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# Public Law Enforcement under Ambiguity \*

Bertrand Chopard<sup>†</sup>    Marie Obidzinski<sup>‡</sup>

January 4, 2021

## Abstract

*In real life situations, potential offenders may only have a vague idea of their own probability of getting caught and possibly, convicted. As they have beliefs regarding this probability, they may exhibit optimism or pessimism. Thus there exists a discrepancy between the objective expected fine and the subjective expected fine. In this context, we investigate how the fact that the choice whether or not to commit a harmful act is framed as a decision under ambiguity can modify the standard Beckerian results regarding the optimal fine and the optimal resources that should be invested in detection and conviction.*

**Keywords:** deterrence, pessimism, optimism, ambiguity.

**JEL Codes:** D81, K42.

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# 1 Introduction

In the public law enforcement framework *à la* Becker (1968), individuals are assumed to perfectly assess the probability of being fined if they commit an offense. However, in real life situations, potential offenders generally only have a vague idea of their own probability of getting caught and possibly, convicted. They have beliefs regarding the probability of detection and may exhibit optimism or pessimism. For instance, in the context of tax compliance, taxpayers tend to overestimate the probability of facing an investigation by the tax authority (Alm et al. 1992; Andreoni et al. 1998). In other situations, people are somewhat optimistic about their chances of not meeting with misfortune such as a car accident or illness (Jolls, 1998).

In our context, the way individuals estimate the probability of being detected will affect their decision whether or not to obey the law. This probability of detection and conviction is ambiguous while potential offenders know full well the amount of the sanction. The main justification for this hypothesis is that sanctions are often detailed in sentencing guidelines or penal codes, while information about the probability of detection cannot be given. Furthermore, uncertainty over the size of sanctions raises issues regarding the principle of equality before the law (Universal Declaration of Human Rights, 1948, article 7). Imagine Mr. A and Mr. B commit the same crime under the same circumstances. If Mr. A is sentenced to 5 years while Mr. B is sentenced to 3 years, the difference seems quite unfair<sup>1</sup>.

Most theoretical contributions based on the standard public law enforcement model (Polinsky and Shavell, 2007) assume that the probability of detection and conviction is known by potential offenders. The aim of our paper is to investigate how ambiguity regarding this probability can modify the classical results regarding the optimal fine and the resources a benevolent public law enforcer should invest in detection and conviction.<sup>2</sup> Ambiguity, as defined by Snow (2010), is uncertainty about probability, created by missing information that is relevant and could be known. There exists several alternative models of choice under ambiguity (Etner *et al.* 2012). In this contribution, we adopt the Choquet expected utility framework and we represent the potential offender's beliefs on the probability of detection with a neoadditive capacity. Our framework is closed to Chateauneuf *et al.*'s (2007).

We consider successively two different objective functions for the authorities, denoted the *populist* and the *paternalistic* social welfare functions. These headings refer to Salanié and Treich (2009). They distinguish the *populist* regulator who “maximizes social welfare computed with citizens’ beliefs” from the *paternalist*

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<sup>1</sup>In practice, circumstances may vary a lot, and plea bargaining or prosecutors’ discretion for instance might alter this principle.

<sup>2</sup>In the basic framework, potential offenders are utility maximizers in the sense of von Neumann and Morgenstern (1944) and Savage (1954). Ellsberg’s experiments (1961) provide evidence that ambiguity induces choices which are incompatible with the SEU model. Since then, many alternative models to frame decision under ambiguity have been provided in the literature.

regulator who “maximizes social welfare computed with the regulator’s own belief”.<sup>3</sup> Whether a regulator should adopt one approach or the other is subject to a debate in public economics (Pollak 1998; Viscusi 2000; Salanié and Treich 2009; Johansson-Stenman 2008).<sup>4</sup> In particular, Johansson-Stenman (2008) supports that the regulator should take into account the discrepancy between the perceived and the objective risk as “the perceived risk affects individual utility directly”. This argument is in favor of the “populist” social welfare function. The author also refers to Becker and Rubinstein (2004) analysis of terror, where fear is inserted in the utility function. On the contrary, one reason to favor the paternalist approach is that the law enforcer may not observe these costs (such as the dis-utility of fear), or that what matters for society is what citizens are actually paying and not what they subjectively expect to pay.

In our public law enforcement setting, we look at the two social welfare functions (populist and paternalist), and we derive, in each case, the optimal enforcement policy.<sup>5</sup> The key difference between the two approaches is whether the discrepancy between the objective and the perceived probability of fine should be taken into account by the law enforcer. Indeed, this discrepancy might generate a perception bias cost (gain) when individuals are pessimistic (optimistic). A paternalistic public law enforcer does not take into account the discrepancy between the expected and the actual expected fine, while the populist law enforcer does.

Our results call for the degree of pessimism of potential offenders in determining deterrence policy to be taken into account. Indeed, recommendations on deterrence policy can be widely affected by beliefs. Assume that individuals are pessimistic: they overestimate the probability of detection and conviction. We find that optimal fines may be lower for two reasons. First, fines may be considered as costly transfers if society takes mental suffering into account. Second, the subjective probability of detection is higher than the objective probability. Regarding the optimal means to invest in detection, our results go in different directions depending on the objective function of the law enforcer. When the law enforcer is populist (he/she takes into account the perception cost), we show that it may be socially desirable to raise the probability and to lower the magnitude of fine accordingly (in order to keep the deterrence level constant) if the marginal cost of detection is sufficiently small. In such a case, the fine is not necessarily maximal. When the law enforcer is paternalistic (he/she ignores the perception cost), the optimal fine is always maximal (as fines are costless transfers). And it is possible that the means invested in detection are lower than those in the absence of ambiguity only under certain conditions. In such a case, the objective probability of detection appears to be a less efficient deterrence tool due to the weight of the beliefs.

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<sup>3</sup>The term “paternalism” refers to the protective attitude of an authority reminiscent of a father (*pater* in Latin).

<sup>4</sup>Note that the case where regulators themselves mis-perceive the risk is developed in Viscusi and Hamilton (1999). Arguably, it may not be the case in our framework since the law enforcer decides about the resources spent on detection.

<sup>5</sup>Note that there cannot be any comparison in terms of welfare since the construction in itself of the social welfare function under the two approaches is different.

The remainder of the paper is organized as follows. Section 2 presents the related literature. In section 3, we present the model of public enforcement of law under ambiguity. In section 4, we study the optimal fine. In section 5, both the monetary sanction and the probability of detection and conviction are endogenous. Section 6 concludes.

## 2 Related literature

To our knowledge, a limited number of papers have addressed the link between crime deterrence and ambiguity.<sup>6</sup> Let us point out the most relevant contributions for our analysis.

Harel and Segal (1999) describe how the legal system actually favors certainty relative to the sanction (for instance, through specifying the penalties in the criminal code or sentencing guidelines) and uncertainty towards the probability of detection and conviction and provide some explanations. They invoke behavioral and psychological insights, namely ambiguity aversion (Ellsberg, 1961). In the analysis developed by Harel and Segal (1999), the authorities' objective is to choose the criminals' most disfavored law enforcement scheme in order to induce more deterrence at the lowest cost. They show that this scheme should consist in a certain sanction and an uncertain probability of getting caught. They base their analysis on contributions and results from the behavioral economic literature and Prospect theory (Kahneman and Tversky, 1979). Our analysis differs in many aspects. We do not aim to determine whether uncertainty should be favored, but instead to examine the consequences of ambiguity for the optimal probability of detection and the amount of the fine. In addition, we consider both the cases where individuals might exhibit optimism and pessimism in a Chateauneuf et al. (2007) framework, while Harel and Segal (1999) consider only the case of ambiguity aversion. Furthermore, in our contribution the benevolent law enforcer aims to maximize social welfare. Conversely, Harel and Segal (1999) aim at determining the law enforcement scheme which maximizes deterrence at the cheapest cost.

Al-Nowaihi and Dhami (2013, 2018) proposes a formalized analysis of what they denote the “Becker proposition” (maximal fine and low probability of sanction) under alternative decision theory settings: expected utility, rank dependant utility and cumulative prospect theory. They show that Becker’s result emerges under each of these settings. In addition to the fact that we use an alternative framework (Choquet expected utility framework), the aim of the law enforcer is different in our contribution.<sup>7</sup> Our

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<sup>6</sup>On the contrary, there is a larger number of articles introducing ambiguity in a tort law framework, such as Teitelbaum (2007), Chappe and Giraud (2013) and Franzoni (2017).

<sup>7</sup>In Al-Nowaihi and Dhami contributions, the law enforcer aims at minimize the social cost of crime, while we consider social welfare functions.

result differs conversely. For instance, with pessimistic individuals, we find that the optimal fine might be reduced and the detection probability raised if the weight of the belief in the subjective probability is large enough and the marginal cost of raising the probability low enough.

A body of literature draws attention to the behavioral analysis of crime control (McAdams and Ulen 2009, Harel 2014, van Winden and Ash 2012). Garoupa (2003) discusses the relevance of a behavioral approach to the theory of public law enforcement. Many biases are presented and discussed in the context of public law enforcement. Jolls (2005) provides a detailed analysis on the negative impact of optimism (of agents with bounded rationality) on deterrence. Teichman (2011) argues that the behavioral analysis of crime control is quite limited by the indeterminacy of the effects of cognitive bias. For instance, the evaluation of the probabilities might depend on whether they concern losses or gains, whether the probability is close to zero or to one, whether or not people are risk-seeking regarding punishment. Moreover, several opposite or “counter” biases might co-exist. For instance, the “availability bias” might recommend making enforcement highly visible, while ambiguity aversion may promote concealing enforcement. Horowitz and Segal (2007) also note that the effect of ambiguity in crime deterrence should be carefully assessed according to whether the ambiguity refers to likely or unlikely events and whether ambiguity concerns gains or losses.

Our contribution is also related to imperfect information regarding the probability of arrest. This issue has been investigated notably by Bebchuk and Kaplow (1992), and more recently by Buechel et al. (2020). Bebchuk and Kaplow (1992) analyze the case where potential offenders get a noisy signal regarding the probability of getting caught. They show that the optimal sanction may be less than the maximal feasible sanction.<sup>8</sup> Buechel et al. (2018) consider two types of potential offenders; the naive ones, who are informed about the resources invested in law enforcement only if the authority decides to reveal that information, and the sophisticated ones, who are perfectly informed. They investigate when it is optimal to hide or reveal the enforcement effort.

## 3 Model and assumptions

### 3.1 Assumptions and notations

Our framework elaborates on the conventional model of public law enforcement (Polinsky and Shavell, 2007). Risk-neutral individuals choose whether or not to commit an act that yields a private benefit  $b$  and generates an external harm per act  $D$ . The public law enforcer does not observe any type  $b$  but knows their distribution

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<sup>8</sup>In their setting, the relative size of the error decreases as the probability of detection rises.

described by a general density function  $f(b)$  with support  $[0, B]$  and a cumulative distribution function  $F(b)$ , with  $D < B$ . The proportion of offenders is equal to  $1 - F(\tilde{b})$ , with  $\tilde{b}$  the deterrence threshold endogenously determined later.

We consider that individuals have difficulty assessing the probability of detection and conviction,  $p$ . They will estimate this probability, and we denote this estimation or belief as  $\alpha$  (with  $0 < \alpha < 1$ ). The parameter  $\alpha$  may be either greater than, less than, or equal to the objective probability of detection and conviction,  $p$ . Following the ambiguity model of Chateauneuf et al. (2007), the parameter  $\alpha$  also represents the weight given to the worst outcome for the offender, that is detection and conviction, while  $1 - \alpha$  is the weight given to the best outcome. We interpret the over or under weighting of these two extreme outcomes with the notions of optimism and pessimism as pessimism corresponds to the weight given to the worst outcome and optimism to the weight given to the best outcome. Thus, we will denote  $\alpha$  as the degree of pessimism. Further, with the probability  $\delta$ , an individual will wrongly estimate that the probability of detection and conviction equals  $\alpha$  instead of  $p$ . With a probability  $1 - \delta$ , the individual will correctly estimate the probability of detection and conviction,  $p$ . As a consequence, the *subjective* probability of being fined is written:

$$\tilde{p} = \delta\alpha + (1 - \delta)p = p + \delta(\alpha - p)$$

where we can interpret  $\delta$  as the degree of ambiguity, that is the weight of the belief  $\alpha$  in the subjective probability of detection and conviction. If  $\delta = 0$ , there is no ambiguity as in the standard Beckerian framework. If  $\delta = 1$ , potential offenders consider themselves to be confronted with complete uncertainty. In the rest of the paper, we will assume that  $0 < \delta < 1$ .

We denote the fine  $s$ ,  $t$  is the tax imposed on each individual to finance detection and conviction, and  $w$  the individual level of (legal) wealth, with  $s \leq w$ . We assume that the external harm per act  $D$  is equally likely to be borne by each individual in the population.<sup>9</sup> The expected utility level of an individual who decides to commit an offense is written:

$$u_c = w + b - \tilde{p}s - t - (1 - F(\tilde{b}))D$$

Conversely, the utility of an individual who decides to abide by the law is:

$$u_a = w - t - (1 - F(\tilde{b}))D$$

Therefore, the individual commits an offense if and only if:

$$b \geq [\delta\alpha + (1 - \delta)]ps = ps + \delta(\alpha - p)s = \tilde{b}(p, s)$$

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<sup>9</sup>This hypothesis is made for instance by Polinsky and Shavell (1979), and later by Langlais and Obidzinski (2017).

where  $\tilde{b}$  denotes the subjective deterrence threshold that verifies  $u_c = u_a$ . Potential offenders compare the benefit of committing the offense  $b$  with two terms: the objective expected fine  $ps$ , and a positive or negative additional term,  $\delta(\alpha - p)s$ . This new term results from the difference between the degree of pessimism and the objective probability of detection and conviction,  $\alpha - p$ , weighted by the degree of ambiguity  $\delta$ .

In the remainder of the paper, we distinguish between two groups of potential offenders. On the one side, pessimistic individuals will overestimate the probability of detection and conviction ( $\alpha > p$ ). On the other side, optimistic individuals will underestimate their probability of being fined ( $\alpha < p$ ). The argument is simply that optimistic behavior overestimates the likelihood of good outcomes while pessimistic attitudes exaggerate the likelihood of bad outcomes. It follows that for pessimistic individuals, the deterrence threshold  $\tilde{b}(p, s)$  is now higher than the objective expected fine  $ps$ , while for optimistic individuals, the deterrence threshold is lower than the objective expected fine. With no ambiguity ( $\delta = 0$ ), the deterrence threshold is written  $\tilde{b}(p, s)|_{\delta=0} = ps$  as in the standard framework. In such a case, individuals are equally affected by a percentage increase in the fine or in the probability of detection and conviction, that is  $e_s^{\tilde{b}}|_{\delta=0} = e_p^{\tilde{b}}|_{\delta=0} = 1$  where  $e_s^{\tilde{b}}|_{\delta=0}$  and  $e_p^{\tilde{b}}|_{\delta=0}$  are the elasticities of the threshold benefit value  $\tilde{b}$  with respect to the two deterrence tools. On the contrary, if there is ambiguity ( $\delta > 0$ ), we have  $e_s^{\tilde{b}} = 1$  and  $e_p^{\tilde{b}} = \frac{(1-\delta)p}{\delta\alpha + (1-\delta)p} < e_s^{\tilde{b}}$ . A potential offender's decision to commit an harmful act is more sensitive to a percentage increase in the fine than to an equal percentage increase in the probability of detection and conviction. And it is all the more true when the degree of ambiguity or the degree of pessimism is high.<sup>10</sup>

Finally, notice that we depart from the Chateauneuf et al (2007) ambiguity model at two levels: the attitude towards risk and the study of both possibility and certainty effects. First, we assume that individuals are risk neutral. Second, we study either pessimistic individuals who will overestimate their probability of being fined (whatever its value), or optimistic individuals who will underestimate their probability of being fined.

### 3.2 The social welfare function

By definition, the social welfare function  $\mathcal{W}$  is the sum of the utilities of all individuals, given their decisions whether or not to commit an offense:

$$\mathcal{W} = \int_0^{\tilde{b}(p,s)} u_a f(b) db + \int_{\tilde{b}(p,s)}^B u_c f(b) db \quad (1)$$

where potential offenders make their decision on the basis of the *subjective* probability of being fined. The per capita cost to achieve the probability of detection and conviction is given by  $m(p)$  where  $m'(p) > 0$  and

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<sup>10</sup>Because  $\frac{\partial e_p^{\tilde{b}}}{\partial \alpha} < 0$  and  $\frac{\partial e_p^{\tilde{b}}}{\partial \delta} < 0$ . The later expression reflects the likelihood insensitivity effect, as the sensibility to increase in resources devoted to law enforcement decreases with  $\delta$ , the lack of confidence in the objective probability.



$m''(p) \geq 0$  (Polinsky and Shavell, 1979). Imposing and collecting fines is costless. Enforcement expenditures are financed through a lump sum tax  $t$  plus the fine  $s$  imposed on the offenders detected. The per capita public budget constraint is written:

$$m(p) = t + \left(1 - F(\tilde{b})\right) ps$$

Only balanced-budget policies are considered.

In our setting, two social welfare functions are plausible according to the value of utility  $u_c$  in (1). We distinguish between the populist law enforcer and the paternalistic one. The law enforcer is said populist when social welfare is based on the subjective expected fine  $\tilde{p}s$  that is when the social welfare function is written as (1) with  $u_c = w + b - \tilde{p}s - t - \left(1 - F(\tilde{b})\right) D$ . At the opposite, a paternalistic benevolent law enforcer takes (only) into account the expected cost  $ps$  actually experienced by individuals who decide to commit harmful acts. This social welfare function is written as (1) with  $u_c = w + b - ps - t - \left(1 - F(\tilde{b})\right) D$  instead of  $u_c = w + b - \tilde{p}s - t - \left(1 - F(\tilde{b})\right) D$ .

### 3.2.1 The paternalistic social welfare function

Substituting the budget constraint in (1) with  $u_c = w + b - ps - t - \left(1 - F(\tilde{b})\right) D$ , the social welfare function simplifies as:

$$\mathcal{W}_a = w + \int_{\tilde{b}(p,s)}^B (b - D) f(b) db - m(p) \quad (2)$$

It equals the individual wealth  $w$ , plus the gains from committing an harmful act net of the harm caused  $\int_{\tilde{b}(p,s)}^B (b - D) f(b) db$ , and less the cost of detection  $m(p)$ .

This expression is very close to the standard formulation of the social welfare considered in the literature (Garoupa, 1997; Polinsky and Shavell, 2007), except for the subjective deterrence threshold.<sup>11</sup> The social welfare is computed with the objective probability of being fined  $p$ , that is the “regulator’s own beliefs” (Salanié and Treich 2009), rather the individuals’ beliefs. As a consequence, a paternalistic social planner does not take into account the discrepancy between how much potential offender subjectively expect to pay and how much they might indeed pay, and the fine is a pure transfer between the potential offenders and the public authority, as in the standard public law enforcement setting with no ambiguity. This is why we have to consider also the populist approach of the social welfare, in order to take into account this discrepancy.

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<sup>11</sup>See in appendix 1 the standard formulation of the social welfare function with no ambiguity.

### 3.2.2 The populist social welfare function

Substituting the budget constraint in (1) with  $u_c = w + b - \tilde{p}s - t - (1 - F(\tilde{b}))D$ , the social welfare function simplifies as:

$$\mathcal{W} = w + \int_{\tilde{b}(p,s)}^B (b - D)f(b)db - (1 - F(\tilde{b}))(\tilde{p} - p)s - m(p) \quad (3)$$

This expression exhibits an additional term by comparison with the paternalistic case (and the standard case with no ambiguity):  $(1 - F(\tilde{b}))(\tilde{p} - p)s$ . This term is referred to as the expected *perception bias*. As  $\tilde{p} - p = (\alpha - p)\delta$ , we have  $\tilde{p} > p$  as long as  $\alpha > p$ , the reverse being true. If individuals are pessimistic, they overestimate the probability of getting fined ( $\tilde{p} > p$ ), and the perception bias is a cost. If individuals are optimistic, then they underestimate the probability of getting caught ( $\tilde{p} < p$ ), and the perception bias is a gain. As a consequence, the fine is no longer a pure transfer.

In summary, if the public law enforcer is populist, monetary sanctions are not a pure transfer, as they induce either a cost if individuals are pessimistic, or a gain if individuals are optimistic. If the public law enforcer is paternalistic, monetary sanctions are seen as a cost-less transfer from the potential offender to the public authority.

## 4 The optimal deterrence policy when resources devoted to detection are given.

In this section, we consider that enforcement expenditures are exogenous, resulting in a given probability of detection and conviction. In the absence of ambiguity surrounding the probability of detection and conviction, we know that the optimal fine equals  $s_n^* = \min\{\frac{D}{p}, w\}$  and that the first-best outcome can be achieved as long as  $\frac{D}{p} \leq w$  (see Appendix 1 for the proof).

Assume now that there is some ambiguity surrounding the probability of being fined. Subsection 4.1 exposes the results regarding the optimal fine under ambiguity. Subsections 4.2 and 4.3 present the impact of pessimism and ambiguity on the optimal fine when individuals over-estimate and under-estimate respectively the probability of being fined. In order to facilitate the reading of the model, we report all the detailed analysis in the appendix (see Appendix 2 for the case with a paternalistic law enforcer and Appendix 3 for the case with a populist law enforcer).

## 4.1 The optimal fine

The paternalistic law enforcer chooses the fine  $s$  that maximizes (2) under the constraint  $s \leq w$ . As the first order condition of this program is given by:

$$f(\tilde{b})\tilde{p}(D - \tilde{p}s^*) = 0$$

we have at equilibrium:

$$s^* = \min\{w; \frac{D}{\tilde{p}}\}$$

As long as the solution is interior, we have:

$$s^* < s_n^* \Leftrightarrow \alpha > p.$$

If individuals are pessimistic, the optimal fine under ambiguity is lower than the optimal fine with no ambiguity ( $s_n^* = \frac{D}{p}$ ). If individual are optimistic, the optimal fine under ambiguity is higher than the optimal fine with no ambiguity. The optimal fine adjusts to the impact of pessimism or optimism on the deterrence threshold. The more pessimistic the offenders, the lower the optimal fine while the more optimistic the offenders, the higher the optimal fine.

The populist social planner maximizes (3) under the constraint  $s \leq w$ . The first-order condition is written:<sup>12</sup>

$$f(\tilde{b})\tilde{p}(D - \tilde{b}) + f(\tilde{b})\tilde{p}(\tilde{p} - p)s^* = (1 - F(\tilde{b}))(\tilde{p} - p) \quad (4)$$

Assume that individuals are pessimistic ( $\tilde{p} > p$ ). The left-hand side in (4) is the social marginal benefit of deterrence. An increase of the fine by one unit reduces the probability of offending by  $f(\tilde{b})\tilde{p}$  units. Thus, it diminishes the occurrence of both the net harm  $D - \tilde{b}$  and the perception bias cost  $(\tilde{p} - p)s$ . The right-hand side equals the marginal cost of fines. Fines are not a cost-less transfer anymore since there exists a perception bias cost of deterrence. Pessimistic offenders do not expect to pay  $ps$  but rather  $\tilde{p}s > ps$ . Thus, a one-unit increase of the fine increases the perception cost  $(\tilde{p} - p)s$ , but also increases deterrence, thereby reducing the proportion of offenders which bears the perception cost. For optimistic individuals, the interpretation is quite the opposite, as increasing the fine raises perception gains, but decreases the proportion of offenders which benefits from it.<sup>13</sup>

<sup>12</sup>The analysis of second-order condition developed for the general case in Appendix 4.

<sup>13</sup>To illustrate, suppose that the private benefits  $b$  follows a uniform distribution with support  $[0, B]$ . We get at equilibrium:

$$s^* = \min\{w; \frac{\tilde{p}D - (\tilde{p} - p)B}{(2p - \tilde{p})\tilde{p}}\}$$

while the optimal fine with no ambiguity equals  $s_n^* = \frac{D}{p}$ .

In order to compare the optimal fine under ambiguity with the optimal fine in the standard case with no ambiguity, it is easier to rewrite equation (4) as:

$$\underbrace{h(\tilde{b})}_{(+)} \tilde{p}(D - ps^*) = \underbrace{(\tilde{p} - p)}_{(+or-)} \quad (5)$$

where  $h(\tilde{b}) = \frac{f(\tilde{b})}{1-F(\tilde{b})}$  is the hazard rate function, that is the relative likelihood that  $b = \tilde{b}$  conditional on  $b \geq \tilde{b}$ . According to the equality above, we find that  $D - ps^* > 0$  or  $s^* < s_n^* = \frac{D}{p}$  if and only if individuals are pessimistic. It is socially desirable to set a lower fine under ambiguity as pessimism deters offenses and sanctions are no longer a costless transfer, as with a paternalistic law enforcer. When individuals are optimistic, the optimal fine under ambiguity is higher than the optimal fine in the standard Beckerian framework.

## 4.2 Individuals over-estimate their probability of being fined

When perception costs are not taken into account (paternalistic law enforcer), and when people overestimate their probability of being fined, the optimal fine under ambiguity is lower than the optimal fine in the standard Beckerian framework with no ambiguity. The deterrent effect of pessimism is at play. The optimal magnitude of fine decreases with the degree of pessimism and the degree of ambiguity. As a consequence, the same level of deterrence might be achieved with a lower fine when facing to an increase of pessimism or ambiguity.

When perception costs are taken into account by a populist law enforcer, the optimal fine under ambiguity is still lower than the optimal fine in the standard Beckerian framework with no ambiguity. We report in Appendix 3 all the comparative statics used to study the effect on the optimal magnitude of fine after changes in degrees of ambiguity or pessimism. We find that results crucially depend on the distribution of the gain obtained by committing the harmful act (see Proposition 1 in Appendix 3). For instance, when the deterrence issue is not severe (in the sense that for any marginal increase in deterrence, we observe a high decrease in the proportion of offenders), we find that the optimal magnitude of the fine may now increase with the degree of pessimism or ambiguity.

## 4.3 Individuals under-estimate their probability of being fined

When individuals are optimistic, the optimal fine under ambiguity is higher than the optimal fine in the standard Beckerian framework whatever the type of law enforcer. It is socially desirable to increase the magnitude of fines beyond the value  $\frac{D}{p}$  because optimistic potential offenders are less sensitive to fines.

Moreover, there is an additional marginal benefit associated with the use of fines due to the perception bias when the law enforcer is populist (this argument is not valid in the paternalistic case).

When the law enforcer is paternalistic, we show in Appendix 2 that the optimal magnitude of fine decreases with the degree of pessimism but now increases with the degree of ambiguity. When the law enforcer is populist, the optimal magnitude of fines decreases with the degree of pessimism under the condition that the benefit  $b$  follows a uniform distribution. However, we show in Appendix 3 that for another type of statistical distribution (for instance such that for an increase in deterrence, the proportion of offenders significantly decreases), the reverse may be true: the optimal magnitude of the fine could increase with the degree of pessimism (See Proposition 2 in Appendix 3).

## 5 The optimal deterrence policy when resources devoted to detection are endogenous

In this section, the public law enforcer determines both the size of the fine and the probability of detection. In the absence of ambiguity, we know since Becker's seminal model that the optimal fine should be maximal  $s_n^* = w$  (See Appendix 1 for the proof). In the remaining of the section, we assume that there is some ambiguity surrounding the probability of being fined, and we consider successively two cases, depending on whether the social welfare function includes the perception costs.

### 5.1 The paternalistic case

The paternalistic law enforcer chooses  $s^*$  and  $p^*$  which maximizes (2). The derivatives of (2) relative to  $s$  and  $p$  respectively are given by:

$$f(\tilde{b})\tilde{p}^*(D - \tilde{p}^*s^*) = 0$$

$$f(\tilde{b})(1 - \delta)s^*(D - \tilde{b}) - m'(p^*) = 0$$

As the fine is cost-free in this setting (as in the standard Beckerian model), the optimal magnitude of fine is set at its maximum  $s^* = w$ . The optimal probability of detection is defined by the second equation whose interpretation is quite similar to the standard framework.

In order to facilitate the reading of the results, we assume that the benefit  $b$  follows a uniform distribution

on  $[0, B]$  and that the detection cost is linear ( $m(p) = mp$ ). Under these assumptions, we get  $s^* = w$  and

$$p^* = \frac{D}{(1-\delta)w} - \frac{m}{(1-\delta)w} \frac{B}{(1-\delta)w} - \alpha \frac{\delta}{(1-\delta)} \quad (6)$$

By comparison with the optimal probability of detection with no ambiguity (equal to  $p_n^* = \frac{D}{w} - \frac{m}{w} \frac{B}{w}$ ), the additional term  $(1-\delta)$  in (6) at each denominator reflects the weight of the objective probability  $p$  in the belief of potential offenders. The additional last term in (6) may be interpreted as: the higher the degree of pessimism  $\alpha$ , the lower the optimal probability of detection, and the more likely the optimal probability of detection under ambiguity is lower than that with no ambiguity. This effect is quite intuitive as the degree of pessimism and the means devoted to detection and conviction are substitutes from a pure deterrence point of view (and we see that the impact of pessimism on the optimal probability of detection positively depends on the degree of ambiguity). As a consequence, there does exist a critical value of the degree of pessimism beyond which the optimal probability of detection under ambiguity will be inferior to the optimal probability of detection with no ambiguity. More precisely, for  $\alpha \in [0, 1]$ , we have:  $p^* > p_n^*$  if  $\alpha \in [0, \frac{D}{w} - \frac{m}{w} \frac{B}{w} \frac{2-\delta}{1-\delta}]$  while  $p^* < p_n^*$  if  $\alpha \in [\frac{D}{w} - \frac{m}{w} \frac{B}{w} \frac{2-\delta}{1-\delta}, \frac{D}{\delta w} - \frac{m}{\delta w} \frac{B}{\delta w}]$ .<sup>14</sup> In words, if individuals are pessimistic enough then the optimal fine under ambiguity is lower than the optimal fine with no ambiguity.

For more sophisticated distributions of the benefit  $b$ , the comparison between the optimal expenditures in detection and conviction in the no ambiguity case versus the ambiguous case is clearly not straightforward. The reason is that the marginal benefit of detection and conviction given by the first order condition above depends not only on the degree of ambiguity but also on the distribution of gains. We give in Appendix 2 the condition under which the means invested in detection should be reduced or increased due to the weight of beliefs, when potential offenders are less sensitive to changes in the probability of detection and conviction (see Proposition 3).

## 5.2 The populist case

The populist law enforcer chooses  $s^*$  and  $p^*$  which maximizes (3). The derivatives of (3) relative to  $s$  and  $p$  respectively are given by:

$$f(\tilde{b})\tilde{p}^*(D - \tilde{p}^*s^*) + f(\tilde{b})\tilde{p}^*s^*(\tilde{p}^* - p^*) = (1 - F(\tilde{b}))(\tilde{p}^* - p^*) \quad (7)$$

$$f(\tilde{b})(1-\delta)s(D - \tilde{b}) + f(\tilde{b})(1-\delta)s(\tilde{p}^* - p^*)s + (1 - F(\tilde{b}))\delta s = m'(p^*) \quad (8)$$

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<sup>14</sup>Notice that  $p^* < 0$  if  $\alpha \in [\frac{D}{\delta w} - \frac{m}{\delta w} \frac{B}{\delta w}, 1]$ . We exclude this case because the probability of detection and conviction must be positive.

We begin with an analysis of the first order condition relative to  $s$  (see equation 7). If individuals are pessimistic, the main difference with the paternalistic case is that the regulator now takes into account the perception bias cost of deterrence (see the marginal cost of fine at the right hand side of equation 7). As a consequence, the optimal fine under ambiguity is now lower than its maximal value  $w$  contrary to the standard framework with no ambiguity.

Next, consider the first order condition relative to  $p$ . At the right-hand side, we have the marginal cost of detection and conviction  $m'(p^*)$ . The left-hand side of equation stands for the social marginal benefit of deterrence, which sums three terms. The two first terms reflects the impact of deterrence. An increase in the probability of detection and conviction by one unit reduces the proportion of offenders by  $-\frac{\partial(1-F(\tilde{b}))}{\partial p} = f(\tilde{b})(1-\delta)s$  units, thereby diminishing both the occurrence of net harm  $(D - \tilde{b})$  and the proportion of the population which suffers the perception cost  $(\tilde{p} - p)s$  when individuals are pessimistic (or which benefits from the perception gain, when individuals are optimistic). The third term represents the direct impact of an increase in the probability of detection on the perception cost or gain. An increase in the probability of conviction diminishes the discrepancy between what the offenders expect to pay and the objectively expected fine denoted as  $(\tilde{p} - p)s = \delta(\alpha - p)s$ , thus reducing this perception cost by  $\delta s$  units at the margin, supported by the proportion  $(1 - F(\tilde{b}))$  of the population.<sup>15</sup>

Contrary to previous sections, applying a uniform distribution does not simplify the analysis. But, as the optimal fine under ambiguity is no longer set to its maximal value  $w$ , it makes sense to study the condition under which it is optimal to replace the investment in ambiguous detection and conviction with the magnitude of fine at the margin (in order to get closer to the standard Beckerian result:  $s^* = w$ ). We show in Appendix 3 that this condition writes:

$$\Delta W = \left[ \underbrace{(1 - F(\tilde{b}))e_{\alpha}^{\tilde{b}}}_{(+)} - \underbrace{m'(p)}_{(-)} \right] \underbrace{dp}_{(-)}$$

where  $e_{\alpha}^{\tilde{b}}$  is the elasticity of the deterrence threshold relative to the degree of pessimism (see Proposition 4 in Appendix 4). A decrease in the probability of detection ( $dp < 0$ ) associated with an increase in the amount of fine ( $ds > 0$ ) such that the level of deterrence remains unchanged (or  $\tilde{b}$  constant) is welfare improving ( $\Delta W > 0$ ) under the condition that the marginal cost of detection and conviction ( $m'(p)$ ) is high enough. But, the reverse may also be true. An increase in the probability of detection ( $dp > 0$ ) associated with a decrease in the amount of fine ( $ds < 0$ ) such that the level of deterrence remains unchanged is welfare

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<sup>15</sup>For optimistic individuals, the discrepancy between how much they expect to pay and the objective expected fine (that is  $(\tilde{p} - p)s = \delta(\alpha - p)s$ ) is positive, and represents the perception bias gain. Thus, a one-unit increase of the probability of detection and conviction raises this gain by  $\delta s$  units, which occurs with probability  $(1 - F(\tilde{b}))$ . In other words, an increase in detection raises the expected sanction more than the subjective expected sanction, and this net marginal difference equals  $\delta s$ .

improving ( $\Delta W > 0$ ) under the condition that the deterrence threshold is sufficiently sensitive to the degree of pessimism (see the term  $e_{\alpha}^{\tilde{b}}$ ). Put differently, it is all the more socially desirable to replace fine with investment in detection and conviction when the deterrent effect of pessimism comes into play.

## 6 Conclusion

The aim of this paper is to provide some insights into how the degree of pessimism influences the socially optimal amount of fines and enforcement expenditures when potential offenders are unable to perfectly estimate the probability of detection and conviction. Two alternatives social welfare functions are considered. The populist social welfare function is computed with citizens' perceived probability of sanction, while the paternalistic one is computed with the objective probability.

By comparison with the standard Beckerian framework (Garoupa, 1997; Polinsky and Shavell, 2000), fines may be no longer a costless transfer if individuals overestimate their probability of being caught with a populist law enforcer. The main condition is that populist law enforcers take into account the discrepancy between the actual expected fine and the subjectively expected fine. This discrepancy, denoted as perception bias, reflects disutility when potential offenders are pessimistic. Consequently, when potential offenders are pessimistic, the optimal fine becomes lower than the maximal one, contrary to the standard Beckerian framework. Under some plausible condition, the higher the degree of pessimism or ambiguity, the lower the optimal fine. Further, it may be socially optimal to raise the probability of detection while decreasing the magnitude of fine under a certain condition. This result, as well as the comparative statics analysis, considerably depends on the distribution of gains.

Let us say a word about the optimistic case. Now, potential offenders underestimate their probability of being caught. Under this behavioral assumption, it is still true that fines are no longer a pure transfer with a populist law enforcer. But the similarities with the pessimistic case end there or almost. As potential offenders underestimate the probability of detection, the deterrence threshold is lower than the objective one. And the perception bias now reflects a gain, as individual expect to pay less than what will actually go into the coffers of the State (to fund deterrence policy). Consequently, the results go in the opposite direction than in the pessimistic case. The optimal fine under ambiguity should be higher than the standard Beckerian fine, and the comparative statics results are reversed.

With a paternalistic law enforcer, the perception cost is no longer taken into account. Fines are a pure transfer between the detected offenders and society, as fines are used to fund detection. In such a case, the first-best outcome can be achieved as long as the optimal fine is lower or equal than the maximal one, as in



the standard Beckerian framework. We show that the optimal fine adjusts to the impact of pessimism (else, optimism) on the deterrence threshold. The more pessimistic (optimistic) the offenders, the lower (higher) the optimal fine. When both the fine and the probability of detection are endogenous, the optimal fine is maximal as in the standard Beckerian framework. We also show that the optimal probability of detection under ambiguity may be either higher or lower than the optimal probability in the absence of ambiguity according to the distribution of benefits of illegal activity.

The populist and the paternalist approaches provide different recommendations regarding the law enforcement policy. For instance, with pessimistic agents and endogenous resource devoted to detection, the fine should not be maximal with a populist law enforcer as the fine is not a costless transfer anymore. On the opposite, the optimal fine is maximal under the paternalist approach. This result adds to traditional public law enforcement literature which provides several explanations on why the fine shall not be maximal (Garoupa, 1997), as observed in reality and in contrast to the *beckerian* result (1968).

Our contribution has several limits. We do not consider the possibility of “debiasing” potential offenders (Jolls and Sunstein, 2006). We could imagine that the law enforcer attempts to educate potential offenders about the risk of being optimistic, or to reduce individuals’ cost of information through public warnings. However, the effect of such attempts on optimistic individuals’ beliefs might be limited. Indeed, the optimistic bias is often associated with a “blind spot bias” (Pronin et al., 2002). The “blind spot bias” is the illusion that one is less prone to bias, and notably optimism. Therefore, it is quite difficult if not impossible to modify optimistic individuals’ beliefs (Luppi and Parisi, 2016).

Another limitation of our contribution is that we consider a homogeneous population in terms of beliefs. Individuals share either the same degree of ambiguity, or the same degree of pessimism. Furthermore, we do not consider risk-aversion or risk-seeking. In a sense, our contribution may be seen as a first step towards producing a formal representation of ambiguity in a public law enforcement model.

## 7 Appendix 1: the no ambiguity case

### 7.1 Optimal fine

In the absence of ambiguity surrounding the probability of detection, the optimal fine solves:

$$\max_s \left\{ \mathcal{W} = w + \int_{ps}^B (b - D)f(b)db - m(p) \right\} \text{ u.c. } s \leq w. \quad (9)$$

The first-order condition of this program is:

$$f(\tilde{b}_n)p(D - ps_n^*) = 0. \quad (10)$$

Thus we have  $s_n^* = \min\{\frac{D}{p}, w\}$  and the first-best outcome can be achieved as long as  $\frac{D}{p} \leq w$ .

## 7.2 Optimal fine and detection

In the absence of ambiguity, when both the fine and the probability of detection and conviction are endogenous, the public law enforcer solves:

$$\max_{s,p} \left\{ \mathcal{W} = w + \int_{ps}^B (b - D)f(b)db - m(p) \right\} \text{ u.c. } s \leq w. \quad (11)$$

The first-order conditions of this program relative to  $s$  and  $p$  respectively are:

$$f(p_n^*s_n^*)p(D - p_n^*s_n^*) = 0 \quad (12)$$

$$f(p_n^*s_n^*)s_n^*(D - p_n^*s_n^*) = m'(p_n^*) \quad (13)$$

We have  $s_n^* = w$  and  $p_n^*$  defined by the second equation just above as in the standard Beckerian framework.

## 8 Appendix 2: the paternalistic case

A paternalistic law enforcer takes into account the expected cost  $ps$  actually experienced by individuals who decide to commit an harmful act. So we replace  $u_c = w + b - ps - t - (1 - F(\tilde{b}))D$  in Equation (1). Thus the social welfare function equals the individual wealth  $w$ , plus the gains net of the harm caused  $\int_{\tilde{b}(p,s)}^B (b - D)f(b)db$ , and less the cost of detection  $m(p)$ :

$$\mathcal{W} = w + \int_{\tilde{b}(p,s)}^B (b - D)f(b)db - m(p) \quad (14)$$

where  $\tilde{b}(p, s) = ps + \delta(\alpha - p)s$ .

### 8.1 Optimal fine

Under ambiguity, the paternalistic law enforcer maximizes  $\mathcal{W}$  with respect to  $s$  under constraint that  $s \leq w$ . The first-order condition of this program is:

$$f(\tilde{b})\tilde{p}(D - \tilde{b}) = 0 \quad (15)$$

Thus, we have at equilibrium:

$$s^* = \frac{D}{\tilde{p}} \quad (16)$$

whatever the individual being optimistic or pessimistic.

First, consider the pessimistic case. As  $\tilde{p} = \delta\alpha + (1 - \delta)p$ , the optimal fine is decreasing with the degree of pessimism and the degree of ambiguity because pessimistic individuals will overestimate the expected fine. Thus the optimal sanction under ambiguity with a paternalistic public law enforcer is lower than the optimal fine in the absence of ambiguity:  $s^* < s_n^* = \frac{D}{p}$ . Finally, the first-best outcome can be achieved as long as  $\frac{D}{\tilde{p}} \leq w$ .

Next, turn to the optimistic case. The optimal magnitude of fine is still decreasing with the degree of pessimism, but is increasing with the degree of ambiguity because optimistic individuals underestimate the probability of detection. We derive that the optimal sanction under ambiguity is higher than the optimal fine in absence of ambiguity  $s^* > s_n^* = \frac{D}{p}$  while the reverse is true if individuals are pessimistic. Finally, the first-best outcome ( $\tilde{b} = D$ ) can still be achieved as long as  $\frac{D}{\tilde{p}} \leq w$ .

## 8.2 Optimal fine and detection

When both deterrence tools are endogenous, the paternalistic law enforcer solves under ambiguity:

$$\max_{s,p} \left\{ \mathcal{W} = w + \int_{\tilde{b}(p,s)}^B (b - D)f(b)db - m(p) \right\} \text{ u.c. } s \leq w. \quad (17)$$

The derivatives of  $\mathcal{W}$  relative to  $s$  and  $p$  respectively are given by:

$$f(\tilde{b})\tilde{p}^*(D - \tilde{p}^*s^*) = 0 \quad (18)$$

$$f(\tilde{b})(1 - \delta)s^*(D - \tilde{b}) - m'(p^*) = 0 \quad (19)$$

As the fine is cost-free in this setting (as in the standard Beckerian model), the fine is set at its maximum:  $s^* = w$ . Note also that we have under-deterrence or  $D > \tilde{b} = \tilde{p}w$  at equilibrium as the first-order condition on  $p$  indicates that:

$$\underbrace{f(\tilde{b})(1 - \delta)w(D - \tilde{b})}_{(+)} = \underbrace{m'(p^*)}_{(+)} \quad (20)$$

**Proposition 3** *With a paternalistic public law enforcer choosing both the fine and the probability of detection, the optimal fine is maximal. The optimal probability of detection and conviction under ambiguity is higher (resp. lower) than the optimal probability of detection in the absence of ambiguity if:*

$$e_{\delta}^{\tilde{b}} \frac{1-\delta}{\delta} \left( e_{\tilde{b}}^{f(\tilde{b})} - \frac{\tilde{b}}{D-\tilde{b}} \right) > 1 \text{ (resp. } < 1 \text{).} \quad (21)$$

*Proof.* In the no ambiguity case, the optimal probability of detection and conviction is given by:

$$f(p_n^* w) w (D - p_n^* w) = m'(p_n^*) \quad (22)$$

while in the paternalistic case (under ambiguity), it is given by:

$$f(\tilde{b})(1-\delta)w(D-\tilde{b}) = m'(p^*) \quad (23)$$

with  $\tilde{b} = [\delta\alpha + (1-\delta)p]w$  and  $D > \tilde{b}$  at equilibrium.

The marginal cost of detection and conviction (at the right-hand side above) does not depend on the degree of ambiguity while the marginal benefit of detection and conviction does in the paternalistic case. Remark that the two values of marginal benefit of detection and conviction depicted above are equal if  $\delta = 0$ . Thus it is sufficient to show that the marginal benefit of detection in the paternalistic case is increasing with the degree of ambiguity ( $\delta$ ) to conclude that the optimal probability of detection and conviction under ambiguity is higher than the optimal probability of detection and conviction in the no ambiguity case. Conversely, if the marginal benefit of detection and conviction in the paternalistic case decreases with the degree of ambiguity, then the optimal probability of detection and conviction under ambiguity is now lower than the optimal probability of detection and conviction in the no ambiguity case.

Using the implicit function theorem and the first-order condition in the paternalistic case, we show that the marginal benefit  $f(\tilde{b})(1-\delta)w(D-\tilde{b})$  is increasing with  $\delta$  if and only if:

$$f'(\tilde{b})(\alpha-p)w(D-\tilde{b})(1-\delta) - f(\tilde{b})(1-\delta)(\alpha-p)w - f(\tilde{b})(D-\tilde{b})w > 0 \quad (24)$$

or

$$f(\tilde{b}) \underbrace{(D-\tilde{b})}_{(+)} \left( \frac{f'(\tilde{b})}{f(\tilde{b})} (\alpha-p)(1-\delta) - \frac{(1-\delta)(\alpha-p)}{D-\tilde{b}} - 1 \right) > 0 \quad (25)$$

or

$$e_{\tilde{b}}^{f(\tilde{b})} \left( \frac{(\alpha-p)(1-\delta)\delta}{\delta\tilde{b}} \right) - \frac{\delta(1-\delta)(\alpha-p)\tilde{b}}{\delta(D-\tilde{b})\tilde{b}} - 1 > 0 \quad (26)$$

where  $e_{\tilde{b}}^{f(\tilde{b})} = \frac{f'(\tilde{b})}{f(\tilde{b})}\tilde{b}$  is the elasticity of the density function with respect to the threshold value of benefit. Next, replacing  $e_{\delta}^{\tilde{b}} = \frac{\delta(\alpha-p)w}{\tilde{b}}$  the elasticity of the threshold value of benefit with respect to the degree of

ambiguity in the inequality above, we have:

$$e_{\tilde{b}}^{f(\tilde{b})} e_{\delta}^{\tilde{b}} \frac{1-\delta}{\delta} - e_{\delta}^{\tilde{b}} \frac{1-\delta}{\delta} \frac{\tilde{b}}{D-\tilde{b}} - 1 > 0 \quad (27)$$

or

$$e_{\delta}^{\tilde{b}} \frac{1-\delta}{\delta} \left( e_{\tilde{b}}^{f(\tilde{b})} - \frac{\tilde{b}}{D-\tilde{b}} \right) > 1. \quad (28)$$

■

We remark that the condition depicted in proposition 3 is the same in optimistic and pessimistic cases. The sole difference is that the element  $e_{\delta}^{\tilde{b}}$  is positive if individuals are pessimistic while the same element  $e_{\delta}^{\tilde{b}}$  is negative if individuals are optimistic. Next, the comparison between the optimal expenditures in detection and conviction in the no ambiguity case versus the pessimistic case is not straightforward and mainly depends on the distribution of benefit of gains. However, in the most simple case, when the gain follows a uniform distribution, the optimal probability of detection and conviction under ambiguity is lower than the optimal probability of detection in the no ambiguity case. For other distributions, the comparative static analysis depends on a large set of factors, such as the elasticity of the threshold benefit value with respect to the degree of ambiguity ( $e_{\delta}^{\tilde{b}}$ ), a relative measure of the degree of ambiguity ( $\frac{1-\delta}{\delta}$ ), the elasticity of the density function with respect to the threshold value of benefit ( $e_{\tilde{b}}^{f(\tilde{b})}$ ) minus a relative measure of the under-deterrence at equilibrium ( $\frac{\tilde{b}}{D-\tilde{b}} > 0$ ).

## 9 Appendix 3: the populist case

When the social planner is populist, the social welfare function is written as:

$$\mathcal{W} = w + \int_{\tilde{b}(p,s)}^B (b-D)f(b)db - \left(1 - F(\tilde{b})\right) (\tilde{p} - p)s - m(p) \quad (29)$$

### 9.1 Optimal fine

The populist law enforcer then maximizes  $\mathcal{W}$  with respect to  $s$  under constraint that  $s \leq w$ . The first-order condition of this program is:<sup>16</sup>

$$f(\tilde{b})\tilde{p}[(D-\tilde{b}) + s^*(\tilde{p}-p)] = \left(1 - F(\tilde{b})\right) (\tilde{p} - p) \quad (30)$$

In order to simplify the total overview and the interpretation, we first analyse the optimal fine under the condition that individuals are pessimistic.

<sup>16</sup>We report the analysis of second-order condition in appendix 4.

### 9.1.1 The pessimistic case

The left-hand side in the first order condition above is the social marginal benefit of deterrence. An increase of the fine by one unit reduces the probability of offending by  $-\frac{\partial(1-F(\tilde{b}))}{\partial s} = f(\tilde{b})\tilde{p}$  units. Thus, it diminishes the occurrence of both the net harm  $D - \tilde{b}$  and the perception bias cost  $(\tilde{p} - p)s$ . The right-hand side equals the marginal cost of fines. Fines are costly because individuals overestimate the probability of getting fined.

We can rewrite the first order condition as:

$$\underbrace{h(\tilde{b})\tilde{p}}_{(+)}(D - ps^*) = \underbrace{(\tilde{p} - p)}_{(+)} \quad (31)$$

where  $h(\tilde{b}) = \frac{f(\tilde{b})}{1-F(\tilde{b})}$  is the hazard rate function, that is the relative likelihood that  $b = \tilde{b}$  conditional on  $b \geq \tilde{b}$ . According to the equality above, we find that  $D - ps^* > 0$  or  $s^* < s_n^* = \frac{D}{p}$ . When individuals are pessimistic, the optimal fine under ambiguity is lower than the optimal fine in the standard Beckerian framework. It is socially desirable to set a lower fine under ambiguity as pessimism deters offenses and sanctions are no longer a costless transfer.

Let us now consider the impact of pessimism and ambiguity on the optimal magnitude of fine. Proposition 1 summarizes our results.

**Proposition 1** *Assume that individuals are pessimistic. Then the optimal magnitude of fines decreases with the degree of pessimism  $\alpha$  and the degree of ambiguity  $\delta$  if and only if:*

$$\underbrace{\frac{p}{(\tilde{p} - p)}}_{(+)} > e_b^{h(\tilde{b})} \quad (32)$$

where  $e_b^{h(\tilde{b})} = \frac{h'(\tilde{b})\tilde{b}}{h(\tilde{b})}$  stands for the elasticity of the hazard rate function with respect to the deterrence threshold.

*Proof.* We start by computing the sign of the derivative  $\frac{ds^*}{d\alpha}$ . The first-order condition can be written:

$$h(\tilde{b})(\delta\alpha + (1 - \delta)p)(D - ps) - \delta(\alpha - p) = 0. \quad (33)$$

Using the implicit function theorem and the second-order condition, we obtain:

$$\frac{ds^*}{d\alpha} > 0 \Leftrightarrow h'(\tilde{b})\tilde{b}(D - ps)\delta + h(\tilde{b})\delta(D - ps) - \delta > 0. \quad (34)$$

Dividing by  $\delta$ :

$$\frac{ds^*}{d\alpha} > 0 \Leftrightarrow h'(\tilde{b})\tilde{b}(D - ps) + h(\tilde{b})(D - ps) > 1. \quad (35)$$

Using the first-order condition, we replace  $h(\tilde{b})(D - ps) = \frac{\delta(\alpha - p)}{\delta\alpha + (1 - \delta)p}$  and put it on the right-hand side. So

$$\frac{ds^*}{d\alpha} > 0 \Leftrightarrow h'(\tilde{b})\tilde{b}(D - ps) > 1 - \frac{\delta(\alpha - p)}{\delta\alpha + (1 - \delta)p} = \frac{p}{\delta\alpha + (1 - \delta)p}. \quad (36)$$

Still using the first-order condition, we now replace  $D - ps = \frac{\delta(\alpha - p)}{(\delta\alpha + (1 - \delta)p)h(\tilde{b})}$  and simplify by  $(\delta\alpha + (1 - \delta)p)$  to have:

$$\frac{ds^*}{d\alpha} > 0 \Leftrightarrow \frac{h'(\tilde{b})\tilde{b}}{h(\tilde{b})}\delta(\alpha - p) > p. \quad (37)$$

If individuals are pessimistic ( $\alpha - p > 0$ ) then  $\frac{ds^*}{d\alpha} > 0 \Leftrightarrow \frac{h'(\tilde{b})\tilde{b}}{h(\tilde{b})} > \frac{p}{\delta(\alpha - p)}$  where  $\frac{h'(\tilde{b})\tilde{b}}{h(\tilde{b})}$  stands for the elasticity of the hazard rate function estimated at the threshold value. If individuals are optimistic ( $\alpha - p < 0$ ) then  $\frac{ds^*}{d\alpha} > 0 \Leftrightarrow \frac{h'(\tilde{b})\tilde{b}}{h(\tilde{b})} < \frac{p}{\delta(\alpha - p)}$ .

Now, we study the sign of the derivative  $\frac{ds^*}{d\delta}$ . Using the implicit function theorem, the first and second-order conditions, we get:

$$\frac{ds^*}{d\delta} > 0 \Leftrightarrow (\alpha - p)((h'(\tilde{b})\tilde{b}(D - ps) + h(\tilde{b})(h - ps) - 1) \quad (38)$$

We have previously shown the conditions under which the expression  $(h'(\tilde{b})\tilde{b}(D - ps) + h(\tilde{b})(h - ps) - 1)$  is either positive or negative. Thus, if  $\alpha - p > 0$  (pessimistic case) then  $\frac{ds^*}{d\delta} > 0 \Leftrightarrow \frac{h'(\tilde{b})\tilde{b}}{h(\tilde{b})} > \frac{p}{\delta(\alpha - p)}$ . If  $\alpha - p < 0$  (optimistic case) then  $\frac{ds^*}{d\delta} > 0 \Leftrightarrow \frac{h'(\tilde{b})\tilde{b}}{h(\tilde{b})} < \frac{p}{\delta(\alpha - p)}$  if  $(\alpha - p) < 0$ .

■

When individuals are pessimistic, the more pessimistic they are (or the more ambiguous the probability of detection and conviction is), the more it is socially desirable to decrease the magnitude of fines if and only if the elasticity of the hazard rate function with respect to the deterrence threshold takes a sufficiently low value. As  $\frac{p}{(\bar{p} - p)} > 0$ , this condition holds if the hazard rate function  $h(\tilde{b})$  is decreasing in the deterrence threshold (or weakly increasing).

The function  $h(\tilde{b})$  divides the density function  $f(\tilde{b})$  by the proportion of offenders  $(1 - F(\tilde{b}))$ , both being estimated at the threshold value. As a consequence, the numerator of  $h(\tilde{b})$  gives the reduction in the proportion of offenders resulting from a one unit increase of deterrence threshold as  $\frac{-d(1 - F(\tilde{b}))}{d\tilde{b}} = f(\tilde{b})$ . Let us consider a numerical example to better interpret this element. Suppose that the proportion of offenders in the population equals to  $(1 - F(\tilde{b})) = 0.20$ . By increasing the deterrence threshold at the margin by one unit, assume that the proportion of offenders decreases by  $f(\tilde{b}) = 0.10$ . At this point, the proportion of offenders has been divided by 2, with  $h(\tilde{b}) = \frac{1}{2}$ . Assume now that the decrease in the proportion of offender is weaker, with  $f(\tilde{b}) = 0.05$ . At this point, the proportion of offenders has decreased by one fourth, with  $h(\tilde{b}) = \frac{1}{4}$ . The lower is  $h(\tilde{b})$ , the more harmful acts are difficult to deter for any given fine and means devoted to detection

and conviction. We thus interpret the function  $h(\tilde{b})$  as a measure of the severity of the deterrence problem. For any marginal increase in deterrence, a higher  $h(\tilde{b})$  implies high degree of variation in the proportion of offenders, while lower  $h(\tilde{b})$  connotes low degree of variation in the proportion of offenders. And we say that the deterrence problem is more severe in the second case in comparison to the first case.

To our knowledge, the law and economics literature does not make any explicit predictions about the shape of function  $h(\tilde{b})$ . But the illegal behavior of agents may suggest the pattern of this function. For instance, one may consider that for higher gains, the deterrence problem is more severe, what is corresponding to lower  $h(\tilde{b})$ . Thus the function  $h(\tilde{b})$  would be rather decreasing. If the hazard rate function  $h(\tilde{b})$  is decreasing, we show in Proposition 1 that the optimal magnitude of fine decreases with the degree of pessimism and the degree of ambiguity under the condition that individuals are pessimistic. The optimal magnitude of fine can be reduced when facing to an increase of pessimism or ambiguity, achieving the same deterrence level. In probability theory, the function  $h(\tilde{b})$  will be decreasing under a Weibull or a Gamma distribution of the gain for a particular range of their shape parameter.

If we expect that the function  $h(\tilde{b})$  is rather constant, then the severity of the deterrence problem is the same whatever the deterrence threshold may be. The assumption of an exponential or uniform distribution would be more suitable. And yet, we show in Proposition 1 that the optimal magnitude of fine still decreases with the degree of pessimism and the degree of ambiguity.

If one expect that the function  $h(\tilde{b})$  is increasing, then the comparative static results can be reversed as the optimal magnitude of fine may now increase with the degree of pessimism and the degree of ambiguity under the condition that individuals are pessimistic. In this case, one may also use a Weibull or a Gamma distribution function but for another range of their shape parameter.

Finally, we rewrite the first order condition in order to better explain how any variation in pessimism or ambiguity has an influence on the optimal magnitude of fine.

$$\underbrace{h(\tilde{b})(D - ps^*)}_{(+)} = \underbrace{\frac{(\tilde{p} - p)}{\tilde{p}}}_{(+)} \iff \underbrace{h(\tilde{b})[\delta\alpha + (1 - \delta)p]}_{(+)}(D - ps^*) = \underbrace{\delta(\alpha - p)}_{(+)} \quad (39)$$

First, consider the marginal perception cost of deterrence on the right-hand side of second equation. If the degree of pessimism or the degree of ambiguity increases, the marginal cost also increases, thereby making it more socially desirable to reduce the magnitude of fines. Next, consider the marginal benefit of deterrence on the left-hand side. We have shown before that a one-unit increase in the fine reduces the probability of offending, thereby reducing the social harm at the margin by  $D$  per capita, net of the objective expected fine,  $ps$ , paid by offenders. Here, two effects play a role. First, an increase of the degree of the pessimism or the degree of ambiguity increases the subjective probability of detection and conviction,



$\delta\alpha + (1 - \delta)p$ , which weights the marginal benefit of deterrence (defined by the difference between the social harm per capita and the objective expected fine,  $D - ps$ ). Intuitively, an increase in pessimism or ambiguity increases the deterrence value of any given fine. Second, this marginal benefit of deterrence is weighted by the value of the hazard rate function at the threshold benefit of offense,  $h(\tilde{b})$ . As a consequence, if the function  $h(\tilde{b})$  is decreasing with the benefit of offense (and is high enough) then it is more plausible that the degree of pessimism or the degree of ambiguity will negatively affect the optimal magnitude of fines. Deterrence becomes less desirable up to a point that when either pessimism or ambiguity increases, the optimal magnitude of fines also decreases. Using the same reasoning, we could also explain how the optimal magnitude of fines rises when confronted with an increase in pessimism or ambiguity, achieving the same deterrence level.

### 9.1.2 The optimistic case

Recall that the optimal magnitude of fines solves:

$$f(\tilde{b})\tilde{p}(D - \tilde{b}) + (1 - F(\tilde{b}))(p - \tilde{p}) = f(\tilde{b})\tilde{p}s^*(p - \tilde{p}) \quad (40)$$

The left-hand side in first order equation is the social marginal benefit of deterrence. It combines two terms. First, an increase of the fine by one unit reduces the probability of offending by  $-\frac{\partial(1-F(\tilde{b}))}{\partial s} = f(\tilde{b})\tilde{p}$  units thereby diminishing the occurrence of the net harm  $D - \tilde{b} > 0$ . Second, the positive term  $(1 - F(\tilde{b}))(p - \tilde{p})$  means that raising the fine increases the expected perception bias gain that is the difference between the expected fine and the subjective expected fine. The right-hand side equals the marginal cost of fines. An increase of the fine by one unit reduces the probability of offending by  $\frac{-\partial(1-F(\tilde{b}))}{\partial s}$  units, thereby preventing individuals from getting the perception gain:  $(p - \tilde{p})s > 0$ .

As before, we can rewrite the first-order condition to have:

$$\underbrace{h(\tilde{b})\tilde{p}(D - ps^*)}_{(+)} = \underbrace{(\tilde{p} - p)}_{(-)} \quad (41)$$

where  $h(\tilde{b})$  is the hazard rate function. We thus find that  $s^* > s_n^* = \frac{D}{p}$ . When individuals are optimistic, the optimal fine under ambiguity is higher than the optimal fine in the standard Beckerian framework while being lower when individuals are pessimistic. It is socially desirable to increase the magnitude of fines beyond the value  $\frac{D}{p}$  for two reasons. First, there is an additional marginal benefit associated with the use of fines due to the perception bias. Second, optimistic potential offenders are less sensitive to fines.

Finally, we turn to the comparative statics analysis.

**Proposition 2** *Assume that individuals are optimistic. Then the optimal magnitude of fines decreases with the degree of pessimism  $\alpha$  and the degree of ambiguity  $\delta$  if and only if:*

$$e_{\tilde{b}}^{h(\tilde{b})} > \underbrace{\frac{p}{(\tilde{p} - p)}}_{(-)} \quad (42)$$

where  $e_{\tilde{b}}^{h(\tilde{b})} = \frac{h'(\tilde{b})\tilde{b}}{h(\tilde{b})}$  stands for the elasticity of the hazard rate function with respect to the deterrence threshold.

*Proof.* See the pessimistic case above. ■

Our result anew depends on the the shape of function  $h(\tilde{b})$ . Assume that the function  $h(\tilde{b})$  is decreasing. In our context, it means that the deterrence problem seems to be more severe. In this case, the optimal magnitude of fine may either decrease or increase with the degree of pessimism or the degree of ambiguity when individuals are optimistic. Without any additional assumptions, for instance on the gap between the subjective (under-estimated) probability of detection and conviction and the correct one, the comparative statics remains unclear. If we expect that the function  $h(\tilde{b})$  is constant, meaning that the severity of the deterrence problem is the same whatever the deterrence threshold may be, then the optimal magnitude of fine decreases with the degree of pessimism and the degree of ambiguity. For instance, this result holds under the condition that the gain follows an uniform distribution. In this case, the optimal magnitude of fine can be reduced when facing to an increase of pessimism or ambiguity, achieving the same deterrence level. Finally, if one expect that the function  $h(\tilde{b})$  is increasing, then the optimal magnitude of fine also decreases with the degree of pessimism and the degree of ambiguity under the condition that individuals are optimistic.

## 9.2 Optimal fine and detection

Assume that the public law enforcer can determine both the size of the fine and the probability of detection. Under ambiguity, she solves:

$$\max_{s,p} \left\{ \mathcal{W} = w + \int_{\tilde{b}(p,s)}^B (b - D)f(b)db - \left(1 - F(\tilde{b})\right) (\tilde{p} - p)s - m(p) \right\} \text{ u.c. } s \leq w. \quad (43)$$

The first-order conditions of this program relative to  $s$  and  $p$  respectively are:

$$f(\tilde{b})\tilde{p}^*(D - \tilde{p}^*s^*) + f(\tilde{b})\tilde{p}^*s^*(\tilde{p}^* - p^*) = \left(1 - F(\tilde{b})\right) (\tilde{p}^* - p^*) \quad (44)$$

$$f(\tilde{b})(1 - \delta)s(D - \tilde{b}) + (1 - F(\tilde{b}))\delta s + f(\tilde{b})(1 - \delta)s(\tilde{p}^* - p^*)s = m'(p^*) \quad (45)$$

In order to simplify the total overview and the interpretation, we anew separate the optimistic case and the pessimistic case.

### 9.2.1 The pessimistic case

The interpretation of the derivative of  $\mathcal{W}$  relative to the probability of detection and conviction deserves more attention. At the right-hand side, we have the marginal cost of detection and conviction  $m'(p^*)$ . The left-hand side of equation stands for the social marginal benefit of deterrence, which sums three terms as we explained in the article. Below, we determine the conditions under which it is socially desirable to replace one deterrence tool by the other. We report our results in Proposition 4.

**Proposition 4** *Assume that individuals are pessimistic. It is socially desirable to increase the probability of detection and conviction while decreasing the magnitude of fines if and only if*

$$e_{\alpha}^{\tilde{b}} s > \frac{m'(p)}{(1 - F(\tilde{b}))} \quad (46)$$

where  $e_{\alpha}^{\tilde{b}}$  is the elasticity of the deterrence threshold relative to the degree of pessimism.

*Proof.* By definition,  $\tilde{b} = (\delta\alpha + (1 - \delta)p) s$ . So,

$$\frac{ds}{dp_{\tilde{b}=cst}} = \frac{-(1 - \delta)s}{\delta\alpha + (1 - \delta)p} < 0. \quad (47)$$

Fully differentiating the social welfare function, we have:

$$d\mathcal{W} = -\left(1 - F(\tilde{b})\right) (\delta(\alpha - p)ds - \delta s dp) - m'(p)dp. \quad (48)$$

Factorizing by  $dp$ , we have:

$$d\mathcal{W} = dp \left( (1 - F(\tilde{b})) \left( -\delta(\alpha - p) \frac{ds}{dp} + \delta s \right) - m'(p) \right). \quad (49)$$

Thus

$$\begin{aligned} d\mathcal{W}_{\tilde{b}=cst} &= dp \left( (1 - F(\tilde{b})) \left( \frac{\delta(\alpha - p)(1 - \delta)s}{\delta\alpha + (1 - \delta)p} + \delta s \right) - m'(p) \right) \\ &= dp \left( \underbrace{(1 - F(\tilde{b}))e_{\alpha}^{\tilde{b}} s}_{(+)} - m'(p) \right) \end{aligned}$$

where  $e_{\alpha}^{\tilde{b}}$  is the elasticity of the deterrence threshold with respect to the degree of pessimism.

As a consequence, if  $e_{\alpha}^{\tilde{b}} s > \frac{m'(p)}{(1 - F(\tilde{b}))}$  then it is socially desirable to increase the probability of detection and conviction while decreasing the magnitude of fines. If  $e_{\alpha}^{\tilde{b}} s < \frac{m'(p)}{(1 - F(\tilde{b}))}$  then it is socially desirable to decrease the probability of detection and conviction while increasing the magnitude of fines, and to choose  $s^* = w$  at equilibrium.

■

Assume that there is no ambiguity ( $\delta = 0$ ). Then, we have  $\tilde{e}_\alpha|_{s=0} = 0$ . It is socially desirable to decrease the probability of detection and conviction while increasing the magnitude of fines. Intuitively, there is neither any perception cost, nor any cost to collect fines, while detection and conviction is costly. We thus find the standard Beckerian result:  $s_n^* = w$ . Next, assume that the probability of detection and conviction becomes ambiguous ( $\delta > 0$ ). The condition under which it is optimal to replace fine with investment in ambiguous detection and conviction at the margin is written  $\tilde{e}_\alpha s > \frac{m'(p)}{1-F(\tilde{b})}$ . The intuition behind this result runs as follows. First, detection and conviction must be cheap enough (see the term  $\frac{m'(p)}{1-F(\tilde{b})}$ ). Second, potential offenders must be sufficiently sensitive to the degree of pessimism (see the term  $\tilde{e}_\alpha$ ). Third, it is all the more socially desirable to increase the investment in detection and conviction while reducing the magnitude of fines when fines are largely used to deter offenses (see the variable  $s$  to the left-hand side of the inequality above).

### 9.2.2 The optimistic case

Recall that the optimal probability of detection and conviction is implicitly defined by:

$$\left(1 - F(\tilde{b})\right) \delta s + f(\tilde{b})(1 - \delta)s(D - \tilde{b}) = -f(\tilde{b})(1 - \delta)s\delta(\alpha - p^*)s + m'(p^*) \quad (50)$$

The left-hand side corresponds to the marginal benefit. If offenders are optimistic ( $\tilde{p} < p$ ), the discrepancy between how much they expect to pay and the objective expected fine (that is  $(\tilde{p} - p)s = \delta(\alpha - p)s$ ) is positive, and represents the perception bias gain. Thus, a one-unit increase of the probability of detection and conviction raises this gain by  $\delta s$  units, which occurs with probability  $(1 - F(\tilde{b}))$ . In other words, an increase in detection raises the expected sanction more than the subjective expected sanction, and this net marginal difference equals  $\delta s$ . Next, the proportion of offenders decreases by  $-\frac{\partial(1-F(\tilde{b}))}{\partial s} = f(\tilde{b})(1 - \delta)s$  units, thereby diminishing the occurrence of the net harm  $(D - \tilde{b})$ .

The right-hand side stands for the marginal cost. There are two terms:  $m'(p)$  and  $-f(\tilde{b})(1 - \delta)s\delta(\alpha - p^*)s$ . The second term means that a one-unit increase of the probability of detection and conviction reduces the probability of offense by  $-\frac{\partial(1-F(\tilde{b}))}{\partial s} = f(\tilde{b})(1 - \delta)s$  units, thereby reducing the occurrence of the perception bias gain  $\delta(\alpha - p^*)s = (\tilde{p} - p)s$ .

Finally, we notice that the condition under which it is socially desirable to replace one deterrence tool by the other is the same in both optimistic and pessimistic cases.

## 10 Appendix 4: Second-order condition

The second-order condition for  $s$  is written:

$$h'(\tilde{b})(\delta\alpha + (1 - \delta)p)(D - ps) < ph(\tilde{b}) \quad (51)$$

where  $h(\tilde{b}) = \frac{f(\tilde{b})}{1-F(\tilde{b})}$  is the hazard rate function estimated at the threshold value  $\tilde{b}$ . The sign of the right-hand side depends on the individuals being either optimistic or pessimistic, while the sign of the left-hand side depends on the monotonicity of the hazard rate function.

Assume that individuals are pessimistic. We have shown that  $D - ps^* > 0$  at equilibrium. Thus  $h'(\tilde{b}) < 0$  (a monotonically decreasing hazard rate function) is a sufficient condition for the second-order condition to hold. Next, using first-order condition, we can replace  $ph(\tilde{b}) = \frac{\delta(\alpha-p)p}{(\delta\alpha+(1-\delta)p)(D-ps)}$  in second order condition above. If the hazard rate function is increasing ( $h'(\tilde{b}) > 0$ ), then the second-order condition can be rewritten:

$$h'(\tilde{b}) < \underbrace{\frac{\delta(\alpha-p)p}{(\delta\alpha + (1-\delta)p)^2(D-ps)^2}}_{(+)} \quad (52)$$

Assume that individuals are optimistic. We now have  $D - ps^* < 0$  at equilibrium because  $\tilde{p} < p$ . Thus,  $h'(\tilde{b}) > 0$  is a sufficient condition for the second-order condition to hold. If  $h'(\tilde{b}) < 0$ , then the second-order condition writes:

$$h'(\tilde{b}) > \underbrace{\frac{\delta(\alpha-p)p}{(\delta\alpha + (1-\delta)p)^2(D-ps)^2}}_{(-)} \quad (53)$$

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