



HAL
open science

A matheuristic for the 2D bounded-size cutting stock problem

Alexis Robbes, Khadija Hadj Salem

► **To cite this version:**

Alexis Robbes, Khadija Hadj Salem. A matheuristic for the 2D bounded-size cutting stock problem. 24ème Congrès Annuel de la Société Française de Recherche Opérationnelle et Aide à la Décision, Feb 2023, Rennes, France. hal-04039120

HAL Id: hal-04039120

<https://hal.science/hal-04039120>

Submitted on 21 Mar 2023

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

A matheuristic for the 2D bounded-size cutting stock problem

Alexis Robbes¹, Khadija Hadj Salem²

¹ Université de Sorbonne, CNRS, UMR 7606 LIP6, 4 place Jussieu, 75005 Paris, France

alexis.robbes@lip6.fr

² Mines Saint-Etienne, Univ Clermont Auvergne, CNRS, UMR 6158 LIMOS, Institut Henri Fayol, F-42023 Saint-Étienne, France




khadija.hadjsalem@emse.fr

Keywords : *Cutting stock, bounded size, strips generation, mat-heuristic, neural networks.*

1 Problem definition

In this paper, we consider a variant of the *two-dimensional Cutting Stock Problem* (2D-CSP). First introduced in 1961 by [1], the 2D-CSP consists of cutting small 2D items in larges plates. Often the 2D-CSP is considered with a fixed number of guillotine cuts to produce rectangular items. Even with these two constraints, the 2D-CSP is \mathcal{NP} -hard. In the typology of [5], a CSP with a strongly heterogeneous assortment of plates is named a *Residual Cutting Stock Problem* as in practice the heterogeneity is due to a previous cutting step. However, some factories can manufacture requested plates with highly customizable sizes using adjustable molds. For this type of production, the plate sizes are bounded and the cutting objective is to minimize the total production. We call this variant as the *two-dimensional Bounded-Size Cutting Stock Problem* (2D-BSCSP).

More formally, we are given a set of rectangular items I , where each item i has a width w_i , a height h_i , and a demand d_i . These items must be cut in a two-stage guillotine manner from plates of width in $[W^{LB}, W^{UB}]$ and height in $[H^{LB}, H^{UB}]$. A manual trimming step is allowed to separate items and waste parts. The objective is to find a cutting plan with the minimum sum of plate area.

An illustrative example of the 2D-BSCSP is given in Figure 1, where a set $I = \{1, 2, 3\}$ of item types has to be cut from plates with bounded sizes. The feasible solution contains only one pattern i.e. a cutting plan on one plate. This pattern has a width of $2 \cdot w_1$ and height of H^{LB} . Three types of waste are produced,  waste produced by first stage guillotine cut,  waste produced by second stage guillotine cut, and  waste produced by handmade trimming.

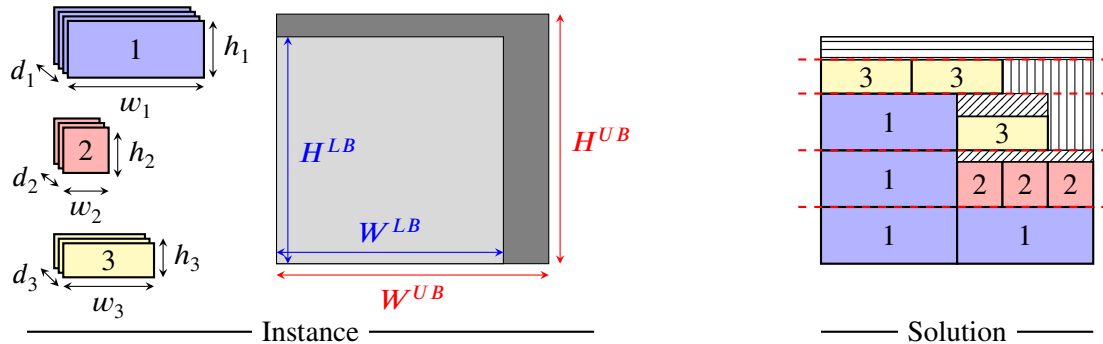


FIG. 1 – An instance of 2D-BSCSP and a feasible solution

2 A strip-based matheuristic

2D-CSPs were tackled through various approaches, mainly based on generating cutting patterns. Just a few methods were considered for the concept of strips, e.g. in [3]. Several approximate algorithms start by generating strips, filling them with pieces, and subsequently searching for the “best” combination of these strips. More recently in [4], a strip-based compact formulation for guillotine 2D-CSPs is proposed, in which a set of two decision variables is considered to define the position of cut of each item type.

To solve the 2D-BSCSP, we first propose a new *strip-based MILP* formulation, where a binary variable represents if a strip is in the same pattern as another one. The formulation is inspired by a recent modeling of the Bin Packing problem proposed in [2]. As the sizes of the plates are bounded, we also consider one continuous variable per strip which represents the wasted height of the pattern defined by this strip.

We then propose a matheuristic based on our *strip-based MILP* formulation. The general flowchart of the proposed matheuristic is shown in Figure 2. In the first step, the set of all possible strip type Θ (respecting W^{LB} and W^{UB}) is generated, then a relaxed version of the problem where $H^{LB} = 0$ is quickly solved with an ILP. With the resulting strip selection S , a compatibility matrix ($c_{\theta\sigma}$) is computed. The compatibility between two strip types θ and σ is based on the gap between the minimum waste pattern containing θ and the minimum waste pattern containing θ and σ together (restrain on S). Thanks to a threshold parameter, the weakest compatibilities are ignored and a sparse binary compatibility matrix ($\delta_{\theta\sigma}$) is constructed. To obtain a feasible heuristic the instance is solved with our *strip-based MILP* restrain on the subset of strip S and the sparse compatibility matrix ($\delta_{\theta\sigma}$). As the S is not necessarily the optimal strip selection, a loop removes the less compatible strip type computed ($\arg \min_{\sigma} (\sum_{\theta} c_{\theta\sigma})$) and the less gainful strip type (i.e. the strip type with the worst use ratio in the best solution).

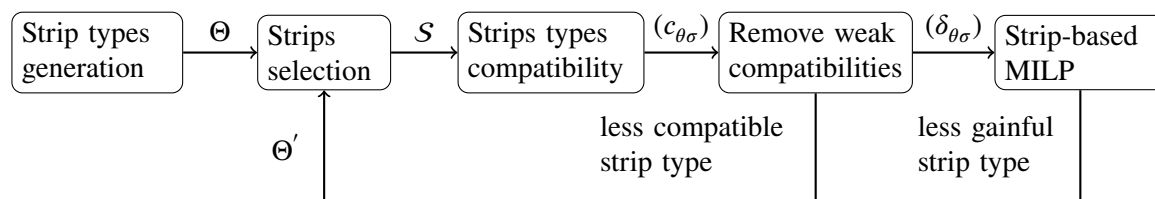


FIG. 2 – An instance of 2D-BSCSP and a feasible solution

We implemented the proposed approaches and conducted our experiments using two sets of instances from the literature, available at <http://or.dei.unibo.it/library/2dpacklib>. Numerical results will be given and analyzed during the presentation.

Références

- [1] Paul C Gilmore and Ralph E Gomory. A linear programming approach to the cutting-stock problem. *Operations research*, 9(6) :849–859, 1961.
- [2] Khadija Hadj Salem and Yann Kieffer. New symmetry-less ilp formulation for the classical one dimensional bin-packing problem. In *International Computing and Combinatorics Conference*, pages 423–434. Springer, 2020.
- [3] Mhand Hifi and Rym M’Hallah. Strip generation algorithms for constrained two-dimensional two-staged cutting problems. *European Journal of Operational Research*, 172(2) :515–527, 2006.
- [4] Carlos Diego Rodrigues, Adriana Cristina Cherri, and Silvio Alexandre de Araujo. Strip based compact formulation for two-dimensional guillotine cutting problems. *Computers & Operations Research*, page 106044, 2022.
- [5] Gerhard Wäscher, Heike Haußner, and Holger Schumann. An improved typology of cutting and packing problems. *European journal of operational research*, 183(3) :1109–1130, 2007.