

Added viscous damping in a finite-size microperforated plate within a nonlinear framework

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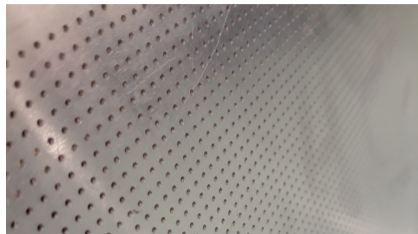
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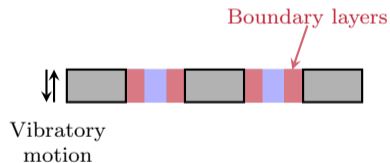
Microperforated plate (MPP)

- ▶ Simple and lightweight structures
- ▶ Sound and vibration absorbers on its own
- ▶ Added viscous damping in low frequency range¹



Dissipation mechanisms

- ▶ Visco-thermic effects in boundary layers
- ▶ Fluid-structure interaction
- ▶ Sensitive to excitation level

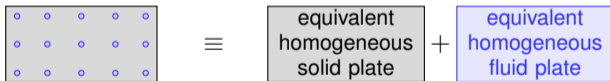
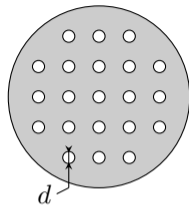


Objective Explore the impact of excitation level on MPP added damping

¹Gallerand et al. "Vibration and damping analysis of a thin finite-size microperforated plate". (2022)

Linear regime – Modeling steps

1. MPP \equiv Porous medium² $\Rightarrow \phi(\mathbf{d}), \sigma_0(\mathbf{d})$ and $\alpha_\infty(\phi, \mathbf{d})$
2. Equivalent mechanical properties³ $\Rightarrow \rho_{\text{eq}}(\phi)$ and $E_{\text{eq}}(\phi)$
3. Dynamics of MPPs \equiv Dynamics of porous plates⁴



²Atalla and Sgard. "Modeling of perforated plates and screens using rigid frame porous models". (2007)

³Boccaccini and Fan. "A new approach for the Young's modulus-porosity correlation of ceramic materials". (1997)

⁴Leclaire, Horoshenkov, and Cummings. "Transverse vibrations of a thin rectangular porous plate saturated by a fluid". (2001)

Governing equations

$$\left(D_{\text{eq}}^*(\phi) + \frac{\alpha^2 M h^3}{12} \right) \nabla^4 w_s + h(\rho_{\text{eq}}(\phi) \ddot{w}_s + \rho_f \ddot{w}) = 0 \quad (1a)$$

$$\alpha M \nabla^2 w_s - \left(\rho_f \ddot{w}_s + \frac{\rho_f \alpha_\infty}{\phi} \ddot{w} \right) - \sigma_0(d) \dot{w} = 0 \quad (1b)$$

- ▶ w_s : solid motion
- ▶ w : relative fluid-solid motion
- ▶ Biot elastic coefficient α and M
- ▶ Isotropic structural damping \Rightarrow complex bending coefficient

$$D_{\text{eq}}^*(\phi) = \frac{E_{\text{eq}}(\phi) h^3}{12(1 - \nu^2)} (1 + j\eta_s) \quad \text{with} \quad E_{\text{eq}}(\phi) = \frac{E(1 - \phi^2)}{1 + (2 - 3\nu^2)} \quad (2)$$

Boundary conditions

- ▶ Free
- ▶ Simply-supported
- ▶ Clamped

Eigenvalue problem

- ▶ Displacement approximation via modal superposition

$$w_s(x, y, t) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} w_{mn}^s e^{\lambda_{mn} t} \psi_{mn}(x, y) \quad (3)$$

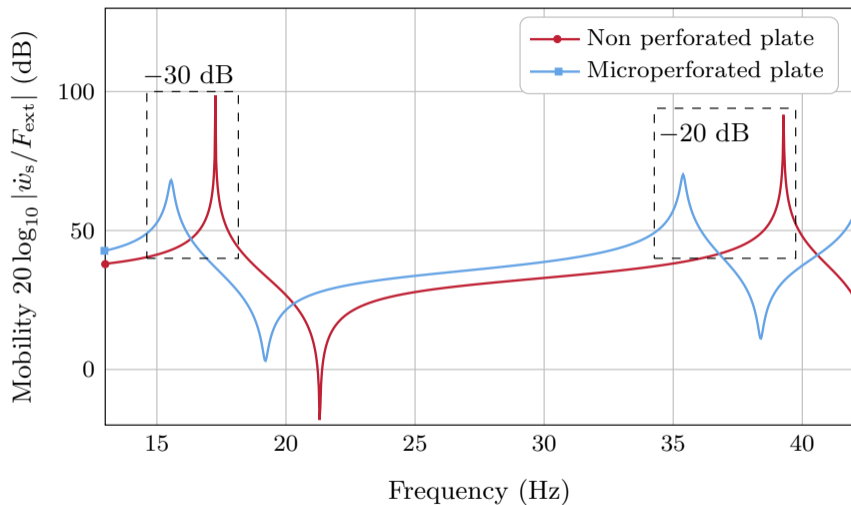
- ▶ Similar formulation for the relative fluid-solid motion $w(x, y, t)$

Resolution

Eigenvalues: $\lambda_{mn} = -\zeta_{mn}\omega_{mn} \pm j\omega_{mn}\sqrt{1 - \zeta_{mn}^2}$

- ▶ ζ_{mn} : modal damping factor of mode mn
- ▶ ω_{mn} : undamped natural frequency

Linear regime – Analytical result



Aluminum simply-supported MPP of size 490 mm × 570 mm × 1 mm with $d = 1.4$ mm and $\phi = 10\%$

Linear regime – Maximum damping

Characteristic frequency

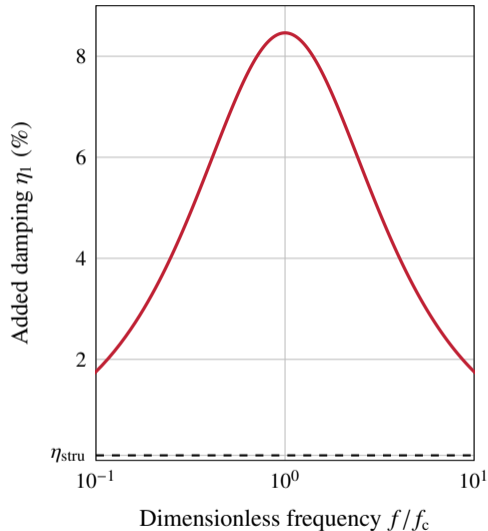
$$f_c(d) = \frac{32\mu_f}{2\pi\rho_f\alpha_\infty d^2} \quad (4)$$

- ▶ f_c independent of the system
- ▶ $f_c \approx f_1 \Rightarrow$ 85 times added damping

Low frequency effect

- ▶ Greater the lower the frequencies
- ▶ Maximum effect on the first MPP mode

Aluminum simply-supported MPP of size 490 mm × 570 mm × 1 mm with $d = 1.4$ mm and $\phi = 10\%$



Acoustic non-linear regime

- ▶ High airflow velocity in perforations \Rightarrow inertial effect
- ▶ Captured by a Forchheimer correction⁵

$$\sigma(d, \dot{w}(x, y, t)) = \sigma_0(1 + \varepsilon|\dot{w}(x, y, t)|) \quad (5)$$

Acoustic and geometric non-linear regimes

- ▶ High airflow velocity in perforations $10^{-2} \text{mm s}^{-1} < V_{\text{fluid}} < 10^3 \text{mm s}^{-1}$
- ▶ Significant structural displacement $\frac{V_{\text{solid}}}{h} > 0.2 \text{s}^{-1}$

⁵Forchheimer. "Wasserbewegung durch Boden". (1901)

Non-Linear regime – Governing equations

$$\left(D_{\text{eq}}^* + \frac{\alpha^2 M h^3}{12}\right) \nabla^4 w_s + h(\rho \ddot{w}_s + \rho_f \ddot{w}) = 0 \quad (6a)$$

$$\alpha M \nabla^2 w_s - \left(\rho_f \ddot{w}_s + \frac{\rho_f \alpha_\infty}{\phi} \ddot{w}\right) - \sigma_0(d) \dot{w} + \sigma_0(d) \varepsilon \dot{w} |\dot{w}| = 0 \quad (6b)$$

- ▶ Space semi-discretization \Rightarrow Matrix system of PDE

$$\mathbf{K}_1 \mathbf{w}_s(t) + \mathbf{M}_1 \ddot{\mathbf{w}}_s(t) + \mathbf{M} \ddot{\mathbf{w}}(t) = \mathbf{F}(t), \quad (7a)$$

$$\mathbf{K}_2 \mathbf{w}_s(t) + \mathbf{M} \ddot{\mathbf{w}}_s(t) + \mathbf{M}_2 \ddot{\mathbf{w}}(t) + \mathbf{C}_2 \dot{\mathbf{w}}(t) + \mathbf{F}_{\text{NL}}(\dot{\mathbf{w}}(t)) = \mathbf{0}, \quad (7b)$$

- ▶ $\mathbf{F}_{\text{NL}}(\dot{\mathbf{w}}(t))$ stores

$$f_r^{\text{NL}}(\dot{\mathbf{w}}(t)) = \sigma_0(d) \varepsilon \int_S \sum_{m=1}^M \sum_{n=1}^N \dot{w}_{mn}(t) \Psi_{mn}(x, y) \left| \sum_{p=1}^M \sum_{q=1}^N \dot{w}_{pq}(t) \Psi_{pq}(x, y) \right| \Psi_r(x, y) dS \quad (8)$$

Non-Linear regime – Harmonic balance method resolution

- ▶ Fourier series truncated at the harmonic N_h

$$\mathbf{w}_s(t) = \sum_{k=1}^{N_h} \mathbf{A}_k \cos k\omega t + \mathbf{B}_k \sin(k\omega t) \quad \text{and} \quad \dot{\mathbf{w}}(t) = \sum_{k=1}^{N_h} \mathbf{C}_k \cos k\omega t + \mathbf{D}_k \sin k\omega t \quad (9)$$

- ▶ Fourier's expansion of Equation (8)

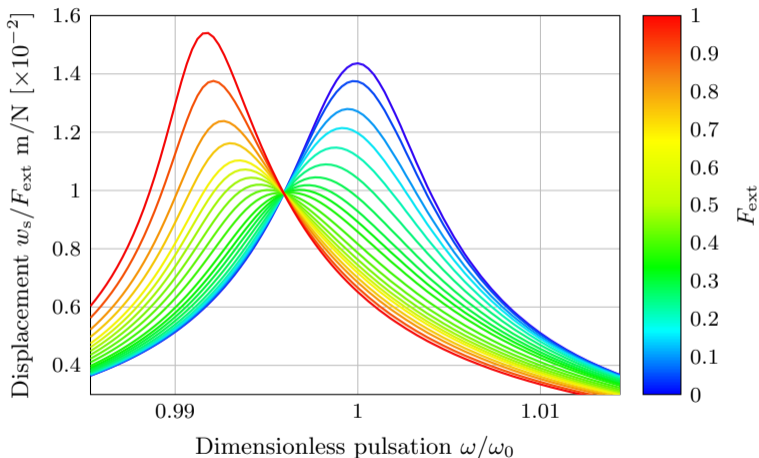
$$f_r^{\text{NL}}(\dot{\mathbf{w}}(t)) = \sigma_0(d)\varepsilon \sum_{m=1}^M \sum_{n=1}^N \sum_{k=1}^{N_h} (\Gamma_{mnkr} \cos k\omega t + \Pi_{mnkr} \sin k\omega t) \quad (10)$$

- ▶ Γ_{mnkr} and Π_{mnkr} : non-linear functions of C_{mnk} and D_{mnk} computed numerically

Non-Linear regime – Steady state response

► F_{ext} softens the system

► Maximum added damping for F_{ext} and relative velocity

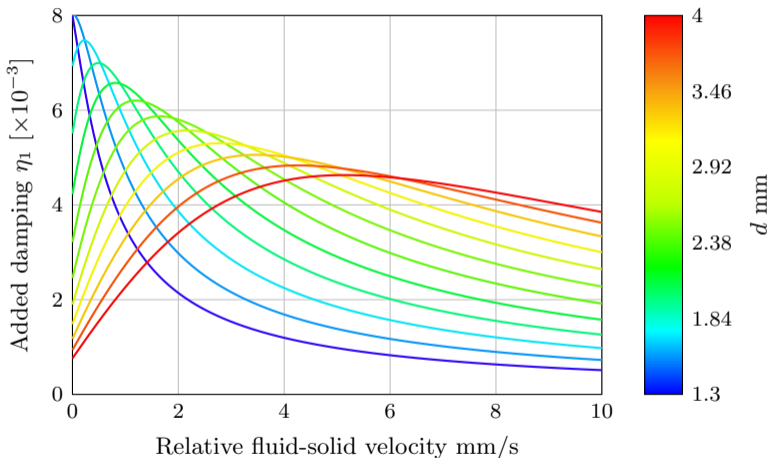


Aluminum simply-supported MPP of size 490 mm \times 570 mm \times 1 mm with $d = 2.2$ mm and $\phi = 10\%$

Non-Linear regime – Added damping

► Fixed ε and MPP dimensions

► Variable d and amplitude of external excitation



Aluminum simply-supported MPP of size 490 mm \times 570 mm \times 1 mm with $\phi = 10\%$

Linear regime

- ▶ Added damping \Rightarrow low frequency range
- ▶ Maximum added damping \Rightarrow characteristic frequency $f_c(d)$

Non-linear acoustic regime

- ▶ Added damping \Rightarrow function of F_{ext} and relative fluid-solid velocity
- ▶ Added damping \Rightarrow maximum for a critical relative fluid-solid velocity
- ▶ Forchheimer \Rightarrow softening the system