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# On the Synthesis of an Integrated Active FTC Scheme for Continuous-Time Switched Systems with Actuator Faults

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**Abstract**—The present paper aims to design an integrated state/fault estimation and fault tolerant control issues for switched systems affected by actuator fault and external disturbances. A switched adaptive state/fault estimation observer is proposed to simultaneously estimate the system state and actuator fault. Based on this observer, we develop a fault tolerant control law not only to stabilize the closed-loop system, but also to compensate the faults effects. Then, by using the switched Lyapunov function and Average Dwell Time (ADT) concept, some sufficient conditions are derived and applied to build the switched adaptive observer-based state feedback controller. The observer and controller gain matrices are also obtained by solving an optimization problem with Linear Matrix Inequality (LMI) constraints. Finally, simulation results of a switched electrical system are presented to illustrate the effectiveness of the proposed design methods.

**Index Terms**—Switched systems, Fault Tolerant Control, Switched Adaptive Observer, Average Dwell Time, LMIs Constraints

## I. INTRODUCTION

In the last few decades, due to the increased demands for maintaining required high system performance, safety, and productivity in different conditions of operation, the realm of Fault Tolerant Control (FTC) has attracted more and more attention for both industry and academic communities. Generally speaking, the FTC system is a control system that is able to maintain the system stability and can preserve such performances in the presence of fault while correctly detected in time [1-4].

On the other hand, switched systems have attracted many scholars' attention because of their large number of applications in practical engineering systems, such as flight control systems, power systems, automotive systems, robotics systems, and biological systems [5-8]. The switched systems are considered a special class of hybrid systems, which consist of a family of subsystems described by continuous-time or discrete-time dynamics, and these subsystems are governed by a switching signal [9].

Recently, such important efforts have been focused on the design of fault tolerant controllers for switched systems. For instance, authors in [10] studied fault estimation and fault tolerant control with time delay for switching systems subject to actuator fault. An adaptive observer was developed for the Fault Detection and Isolation (FDI) task to detect and estimate the fault signal. Based on the fault estimation information an observer-based fault-tolerant controller is proposed in order to guarantee the closed loop system stability. Authors in [11] have dealt with the problem of fault estimation and accommodation for a class of discrete-time switched systems vulnerable to a sensor fault. Based on the descriptor observer design scheme, the sensor fault has to be detected and based on the information about the fault, an online controller re-design took place such that closed-loop stability with Lyapunov theory. In [12], a reduced-order observer has been designed in order to ensure fault detection for a class of uncertain switched systems affected by both sensor and actuator faults. After this, a state feedback robust controller is developed and the stability of the observer- fault tolerant controller closed-loop system is analysed in a sense of  $H_\infty$  performance index.

During the last decade, literature shows a steady increase in interest in using the Average Dwell Time (ADT) technique to deal with fault tolerant control problems for switched systems is developed. Authors in [13] proposed a robust fault detection and fault tolerant control for switched systems with ADT. The synthesis problem of the observer and controller is solved by using multiple Lyapunov function and ADT techniques. The proposed control system guaranteed the  $H_\infty$  performance and the asymptotic stability. Sufficient conditions for the existence of the designed observer and controller have been provided in terms of Linear Matrix Inequalities (LMI). In [14], authors have dealt with the problem of fault-tolerant control design for a class of switched time-delay linear systems with structural uncertainties by using the ADT approach. Authors in [15] developed an integrated sensor Fault Estimation (FE) and FTC method for a class of continuous-time switched

systems. By mean the state/fault estimation observer, the fault amplitude and system states were simultaneously estimated, and then exploiting information provided by the observer, a fault tolerant state feedback controller was designed. The stability of the overall system was fully studied by applying the ADT concept and the Lyapunov theory and the problem was solved in terms of LMIs.

Motivated by the above observations, in the present paper we are dedicated to the FE/FTC designs for a class of switched systems with actuator faults and external disturbances. The major contributions of this work can be summarized as the following aspects:

- (i) A Switched Adaptive Observer (SAO) is proposed by using ADT method, such that the states and actuator faults can be simultaneously estimated.
- (ii) Based on the states and actuator faults estimation information, a SAO-based state feedback fault tolerant controller is designed, under the average dwell time switching to tolerate the faults, which can guarantee that the closed-loop system is asymptotically stable.
- (iii) A sufficient condition is provided to guarantee the existence of the observer-based fault tolerant controller by the feasibility of an optimization problem with LMI constraint.
- (iv) The obtained results are successfully illustrated to a switched electrical system.

The remaining part of this paper is organized as follows. Section II describes preliminaries and problem formulation. Section III is dedicated to the design of an observer-based FTC to robustly stabilize the closed-loop system in the presence of actuator faults. In section IV, an example is given to illustrate the effectiveness of the proposed approach. Finally, section V draws some conclusions.

## II. PRELIMINARIES AND PROBLEM FORMULATION

Let's consider a class of dynamical systems modeled as switched linear systems with actuator faults and unknown external disturbance described as follows:

$$\begin{cases} \dot{x}(t) = A_{\sigma(t)}x(t) + B_{\sigma(t)}u(t) + F_{\sigma(t)}f(t) + R_{\sigma(t)}d(t) \\ y(t) = C_{\sigma(t)}x(t) \end{cases} \quad (1)$$

where  $x(t) \in \mathbb{R}^n$  is the state vector,  $u(t) \in \mathbb{R}^p$  is the FTC law vector to be designed,  $y(t) \in \mathbb{R}^m$  represents the output measurement vectors,  $f(t) \in \mathbb{R}^q$  is the actuator fault vector and  $d(t) \in \mathbb{R}^r$  represents the unknown bounded disturbance vector. The switching signal  $\sigma(t) : \mathbb{R}^+ \rightarrow \Theta$  is assumed to be a piecewise constant function and continuous from the right everywhere. Where  $\Theta$  is a finite set described by  $\Theta = \{1, 2, \dots, N\}$  and  $N$  denotes the number of subsystems. When  $\sigma(t) = i$ , the  $i$ -th subsystem is activated for some  $i \in \Theta$ . In this case, the system (1) can be simplified in the form below:

$$\begin{cases} \dot{x}(t) = A_i x(t) + B_i u(t) + F_i f(t) + R_i d(t) \\ y(t) = C_i x(t) \end{cases} \quad (2)$$

It should be indicated that the parameter matrices in the abovementioned system are assumed to be constant real matrices with appropriate dimensions.

Before starting the main results of the present paper, we will make the following assumptions which are typically required in the switched adaptive observer and fault tolerant control design for switched systems.

**Assumption 1.** We assume that the actuator fault distribution matrices  $F_i$  are of full column rank.

$$\text{rank}(C_i F_i) = \text{rank}(F_i) = q \quad (3)$$

**Assumption 2.** The pair  $(A_i, C_i)$  is observable  $\forall i \in \Theta$ .

$$\text{rank} \begin{bmatrix} C_i \\ C_i A_i \\ \vdots \\ C_i A_i^{n-1} \end{bmatrix} = n \quad (4)$$

**Assumption 3.** The fault  $f(t)$  satisfies  $\|f(t)\| \leq \alpha_1$ , its derivative is bounded such that  $\|\dot{f}(t)\| \leq \alpha_2$  and the unknown input vector verifies  $d(t)$  verifies  $\|d(t)\| \leq \alpha_3$ , where  $\alpha_1, \alpha_2, \alpha_3 \geq 0$ .

The following definition and lemmas are added for the convenes of later proof.

**Definition 1.** [16] For any switching signal  $\sigma(t)$  and any  $\tau_2 \geq \tau_1 \geq 0$ , let  $N_{\sigma(t)}(\tau_1, \tau_2)$  denote the number of switchings  $\sigma(t)$  on an interval  $(\tau_1, \tau_2)$ . If

$$N_{\sigma(t)}(\tau_1, \tau_2) \leq N_0 + \frac{\tau_2 - \tau_1}{\tau_a} \quad (5)$$

holds for a given two positive numbers  $N_0$  and  $\tau_a$ , then the constant  $\tau_a$  is called the average dwell time and  $N_0$  is the chattering bound.

**Lemma 1.** [17] Suppose that there exist  $C^1$  functions  $V_{\sigma(t)} : \mathbb{R}^n \rightarrow \mathbb{R}$  and two class  $K_\infty$  functions  $\kappa_1$  and  $\kappa_2$  and two positive numbers  $\alpha > 0$ ,  $\mu > 1$  such that we have

$$\kappa_1(|x(t)|) \leq V_{\sigma(t)}(x(t)) \leq \kappa_2(|x(t)|) \quad (6)$$

$$\dot{V}_{\sigma(t)}(x(t)) \leq -\alpha V_{\sigma(t)}(x(t)) \quad (7)$$

and  $\forall(\sigma(t_l) = i, \sigma(t_l^-) = j) \in N \times N, i \neq j$ ,

$$V_i(x(t)) \leq \mu V_j(x(t)) \quad (8)$$

then the switched linear system  $\dot{x}(t) = f_{\sigma(t)}(x(t))$  is Globally Uniformly Asymptotically Stable (GUAS) for any switching signal with average dwell time:

$$\tau_a \geq \tau_a^* = \frac{\ln \mu}{\alpha} \quad (9)$$

**Lemma 2.** [18] For a positive definite matrix  $Q$  that is:  $Q = Q^T > 0$  and a positive scalar  $\varepsilon$ ; the following inequality is true:

$$2x^T y \leq \frac{1}{\varepsilon} x^T Q x + \varepsilon y^T Q^{-1} y; \quad x, y \in \mathbb{R}^n \quad (10)$$

### III. OBSERVER-BASED FTC DESIGN

In order to study the state feedback fault tolerant control design problem, we have developed a robust FTC law for switched systems in presence of actuator faults. Thus, we require to estimate simultaneously system states and actuator faults. To do this, we have designed the following switched adaptive observer:

$$\begin{cases} \dot{z}(t) = N_i z(t) + G_i u_f(t) + L_i y(t) + H_i \hat{f}(t) \\ \hat{x}(t) = z(t) + T_2 y(t) \\ \hat{y}(t) = C_i \hat{x}(t) \\ \dot{\hat{f}}(t) = \Gamma \Phi_i(\dot{e}_y(t) + \eta e_y(t)) \\ e_y(t) = y(t) - \hat{y}(t) \end{cases} \quad (11)$$

where  $z(t) \in \mathbb{R}^n$  is the observer state vector,  $\hat{x}(t) \in \mathbb{R}^n$  the estimated state vector,  $\hat{y}(t) \in \mathbb{R}^m$  is the estimated output vector,  $\hat{f}(t) \in \mathbb{R}^p$  is the estimated actuator faults and  $N_i \in \mathbb{R}^{n \times n}$ ,  $G_i \in \mathbb{R}^{n \times p}$ ,  $L_i \in \mathbb{R}^{n \times m}$ ,  $\Phi_i \in \mathbb{R}^{p \times m}$  and  $T_2 \in \mathbb{R}^{n \times m}$  are the observer gain matrices to be determined. The matrix  $\Gamma \in \mathbb{R}^{p \times p}$  is a symmetric positive definite learning rate matrix and  $\eta$  is a positive scalar.

Based on both actuator fault and system states estimation, we propose to conceive a robust controller as:

$$u_{FTC}(t) = -K_i \hat{x}(t) - q_i \hat{f}(t) \quad (12)$$

where  $K_i \in \mathbb{R}^{p \times n}$  and  $q_i \in \mathbb{R}^{p \times p}$  represent, respectively, the state feedback control and fault compensation gains. We assume that  $q_i = B_i^+ F_i$ . Based on this observer, the fault tolerant controller can compensate faults effects and guarantee the stabilization of the closed-loop system.

Now, let us define the following state estimation error  $e_s(t)$  from (2) and (11) such that:

$$e_s(t) = x(t) - \hat{x}(t) = (I_n - T_2 C_i)x(t) - z(t) \quad (13)$$

Since that for  $\text{rank} \begin{bmatrix} I_n & C_i \end{bmatrix}^T = n$ , there exists nonsingular matrices  $T_1 \in \mathbb{R}^{n \times n}$  and  $T_2 \in \mathbb{R}^{n \times m}$  such that:

$$\begin{bmatrix} T_1 & T_2 \end{bmatrix} \begin{bmatrix} I_n \\ C_i \end{bmatrix} = I_n \quad (14)$$

Then, the state estimation error (13) is described by:

$$e_s(t) = T_1 x(t) - z(t) \quad (15)$$

From (15), the dynamic of the state estimation error obey the differential equation:

$$\dot{e}_s(t) = T_1 \dot{x}(t) - \dot{z}(t) \quad (16)$$

Substituting the equations of  $\dot{x}(t)$  and  $\dot{z}(t)$  respectively from (2) and (11), the equation (16) becomes after some calculations:

$$\begin{aligned} \dot{e}_s(t) &= (T_1 A_i + E_i C_i - N_i)x(t) + (T_1 B_i - G_i)u(t) \\ &\quad + N_i e_s(t) + H_i e_f(t) + \bar{R}_i d(t) \end{aligned} \quad (17)$$

where

$$E_i = N_i T_2 - L_i \quad (18)$$

$$\bar{R}_i = T_1 R_i \quad (19)$$

If the following conditions hold  $\forall i \in \Theta$ :

$$N_i = T_1 A_i + E_i C_i \quad (20)$$

$$G_i = T_1 B_i \quad (21)$$

$$H_i = T_1 F_i \quad (22)$$

The dynamic of the state estimation error reduces to:

$$\dot{e}_s(t) = N_i e_s(t) + H_i e_f(t) + \bar{R}_i d(t) \quad (23)$$

The dynamic of the closed-loop state system with the control law (12) is defined as follows:

$$\dot{x}(t) = \Pi_i x(t) + B_i K_i e_s(t) + F_i e_f(t) + R_i d(t) \quad (24)$$

with  $\Pi_i = A_i - B_i K_i$ .

Finally, the fault estimation error dynamic is defined as follows:

$$\dot{e}_f(t) = \dot{f}(t) - \hat{\dot{f}}(t) \quad (25)$$

**Theorem 1.** For given positive scalars  $\eta, \varepsilon, \beta, \alpha, \mu > 1$  and a positive definite matrix  $\Gamma$ , if there exist symmetric and positive definite matrices  $X_i = P_i^{-1}$ ,  $Q_i$ ,  $Q_{1i}$ ,  $Q_{2i}$  and  $Q_{3i}$  and matrices  $S_i = Q_i E_i$ ,  $W_i = K_i X_i$  and  $\Phi_i$  such that the following LMIs are satisfied  $\forall (i, j) \in \Theta \times \Theta, i \neq j$ :

$$\begin{pmatrix} \bar{\Xi}_i & B_i W_i & F_i & 0 & 0 & X_i^T \\ * & -2\beta X_i & 0 & \beta I & 0 & 0 \\ * & * & -2\beta I & 0 & \beta I & 0 \\ * & * & * & \bar{\Omega}_i & \Sigma_i & 0 \\ * & * & * & * & \Upsilon_i & 0 \\ * & * & * & * & * & -\frac{1}{\varepsilon} Q_{1i} \end{pmatrix} < 0 \quad (26)$$

$$X_j < \mu X_i \quad (27)$$

such that

$$H_i^T Q_i - \Phi_i C_i = 0 \quad (28)$$

where

$$\bar{\Xi}_i = A_i X_i + X_i A_i^T - B_i W_i - W_i^T B_i^T + \alpha X_i \quad (29)$$

$$\bar{\Omega}_i = (T_1 A_i)^T Q_i + Q_i (T_1 A_i) + S_i C_i + C_i^T S_i^T + \alpha Q_i + \frac{1}{\varepsilon} Q_{2i} \quad (30)$$

$$\Sigma_i = -\frac{1}{\eta} C_i^T S_i^T H_i - \frac{1}{\eta} (T_1 A_i)^T Q_i H_i \quad (31)$$

$$\Upsilon_i = -\frac{1}{\eta} (H_i^T Q_i H_i + H_i^T Q_i H_i + \Gamma^{-1}) + \frac{2}{\eta \varepsilon} Q_{3i} \quad (32)$$

Then the state vector  $x(t)$ , the state estimation error  $e_s(t)$  and the fault estimation error  $e_f(t)$  are bounded if the ADT satisfies the following condition  $\tau_a \geq \tau_a^* = \frac{\ln \mu}{\alpha}$ . The matrix gains of the switched adaptive observer (11) and the controller (12) are given by  $E_i = Q_i^{-1} S_i$  and  $K_i = W_i X_i^{-1}$ .

**Proof of Theorem 1.** In this proof, we use the theory of Lyapunov to demonstrate the stability of the closed loop system. Then, the problem is turned into an optimization problem through the use of LMI so as to determinate the unknown matrices of both the proposed FTC controller and the switched adaptive observer.

Thus, let us consider the following switched Lyapunov function candidate as:

$$V_i(t) = x^T(t)P_i x(t) + e_s^T(t)Q_i e_s(t) + \frac{1}{\eta} e_f^T(t)\Gamma^{-1} e_f(t) \quad (33)$$

where  $P_i$ ,  $Q_i$  and  $\Gamma$  are symmetric positive definite matrices with appropriate dimensions. The time derivative of the switched Lyapunov function (33) leads to:

$$\begin{aligned} \dot{V}_i(t) &= \dot{x}^T(t)P_i x(t) + x^T(t)P_i \dot{x}(t) + \dot{e}_s^T(t)Q_i e_s(t) \\ &+ e_s^T(t)Q_i \dot{e}_s(t) + \frac{1}{\eta} \dot{e}_f^T(t)\Gamma^{-1} e_f(t) \\ &+ \frac{1}{\eta} e_f^T(t)\Gamma^{-1} \dot{e}_f(t) \end{aligned} \quad (34)$$

By considering the expressions (24) of  $\dot{x}(t)$ , (23) of  $\dot{e}_s(t)$ , (25) of  $\dot{e}_f(t)$  and by taking into account the fault estimation of  $\hat{f}(t)$  in (11),  $\dot{V}_i(t)$  becomes:

$$\begin{aligned} \dot{V}_i(t) &= x^T(t)\Xi_i x(t) + 2x^T(t)P_i B_i K_i e_s(t) \\ &+ 2x^T(t)P_i B_i e_f(t) + 2x^T(t)P_i R_i d(t) \\ &+ e_s^T(t)\Omega_i e_s(t) + 2e_s^T(t)Q_i H_i e_f(t) \\ &+ 2e_s^T(t)Q_i \bar{R}_i d(t) + \frac{2}{\eta} e_f^T(t)\Gamma^{-1} \dot{f}(t) \\ &- \frac{2}{\eta} e_f^T(t)\Phi_i C_i \dot{e}_s(t) - 2e_s^T(t)C^T \Phi_i^T e_f(t) \end{aligned} \quad (35)$$

where

$$\Xi_i = \Pi_i^T P_i + P_i \Pi_i \quad (36)$$

$$\Omega_i = N_i^T Q_i + Q_i N_i \quad (37)$$

By using the equation (23) and the equality (28), it follows that

$$\begin{aligned} \dot{V}_i(t) &= x^T(t)\Xi_i x(t) + 2x^T(t)P_i B_i K_i e_s(t) + 2x^T(t)P_i B_i e_f(t) \\ &+ 2x^T(t)P_i R_i d(t) + e_s^T(t)\Omega_i e_s(t) + 2e_s^T(t)Q_i \bar{R}_i d(t) \\ &+ \frac{2}{\eta} e_f^T(t)\Gamma^{-1} \dot{f}(t) - \frac{2}{\eta} e_f^T(t)H_i^T Q_i N_i e_s(t) \\ &- \frac{2}{\eta} e_f^T(t)H_i^T Q_i H_i e_f(t) - \frac{2}{\eta} e_f^T(t)H_i^T Q_i \bar{R}_i d(t) \end{aligned} \quad (38)$$

Under Assumption 3, we apply Lemma 2 so as to get the following term inequalities from (38):

$$2x^T(t)P_i R_i d(t) \leq \frac{1}{\varepsilon} x^T(t)Q_{1i} x(t) + \eta_{1i} \quad (39)$$

$$2e_s^T(t)Q_i \bar{R}_i d(t) \leq \frac{1}{\varepsilon} e_s^T(t)Q_{2i} e_s(t) + \eta_{2i} \quad (40)$$

$$\frac{2}{\eta} e_f^T(t)\Gamma^{-1} \dot{f}(t) \leq \frac{1}{\eta\varepsilon} e_f^T(t)Q_{3i} e_f(t) + \eta_{3i} \quad (41)$$

$$-\frac{2}{\eta} e_f^T(t)H_i^T Q_i \bar{R}_i d(t) \leq \frac{1}{\eta\varepsilon} e_f^T(t)Q_{3i} e_f(t) + \eta_{4i} \quad (42)$$

where the scalars  $\eta_{1i}$ ,  $\eta_{2i}$ ,  $\eta_{3i}$  and  $\eta_{4i}$  are expressed as follows:

$$\eta_{1i} = \varepsilon\alpha_3^2 \lambda_{\max}(R_i^T P_i Q_{1i}^{-1} P_i R_i) \quad (43)$$

$$\eta_{2i} = \varepsilon\alpha_2^2 \lambda_{\max}(\bar{R}_i^T Q_i Q_{2i}^{-1} Q_i \bar{R}_i) \quad (44)$$

$$\eta_{3i} = \frac{\varepsilon}{\sigma} \alpha_1^2 \lambda_{\max}(\Gamma^{-1} Q_{3i}^{-1} \Gamma^{-1}) \quad (45)$$

$$\eta_{4i} = \frac{\varepsilon}{\sigma} \alpha_2^2 \lambda_{\max}(H_i^T Q_i Q_{3i}^{-1} Q_i H_i) \quad (46)$$

By taking into consideration the inequalities (43)-(46) and the equation (38),  $\dot{V}_i(t)$  can be bounded as follows:

$$\begin{aligned} \dot{V}_i(t) &\leq x^T(t)(\Xi_i + \frac{1}{\varepsilon} Q_{1i})x(t) + 2x^T(t)P_i B_i K_i e_s(t) \\ &+ 2x^T(t)P_i B_i e_f(t) + e_s^T(t)(\Omega_i + \frac{1}{\varepsilon} Q_{2i})e_s(t) \\ &- \frac{2}{\eta} e_f^T(t)(H_i^T Q_i H_i + \frac{2}{\eta\varepsilon} Q_{3i})e_f(t) \\ &- \frac{2}{\eta} e_s^T(t)N_i^T Q_i H_i e_f(t) + \delta \end{aligned} \quad (47)$$

where the scalar  $\delta$  is the maximum value over  $i$  such that:

$$\delta = \max_i(\eta_{1i} + \eta_{2i} + \eta_{3i} + \eta_{4i}) \quad (48)$$

By Lemma 1, one can get that:

$$\dot{V}_i(t) + \alpha V_i(t) \leq \tilde{x}^T(t)\Lambda_i \tilde{x}(t) + \delta \quad (49)$$

where  $\tilde{x}^T(t) = [x^T(t) \ e_s^T(t) \ e_f^T(t)]$  and  $\Lambda_i$  is a matrix defined as follows:

$$\Lambda_i = \begin{pmatrix} \tilde{\Xi}_i & P_i B_i K_i & P_i H_i \\ * & \tilde{\Omega}_i & -\frac{1}{\sigma} N_i^T Q_i H_i \\ * & * & \Upsilon_i \end{pmatrix} \quad (50)$$

with

$$\tilde{\Xi}_i = \Xi_i + \alpha P_i + \frac{1}{\varepsilon} Q_{1i} \quad (51)$$

$$\tilde{\Omega}_i = \Omega_i + \alpha Q_i + \frac{1}{\varepsilon} Q_{2i} \quad (52)$$

$$\Upsilon_i = -\frac{1}{\eta}(H_i^T Q_i H_i + H_i^T Q_i H_i + \Gamma^{-1}) + \frac{2}{\eta\varepsilon} Q_{3i} \quad (53)$$

If the following inequality holds:

$$\Lambda_i < 0 \quad (54)$$

We can obtain that:

$$\dot{V}_i(\tilde{x}(t)) + \alpha V_i(\tilde{x}(t)) \leq -\zeta \|\tilde{x}(t)\|^2 + \delta \quad (55)$$

where  $\zeta > 0$  is given by

$$\zeta = \min \lambda_{\min}(-\Lambda_i) < 0 \quad (56)$$

To complete the proof by considering the constraint (54), we define a matrix:

$$\Delta_i = \begin{pmatrix} \tilde{\Xi}_i & Z_i \\ Z_i^T & \Psi_i \end{pmatrix} < 0 \quad (57)$$

with

$$Z_i = \begin{pmatrix} P_i B_i K_i & P_i H_i \end{pmatrix} \quad (58)$$

$$\Psi_i = \begin{pmatrix} \tilde{\Omega}_i & -\frac{1}{\sigma} N_i^T Q_i H_i \\ * & \Upsilon_i \end{pmatrix} \quad (59)$$

Consider a symmetric matrix  $\chi$  defined as

$$\chi = \begin{pmatrix} P_i^{-1} & 0 \\ 0 & \chi_1 \end{pmatrix} \quad (60)$$

with

$$\chi_1 = \begin{pmatrix} P_i^{-1} & 0 \\ 0 & I \end{pmatrix} \quad (61)$$

Then, by post and pre-multiplying the inequality (57) by  $\chi$ , we can obtain that

$$\begin{pmatrix} P_i^{-1}\tilde{\Xi}_i P_i^{-1} & P_i^{-1}Z_i\chi_1 \\ * & \chi_1\Psi_i\chi_1 \end{pmatrix} < 0 \quad (62)$$

The term  $\chi_1\Psi_i\chi_1$  can be replaced by considering the following inequality which holds any scalar  $\beta$  such that

$$\begin{aligned} (\chi_1 + \beta\Psi_i^{-1})^T \Psi_i (\chi_1 + \beta\Psi_i^{-1}) &\leq 0 \\ \Leftrightarrow \chi_1\Psi_i\chi_1 &\leq -2\beta\chi_1 - \beta^2\Psi_i^{-1} \end{aligned} \quad (63)$$

Considering (63) and with the Schur Complement, the inequality (62) becomes

$$\begin{pmatrix} P_i^{-1}\tilde{\Xi}_i P_i^{-1} & P_i^{-1}Z_i\chi_1 & 0 \\ * & -2\beta\chi_1 & \beta I \\ * & * & \Psi_i \end{pmatrix} < 0 \quad (64)$$

Using the definition of matrices  $\tilde{\Xi}_i$ ,  $Z_i$  and  $\Psi_i$  and considering the change of variable  $X_i = P_i^{-1}$  we can obtain that

$$\begin{pmatrix} \hat{\Xi}_i & B_i K_i X_i & F_i & 0 & 0 \\ * & -2\beta X_i & 0 & \beta I & 0 \\ * & * & -2\beta I & 0 & \beta I \\ * & * & * & \tilde{\Omega}_i & -\frac{1}{\sigma} N_i^T Q_i H_i \\ * & * & * & * & \Upsilon_i \end{pmatrix} < 0 \quad (65)$$

with

$$\hat{\Xi}_i = X_i \Pi_i^T + \Pi_i X_i + \alpha X_i + \frac{1}{\varepsilon} X_i Q_{1i} X_i \quad (66)$$

Now, we dissociate the term  $\frac{1}{\varepsilon} X_i Q_{1i} X_i$  from the inequality (65) in order to reformulate it as follows

$$\begin{pmatrix} \bar{\Xi}_i & B_i K_i X_i & F_i & 0 & 0 \\ * & -2\beta X_i & 0 & \beta I & 0 \\ * & * & -2\beta I & 0 & \beta I \\ * & * & * & \tilde{\Omega}_i & -\frac{1}{\sigma} N_i^T Q_i H_i \\ * & * & * & * & \Upsilon_i \end{pmatrix} - \bar{X}_i^T \left( -\frac{1}{\varepsilon} Q_{1i} \right) \bar{X}_i < 0 \quad (67)$$

where

$$\bar{\Xi}_i = X_i \Pi_i^T + \Pi_i X_i + \alpha X_i \quad (68)$$

and

$$\bar{X}_i = \begin{pmatrix} X_i & 0 & 0 & 0 & 0 \end{pmatrix} \quad (69)$$

Now, we apply the modified Schur Lemma in the above inequality (67), changing  $\Pi_i$  and  $N_i$  by their expressions and taking into account the following change of variables  $W_i = K_i X_i$  and  $S_i = Q_i E_i$  the inequality (67) becomes reformulated as a Linear Matriciel Inequality which can be rewritten as in the **Theorem 1**. This one ends the proof of the theorem.

#### IV. ILLUSTRATIVE EXAMPLE

In this section, the proposed example considers a switched electrical circuit model borrowed from [19]. In the system shown in Fig. 1, Sw1 is a bipolar transistor and Sw2 is a diode. This circuit has two switching modes, that is,  $N = 2$ ,  $\sigma(t) : [0, \infty) \rightarrow \{1, 2\}$ : mode 1, Sw1 is closed and Sw2 is off; mode 2, Sw1 is off and Sw2 is closed. Selecting the state variable  $\begin{bmatrix} x_1(t) & x_2(t) \end{bmatrix}^T$  where  $x_1(t)$  is the inductance current  $i_L(t)$ ,  $x_2(t)$  is the capacitor voltage  $V_c(t)$  and  $V_s(t)$  is the power supply voltage. Under the different modes, the system matrices of the switched electrical system are given by:

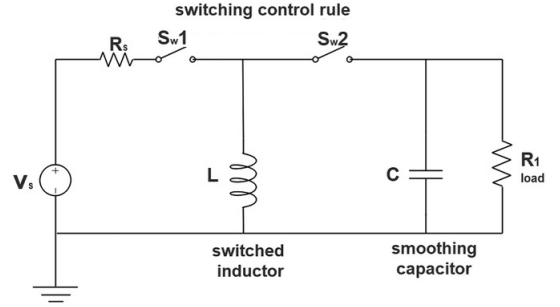


Fig. 1. A switched electrical circuit

##### Subsystem 1:

$$A_1 = \begin{bmatrix} -0.54 & 1.02 \\ 0.17 & -0.31 \end{bmatrix}, B_1 = \begin{bmatrix} 0.1 \\ 0.2 \end{bmatrix}, F_1 = \begin{bmatrix} 0.1 \\ 0.2 \end{bmatrix}$$

$$C_1 = \begin{bmatrix} 0.1 & 0.2 \end{bmatrix}, R_1 = \begin{bmatrix} 0.01 \\ 0.03 \end{bmatrix}$$

##### Subsystem 2:

$$A_2 = \begin{bmatrix} -0.01 & 0.1 \\ 0.01 & 0.04 \end{bmatrix}, B_2 = \begin{bmatrix} 0.2 \\ 0.4 \end{bmatrix}, F_2 = \begin{bmatrix} 0.2 \\ 0.4 \end{bmatrix},$$

$$C_2 = \begin{bmatrix} 0.2 & 0.3 \end{bmatrix}, R_2 = \begin{bmatrix} 0.01 \\ 0.03 \end{bmatrix}$$

By solving the LMIs condition of **Theorem 1** for the following choice of parameters:  $\eta = 1$ ,  $\varepsilon = 1$ ,  $\beta = 0.5$  and  $\Gamma = 200$ . The designed observer and controller gains are obtained as:

##### For i=1

$$N_1 = \begin{bmatrix} -1.2280 & -0.3637 \\ 0.5481 & 0.2509 \end{bmatrix}, G_1 = \begin{bmatrix} 0.0952 \\ 0.1905 \end{bmatrix}$$

$$L_1 = \begin{bmatrix} 6.7131 \\ -2.7512 \end{bmatrix}, \Phi_1 = [17.0262]$$

$$K_1 = \begin{bmatrix} 1.8570 & 2.9636 \end{bmatrix}$$

##### For i=2

$$N_2 = \begin{bmatrix} 0.2349 & 0.5883 \\ -0.1575 & -0.2980 \end{bmatrix}, G_2 = \begin{bmatrix} 0.1905 \\ 0.3810 \end{bmatrix}$$

$$L_2 = \begin{bmatrix} -2.3156 \\ 1.6010 \end{bmatrix}, \Phi_2 = [34.0524]$$

$$K_2 = \begin{bmatrix} 1.2992 & 1.2949 \end{bmatrix}$$

For the switching signal, choose the parameter value as  $\mu = 2.5$  and  $\alpha = 0.8$ , to calculate average dwell time as  $\tau_a^* = \frac{\ln \mu}{\alpha} = 1.1454$ . Hence, the switching interval from one mode to the other is greater than 1.1454. For simulation purposes, we apply the switching signal presented in Fig. 2 specifies the switching between the two subsystems.

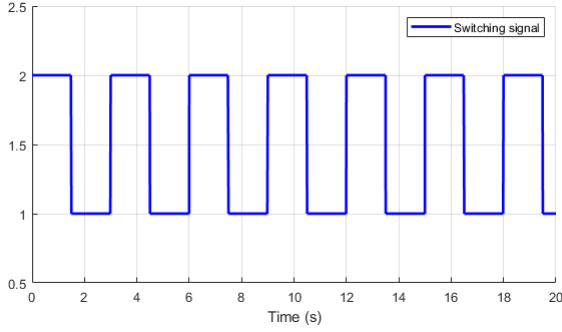


Fig. 2. Switching signal evolution

For the simulation, the external disturbance  $d(t)$  is chosen as the band-limited white noise, which is shown in Fig. 3.

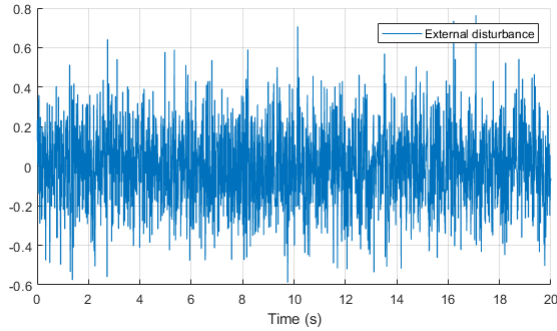


Fig. 3. The external disturbance  $d(t)$

In order to show the performance and capability of actuator fault estimation effect by using the switched adaptive observer, two cases of actuators faults are considered.

If the actuator faults type is a constant value described as:

$$f_1(t) = \begin{cases} 0, & 0s \leq t < 5s \\ 1, & 5s \leq t \leq 20s \end{cases} \quad (70)$$

In this case, the simulation result is shown in Fig. 4.

If the actuator fault is a time-varying function as follows:

$$f_2(t) = \begin{cases} 0, & 0s \leq t < 5s \\ 1.8 \cos(1.6t + 3.25), & 5s \leq t \leq 20s \end{cases} \quad (71)$$

In this case, the simulation result is described in Fig. 5.

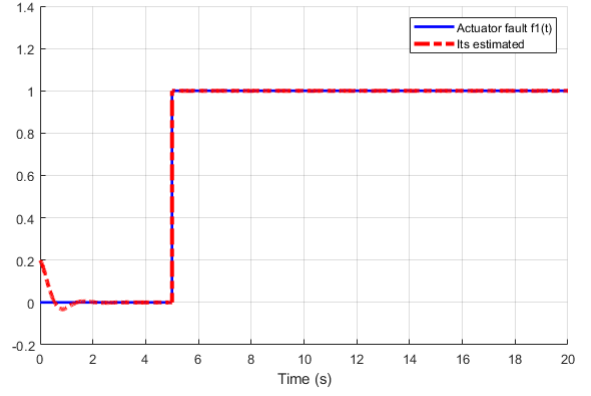


Fig. 4. Actuator fault  $f_1(t)$  and its estimate  $\hat{f}_1(t)$

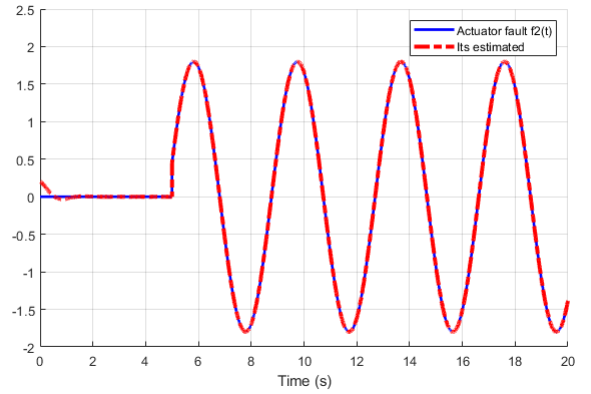


Fig. 5. Actuator fault  $f_2(t)$  and its estimate  $\hat{f}_2(t)$

Form the simulation results shown in Figs. 4 and 5, one can conclude that the technique proposed in this paper has a good actuator fault estimation performance with accuracy and rapidly for both constant faults and time-varying faults by use of the switched adaptive observer despite an additive noise.

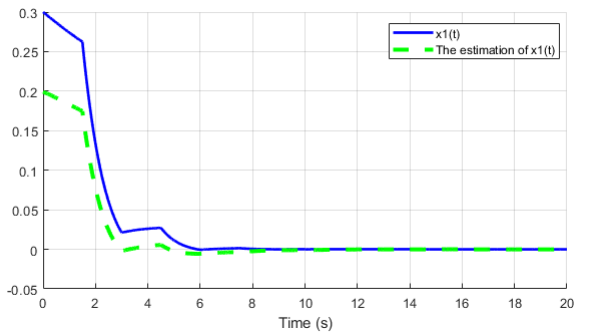


Fig. 6. The state  $x_1(t)$  and its estimate  $\hat{x}_1(t)$

In simulation, the initial state value of original switched system (2) and the initial estimated switched system states (11) are set as  $x(0) = [0.3 \ -0.2]^T$  and  $\hat{x}(0) = [0.2 \ -0.1]^T$  respectively.

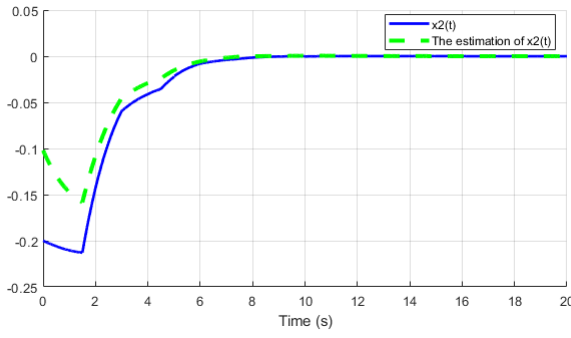


Fig. 7. The state  $x_2(t)$  and its estimate  $\hat{x}_2(t)$

The evolution of states trajectories are plotted in Figs. 6 and 7, showing that the system is stablized and that actuator faults are completely compensated even in the presence of the disturbance signal  $d(t)$ .

The state estimation errors are shown in the Fig. 8, from which we can seen that the proposed Switched Adaptive Observer can estimate the states very accurately, even after the occurrence of an actuator fault.

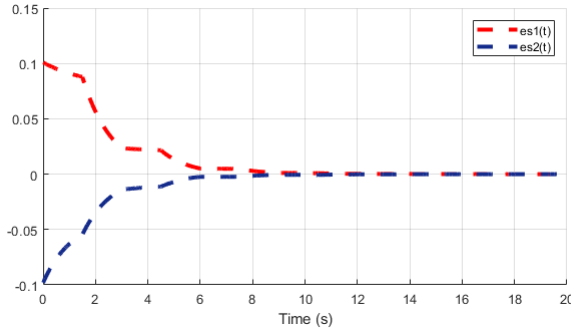


Fig. 8. State estimation errors  $e_s(t)$

The example studied shows that the integrated FE/FTC design scheme can accurately estimate the actuator fault and ensure the stability of the closed-loop system.

## V. CONCLUSION

This paper has studied the problem of fault estimation and fault tolerant control for continuous-time switched according to the ADT concept. By using switched adaptive observer a state/fault estimation algorithm is achieved rapidly and accurately. Then based on the obtained online fault estimation information, a switched state feedback fault tolerant controller is employed to compensate the effect of faults by stabilizing the closed-loop systems. Sufficient stability conditions, in terms of linear matrix inequalities, for the existence of both switched adaptive observer and robust active fault tolerant control law are given in an integrated way. Finally, the developed scheme has been applied to an electrical circuit system with two types of actuator faults so as to illustrate the effectiveness of the proposed approach. As future work, we will focus on FTC design problem for switched nonlinear systems.

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