

On the Synthesis of an Integrated Active FTC Scheme for Continuous-Time Switched Systems with Actuator Faults

Mohamed Elouni, Hamdi Habib, Bouali Rabaoui, Naceur Benhadj Braiek, Mickael Rodrigues

▶ To cite this version:

Mohamed Elouni, Hamdi Habib, Bouali Rabaoui, Naceur Benhadj Braiek, Mickael Rodrigues. On the Synthesis of an Integrated Active FTC Scheme for Continuous-Time Switched Systems with Actuator Faults. 2023 IEEE International Conference on Advanced Systems and Emergent Technologies, Apr 2023, Hammamet, Tunisia. hal-04035444

HAL Id: hal-04035444 https://hal.science/hal-04035444

Submitted on 17 Mar 2023

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers. L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

On the Synthesis of an Integrated Active FTC Scheme for Continuous-Time Switched Systems with Actuator Faults

Mohamed Elouni*, Habib Hamdi*, Bouali Rabaoui*, Naceur BenHadj Braiek* and Mickael Rodrigues[†]

*Advanced Systems Laboratory

Polytechnic School of Tunisia, University of Carthage, La Marsa 2078, Tunisia

Email: mohamed.elouni@isetb.rnu.tn

[†]Automation and Process Engineering Laboratory

University of Lyon, Lyon, Villeurbanne, F-69622, France

Email: mickael.rodrigues@univ-lyon1.fr

Abstract-The present paper aims to design an integrated state/fault estimation and fault tolerant control issues for switched systems affected by actuator fault and external disturbances. A switched adaptive state/fault estimation observer is proposed to simultaneously estimate the system state and actuator fault. Based on this observer, we develop a fault tolerant control law not only to stabilize the closed-loop system, but also to compensate the faults effects. Then, by using the switched Lyapunov function and Average Dwell Time (ADT) concept, some sufficient conditions are derived and applied to build the switched adaptive observer-based state feedback controller. The observer and controller gain matrices are also obtained by solving an optimization problem with Linear Matrix Inequality (LMI) constraints. Finally, simulation results of a switched electrical system are presented to illustrate the effectiveness of the proposed design methods.

Index Terms—Switched systems, Fault Tolerant Control, Switched Adaptive Observer, Average Dwell Time, LMIs Constraints

I. INTRODUCTION

In the last few decades, due to the increased demands for maintaining required high system performance, safety, and productivity in different conditions of operation, the realm of Fault Tolerant Control (FTC) has attracted more and more attention for both industry and academic communities. Generally speaking, the FTC system is a control system that is able to maintain the system stability and can preserve such performances in the presence of fault while correctly detected in time [1-4].

On the other hand, switched systems have attracted many scholars' attention because of their large number of applications in practical engineering systems, such as flight control systems, power systems, automotive systems, robotics systems, and biological systems [5-8]. The switched systems are considered a special class of hybrid systems, which consist of a family of subsystems described by continuous-time or discrete-time dynamics, and these subsystems are governed by a switching signal [9].

Recently, such important efforts have been focused on the design of fault tolerant controllers for switched systems. For instance, authors in [10] studied fault estimation and fault tolerant control with time delay for switching systems subject to actuator fault. An adaptive observer was developed for the Fault Detection and Isolation (FDI) task to detect and estimate the fault signal. Based on the fault estimation information an observer-based fault-tolerant controller is proposed in order to guarantee the closed loop system stability. Authors in [11] have dealt with the problem of fault estimation and accommodation for a class of discrete-time switched systems vulnerable to a sensor fault. Based on the descriptor observer design scheme, the sensor fault has to be detected and based on the information about the fault, an online controller re-design took place such that closed-loop stability with Lyapunov theory. In [12], a reduced-order observer has been designed in order to ensure fault detection for a class of uncertain switched systems affected by both sensor and actuator faults. After this, a state feedback robust controller is developed and the stability of the observer- fault tolerant controller closed-loop system is analysed in a sense of H_{∞} performance index.

During the last decade, literature shows a steady increase in interest in using the Average Dwell Time (ADT) technique to deal with fault tolerant control problems for switched systems is developed. Authors in [13] proposed a robust fault detection and fault tolerant control for switched systems with ADT. The synthesis problem of the observer and controller is solved by using multiple Lyapunov function and ADT techniques. The proposed control system guaranteed the H_{∞} performance and the asymptotic stability. Sufficient conditions for the existence of the designed observer and controller have been provided in terms of Linear Matrix Inequalities (LMI). In [14], authors have dealt with the problem of fault-tolerant control design for a class of switched time-delay linear systems with structural uncertainties by using the ADT approach. Authors in [15] developed an integrated sensor Fault Estimation (FE) and FTC method for a class of continuous-time switched systems. By mean the state/fault estimation observer, the fault amplitude and system states were simultaneously estimated, and then exploiting information provided by the observer, a fault tolerant state feedback controller was designed. The stability of the overall system was fully studied by applying the ADT concept and the Lyapunov theory and the problem was solved in terms of LMIs.

Motivated by the above observations, in the present paper we are dedicated to the FE/FTC designs for a class of switched systems with actuator faults and external disturbances. The major contributions of this work can be summarized as the following aspects:

- (i) A Switched Adaptive Observer (SAO) is proposed by using ADT method, such that the states and actuator faults can be simultaneously estimated.
- (ii) Based on the states and actuator faults estimation information, a SAO-based state feedback fault tolerant controller is designed, under the average dwell time switching to tolerate the faults, which can guarantee that the closed-loop system is asymptotically stable.
- (iii) A sufficient condition is provided to guarantee the existence of the observer-based fault tolerant controller by the feasibility of an optimization problem with LMI constraint.
- (iv) The obtained results are successfully illustrated to a switched electrical system.

The remaining part of this paper is organized as follows. Section II describes preliminaries and problem formulation. Section III is dedicated to the design of an observer-based FTC to robustly stabilize the closed-loop system in the presence of actuator faults. In section IV, an example is given to illustrate the effectiveness of the proposed approach. Finally, section V draws some conclusions.

II. PRELIMINARIES AND PROBLEM FORMULATION

Let's consider a class of dynamical systems modeled as switched linear systems with actuator faults and unknown external disturbance described as follows:

$$\begin{cases} \dot{x}(t) = A_{\sigma(t)}x(t) + B_{\sigma(t)}u(t) + F_{\sigma(t)}f(t) + R_{\sigma(t)}d(t) \\ y(t) = C_{\sigma(t)}x(t) \end{cases}$$
(1)

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^p$ is the FTC law vector to be designed, $y(t) \in \mathbb{R}^m$ represents the output measurement vectors, $f(t) \in \mathbb{R}^q$ is the actuator fault vector and $d(t) \in \mathbb{R}^r$ represents the unknown bounded disturbance vector. The switching signal $\sigma(t) : \mathbb{R}^+ \to \Theta$ is assumed to be a piecewise constant function and continuous from the right everywhere. Where Θ is a finite set described by $\Theta = \{1, 2, ..., N\}$ and N denotes the number of subsystems. When $\sigma(t) = i$, the i-th subsystem is activated for some $i \in \Theta$. In this case, the system (1) can be simplified in the form below:

$$\begin{cases} \dot{x}(t) = A_i x(t) + B_i u(t) + F_i f(t) + R_i d(t) \\ y(t) = C_i x(t) \end{cases}$$
(2)

It should be indicated that the parameter matrices in the abovementioned system are assumed to be constant real matrices with appropriate dimensions.

Before starting the main results of the present paper, we will make the following assumptions which are typically required in the switched adaptive observer and fault tolerant control design for switched systems.

Assumption 1. We assume that the actuator fault distribution matrices F_i are of full column rank.

$$rank(C_iF_i) = rank(F_i) = q \tag{3}$$

Assumption 2. The pair (A_i, C_i) is observable $\forall i \in \Theta$.

$$rank \begin{bmatrix} C_i \\ C_i A_i \\ \vdots \\ C_i A_i^{n-1} \end{bmatrix} = n$$
(4)

Assumption 3. The fault f(t) satisfies $||f(t)|| \leq \alpha_1$, its derivative is bounded such that $||\dot{f}(t)|| \leq \alpha_2$ and the unknown input vector verifies d(t) verifies $||d(t)|| \leq \alpha_3$, where $\alpha_1, \alpha_2, \alpha_3 \geq 0$.

The following definition and lemmas are added for the convenes of later proof.

Definition 1. [16] For any switching signal $\sigma(t)$ and any $\tau_2 \ge \tau_1 \ge 0$, let $N_{\sigma(t)}(\tau_1, \tau_2)$ denote the number of switchings $\sigma(t)$ on an interval (τ_1, τ_2) . If

$$N_{\sigma(t)}(\tau_1, \tau_2) \leqslant N_0 + \frac{\tau_2 - \tau_1}{\tau_a} \tag{5}$$

holds for a given two positive numbers N_0 and τ_a , then the constant τ_a is called the average dwell time and N_0 is the chattering bound.

Lemma 1. [17] Suppose that there exist C^1 functions $V_{\sigma(t)}$: $\mathbb{R}^n \to \mathbb{R}$ and two class K_∞ functions κ_1 and κ_2 and two positive numbers $\alpha > 0$, $\mu > 1$ such that we have

$$\kappa_1(|x(t)|) \leqslant V_{\sigma(t)}(x(t)) \leqslant \kappa_2(|x(t)|) \tag{6}$$

$$\dot{V}_{\sigma(t)}(x(t)) \leqslant -\alpha V_{\sigma(t)}(x(t)) \tag{7}$$

and $\forall (\sigma(t_l) = i, \sigma(t_l^-) = j) \in N \times N, i \neq j,$

$$V_i(x(t)) \leqslant \mu V_j(x(t)) \tag{8}$$

then the switched linear system $\dot{x}(t) = f_{\sigma(t)}(x(t))$ is Globally Uniformly Asymptotically Stable (GUAS) for any switching signal with average dwell time:

$$\tau_a \geqslant \tau_a^* = \frac{\ln \mu}{\alpha} \tag{9}$$

Lemma 2. [18] For a positive definite matrix Q that is: $Q = Q^T > 0$ and a positive scalar ε ; the following inequality is true:

$$2x^T y \leqslant \frac{1}{\varepsilon} x^T Q x + \varepsilon y^T Q^{-1} y; \quad x, y \in \mathbb{R}^n$$
 (10)

III. OBSERVER-BASED FTC DESIGN

In order to study the state feedback fault tolerant control design problem, we have developed a robust FTC law for switched systems in presence of actuator faults. Thus, we require to estimate simultaneously system states and actuator faults. To do this, we have designed the following switched adaptive observer:

$$\begin{aligned}
\dot{z}(t) &= N_i z(t) + G_i u_f(t) + L_i y(t) + H_i f(t) \\
\hat{x}(t) &= z(t) + T_2 y(t) \\
\hat{y}(t) &= C_i \hat{x}(t) \\
\dot{f}(t) &= \Gamma \Phi_i (\dot{e}_y(t) + \eta e_y(t)) \\
e_y(t) &= y(t) - \hat{y}(t)
\end{aligned} \tag{11}$$

where $z(t) \in \mathbb{R}^n$ is the observer state vector, $\hat{x}(t) \in \mathbb{R}^n$ the estimated state vector, $\hat{y}(t) \in \mathbb{R}^m$ is the estimated output vector, $\hat{f}(t) \in \mathbb{R}^p$ is the estimated actuator faults and $N_i \in \mathbb{R}^{n \times n}$, $G_i \in \mathbb{R}^{n \times p}$, $L_i \in \mathbb{R}^{n \times m}$, $\Phi_i \in \mathbb{R}^{p \times m}$ and $T_2 \in \mathbb{R}^{n \times m}$ are the observer gain matrices to be determined. The matrix $\Gamma \in \mathbb{R}^{p \times p}$ is a symmetric positive definite learning rate matrix and η is a positive scalar.

Based on both actuator fault and system states estimation, we propose to conceive a robust controller as:

$$u_{FTC}(t) = -K_i \hat{x}(t) - q_i \hat{f}(t) \tag{12}$$

where $K_i \in \mathbb{R}^{p \times n}$ and $q_i \in \mathbb{R}^{p \times q}$ represent, respectively, the state feedback control and fault compensation gains. We assume that $q_i = B_i^+ F_i$. Based on this observer, the fault tolerant controller can compensate faults effects and guarantee the stabilization of the closed-loop system.

Now, let us define the following state estimation error $e_s(t)$ from (2) and (11) such that:

$$e_s(t) = x(t) - \hat{x}(t) = (I_n - T_2 C_i) x(t) - z(t)$$
(13)

Since that for $rank \begin{bmatrix} I_n & C_i \end{bmatrix}^T = n$, there exists nonsingular matrices $T_1 \in \mathbb{R}^{n \times n}$ and $T_2 \in \mathbb{R}^{n \times m}$ such that:

$$\begin{bmatrix} T_1 & T_2 \end{bmatrix} \begin{bmatrix} I_n \\ C_i \end{bmatrix} = I_n \tag{14}$$

Then, the state estimation error (13) is described by:

$$e_s(t) = T_1 x(t) - z(t)$$
 (15)

From (15), the dynamic of the state estimation error obey the differential equation:

$$\dot{e}_s(t) = T_1 \dot{x}(t) - \dot{z}(t) \tag{16}$$

Substituting the equations of $\dot{x}(t)$ and $\dot{z}(t)$ respectively from (2) and (11), the equation (16) becomes after some calculations:

$$\dot{e}_s(t) = (T_1 A_i + E_i C_i - N_i) x(t) + (T_1 B_i - G_i) u(t) + N_i e_s(t) + H_i e_f(t) + \bar{R}_i d(t)$$
(17)

where

$$E_i = N_i T_2 - L_i \tag{18}$$

$$\bar{R}_i = T_1 R_i \tag{19}$$

If the following conditions hold $\forall i \in \Theta$:

$$N_i = T_1 A_i + E_i C_i \tag{20}$$

$$G_i = T_1 B_i \tag{21}$$

$$H_i = T_1 F_i \tag{22}$$

The dynamic of the state estimation error reduces to:

$$\dot{e}_s(t) = N_i e_s(t) + H_i e_f(t) + \bar{R}_i d(t)$$
 (23)

The dynamic of the closed-loop state system with the control law (12) is defined as follows:

$$\dot{x}(t) = \Pi_i x(t) + B_i K_i e_s(t) + F_i e_f(t) + R_i d(t)$$
(24)

with $\Pi_i = A_i - B_i K_i$.

Finally, the fault estimation error dynamic is defined as follows:

$$\dot{e}_f(t) = \dot{f}(t) - \hat{f}(t)$$
 (25)

Theorem 1. For given positive scalars η , ε , β , α , $\mu > 1$ and a positive definite matrix Γ , if there exist symmetric and positive definite matrices $X_i = P_i^{-1}$, Q_i , Q_{1i} , Q_{2i} and Q_{3i} and matrices $S_i = Q_i E_i$, $W_i = K_i X_i$ and Φ_i such that the following LMIs are satisfied $\forall (i, j) \in \Theta \times \Theta, i \neq j$:

$$\begin{pmatrix} \bar{\Xi}_{i} & B_{i}W_{i} & F_{i} & 0 & 0 & X_{i}^{T} \\ * & -2\beta X_{i} & 0 & \beta I & 0 & 0 \\ * & * & -2\beta I & 0 & \beta I & 0 \\ * & * & * & \tilde{\Omega}_{i} & \Sigma_{i} & 0 \\ * & * & * & * & \Upsilon_{i} & 0 \\ * & * & * & * & * & -\frac{1}{\varepsilon}Q_{1i} \end{pmatrix} < 0$$
(26)
$$X_{j} < \mu X_{i}$$
(27)

such that

where

$$H_i^T Q_i - \Phi_i C_i = 0 \tag{28}$$

(29)

$$\bar{\Xi}_i = A_i X_i + X_i A_i^T - B_i W_i - W_i^T B_i^T + \alpha X_i$$

$$\tilde{\Omega}_{i} = (T_{1}A_{i})^{T}Q_{i} + Q_{i}(T_{1}A_{i}) + S_{i}C_{i} + C_{i}^{T}S_{i}^{T} + \alpha Q_{i} + \frac{1}{\varepsilon}Q_{2i}$$
(30)

$$\Sigma_{i} = -\frac{1}{\eta} C_{i}^{T} S_{i}^{T} H_{i} - \frac{1}{\eta} (T_{1} A_{i})^{T} Q_{i} H_{i}$$
(31)

$$\Upsilon_i = -\frac{1}{\eta} (H_i{}^T Q_i H_i + H_i{}^T Q_i H_i + \Gamma^{-1}) + \frac{2}{\eta \varepsilon} Q_{3i} \quad (32)$$

Then the state vector x(t), the state estimation error $e_s(t)$ and the fault estimation error $e_f(t)$ are bounded if the ADT satisfies the following condition $\tau_a \ge \tau_a^* = \frac{\ln \mu}{\alpha}$. The matrix gains of the switched adaptive observer (11) and the controller (12) are given by $E_i = Q_i^{-1}S_i$ and $K_i = W_i X_i^{-1}$.

Proof of Theorem 1. In this prof, we use the theory of Lyapunov to demonstrate the stability of the closed loop system. Then, the problem is turned into an optimization problem through the use of LMI so as to determinate the unknown matrices of both the proposed FTC controller and the switched adaptive observer.

Thus, let us consider the following switched Lyapunov function candidate as:

$$V_{i}(t) = x^{T}(t)P_{i}x(t) + e_{s}^{T}(t)Q_{i}e_{s}(t) + \frac{1}{\eta}e_{f}^{T}(t)\Gamma^{-1}e_{f}(t)$$
(33)

where P_i , Q_i and Γ are symmetric positive definite matrices with appropriate dimensions. The time derivative of the switched Lyapunov function (33) leads to:

$$\dot{V}_{i}(t) = \dot{x}^{T}(t)P_{i}x(t) + x^{T}(t)P_{i}\dot{x}(t) + \dot{e}_{s}^{T}(t)Q_{i}e_{s}(t) + e_{s}^{T}(t)Q_{i}\dot{e}_{s}(t) + \frac{1}{\eta}\dot{e}_{f}^{T}(t)\Gamma^{-1}e_{f}(t) + \frac{1}{\eta}e_{f}^{T}(t)\Gamma^{-1}\dot{e}_{f}(t)$$
(34)

By considering the expressions (24) of $\dot{x}(t)$, (23) of $\dot{e}_s(t)$, (25) of $\dot{e}_f(t)$ and by taking into account the fault estimation of $\hat{f}(t)$ in (11), $\dot{V}_i(t)$ becomes:

$$\dot{V}_{i}(t) = x^{T}(t)\Xi_{i}x(t) + 2x^{T}(t)P_{i}B_{i}K_{i}e_{s}(t) + 2x^{T}(t)P_{i}B_{i}e_{f}(t) + 2x^{T}(t)P_{i}R_{i}d(t) + e_{s}^{T}(t)\Omega_{i}e_{s}(t) + 2e_{s}^{T}(t)Q_{i}H_{i}e_{f}(t) + 2e_{s}^{T}(t)Q_{i}\bar{R}_{i}d(t) + \frac{2}{\eta}e_{f}^{T}(t)\Gamma^{-1}\dot{f}(t) - \frac{2}{\eta}e_{f}^{T}(t)\Phi_{i}C_{i}\dot{e}_{s}(t) - 2e_{s}^{T}(t)C^{T}_{i}\Phi^{T}_{i}e_{f}(t)$$
(35)

where

$$\Xi_i = \Pi_i{}^T P_i + P_i \Pi_i \tag{36}$$

$$\Omega_i = N_i^T Q_i + Q_i N_i \tag{37}$$

By using the equation (23) and the equality (28), it follows that

$$\dot{V}_{i}(t) = x^{T}(t)\Xi_{i}x(t) + 2x^{T}(t)P_{i}B_{i}K_{i}e_{s}(t) + 2x^{T}(t)P_{i}B_{i}e_{f}(t)
+ 2x^{T}(t)P_{i}R_{i}d(t) + e_{s}^{T}(t)\Omega_{i}e_{s}(t) + 2e_{s}^{T}(t)Q_{i}\bar{R}_{i}d(t)
+ \frac{2}{\eta}e_{f}^{T}(t)\Gamma^{-1}\dot{f}(t) - \frac{2}{\eta}e_{f}^{T}(t)H_{i}^{T}Q_{i}N_{i}e_{s}(t)
- \frac{2}{\eta}e_{f}^{T}(t)H_{i}^{T}Q_{i}H_{i}e_{f}(t) - \frac{2}{\eta}e_{f}^{T}(t)H_{i}^{T}Q_{i}\bar{R}_{i}d(t)$$
(38)

Under Assumption 3, we apply Lemma 2 so as to get the following term inequalities from (38):

$$2x^{T}(t)P_{i}R_{i}d(t) \leqslant \frac{1}{\varepsilon}x^{T}(t)Q_{1i}x(t) + \eta_{1i}$$
(39)

$$2e_s^T(t)Q_i\bar{R}_id(t) \leqslant \frac{1}{\varepsilon}e_s^T(t)Q_{2i}e_s^T(t) + \eta_{2i} \qquad (40)$$

$$\frac{2}{\eta}e_f^{T}(t)\Gamma^{-1}\dot{f}(t) \leqslant \frac{1}{\eta\varepsilon}e_f^{T}(t)Q_{3i}e_f^{T}(t) + \eta_{3i}$$
(41)

$$-\frac{2}{\eta}e_f^{T}(t)H_i^{T}Q_i\bar{R}_id(t) \leqslant \frac{1}{\eta\varepsilon}e_f^{T}(t)Q_{3i}e_f^{T}(t) + \eta_{4i} \quad (42)$$

where the scalars η_{1i} , η_{2i} , η_{3i} and η_{4i} are expressed as follows:

$$\eta_{1i} = \varepsilon \alpha_3^2 \lambda_{\max} (R_i^T P_i Q_{1i}^{-1} P_i R_i)$$
(43)

$$\eta_{2i} = \varepsilon \alpha_2^2 \lambda_{\max}(\bar{R}_i^T Q_i Q_{2i}^{-1} Q_i \bar{R}_i) \tag{44}$$

$$\eta_{3i} = \frac{\varepsilon}{\sigma} \alpha_1^{\ 2} \lambda_{\max} (\Gamma^{-1} Q_{3i}^{\ -1} \Gamma^{-1}) \tag{45}$$

$$\eta_{4i} = \frac{\varepsilon}{\sigma} \alpha_2^{\ 2} \lambda_{\max} (H_i^{\ T} Q_i Q_{3i}^{\ -1} Q_i H_i) \tag{46}$$

By taking into consideration the inequalities (43)-(46) and the equation (38), $\dot{V}_i(t)$ can be bounded as follows:

$$\dot{V}_{i}(t) \leq x^{T}(t)(\Xi_{i} + \frac{1}{\varepsilon}Q_{1i})x(t) + 2x^{T}(t)P_{i}B_{i}K_{i}e_{s}(t)$$

$$+ 2x^{T}(t)P_{i}B_{i}e_{f}(t) + e_{s}^{T}(t)(\Omega_{i} + \frac{1}{\varepsilon}Q_{2i})e_{s}(t)$$

$$- \frac{2}{\eta}e_{f}^{T}(t)(H_{i}^{T}Q_{i}H_{i} + \frac{2}{\eta\varepsilon}Q_{3i})e_{f}(t)$$

$$- \frac{2}{\eta}e_{s}^{T}(t)N_{i}^{T}Q_{i}H_{i}e_{f}(t) + \delta$$

$$(47)$$

where the scalar δ is the maximum value over *i* such that:

$$\delta = \max_{i} (\eta_{1i} + \eta_{2i} + \eta_{3i} + \eta_{4i}) \tag{48}$$

By Lemma 1, one can get that:

$$\dot{V}_i(t) + \alpha V_i(t) \leqslant \tilde{x}^T(t) \Lambda_i \tilde{x}(t) + \delta$$
(49)

where $\tilde{x}^T(t) = \begin{bmatrix} x^T(t) & e_s^T(t) & e_f^T(t) \end{bmatrix}$ and Λ_i is a matrix defined as follows:

$$\Lambda_{i} = \begin{pmatrix} \tilde{\Xi}_{i} & P_{i}B_{i}K_{i} & P_{i}H_{i} \\ * & \tilde{\Omega}_{i} & -\frac{1}{\sigma}N_{i}^{T}Q_{i}H_{i} \\ * & * & \Upsilon_{i} \end{pmatrix}$$
(50)

with

$$\tilde{\Xi}_i = \Xi_i + \alpha P_i + \frac{1}{\varepsilon} Q_{1i} \tag{51}$$

$$\tilde{\Omega}_i = \Omega_i + \alpha Q_i + \frac{1}{\varepsilon} Q_{2i}$$
(52)

$$\Upsilon_{i} = -\frac{1}{\eta} (H_{i}^{T} Q_{i} H_{i} + H_{i}^{T} Q_{i} H_{i} + \Gamma^{-1}) + \frac{2}{\eta \varepsilon} Q_{3i} \quad (53)$$

If the following inequality holds:

$$\Lambda_i < 0 \tag{54}$$

We can obtain that:

$$\dot{V}_i(\tilde{x}(t)) + \alpha V_i(\tilde{x}(t)) \leqslant -\zeta \|\tilde{x}(t)\|^2 + \delta$$
(55)

where $\zeta > 0$ is given by

$$\zeta = \min \ \lambda_{\min} \left(-\Lambda_i \right) < 0 \tag{56}$$

To complete the prof by considering the constraint (54), we define a matrix:

$$\Delta_{i} = \begin{pmatrix} \tilde{\Xi}_{i} & Z_{i} \\ Z_{i}^{T} & \Psi_{i} \end{pmatrix} < 0$$
(57)

with

$$Z_i = \left(\begin{array}{cc} P_i B_i K_i & P_i H_i \end{array}\right) \tag{58}$$

$$\Psi_{i} = \begin{pmatrix} \tilde{\Omega}_{i} & -\frac{1}{\sigma} N_{i}^{T} Q_{i} H_{i} \\ * & \Upsilon_{i} \end{pmatrix}$$
(59)

Consider a symmetric matrix χ defined as

$$\chi = \left(\begin{array}{cc} P_i^{-1} & 0\\ 0 & \chi_1 \end{array}\right) \tag{60}$$

with

$$\chi_1 = \left(\begin{array}{cc} P_i^{-1} & 0\\ 0 & I \end{array}\right) \tag{61}$$

Then, by post and pre-multiplying the inequality (57) by χ , we can obtain that

$$\begin{pmatrix}
P_i^{-1}\tilde{\Xi}_i P_i^{-1} & P_i^{-1} \mathbf{Z}_i \chi_1 \\
* & \chi_1 \Psi_i \chi_1
\end{pmatrix} < 0$$
(62)

The term $\chi_1 \Psi_i \chi_1$ can be replaced by considering the following inequality which holds any scalar β such that

$$(\chi_1 + \beta \Psi_i^{-1})^T \Psi_i (\chi_1 + \beta \Psi_i^{-1}) \leq 0$$

$$\Leftrightarrow \chi_1 \Psi_i \chi_1 \leq -2\beta \chi_1 - \beta^2 \Psi_i^{-1}$$
 (63)

Considering (63) and with the Schur Complement, the inequality (62) becomes

$$\begin{pmatrix} P_i^{-1} \tilde{\Xi}_i P_i^{-1} & P_i^{-1} Z_i \chi_1 & 0 \\ * & -2\beta \chi_1 & \beta I \\ * & * & \Psi_i \end{pmatrix} < 0$$
 (64)

Using the definition of matrices $\tilde{\Xi}_i$, Z_i and Ψ_i and considering the change of variable $X_i = P_i^{-1}$ we can obtain that

$$\begin{pmatrix} \Xi_{i} & B_{i}K_{i}X_{i} & F_{i} & 0 & 0 \\ * & -2\beta X_{i} & 0 & \beta I & 0 \\ * & * & -2\beta I & 0 & \beta I \\ * & * & * & \tilde{\Omega}_{i} & -\frac{1}{\sigma}N_{i}^{T}Q_{i}H_{i} \\ * & * & * & * & \Upsilon_{i} \end{pmatrix} < 0$$
(65)

with

$$\hat{\Xi}_i = X_i \Pi_i^{\ T} + \Pi_i X_i + \alpha X_i + \frac{1}{\varepsilon} X_i Q_{1i} X_i \qquad (66)$$

Now, we dissociate the term $\frac{1}{\varepsilon}X_iQ_{1i}X_i$ from the inequality (65) in order to reformulate it as follows

$$\begin{pmatrix} \bar{\Xi}_{i} & B_{i}K_{i}X_{i} & F_{i} & 0 & 0 \\ * & -2\beta X_{i} & 0 & \beta I & 0 \\ * & * & -2\beta I & 0 & \beta I \\ * & * & * & \tilde{\Omega}_{i} & -\frac{1}{\sigma}N_{i}^{T}Q_{i}H_{i} \\ * & * & * & * & \Upsilon_{i} \end{pmatrix}$$
(67)
$$-\bar{X}_{i}^{T} \left(-\frac{1}{\varepsilon}Q_{1i}\right) \bar{X}_{i} < 0$$

where

$$\bar{\Xi}_i = X_i \Pi_i^{\ T} + \Pi_i X_i + \alpha X_i \tag{68}$$

and

$$\bar{X}_i = \left(\begin{array}{cccc} X_i & 0 & 0 & 0 \end{array}\right) \tag{69}$$

Now, we apply the modified Schur Lemma in the above inequality (67), changing Π_i and N_i by their expressions and taking into account the following change of variables $W_i = K_i X_i$ and $S_i = Q_i E_i$ the inequality (67) becomes reformulated as a Linear Matriciel Inequality which can be rewritten as in the **Theorem 1**. This one ends the proof of the theorem.

IV. ILLUSTRATIVE EXAMPLE

In this section, the proposed example considers a switched electrical circuit model borrowed from [19]. In the system shown in Fig. 1, Sw1 is a bipolar transistor and Sw2 is a diode. This circuit has two switching modes, that is, N = 2, $\sigma(t) : [0, \infty) \rightarrow \{1, 2\}$: mode 1, Sw1 is closed and Sw2 is off; mode 2, Sw1 is off and Sw2 is closed. Selecting the state variable $\begin{bmatrix} x_1(t) & x_2(t) \end{bmatrix}^T$ where $x_1(t)$ is the inductance current $i_L(t)$, $x_2(t)$ is the capacitor voltage $V_c(t)$ and $V_s(t)$ is the power supply voltage. Under the different modes, the system matrices of the switched electrical system are given by:



Fig. 1. A switched electrical circuit

Subsystem 1:

$$A_{1} = \begin{bmatrix} -0.54 & 1.02 \\ 0.17 & -0.31 \end{bmatrix}, B_{1} = \begin{bmatrix} 0.1 \\ 0.2 \end{bmatrix}, F_{1} = \begin{bmatrix} 0.1 \\ 0.2 \end{bmatrix}$$
$$C_{1} = \begin{bmatrix} 0.1 & 0.2 \end{bmatrix}, R_{1} = \begin{bmatrix} 0.01 \\ 0.03 \end{bmatrix}$$
Subsystem 2:

$$A_{2} = \begin{bmatrix} -0.01 & 0.1 \\ 0.01 & 0.04 \end{bmatrix}, B_{2} = \begin{bmatrix} 0.2 \\ 0.4 \end{bmatrix}, F_{2} = \begin{bmatrix} 0.2 \\ 0.4 \end{bmatrix}, C_{2} = \begin{bmatrix} 0.2 & 0.3 \end{bmatrix}, R_{2} = \begin{bmatrix} 0.01 \\ 0.03 \end{bmatrix}$$

By solving the LMIs condition of **Theorem 1** for the following choice of parameters: $\eta = 1$, $\varepsilon = 1$, $\beta = 0.5$ and $\Gamma = 200$. The designed observer and controller gains are obtained as:

For i=1

$$N_{1} = \begin{bmatrix} -1.2280 & -0.3637\\ 0.5481 & 0.2509 \end{bmatrix}, G_{1} = \begin{bmatrix} 0.0952\\ 0.1905 \end{bmatrix}$$
$$L_{1} = \begin{bmatrix} 6.7131\\ -2.7512 \end{bmatrix}, \Phi_{1} = \begin{bmatrix} 17.0262 \end{bmatrix}$$
$$K_{1} = \begin{bmatrix} 1.8570 & 2.9636 \end{bmatrix}$$

For i=2

$$N_2 = \begin{bmatrix} 0.2349 & 0.5883 \\ -0.1575 & -0.2980 \end{bmatrix}, G_2 = \begin{bmatrix} 0.1905 \\ 0.3810 \end{bmatrix}$$

$$L_2 = \begin{bmatrix} -2.3156\\ 1.6010 \end{bmatrix}, \ \Phi_2 = \begin{bmatrix} 34.0524 \end{bmatrix}$$

 $K_2 = \begin{bmatrix} 1.2992 & 1.2949 \end{bmatrix}$

For the switching signal, choose the parameter value as $\mu = 2.5$ and $\alpha = 0.8$, to calculate average dwell time as $\tau_a^* = \frac{\ln \mu}{\alpha} = 1.1454$. Hence, the switching interval from one mode to the other is greater than 1.1454. For simulation purposes, we apply the switching signal presented in Fig. 2 specifies the switching between the two subsystems.



Fig. 2. Switching signal evolution

For the simulation, the external disturbance d(t) is chosen as the band-limited white noise, which is shown in Fig. 3.



Fig. 3. The external disturbance d(t)

In order to show the performance and capability of actuator fault estimation effect by using the switched adaptive observer, two cases of actuators faults are considered.

If the actuator faults type is a constant value described as:

$$f_1(t) = \begin{cases} 0, & 0s \le t < 5s \\ 1, & 5s \le t \le 20s \end{cases}$$
(70)

In this case, the simulation result is shown in Fig. 4.

If the actuator fault is a time-varying function as follows:

$$f_2(t) = \begin{cases} 0, & 0s \le t < 5s \\ 1.8\cos(1.6t + 3.25), & 5s \le t \le 20s \end{cases}$$
(71)

In this case, the simulation result is described in Fig. 5.



Fig. 4. Actuator fault $f_1(t)$ and its estimate $\hat{f}_1(t)$



Fig. 5. Actuator fault $f_2(t)$ and its estimate $\hat{f}_2(t)$

Form the simulation results shown in Figs. 4 and 5, one can conclude that the technique proposed in this paper has a good actuator fault estimation performance with accuracy and rapidly for both constant faults and time-varying faults by use of the switched adaptive observer despite an additive noise.



Fig. 6. The state $x_1(t)$ and its estimate $\hat{x}_1(t)$

In simulation, the initial state value of original switched system (2) and the initial estimated switched system states (11) are set as $x(0) = \begin{bmatrix} 0.3 & -0.2 \end{bmatrix}^T$ and $\hat{x}(0) = \begin{bmatrix} 0.2 & -0.1 \end{bmatrix}^T$ respectively.



Fig. 7. The state $x_2(t)$ and its estimate $\hat{x}_2(t)$

The evolution of states trajectories are plotted in Figs. 6 and 7, showing that the system is stabibized and that actuator faults are comletely compensated even in the presence of the disturbance signal d(t).

The state estimation errors are shown in the Fig. 8, from which we can seen that the proposed Switched Adaptive Observer can estimate the states very accurately, even after the occurrence of an actuator fault.



Fig. 8. State estimation errors $e_s(t)$

The example studied shows that the integrated FE/FTC design scheme can accurately estimate the actuator fault and ensure the stability of the closed-loop system.

V. CONCLUSION

This paper has studied the problem of fault estimation and fault tolerant control for continuous-time switched according to the ADT concept. By using switched adaptive observer a state/fault estimation algorithm is achieved rapidly and accurately. Then based on the obtained online fault estimation information, a switched state feedback fault tolerant controller is employed to compensate the effect of faults by stabilizing the closed-loop systems. Sufficient stability conditions, in terms of linear matrix inequalities, for the existence of both switched adaptive observer and robust active fault tolerant control law are given in an integrated way. Finally, the developed scheme has been applied to an electrical circuit system with two types of actuator faults so as to illustrate the effectiveness of the proposed approach. As future work, we will focus on FTC design problem for switched nonlinear systems.

REFERENCES

- H. Yang, B. Jiang and V. Cocquempot, "Fault tolerant control and hybrid systems," In *Fault Tolerant Control Design for Hybrid Systems*, pp. 1-9, Springer, Berlin, Heidelberg, 2010.
- [2] X. Yu, J. Jiang, "A survey of fault-tolerant controllers based on safetyrelated issues," *Annual Reviews in Control*, vol. 39, pp. 46-57, January 2015.
- [3] A. A. Amin and K. M. Hasan, "A review of fault tolerant control systems: advancements and applications," *Measurement*, vol. 143, pp. 58-68, September 2019.
- [4] A. Abbaspour, S. Mokhtari, A. Sargolzaei and K.K. Yen, "A survey on active fault-tolerant control systems," *Electronics*, vol. 9, no 9, pp. 1513, September 2020.
- [5] Z. Sun and S. S. Ge, "Analysis and synthesis of switched linear control systems," *Automatica*, vol. 41, no 2, pp. 181-195, February 2005.
- [6] Y. Eddoukali, A. Benzaouia and M. Ouladsine, "Integrated fault detection and control design for continuous-time switched systems under asynchronous switching," *ISA transactions*, vol. 84, pp. 12-19, October 2018.
- [7] J. Li, X. Guo, C. Chen and Q. Su, "Robust fault diagnosis for switched systems based on sliding mode observer," *Applied Mathematics and Computation*, vol. 341, pp. 193-203, January 2019.
- [8] M. Elouni, H. Hamdi, B. Rabaoui, N. Benhadj Braiek and M. Rodrigues, "Robust sensor faults observer design for continuous-time switched systems using descriptor approach," 2023 20th International Multi-Conference on Systems, Signals and Devices (SSD), Mahdia, Tunisia, pp. 802-808, February 2023.
- [9] R. Goebel, R. G. Sanfelice and A. R. Teel, "Hybrid dynamical systems," in *IEEE Control Systems Magazine*, vol. 29, no. 2, pp. 28-93, April 2009.
- [10] D. Du, B. Jiang and P. Shi, "Fault Estimation and Accommodation for Switched Systems with Time-Varying Delay," *International Journal of Control Automation and Systems*, vol. 9, no 3, pp. 442-451, June 2011.
- [11] D. Du, B. Jiang and P. Shi, "Sensor fault estimation and compensation for time-delay switched systems," *International Journal of Systems Science*, vol. 43, no 4, pp. 629-640, April 2012.
- [12] F. Zhu, Y. Shan, J. Zhang and F. Wang, "Observer-based fault reconstructions and fault-tolerant control designs for uncertain switched systems with both actuator and sensor faults," *IET Control Theory and Applications*, vol. 14, no 15, pp. 2017-2029, March 2020.
- [13] A. Benzaouia, Y. Eddoukali, "Robust fault detection and control for continuous-time switched systems with Average Dwell Time," *Circuits Systems and Signal Processing*, vol. 37, no 6, pp. 2357-2373, June 2018.
- [14] Y. Jin, Y. Zhang, Y. Jing and J. Fu, "An Average Dwell-Time Method for Fault-Tolerant Control of Switched Time-Delay Systems and Its Application," in *IEEE Transactions on Industrial Electronics*, vol. 66, no. 4, pp. 3139-3147, April 2019.
- [15] A. A. Ladel, A. Benzaouia, R. Outbib and M. Ouladsine, "Integrated sensor fault estimation and control for continuous-time switched systems: A new separation principle," *ISA transactions*, vol. 112, pp. 224-233, June 2021.
- [16] P. Gahinet and P. Apkarian, "An linear matrix inequality approach to H_{∞} control," *International journal of robust and nonlinear control*, vol. 4, no 4, pp. 421-448, July 1994.
- [17] L. Zhou, D. W. Ho and G. Zhai, "Stability analysis of switched linear singular systems," *Automatica*, vol. 49, no 5, pp. 1481-1487, May 2013.
- [18] B. Jiang, J. L. Wang and Y. C. Soh, "An adaptive technique for robust diagnosis of faults with independent effects on system outputs," *International Journal of control*, vol. 75, no 11, pp. 792-802, November 2002.
- [19] K. Telbissi and A. Benzaouia, "Actuator fault estimation based on proportional-integral observer for delayed continuous-time switched systems," *Circuits, Systems, and Signal Processing*, vol. 38, no 10, pp. 4405-4419, October 2019.