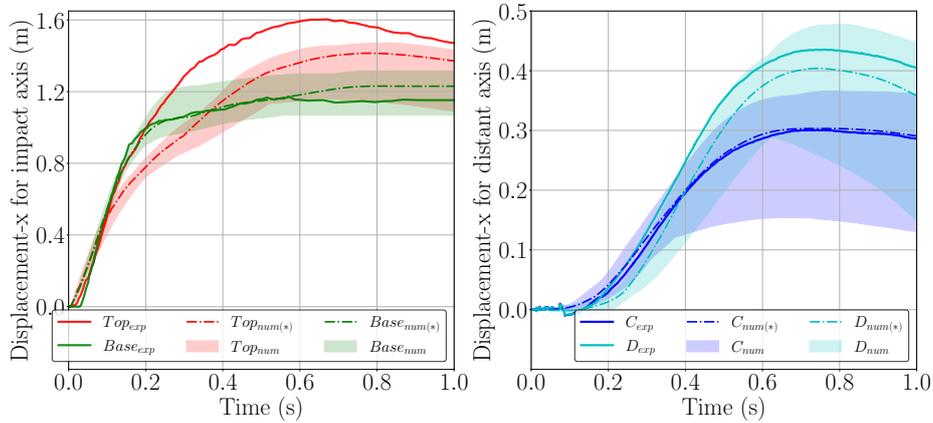
Figure 18: Posterior sample distribution of five model parameters obtained from Bayesian interface application as per ‘all’ approach. The mean point estimate (in red) highlights the calibrated values of the respective model parameters

563 Further, as the counterpart of the displacement response before the calibration process (see Figure 13),
 564 the post-calibration response (from ‘all’ approach) is presented in Figure 19. Overall, a good qualitative
 565 agreement between experimental and numerical observations is reported favoring the calibrated set of

566 input parameters. From the quantitative comparison of the collective experimental and numerical (pos-
567 terior mean) displacement measurements at three time representative, a root mean square error (RMSE)
568 of 9.9cm and 2.3cm is reported for impact and distant axis respectively which account for about 8.6%
569 and 5.6% of representative displacements from the respective axis. A relatively better accuracy to repro-
570 duce the distant axis response favors the model practical utility. Continuing with the loss of mechanical
571 continuity argument for the top point during the 1020-kJ test, the reported relatively large quantitative
572 and qualitative difference is accepted and also is not considered in RMSE estimation.



(a) posterior for 520kJ



(b) posterior for 1020kJ

Figure 19: Comparison of the experimental displacement response against the NSCD model response obtained from the best calibrated set of input parameters along with the band of possible posterior sample based outcomes for (a) 520kJ and (b) 1020kJ impact tests respectively

573 Moreover, the range of possible model outputs is also presented using all set of input parameters from
574 the posterior distribution. The relatively narrow width of the band in comparison to the prior sample

575 validates the computational efficiency of the Bayesian interface. Besides, in co-relation to the input
576 sample distribution, most of the output trends in the band are likely to be closer to the posterior-mean
577 response. Nonetheless, such co-relation is not made as the eventual goal is to obtain a deterministic set
578 of input parameters for further utility of NSCD model.

579 Globally, the NSCD model is well-calibrated as a positive consequence of the Bayesian interface
580 approach where a large number of model computations are favored by the usage of the PCE based
581 meta-models. As an extension, for a better and more reliable model calibration, the diversity in the
582 experimental data both in time and space is favorable.

583 **5. Discussion**

584 *5.1. On the calibration strategy*

585 The results presented in section 4.4 revealed that very different sets of model parameters may result
586 in similar responses over time at a given point in the structure. This is evidenced comparing the NSCD
587 model results when calibrated following the ‘point’ approach to that following the ‘all’ approach. The
588 coherent response obtained through the presented three approaches also expresses the advantage of not
589 prioritizing the response recorded at a particular point (using ‘point’ approach) to represent the overall
590 model behavior. This is because of the large variability in the reported input parameters at different
591 points where a particular set corresponding to a particular point (if prioritized) may not necessarily
592 provide a good response at other point and by extension at a different energy level.

593 In addition, these results also revealed that the parameter values for a calibrated model are extremely
594 dependent on the calibration strategy. Ratios of about two are observed between extreme values of a
595 given model parameter from one strategy to the other. This is consistent with the idea that the intended
596 use of the model should be taken into account when calibrating the model. Indeed, a calibration based
597 on the displacement with time at a single point in the structure will result in a set of model best fit
598 parameters that may not be reliable for estimating the structure response at another location.

599 Overall, a relatively low quantitative difference in the displacement output responses between all three
600 approaches is observed, demonstrating that calibrating the model considering a large number of data is
601 not at the expense of a reduction in prediction capacity as compared to models calibrated for a more
602 specific purpose. In other words, the agreement with the experimental data at a given point of a model

603 calibrated according to the ‘all’ approach is nearly similar to that obtained with the model calibrated for
 604 each specific point, following the ‘point’ approach.

605 To demonstrate this, the limitation of ‘point’ calibration approach to correctly represent spatial-
 606 temporal displacement response is demonstrated in Figure 20. Here, the calibration set corresponding
 607 to the point Base of 520kJ impact test is presumed representative of the whole structure’s response. It
 608 is clearly seen that, despite a very good qualitative and quantitative agreement with the experimental
 609 response at it’s own identity, it fails to correctly reproduce the responses at nearly all remaining locations
 610 for 520kJ test and by extension to all four locations of the 1020kJ energy test. Hence, the significance of
 611 using all spatio-temporal database is justified.

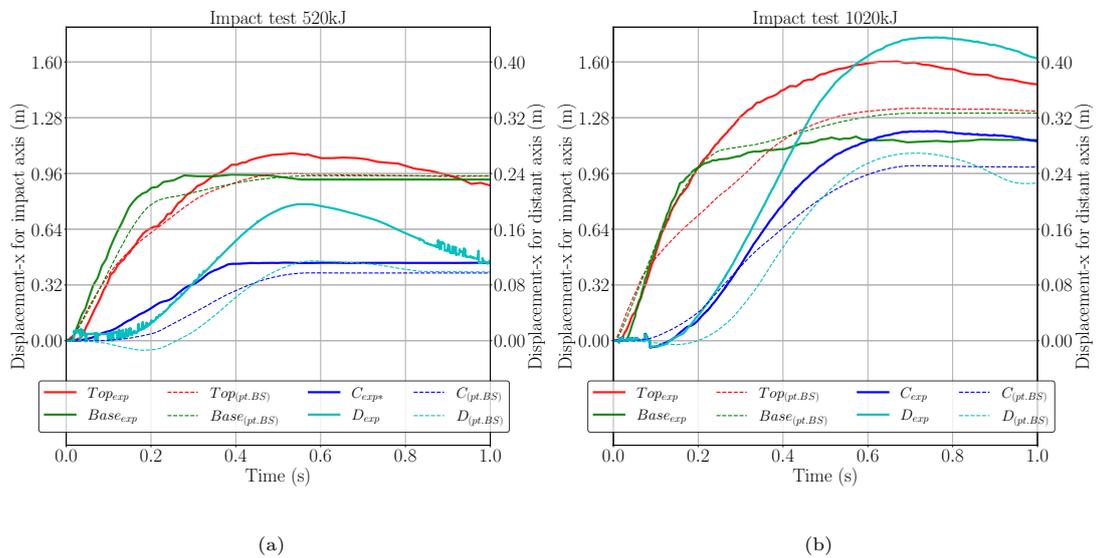


Figure 20: Comparison of experimental displacement response with the NSCD model response obtained from the ‘point’ Base calibration set from 520kJ for (a) 520 kJ and (b) 1020kJ impact test

612 The successful model calibration also justified the usage of the Bayesian interface approach in place of
 613 a manual model calibration where some emphasis is typically given to a particular point and/or particular
 614 time instance. In case the model is to be calibrated based on a simplest approach, such as Monte Carlo
 615 simulations, would fail in satisfactorily describing the complex spatio-temporal response of the structure.
 616 Besides, these procedures do not provide the distribution of the calibrated parameters which help in
 617 understanding the uncertainty in calibration and also the possibility of finding more than one calibration
 618 set. This limitations justify considering an extensive stochastic calibration approach through Bayesian
 619 interface.

620 5.2. Model efficacy

621 The five model parameters are calibrated so that the simulation results conformed with the eight
622 displacement evolution experimental database describing the spatio-temporal response of the structure
623 when exposed to two impacts with different kinetic energies (520 and 1020 kJ). This way to proceed is
624 particularly well adapted for this model for which parameter values can not be precisely measured or
625 determined a priori. The model parameter e can not be directly compared to a measurable mechanical
626 characteristic. Besides, the contact disk position is a parameter associated with the approach considered
627 for modeling the connectors passing through the blocks. Also, the uncertainty and variability associated
628 with the vertical play and friction angles justify such a calibration approach. This calibration approach,
629 which makes use of PCE based meta-model, could advantageously be used for calibrating any relevant
630 model where parameters cannot be precisely determined a priori.

631 The other asset of this calibration approach is that the best set of model parameters is defined from
632 a quantitative evaluation of the deviation of the simulation results with the experimental data. When
633 considering many output data, model parameters calibration most often relies on a qualitative evaluation
634 of the simulated response with time, without any quantification of the deviation. By contrast, the
635 proposed calibration approach guaranties that the set of determined parameter values is the optimal for
636 describing the structure response with time and space.

637 Also, the presented simple NSCD model has limited assumptions and is immune to the bias of a typical
638 FE model definition (constitutive laws, interface properties and so on) making the complex calibration
639 procedure reliable. Besides, the combination of the meta-model and Bayesian interface presented a
640 practical route to surpass the computational time constraint. The average computational time of a
641 NSCD model simulation is about 20 minutes. Thereby, the initial investment in the stochastic analysis
642 is 300 NSCD model runs to retrieve the essential database. In addition, a similar numerical modeling
643 conducted by Furet et al. [27] using the finite element based software reported the computation time of 10
644 hours for one simulation. This highly significant difference further highlights the practical implementation
645 potential of the NSCD model given the output of interest.

646 5.3. Energy dissipation

647 Estimating the energy dissipative capacities of the protective structures exposed to a localized dynamic
648 loading is of paramount importance for understanding their mechanical response and improving their

649 design. Numerous research related to rockfall protection have considered the energy issue based on
650 simulation results, with application to flexible barriers [66, 67, 68], embankments and walls [14, 27] and
651 dissipative materials or components [69, 70, 71]. The provided data mainly aimed at explaining how the
652 incident block kinetic energy is transferred and dissipated in the impacted structure and its foundation.

653 As for articulated concrete blocks structures, finite difference numerical simulations presented in [27]
654 suggested that 50% of the projectile kinetic energy is dissipated by concrete plasticization due to shear
655 and tension. The NSCD model proposed in the present study accounts for energy dissipation via e , μ_{cc}
656 and μ_{cs} . The former indirectly accounted for damage to concrete blocks, while the two latter accounted
657 for dissipation by friction between structure components. From the Bayesian interace results, it is evident
658 that different sets of model parameters could result in very similar results, in particular when focusing on
659 some specific locations of the structure. Ratios up to two are observed between parameter values from
660 one calibrated model to the other, including the parameters governing energy dissipation (e , μ_{cc} and μ_{cs}).
661 Obviously, the difference in model parameters has an influence on the contribution of each dissipative
662 mechanism.

663 In the end, this highlights that simulation results interpreted in terms of energy dissipation should
664 be considered with caution. The contribution by the various dissipative mechanisms (plasticization,
665 friction...) strongly depends on the modeling strategy, on the constitutive laws and on the value attributed
666 to the model parameters. This comes in addition to the classical energy conservation issue associated with
667 some numerical schemes. These general but critical comments undoubtedly holds for any numerical model
668 of structures exposed to dynamic loading, such as rockfall protective structures (e.g. [14, 27, 66, 68]).
669 Further investigation is necessary for evaluating the influence of the model parameters on the contribution
670 by each mechanisms to energy dissipation.

671 **6. Conclusions and perspectives**

672 In this work, a numerical model of a structure consisting of concrete blocks connected one to the
673 others via metallic components and designed to intercept rockfall is presented and successfully calibrated.
674 The model is kept simple and developed in SICONOS software package based on NSCD approach for
675 computation time saving purpose. NSCD revealed efficient for modeling such a complex structure where
676 many components interact one with each other and some are invisible to each others. Each component

677 in the real structure is directly and indirectly reproduced in the NSCD model considering the interaction
678 mechanisms and the design framework of objects with contactors in SICONOS. An average computation
679 time of 20 mins is reported for the NSCD model in comparison to highly expensive FEM computations
680 lasting for about 10 hours for the same computation.

681 Five model parameters required calibration. Three parameters governed the mechanical interaction
682 between the various model components and two concerned the structure description. The calibration of
683 these five model parameters is conducted based on the displacement with time at four different locations
684 in the structure measured during real-scale experiments considering two projectile kinetic energies. The
685 calibration is conducted considering Bayesian interface statistical learning method, accompanied by the
686 meta-modeling techniques. This is done in view of accounting for the spatial and temporal displace-
687 ment response of the structure upon a projectile impact. The meta-modeling techniques presented a
688 surrogate of the NSCD model which represented the similar response albeit negligible computation time
689 (microseconds) in comparison to the approximated 20 minutes time for one NSCD model computation.
690 Subsequently, the large number of model computations are made possible by meta-models for the efficient
691 usage of Bayesian interface for model calibration. Besides, the Sobol sensitivity analysis is made possible
692 through the surrogate model which presented the relative influence of one parameter to the other both
693 in space and time.

694 In the calibration process different calibration approaches, ranging from local point to the global
695 structure's response are presented. It is evidenced that the model parameters value significantly depended
696 on the calibration strategy, meaning on the number and variety of data used in this purpose. The final
697 set of parameters value is reported to nearly mimic the spatial-temporal response of the real structure for
698 both impact tests. An overall quantitative deviation of numerical results with the experimental evolution
699 is reported to be 8.6% for the impact axis and 5.6% for the distant axis.

700 As a perspective, the developed NSCD model will be used to investigate the response of articulated
701 concrete blocks structures when exposed to impacts under different conditions, in view of quantifying
702 their real efficiency, in a similar approach as in Mentani et al. [72], Toe et al. [29] and Lambert et al. [30].

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708 **References**

- 709 [1] M. Jean, J. J. Moreau, Dynamics in the presence of unilateral contacts and dry friction: a numerical
710 approach, in: G. Del Pietro, F. Maceri (Eds.), Unilateral problems in structural analysis. II, CISM
711 304, Springer Verlag, 1987, pp. 151–196.
- 712 [2] M. Jean, The non smooth contact dynamics method, *Computer Methods in Applied Mechanics and*
713 *Engineering* 177 (1999) 235–257. Special issue on computational modeling of contact and friction,
714 J.A.C. Martins and A. Klarbring, editors.
- 715 [3] F. Dubois, V. Acary, M. Jean, The Contact Dynamics method: A nonsmooth story , *Comptes*
716 *Rendus Mécanique* 346 (2018) 247–262.
- 717 [4] V. Acary, B. Brogliato, Numerical methods for nonsmooth dynamical systems. Applications in
718 mechanics and electronics., *Lecture Notes in Applied and Computational Mechanics* 35. Berlin:
719 Springer. xxi, 525 p. , 2008.
- 720 [5] V. Acary, M. Jean, Numerical simulation of monuments by the contact dynamics method, in:
721 DGEMN-LNEC-JRC (Ed.), Monument-98, Workshop on seismic performance of monuments, Laboratório Nacional de engenharia Civil (LNEC), Lisboa, Portugal, Lisbon, Portugal, 1998, pp. 69–78.
722 URL: <https://hal.inria.fr/inria-00425359>.
- 723
- 724 [6] V. Acary, J. Blaise, P. Drap, M. Florenzano, S. Garrec, M. Jean, D. Merad, Nscd method applied to
725 mechanical simulation of masonry in historical buildings using moma, in: *International Symposium*
726 *WG3—Simple methods for architectural photogrammetry*, 1999, pp. 1–8.
- 727 [7] A. Rafiee, M. Vinches, C. Bohatier, Application of the nscd method to analyse the dynamic behaviour
728 of stone arched structures, *International Journal of Solids and Structures* 45 (2008) 6269–6283.

- 729 [8] F. Radjai, M. Jean, J. J. Moreau, S. Roux, Force Distributions in Dense Two-Dimensional Granular
730 Systems, *Physical Review Letters* 77 (1996) 274 – 277.
- 731 [9] M. Jean, V. Acary, Y. Monerie, Non-smooth contact dynamics approach of cohesive materials,
732 *Philosophical Transactions of the Royal Society of London Series A Mathematical and Physical*
733 *Sciences (1934-1990)* 359 (2001) 2497–2518.
- 734 [10] F. Bourrier, V. Acary, Predictive Capabilities of 2D and 3D Block Propagation Models Integrating
735 Block Shape Assessed from Field Experiments, *Rock Mechanics and Rock Engineering* 55 (2022)
736 591–609.
- 737 [11] N. Akhadkar, V. Acary, B. Brogliato, Multibody systems with 3D revolute joints with clearances:
738 an industrial case study with an experimental validation, *Multibody System Dynamics* 42 (2018)
739 249–282.
- 740 [12] M. Jean, G. Touzot, Implementation of unilateral contact and dry friction in computer codes dealing
741 with large deformations problems., *Journal of theoretical and applied Mechanics* (1988) 145–160.
- 742 [13] M. Wronski, M. Jean, Some computational aspects of structural dynamics problems with frictional
743 contact, *Contact Mechanics* (1995) 137–144.
- 744 [14] C. Ronco, C. Oggeri, D. Peila, Design of reinforced ground embankments used for rockfall protection,
745 *Natural Hazards and Earth System Science* (2009).
- 746 [15] M. Simmons, S. Pollak, B. Peirone, High energy rock fall embankment constructed using a free-
747 standing woven wire mesh reinforced soil structure, in: *60th Highway Geology Symposium, 2009*,
748 pp. 290–301.
- 749 [16] S. Lambert, F. Bourrier, Design of rockfall protection embankments: A review, *Engineering Geology*
750 154 (2013) 77–88.
- 751 [17] O. Korini, M. Bost, J.-P. Rajot, Y. Bennani, N. Freitag, The influence of geosynthetics design on the
752 behavior of reinforced soil embankments subjected to rockfall impacts, *Engineering Geology* (2021).
- 753 [18] N. T. K. Lam, A. C. Y. Yong, C. Lam, J. S. H. Kwan, J. S. Perera, M. M. Disfani, E. Gad,
754 Displacement-based approach for the assessment of overturning stability of rectangular rigid barriers
755 subjected to point impact, *Journal of Engineering Mechanics* (2018).

- 756 [19] A. Yong, C. Lam, N. Lam, J. Perera, J. Kwan, Analytical solution for estimating sliding displacement
757 of rigid barriers subjected to boulder impact, *Journal of Engineering Mechanics* (2019).
- 758 [20] M. Çelik, Investigating the performance of passageway corridor for ground reinforced embankments
759 against rockfall , *Journal of Mountain Science* (2023).
- 760 [21] T. Colgan, E. Ewe, Design of a large scale rockfall protection bund for coastal transport corridor
761 recovery following the 2016 kaikōura earthquake, in: *Pacific conference on earthquake engineering*,
762 2019, pp. 1–12.
- 763 [22] R. Green, J. S. Finlan, Rapid Design of a Modular Rockfall Protection Wall in Response to the 2016
764 KaiKoura Earthquake, 2021, pp. 151–165. doi:[10.1061/9780784483688.015](https://doi.org/10.1061/9780784483688.015).
- 765 [23] J. P. Perera, N. Lam, Rockfall protection wall that can withstand multiple strikes without needing
766 to be repaired, *International Journal of Impact Engineering* 173 (2023) 104476.
- 767 [24] S. Lambert, F. Bourrier, P. Gotteland, F. Nicot, An experimental investigation of the response of
768 slender protective structures to rockfall impacts, *Canadian geotechnical journal* (2019).
- 769 [25] C. Williams, J. Morkeh, K. Dorfschmidt, C. Poon, P. Matlashewski, J. Carvalho, Innovative rockfall
770 solutions based on calibration and field testing, *Mining, Metallurgy & Exploration* (2021).
- 771 [26] A. Furet, S. Lambert, P. Villard, J. P. Jarrin, J. Lorentz, Response of rockfall protection walls
772 subjected to impact, *Revue française de géotechnique* (2020).
- 773 [27] A. Furet, S. Lambert, P. Villard, J. J. Philippe, Experimental and numerical impact responses of an
774 innovative rockfall protection structure made of articulated concrete blocks , *Rock Mechanics and*
775 *Rock Engineering* (2022).
- 776 [28] F. Bourrier, S. Lambert, J. Baroth, A reliability-based approach for the design of rockfall protection
777 fences , *Rock Mechanics and Rock Engineering* (2015).
- 778 [29] D. Toe, A. Mentani, L. Govoni, F. Bourrier, G. Gottardi, S. Lambert, Introducing meta-models for
779 a more efficient hazard mitigation strategy with rockfall protection barriers, *Rock Mechanics and*
780 *Rock Engineering* (2018).
- 781 [30] S. Lambert, D. Toe, A. Mentani, F. Bourrier, A Meta-Model-Based Procedure for Quantifying the
782 On-Site Efficiency of Rockfall Barriers, *Rock Mechanics and Rock Engineering* 54 (2021) 487–500.

- 783 [31] V. Acary, F. Perignon, Siconos: A Software Platform for Modeling, Simulation, Analysis and Control
784 of Nonsmooth Dynamical Systems, SIMULATION NEWS EUROPE, ArgeSIM/ASIM 17 (2007) 19–
785 26.
- 786 [32] V. Acary, O. Bonnefon, M. Brémond, O. Huber, S. Sinclair, V. Acary, O. Bonnefon, M. Brémond,
787 O. Huber, F. Pérignon, V. Acary, O. Bonnefon, M. Brémond, O. Huber, An introduction to Siconos,
788 Technical report (2019).
- 789 [33] V. Acary, Energy conservation and dissipation properties of time-integration methods for nonsmooth
790 elastodynamics with contact, Journal of Applied Mathematics and Mechanics / Zeitschrift für
791 Angewandte Mathematik und Mechanik 96 (2016) 585–603.
- 792 [34] J. Escallón, C. Wendeler, E. Chatzi, P. Bartelt, Parameter identification of rockfall protection barrier
793 components through an inverse formulation, Engineering Structures 77 (2014) 1–16.
- 794 [35] D. Rossat, J. Baroth, M. Briffaut, F. Dufour, A. Monteil, B. Masson, S. Michel-Ponnelle, Bayesian
795 inference with correction of model bias for thermo-hydro-mechanical models of large concrete struc-
796 tures, Engineering Structures 278 (2023) 115433.
- 797 [36] M. B. Heredia, N. Eckert, C. Prieur, E. Thibert, Bayesian calibration of an avalanche model from au-
798 tocorrelated measurements along the flow: application to velocities extracted from photogrammetric
799 images, Journal of Glaciology 66 (2020) 373–385.
- 800 [37] M. Asch, M. Bocquet, M. Nodet, Data Assimilation, Society for Indus-
801 trial and Applied Mathematics, Philadelphia, PA, 2016. URL: <https://epubs.siam.org/doi/abs/10.1137/1.9781611974546>. doi:10.1137/1.9781611974546.
802 [arXiv:https://epubs.siam.org/doi/pdf/10.1137/1.9781611974546](https://epubs.siam.org/doi/pdf/10.1137/1.9781611974546).
- 803
- 804 [38] M. A. Hariri-Ardebili, B. Sudret, Polynomial chaos expansion for uncertainty quantification of dam
805 engineering problems, Engineering Structures 203 (2020) 109631.
- 806 [39] X. Guo, D. Dias, C. Carvajal, L. Peyras, P. Breul, Reliability analysis of embankment dam sliding
807 stability using the sparse polynomial chaos expansion, Engineering Structures 174 (2018) 295–307.
- 808 [40] R. Gupta, D. Rossat, X. Dérobert, J. Baroth, M. Briffaut, G. Villain, F. Dufour, Blind comparison

- 809 of saturation ratio profiles on large rc structures by means of ndt and sfe—application to the vercors
810 mock-up, *Engineering Structures* 258 (2022) 114057.
- 811 [41] G. D. Saxce, Z. Q. Feng, New inequality and functional for contact with friction: The implicit
812 standard material approach, *Mechanics of Structures and Machines* 19 (1991) 301–325.
- 813 [42] V. Acary, M. Brémond, O. Huber, *Advanced Topics in Nonsmooth Dynamics.*, Acary, V. and Brüls.
814 O. and Leine, R. (eds). Springer Verlag, 2018, p. 375–457.
- 815 [43] P. A. Cundall, O. D. Strack, A discrete numerical model for granular assemblies, *geotechnique* 29
816 (1979) 47–65.
- 817 [44] N. Bićanić, Discrete element methods, *Encyclopedia of computational mechanics* (2004).
- 818 [45] Q. Z. Chen, V. Acary, G. Virlez, O. Brüls, A nonsmooth generalized- α scheme for flexible multibody
819 systems with unilateral constraints, *International Journal for Numerical Methods in Engineering* 96
820 (2013) 487–511.
- 821 [46] O. Brüls, V. Acary, A. Cardona, On the Constraints Formulation in the Nonsmooth Generalized- α
822 Method, in: S. I. Publishing (Ed.), *Advanced Topics in Nonsmooth Dynamics. Transactions of the*
823 *European Network for Nonsmooth Dynamics*, 2018, pp. 335–374. URL: <https://hal.inria.fr/hal-01878550>. doi:10.1007/978-3-319-75972-2_9.
- 824
- 825 [47] O. Brüls, V. Acary, A. Cardona, Simultaneous enforcement of constraints at position and veloc-
826 ity levels in the nonsmooth generalized- α scheme, *Computer Methods in Applied Mechanics and*
827 *Engineering* 281 (2014) 131–161.
- 828 [48] T. Schindler, S. Rezaei, J. Kursawe, V. Acary, Half-explicit timestepping schemes on velocity level
829 based on time-discontinuous Galerkin methods, *Computer Methods in Applied Mechanics and En-*
830 *gineering* 290 (2015) 250–276.
- 831 [49] T. Schindler, V. Acary, Timestepping schemes for nonsmooth dynamics based on discontinuous
832 Galerkin methods: Definition and outlook, *Mathematics and Computers in Simulation* 95 (2014)
833 180–199.
- 834 [50] C. B. Barber, D. P. Dobkin, H. Huhdanpaa, The quickhull algorithm for convex hulls, *ACM Trans.*
835 *Math. Softw.* 22 (1996) 469–483.

- 836 [51] EOTA, Ead 340059-00-0106: Falling rock protections kits, 2018.
- 837 [52] M. Taklas, M. Leblouba, S. Barakat, A. Fageeri, F. Mohamad, Concrete-to-concrete shear friction
838 behavior under cyclic loading: experimental investigation, *Scientific Reports* 12 (2022) 9451.
- 839 [53] H. Niederreiter, 1. Monte Carlo Methods and Quasi-Monte Carlo Methods, 1992, pp. 1–
840 12. URL: <https://epubs.siam.org/doi/abs/10.1137/1.9781611970081.ch1>. doi:10.1137/1.
841 9781611970081.ch1. arXiv:<https://epubs.siam.org/doi/pdf/10.1137/1.9781611970081.ch1>.
- 842 [54] C. Lataniotis, S. Marelli, B. Sudret, UQLab user manual – The Input module, Technical Report,
843 Chair of Risk, Safety & Uncertainty Quantification, ETH Zurich, 2015. Report UQLab-V0.9-102.
- 844 [55] S. Marelli, B. Sudret, UQLab: A Framework for Uncertainty Quantification in Matlab, 2014, pp.
845 2554–2563. doi:10.1061/9780784413609.257.
- 846 [56] I. M. Sobol, A screening design for factorial experiments with interactions, *Mathematical and*
847 *Computer Modelling* 1 (1993) 407–414.
- 848 [57] B. Sudret, Global sensitivity analysis using polynomial chaos expansions, *Reliability Engineering &*
849 *System Safety* 93 (2008) 964–979. Bayesian Networks in Dependability.
- 850 [58] E. Rouzies, C. Lauvernet, B. Sudret, A. Vidard, How to perform global sensitivity analysis of a
851 catchment-scale, distributed pesticide transfer model? application to the peshmelba model, *Geosci-*
852 *entific Model Development* (under review).
- 853 [59] S. Marelli, C. Lamas, B. Sudret, UQLab user manual - sensitivity analysis, 2015. doi:10.13140/RG.
854 2.1.1419.4404.
- 855 [60] S. Dobre, T. Bastogne, C. Profeta, M. Barberi-Heyob, A. Richard, Limits of variance-based sensi-
856 tivity analysis for non-identifiability testing in high dimensional dynamic models, *Automatica* 48
857 (2012) 2740–2749.
- 858 [61] T. Bayes, n. Price, Lii. an essay towards solving a problem in the doctrine of chances. by the late rev.
859 mr. bayes, f. r. s. communicated by mr. price, in a letter to john canton, a. m. f. r. s, *Philosophical*
860 *Transactions of the Royal Society of London* 53 (1763) 370–418.
- 861 [62] Y. Marzouk, D. Xiu, A stochastic collocation approach to bayesian inference in inverse problems,
862 *Communications in Computational Physics* (2009).

- 863 [63] Y. M. Marzouk, H. N. Najm, Dimensionality reduction and polynomial chaos acceleration of bayesian
864 inference in inverse problems, *Journal of Computational Physics* 228 (2009) 1862–1902.
- 865 [64] L. Yan, L. Guo, Stochastic collocation algorithms using L₁-minimization for bayesian solution of
866 inverse problems, *SIAM Journal on Scientific Computing* 37 (2015) A1410–A1435.
- 867 [65] P.-R. Wagner, J. Nagel, S. Marelli, B. Sudret, UQLab user manual – Bayesian inversion for model
868 calibration and validation, Technical Report, Chair of Risk, Safety and Uncertainty Quantification,
869 ETH Zurich, Switzerland, 2022. Report UQLab-V2.0-113.
- 870 [66] H. Xu, C. Gentilini, Z. Yu, X. Qi, S. Zhao, An energy allocation based design approach for flexible
871 rockfall protection barriers, *Engineering Structures* 173 (2018) 831–852.
- 872 [67] L. Castanon-Jano, E. Blanco-Fernandez, D. Castro-Fresno, Design of a new energy dissipating device
873 and verification for use in rockfall protection barriers, *Engineering Structures* 199 (2019) 109633.
- 874 [68] D. Di Giacinto, L. Grassia, G. Capriello, E. Ruocco, A novel steel damping system for rockfall
875 protection galleries, *Journal of Constructional Steel Research* 175 (2020) 106360.
- 876 [69] L. Zhang, S. Lambert, F. Nicot, Discrete dynamic modelling of the mechanical behaviour of a
877 granular soil, *International Journal of Impact Engineering* 103 (2017) 76–89.
- 878 [70] M. Previtali, M. O. Ciantia, S. Spadea, R. P. Castellanza, G. B. Crosta, Multiscale modelling of
879 dynamic impact on highly deformable compound rockfall fence nets, *Proceedings of the Institution
880 of Civil Engineers - Geotechnical Engineering* 174 (2021) 498–511.
- 881 [71] S. Yan, Y. Wang, D. Wang, S. He, Application of eps geofoam in rockfall galleries: Insights from
882 large-scale experiments and fdem simulations, *Geotextiles and Geomembranes* 50 (2022) 677–693.
- 883 [72] A. Mentani, L. Govoni, G. Gottardi, S. Lambert, F. Bourrier, D. Toe, A new approach to evaluate
884 the effectiveness of rockfall barriers, *Procedia Engineering* 158 (2016) 398–403. VI Italian Conference
885 of Researchers in Geotechnical Engineering, CNRIG2016 - Geotechnical Engineering in Multidisciplinary
886 Research: from Microscale to Regional Scale, 22-23 September 2016, Bologna (Italy).
- 887 [73] S. Marelli, B. Sudret, Uqlab user manual - polynomial chaos expansions, 2015. doi:[10.13140/RG.2.
888 1.3778.7366](https://doi.org/10.13140/RG.2.1.3778.7366).

889 **Appendix A. PCE based meta-model**

890 Consider the articulated structure model represented by a $\mathcal{M}(\mathbf{X})$ as an equivalent mathematical
 891 model. Here, $\mathbf{X} \in \mathbb{R}^M$ is a random vector with independent components described by the joint probability
 892 density function (PDF) $f_{\mathbf{X}}$. Consider also a finite variance computational model as a map $Y = \mathcal{M}(\mathbf{X})$,
 893 with $Y \in \mathbb{R}$ such that:

$$\mathbb{E} [Y^2] = \int_{\mathcal{D}_{\mathbf{X}}} \mathcal{M}(\mathbf{x})^2 f_{\mathbf{X}}(\mathbf{x}) d\mathbf{x} < \infty \quad (\text{A.1})$$

894 Then, under the assumption of Equation A.1, the PCE of $\mathcal{M}(\mathbf{X})$ is defined as:

$$Y = \mathcal{M}(\mathbf{X}) = \sum_{\alpha \in \mathbb{N}^M} y_{\alpha} \Psi_{\alpha}(\mathbf{X}) \quad (\text{A.2})$$

895 where, the $\Psi_{\alpha}(\mathbf{X})$ are multivariate polynomials orthonormal with respect to $f_{\mathbf{X}}$, $\alpha \in \mathbb{N}^M$ is a multi-
 896 index that identifies the components of the multivariate polynomials Ψ_{α} and the $y_{\alpha} \in \mathbb{R}$ are the corre-
 897 sponding coefficients. In practical applications, the sum in Equation A.2 needs to be truncated to a finite
 898 sum by introducing the truncated polynomial chaos expansion:

$$\mathcal{M}(\mathbf{X}) \approx \mathcal{M}^{PC}(\mathbf{X}) = \sum_{\alpha \in \mathcal{A}} y_{\alpha} \Psi_{\alpha}(\mathbf{X}) \quad (\text{A.3})$$

899 where, $\mathcal{A} \subset \mathbb{N}^M$ is the set of selected multi-indices of multivariate polynomials.

900 In this work, the least-angle regression (LARS) method is used to create the PCE meta-model trun-
 901 cated to the maximum polynomial degree (p) ranging from 1 to 20, and using hyperbolic truncation
 902 scheme (q) ranging from 0.5 to 1.

$$\mathcal{A}^{M,p,q} = \{\alpha \in \mathcal{A}^{M,p} : \|\alpha\|_q \leq p\}, \text{ where } \|\alpha\|_q = \left(\sum_{i=1}^M \alpha_i^q \right)^{1/q} \quad (\text{A.4})$$

903 The accuracy of the constructed PCE is estimated by computing the leave-one-out (LOO) cross-
 904 validation error (ϵ_{LOO}). It consists in building N meta-models $\mathcal{M}^{PC \setminus i}$, each one created on a reduced
 905 experimental design $\mathfrak{X} \setminus \mathbf{x}^{(i)} = \{\mathbf{x}^{(j)}, j = 1, \dots, N, j \neq i\}$ and comparing its prediction on the excluded
 906 point $\mathbf{x}^{(i)}$ with the real value $y^{(i)} = \mathcal{M}(\mathbf{x}^{(i)})$ [73]. The leave-one-cross-validation error can be written as:

$$\epsilon_{LOO} = \frac{\sum_{i=1}^N \left(\mathcal{M}(\mathbf{x}^{(i)}) - \mathcal{M}^{PC \setminus i}(\mathbf{x}^{(i)}) \right)^2}{\sum_{i=1}^N \left(\mathcal{M}(\mathbf{x}^{(i)}) - \hat{\mu}_Y \right)^2} \quad (\text{A.5})$$

907 where, $\hat{\mu}_Y$ is the mean of the experimental design sample.

908 **Appendix B. Bayesian interface for model calibration**

909 Consider the computational model \mathcal{M} that allows the analyst to predict certain quantities of interest
 910 gathered in a vector $\mathbf{y} \in \mathbb{R}^{N_{out}}$ as a function of input parameters \mathbf{x} :

$$\mathcal{M} : \mathbf{x} \in \mathcal{D}_{\mathbf{X}} \subset \mathbb{R}^M \mapsto \mathbf{y} = \mathcal{M}(\mathbf{x}) \in \mathbb{R}^{N_{out}} \quad (\text{B.1})$$

911 The Bayesian interface for model calibration focus on identifying the input parameters of a computa-
 912 tional model to recover the observations in the collected output data-set. It comprises of a computational
 913 forward model \mathcal{M} , a set of input parameters $\mathbf{x} \in \mathcal{D}_{\mathbf{X}}$ that need to be inferred, and a set of experimental
 914 data \mathcal{Y} . Here, $\mathcal{Y} \stackrel{\text{def}}{=} \{\mathbf{y}_1, \dots, \mathbf{y}_N\}$ is a global data set of N independent measured quantities of interest
 915 (\mathbf{y}_i).

916 The forward model $\mathbf{x} \mapsto \mathcal{M}(\mathbf{x})$ is a mathematical representation of the system under consideration.
 917 The lack of knowledge on the input parameters is modeled by considering them as a random vector,
 918 denoted by \mathbf{X} which is assumed to follow a so-called prior distribution (with support $\mathcal{D}_{\mathbf{X}}$), as presented
 919 in Figure 12 in the present work.

$$\mathbf{X} \sim \pi(\mathbf{x}) \quad (\text{B.2})$$

920 The Bayesian statistics combines this prior knowledge on the parameters with the few observed data
 921 points to obtain a statistical model called posterior distribution ($\pi(\mathbf{x} | \mathbf{y})$) of the input parameters, using
 922 Bayes' theorem [61], expressed as:

$$\pi(\mathbf{x} | \mathbf{y}) = \frac{\pi(\mathbf{y} | \mathbf{x})\pi(\mathbf{x})}{\pi(\mathbf{y})} \quad (\text{B.3})$$

923 Now, considering the available data set (\mathcal{Y}) as independent realizations of $\mathbf{Y} | \mathbf{x} \sim \pi(\mathbf{y} | \mathbf{x})$, the
 924 collected measurements result in the definition of the likelihood function $\mathcal{L}(\mathbf{x}; \mathcal{Y})$, which is a function of
 925 input parameters \mathbf{x} :

$$\mathcal{L} : \mathbf{x} \mapsto \mathcal{L}(\mathbf{x}; \mathcal{Y}) \stackrel{\text{def}}{=} \prod_{i=1}^N \pi(\mathbf{y}_i | \mathbf{x}) \quad (\text{B.4})$$

926 This implicitly assumes independence between individual measurements in \mathcal{Y} . Intuitively the like-
 927 lihood function for a given \mathbf{x} returns the relative likelihood of observing the data at hand, under the
 928 assumption that it follows the prescribed parametric distribution $\pi(\mathbf{y} | \mathbf{x})$.

929 As all models are simplification of the real world, a discrepancy term (ϵ) is introduced to connect real
 930 world observations (\mathcal{Y}) to the predictions by the model. In practice, the discrepancy term represents the
 931 effects of the measurement error and model inaccuracy. The discrepancy term introduced here reads:

$$\mathbf{y} = \mathcal{M}(x) + \epsilon \quad (\text{B.5})$$

932 Here, the ϵ is assumed as an additive Gaussian discrepancy [65] with a zero mean and given covariance
 933 matrix (Σ):

$$\epsilon \sim \mathcal{N}(\epsilon | \mathbf{0}, \Sigma) \quad (\text{B.6})$$

934 Taking insights from the discrepancy term definition, a particular measurement point ($\mathbf{y}_i \in \mathcal{Y}$, is a
 935 realization of the Gaussian distribution with mean value $\mathcal{M}(x)$ and covariance matrix Σ . This distribution
 936 is named as discrepancy model and is expressed as:

$$\pi(\mathbf{y} | \mathbf{x}) = \mathcal{N}(\mathbf{y} | \mathcal{M}(\mathbf{x}), \Sigma) \quad (\text{B.7})$$

937 In application, the discrepancy model defines the connection between the supplied data (\mathcal{Y}) and the
 938 forward model. In the present work, as the model calibration relies on a single experimental measurement,
 939 the discrepancy model with known residual variance is assigned.

940 Afterwards, the N independent available measurements gathered in the data-set \mathcal{Y} are used to define
 941 the likelihood function as:

$$\begin{aligned} \mathcal{L}(\mathbf{x}; \mathcal{Y}) &= \prod_{i=1}^N \mathcal{N}(\mathbf{y}_i | \mathcal{M}(\mathbf{x}), \Sigma) \\ &= \prod_{i=1}^N \frac{1}{\sqrt{(2\pi)^{N_{out}} \det(\Sigma)}} \exp\left(-\frac{1}{2}(\mathbf{y}_i - \mathcal{M}(\mathbf{x}))^\top \Sigma^{-1}(\mathbf{y}_i - \mathcal{M}(\mathbf{x}))\right) \end{aligned} \quad (\text{B.8})$$

942 Moreover, in the present work, the experimental data is retrieved through various points and for
 943 different impact energies (*i.e.*, $\mathcal{Y} = \{\mathbf{y}_1, \dots, \mathbf{y}_N\}$). The Bayesian interface allows to incorporate all these
 944 data points together for the model calibration by arranging the elements of \mathcal{Y} in disjoint data-groups and
 945 define different likelihood functions for each data group [65].

946 Denoting the g -th data group by $\mathcal{G}^g = \{\mathbf{y}_i\}_{i \in \mathbf{u}}$, where $\mathbf{u} \subseteq \{1, \dots, N\}$, the full data-set can be combined
 947 by

$$\mathcal{Y} = \bigcup_{g=1}^{N_{gr}} \mathcal{G}^{(g)} \quad (\text{B.9})$$

948 Here, each of the N_{gr} data groups contains measurements collected at the same measurement point.
 949 In the context of present work, these measurements are experimental data recorded at ‘init’, ‘max’ and
 950 ‘rest’ time instances at a particular point. This makes it evident to have a different likelihood function
 951 $\mathcal{L}^{(g)}$ describing the experimental conditions that led to measuring $\mathcal{G}^{(g)}$. Assuming the Independence
 952 between N_{gr} measurement conditions, the full likelihood function can then be written as

$$\mathcal{L}(\mathbf{x}_{\mathcal{M}}, \mathbf{x}_{\epsilon}; \mathcal{Y}) = \prod_{g=1}^{N_{gr}} \mathcal{L}^{(g)}(\mathbf{x}_{\mathcal{M}}, \mathbf{x}_{\epsilon}^{(g)}; \mathcal{G}^{(g)}) \quad (\text{B.10})$$

953 where, $\mathbf{x}_{\epsilon}^{(g)}$ are the parameters of the g -th discrepancy group. The different model output groups are
 954 assigned through a model output map (MOMap) vector [65].

955 Thereby, following the Bayes’ theorem, the posterior distribution $\pi(\mathbf{x} | \mathcal{Y})$ of the parameters (\mathbf{x}) given
 956 the observations in \mathcal{Y} can be written as:

$$\pi(\mathbf{x} | \mathcal{Y}) = \frac{\mathcal{L}(\mathbf{x}; \mathcal{Y})\pi(\mathbf{x})}{Z} \quad (\text{B.11})$$

957 Here, Z is a normalizing factor, known as the marginal likelihood or evidence, is added to ensure that
 958 this distribution integrates to 1:

$$Z \stackrel{\text{def}}{=} \int_{\mathcal{D}_{\mathbf{x}}} \mathcal{L}(\mathbf{x}; \mathcal{Y})\pi(\mathbf{x})d\mathbf{x} \quad (\text{B.12})$$

959 The closed form solutions do not exist in practice the posterior distribution is obtained through
 960 Markov chain Monte Carlo (MCMC) simulations. In the present work, among many proposed algorithms
 961 in [65], the Adaptive Metropolis (AM) algorithm is used and 100 parallel chains with 1000 steps are
 962 assigned to the MCMC solver. The start of the covariance adaptation in AM algorithm is assigned at
 963 100th step (see Wagner et al. [65] for more details).

964 The output (\mathbf{y}) predictive capabilities of the Bayesian interface is assessed through the comparison of
 965 prior ($\pi(\mathbf{y})$) and posterior ($\pi(\mathbf{y} | \mathcal{Y})$) output distributions as:

$$\pi(\mathbf{y}) = \int_{\mathcal{D}_{\mathbf{x}}} \pi(\mathbf{y} | \mathbf{x})\pi(\mathbf{x})d\mathbf{x} \quad (\text{B.13})$$

$$\pi(\mathbf{y} | \mathcal{Y}) = \int_{\mathcal{D}_{\mathbf{x}}} \pi(\mathbf{y} | \mathbf{x})\pi(\mathbf{x} | \mathcal{Y})d\mathbf{x} \quad (\text{B.14})$$

966 Lastly, in the present studies, the purpose of Bayesian interface is to obtain the ‘best set of input
 967 parameters’. Given the posterior distribution ($\pi(\mathbf{x} | \mathcal{Y})$), we are interested in finding a suitable set among
 968 the posterior computed set *i.e.*, $\mathbf{X} | \mathcal{Y}$. This is done through a point estimator ($\hat{\mathbf{x}}$) computed from:

$$\pi(\mathbf{y} | \mathcal{Y}) \stackrel{\text{def}}{=} \pi(\mathbf{y} | \hat{\mathbf{x}}) \quad (\text{B.15})$$

969 This point estimator can be a mean or mode (maximum a posteriori ‘MAP’) [65] of the posterior
 970 distribution as per user’s choice.