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# **Lattice strain development in an alpha titanium alloy studied using synchrotron and neutron diffraction**

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## **Abstract**

Neutron and high-energy X-ray diffraction methods have been used to investigate deformation mechanisms (slip and twinning activities) during uniaxial tensile tests of Grade 1 commercially pure titanium (Ti- $\alpha$ ). These two diffraction techniques have provided valuable and complementary information. The predictions made by an Elasto-Plastic Self-Consistent (EPSC) model, taking into account grain reorientation and stress relaxation induced by twinning activity, have been compared with the experimental data. Results clearly demonstrate that there is a good agreement between simulations and diffraction measurements. The EPSC model enables accurate predictions of: (i) the lattice strains developed in variously oriented  $\{hk.l\}$  grain families within the bulk material, (ii) the grain reorientation induced by twinning activity and (iii) the macroscopic stress-strain response.

Two different textured products were studied in order to evaluate the influence of texture on the mesoscopic and macroscopic responses of Ti- $\alpha$  alloy.

**Keywords:** titanium alloys, strain measurements, plasticity, modelling, anisotropy, diffraction

## 1. Introduction

Due to their high specific strength, good corrosion resistance and biocompatibility, titanium and its alloys are of the utmost importance for aeronautic applications and are also widely used in naval and biomedical industries [1–3]. Hexagonal Close Packed (HCP) metals present a low crystalline symmetry as compared to cubic materials. They are then governed by a heterogeneous plastic deformation, and the way it is accommodated is still not well understood. At room temperature, it is well experimentally established that prismatic slip  $\{10.0\}\langle 11.0 \rangle$  is the easiest deformation mode to be activated in HCP titanium. Pyramidal  $\langle c + a \rangle$  slip  $\{10.1\}\langle 11.3 \rangle$ , pyramidal  $\langle a \rangle$  slip  $\{10.1\}\langle 11.0 \rangle$  and basal slip  $\{00.2\}\langle 11.0 \rangle$  are generally presented as secondary slip modes [4,5] that are more difficult to activate. In particular,  $\{10.1\}\langle 11.3 \rangle$  slip produces plastic deformation with a component out of the basal plane (i.e. along the  $\vec{c}$ -axis). Deformation along  $\vec{c}$ -axis can also be accommodated by twinning modes  $\{10.2\}\langle 10.\bar{1} \rangle$  or  $\{11.2\}\langle 11.\bar{3} \rangle$  that induce a reorientation of the crystal lattice [4,6,7]. The relative activity of these multiple deformation mechanisms strongly depends on the chemical composition, grain size, morphological and crystallographic textures, straining direction and strain rate. Understanding how the different deformation mechanisms operate during plastic deformation of titanium alloys require thus further investigations. A deeper knowledge of the mechanical behavior of HCP materials during plastic deformation calls for the use of advanced techniques such as Time-Of-Flight (TOF) neutron or High Energy synchrotron X-Ray Diffraction (HEXRD) enabling to monitor the evolution of internal elastic strains. With a high spatial resolution, X-ray synchrotron radiation allows a very small gauge

volume ( $200\text{ }\mu\text{m} \times 200\text{ }\mu\text{m}$  spot size for the present study) as compared to the neutron diffraction one ( $4\text{ mm} \times 4\text{ mm}$  spot size for the present study), but offers fast rate data acquisition (few seconds for synchrotron radiation and around 40 minutes for neutron diffraction in this work). This technique enables to measure online strain distributions and *in situ* during a continuous mechanical loading. In contrast, for materials characterized by a substantial creep behavior such as titanium alloys, the samples investigated by neutron techniques have to be unloaded once the desired macroscopic strain has been reached. Analyses relate thus only to the residual lattice strains remaining after a given increment of plastic deformation [8–10]. These two complementary non-destructive methods have been adopted in this work to measure the change in the  $\{hk.l\}$  d-spacing during tensile loadings in order to characterize the internal elastic strains (or intergranular strains) distribution. Experimental results can be explained and interpreted using a polycrystalline modeling approach like the Elasto-Plastic Self-Consistent (EPSC) model [11–15]. This micromechanical modelling is a helpful technique to characterize the mechanical state of polycrystalline materials at the grain level and to understand the role played by plastic anisotropy, slip-slip (and slip-twin) competitions in the macroscopic behavior. For example, Warwick et al. [8] have examined a textured Grade 1 Commercially Pure titanium (CP Ti) by coupling neutron diffraction to self-consistent modelling. A good agreement was found between neutron diffraction and the simulated results. The authors found that twinning produces a greater hardening than slip and that the residual lattice strains developed in the  $\{00.2\}$  grains are higher than in other orientations for both tensile and compression straining. A similar finding has been achieved by Gloaguen et al. [9], who analyzed the deformation mechanisms involved in the plasticity of Grade 2 CP Ti during compressive loadings. The residual lattice strain of different  $\{hk.l\}$  orientations was tracked in both parallel and transverse directions to the straining axis and the diffraction data were investigated to

highlight the role of tensile twin  $\{10.2\}$  in the  $\alpha$ -titanium alloy. Cho et al. [10] used a similar methodology to characterize the intergranular residual strains accumulation in a near  $\alpha$ -titanium alloy and a Ti-834 alloy during tensile tests. It has been found that it was possible to successfully reproduce the trends and magnitudes of the observed microstrains in both loading and transverse directions. The plastic behavior of a two phase titanium alloy, Ti-6Al-4V, has also been studied [16] thanks to this coupled approach. The X-ray synchrotron diffraction has likewise been used with EPSC modelling by Stapleton et al. [17] to analyze the evolution of internal lattice strains in a Ti-6Al-4V alloy during *in situ* tensile tests. A similar study has been carried out in a near  $\beta$ -titanium alloy [18]. The main advantage of high energy X-rays is the possibility to have short measurements during a continuous mechanical test. The number of diffracting grains is nevertheless smaller as compared to neutron diffraction experiments due to the smaller gauge volume (respectively  $200\text{ }\mu\text{m} \times 200\text{ }\mu\text{m} \times 2\text{ mm}$  and  $4\text{ mm} \times 4\text{ mm} \times 4\text{ mm}$ ). Therefore, the lattice strains determined with synchrotron X-ray radiation are less representative due to a smaller number of crystallites contributing to the diffracting volume. The comparison with a homogenization approach is then more problematic [14].

The purpose of the work presented in this paper is to understand the internal strain development of a commercially pure titanium under uniaxial tensile loadings. Experimental observations are coupled with the predictions of the EPSC model in order to explain the plastic behavior of Ti- $\alpha$  and to identify the slip systems and twinning modes responsible for the plastic activity within the material. One of the objectives of this study is to evaluate the influence of crystallographic texture and twin activity on the mechanical response. An EPSC model, taking into account grain reorientation and stress relaxation induced by twinning activity, has been developed for this study. Two different textured samples (a unidirectional rolled plate and a forged bar) for the same alloy were studied in order to evaluate the influence of texture on the mesoscopic and macroscopic responses of a Grade 1 CP Ti- $\alpha$  alloy.

A comparison between these two techniques and the experimental internal (X-ray) and residual (neutron) strain values obtained with an EPSC model has been made to analyze the potential difference.

## 2. Experimental methods

### 2.1. Material

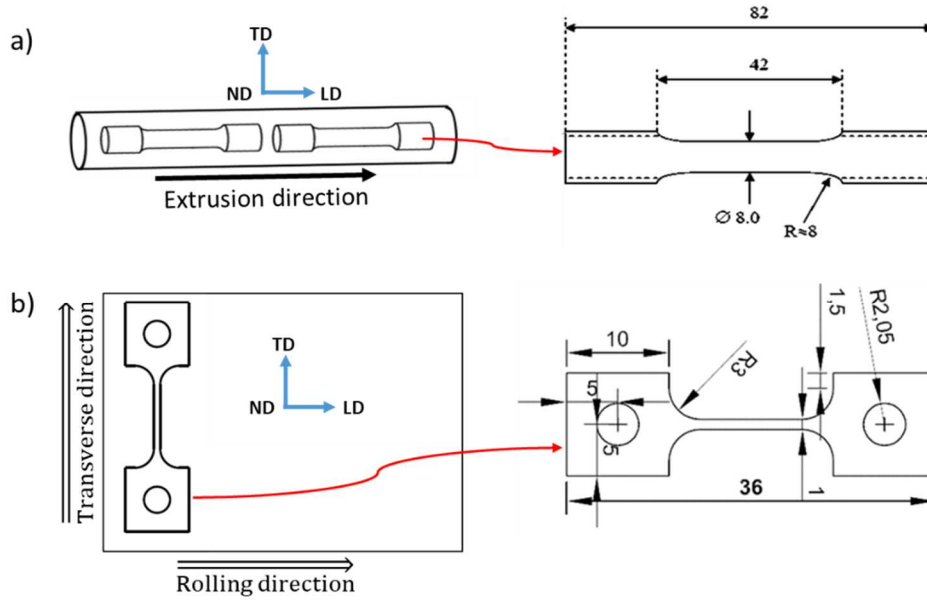
The investigated material is a Commercially Pure  $\alpha$ -titanium (denoted CP Ti- $\alpha$ ). Two samples, produced by two different forming processes, are studied: a unidirectional rolled plate and a forged bar. The samples are fully hexagonal compact phased at room temperature and their chemical compositions are given in Table 1. The tensile specimens have different geometries (see Fig. 1).

**Table 1:** Chemical composition data (wt.%) of the materials

Specimen	C	N	O	H	Fe	Ti
N-Sample	0.07	0.008	0.12	0.0013	-	Balance
S-Sample	0.012	0.0078	0.137	0.001	0.164	Balance

Cylindrical specimens (denoted below as N-sample) with 8 mm diameter and 42 mm gauge length were cut along the extrusion direction (represented by the Longitudinal Direction: LD, Fig. 1.a) for neutron diffraction experiments. This geometry was adopted in order to access to a sufficiently large gauge volume. However, *in situ* synchrotron experiment specimens, denoted as S-samples, have been machined along the Transverse Direction (TD) of the rolled plate, as shown on Fig. 1.b. The sample thickness was small enough (2 mm) to enable transmission measurements [19]. The metallographic characterization has revealed that the

grains are generally equiaxed with an average grain size of 40 and 45  $\mu\text{m}$  in S-samples and N-samples, respectively.

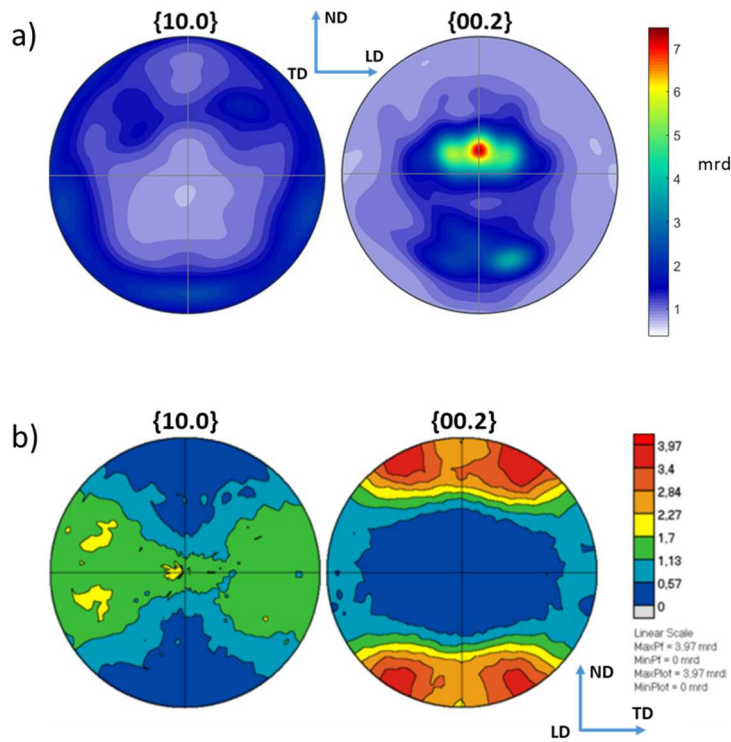


**Fig. 1:** Schematic view showing dimensions (in mm) of the samples used in a) neutron and b) synchrotron experiments. LD: Longitudinal (loading) Direction, TD: Transverse Direction, ND: Normal Direction

N-sample texture measurements were performed on the GEM beamline at the ISIS neutron spallation source (United Kingdom). Information about the instrument and the measurement technique can be found in [20,21]. Measurements of the initial texture (Fig. 2.a) have revealed that the samples have a strong asymmetric texture in the TD-ND plane, perpendicular to the loading axis (LD). This texture is consistent with the analyses made on extruded hexagonal polycrystalline samples found in literature [8,9,22].

The S-sample initial texture, shown in (Fig. 2.b), was measured using the neutron diffraction on the 6T1 diffractometer from the French national Orpheus nuclear reactor (at Léon Brillouin Laboratory, Saclay, Gif-sur-Yvette, France). As revealed in Fig. 2.b, basal poles are tilted +

30° away from the Normal Direction axis (corresponding to the plate normal direction) toward the Transverse Direction. The observed texture is typical of rolled titanium plates followed by recrystallization.

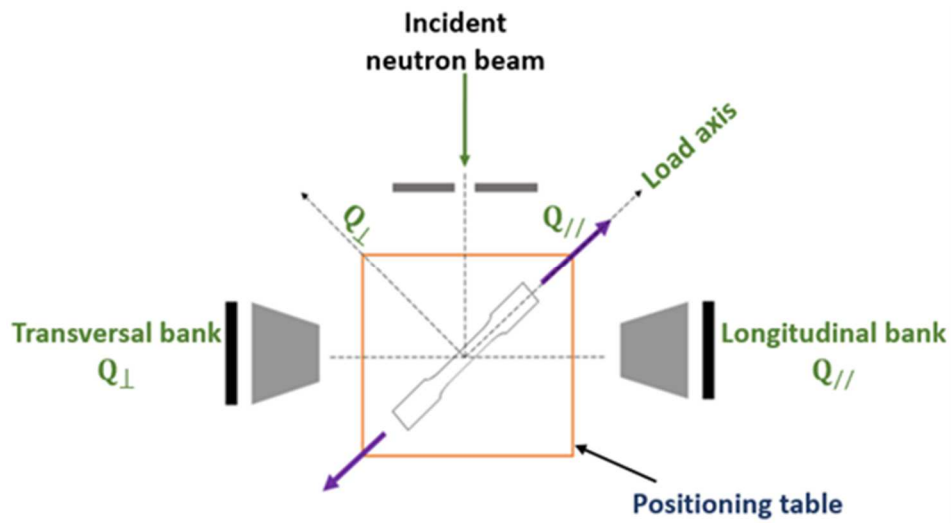


**Fig. 2:** Initial crystallographic textures of the studied specimens a) N-sample and b) S-sample

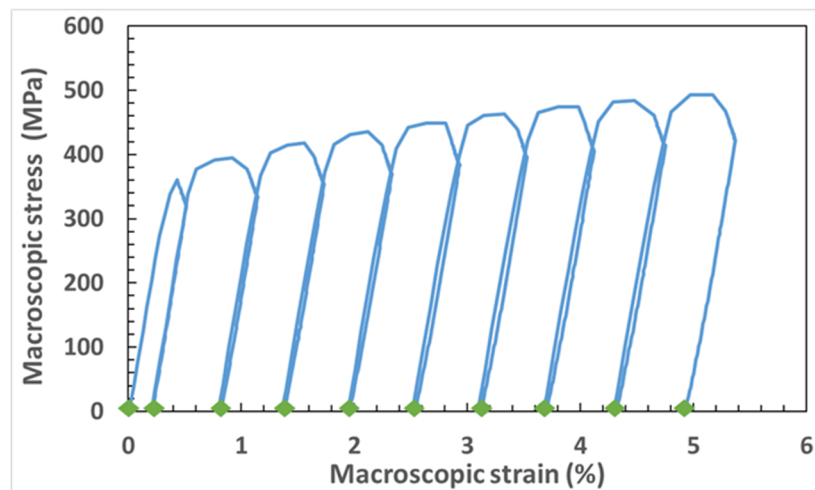
## 2.2. Neutron diffraction measurements

Tensile tests were performed at room temperature using the ENGIN-X neutron diffractometer (Fig. 3) at the ISIS spallation neutron source. The measurement steps were applied by a 50 kN Instron testing machine with a strain rate of  $10^{-5} \text{ s}^{-1}$  up to 6 % of macroscopic strain. The specimen was subjected to a displacement controlled regime. The incident slits and radial collimators defined a scattering volume of  $4 \times 4 \times 4 \text{ mm}^3$  with a counting time of 43 minutes per measurement. The macroscopic strain was determined using an extensometer that spanned the irradiated region. The sample was first deformed to the desired macroscopic strain and

then immediately unloaded, as shown in Fig. 4, to avoid any strain relaxation due to the creep behavior.



**Fig. 3:** Schematic top-view of the set-up used on the neutron diffraction instrument ENGIN-X



**Fig. 4:** Measured macroscopic stress– strain curve obtained with ENGIN-X. The green points indicate the macroscopic strain during neutron measurements after unloading, i.e. at zero applied load.

Successive steps have been achieved at room temperature, in the plastic regime, studying the remaining residual strains resulting from a given increment of plastic deformation at zero

applied load. The schematic view of the ENGIN-X instrument (Fig. 3) shows a horizontal loading axis positioned at 45° with respect to the incident beam. Two detector banks are set up horizontally at ± 90° from the incident beam, enabling simultaneous measurements of lattice strains from {hk.l} plane families with parallel (i.e. along the loading direction) and perpendicular (i.e. along the transverse direction) directions, as regard to the loading applied direction (Fig. 3). The Time-Of-Flight (TOF) technique enables to collect simultaneously an entire diffraction pattern (the effective d-spacing range from 0.88 to 2.63 Å) on each detector [23]. Single peak fitting was performed using the Open Genie code [24] with Gaussian functions. The lattice strain  $\langle \varepsilon(hk.l, \varphi, \psi) \rangle_{V_d}$  of a grain group having a common {hk.l} plane-normal parallel to the diffraction vector  $\vec{Q}$  (defined by azimuthal and inclination angles, respectively  $\varphi$  and  $\psi$ ) can be calculated by the following formula:

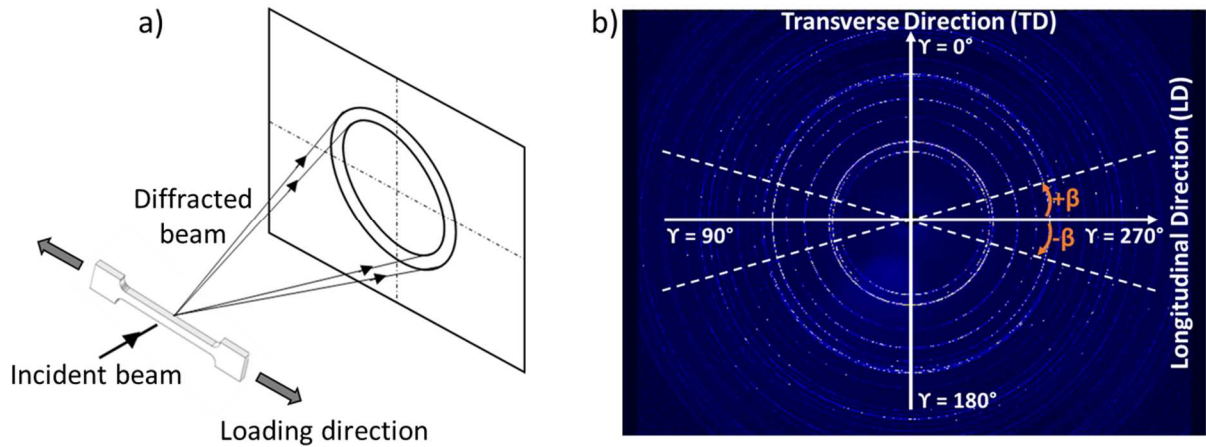
$$\langle \varepsilon(hk.l, \varphi, \psi) \rangle_{V_d} = \ln \left( \frac{\langle d(hk.l, \varphi, \psi) \rangle_{V_d}}{d_0(hk.l)} \right) \quad Eq. 1$$

The initial lattice parameter  $d_0(hk.l)$ , measured before deformation of the material, is used as the reference point for this calculation.  $\langle X \rangle_{V_d}$  ( $V_d$ : diffracting volume,  $X = \varepsilon(hk.l, \varphi, \psi)$  or  $d(hk.l, \varphi, \psi)$ ) stands for the averaging over the diffracting grains for the considered {hk.l} reflection.  $\langle d(hk.l, \varphi, \psi) \rangle_{V_d}$  is the measured lattice spacing and resulting lattice strain values are given as micro-strain (denote  $\mu\varepsilon$ , i.e. units of  $10^{-6}$ ). In the remainder of the paper, // and  $\perp$  symbols respectively refer to the lattice strain in the parallel and perpendicular directions to the stress axis (corresponding to the extrusion direction).

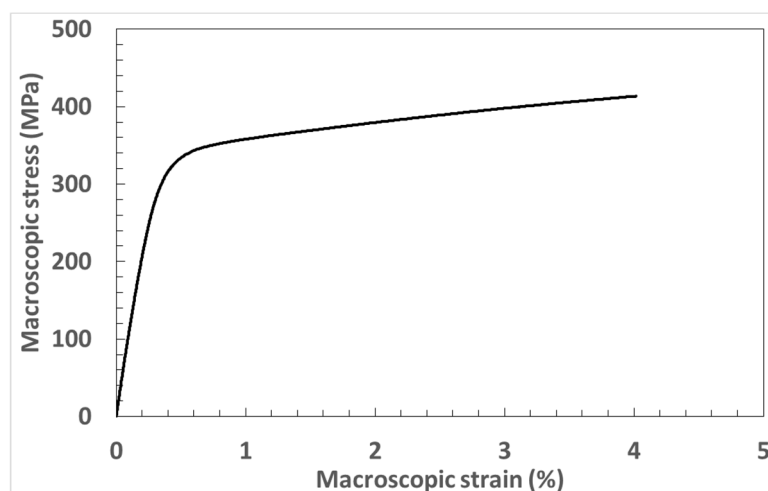
### 2.3. Synchrotron X-ray measurements

ID15B beamline at the ESRF (the European Synchrotron Radiation Facility, Grenoble, France) has been used to perform *in situ* tensile tests in a transmission mode, with a monochromatic X-ray radiation of 87 keV ( $\lambda = 0.1425$  Å wavelength). The configuration of

the experimental setup is shown in Fig. 5.a. The tests were performed on an Instron 8800 electrothermomechanical test machine with a maximum load capacity of 3 kN under a constant strain rate  $4 \cdot 10^{-4} \text{ s}^{-1}$  up to 4 % macroscopic strain. The load was applied perpendicular to the rolling direction (Fig. 1.b). Fig. 6 shows the macroscopic curve obtained on ID15B beamline.



**Fig. 5:** a) Experimental setup showing S-sample positioning, b) example of image recorded on the 2D detector with  $\gamma$  and  $\beta$  angle definitions



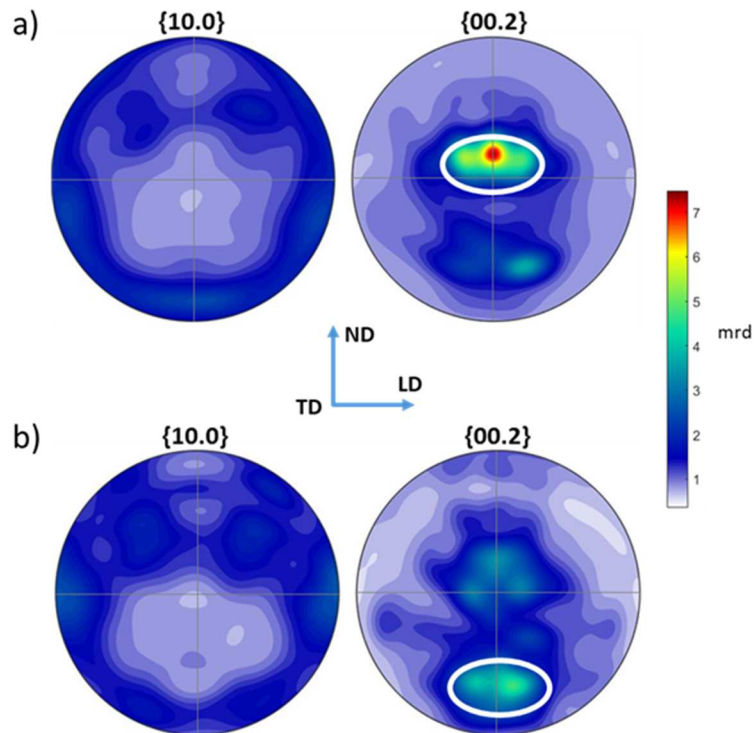
**Fig. 6:** Measured macroscopic stress – strain curve obtained on the ID15B beamline.

The diffraction patterns (example presented in Fig. 5.b), have been recorded using a 2D Pixium 4700 flat-panel detector [25], positioned in agreement with the targeted  $\{hk.l\}$  diffracting planes and signal optimization. Each ring recorded with the 2D detector corresponds to a  $\{hk.l\}$  crystallographic family plane. Regarding high  $\alpha$ -Ti creep rates, high speed data acquisition is mandatory for online measurements (2s counting time every 7s). Considering the sample dimensions (1 mm  $\times$  2 mm) and the beam size (200  $\mu$ m  $\times$  200  $\mu$ m), the irradiated volume is 0.2  $\times$  0.2  $\times$  2 mm<sup>3</sup>. It has to be underlined that this latter volume is much smaller than the gauge volume obtained in neutron diffraction experiments (4  $\times$  4  $\times$  4 mm<sup>3</sup>). The recorded images were processed using the FIT2D pattern analysis software [26], which integrates the two-dimensional images (Fig. 5.b) into one-dimensional spectra (intensity  $I(\gamma, hk.l)$  vs. scattering angle  $\langle 2\theta(hk.l) \rangle_{V_d}$ ). ( $\langle d(hk.l) \rangle_{V_d}$ ,  $I(\gamma, hk.l)$ ) couples have been calculated integrating the 2D patterns over a given  $\pm \beta$  range for the two directions : longitudinal (i.e. LD, corresponding to the symmetrical azimuth angles  $\gamma = 90$  and  $270^\circ$ ) and transverse (i.e. TD, corresponding to  $\gamma = 0$  and  $180^\circ$ ). The integration range over the ring (defined by  $\pm \beta$ ) has to be reasonably large for a statistically representative behavior of the material according to the considered orientation i.e. sufficient amount of the diffractive crystallites contributing to the signal. In addition, the  $\beta$  value must be small enough to cover only crystallites oriented relatively to the considered direction. For these reasons and based on [14,19] studies, the intensities of the diffraction rings were integrated, as a compromise, within "cake shape" sectors defined by  $\beta \pm 10^\circ$ . Finally the arithmetic mean values of the d-spacings  $\langle d(hk.l) \rangle_{V_d}$  were calculated from the averaging of the symmetrically equivalent  $\gamma$  and  $\gamma + 180^\circ$  angles.

The  $\langle 2\theta(hk.l) \rangle_{V_d}$  positions of the diffraction peaks were determined using pseudo-Voigt function fitting [27–29]. The lattice spacing  $\langle d(hk.l) \rangle_{V_d}$  was then calculated from the peak positions using the Bragg's law [30].

#### 2.4. Twinning activity in both N-sample and S-sample

Fig. 7.a shows the initial N-sample texture measured on the GEM beamline. The specimen has a strong asymmetric texture in the TD-ND plane with the majority of grains with their basal normal close to the transverse direction and perpendicular to the loading axis. After 6 % of macroscopic deformation, a reorientation of the  $\{00.2\}$  poles was observed (presented by a white framed area in Fig. 7.b). This can be associated with twinning deformation. Actually, the activation of compression twinning  $\{11.2\}\{11.\bar{3}\}$  produces a reorientation of the basal poles of approximately  $65^\circ$  [6,31].

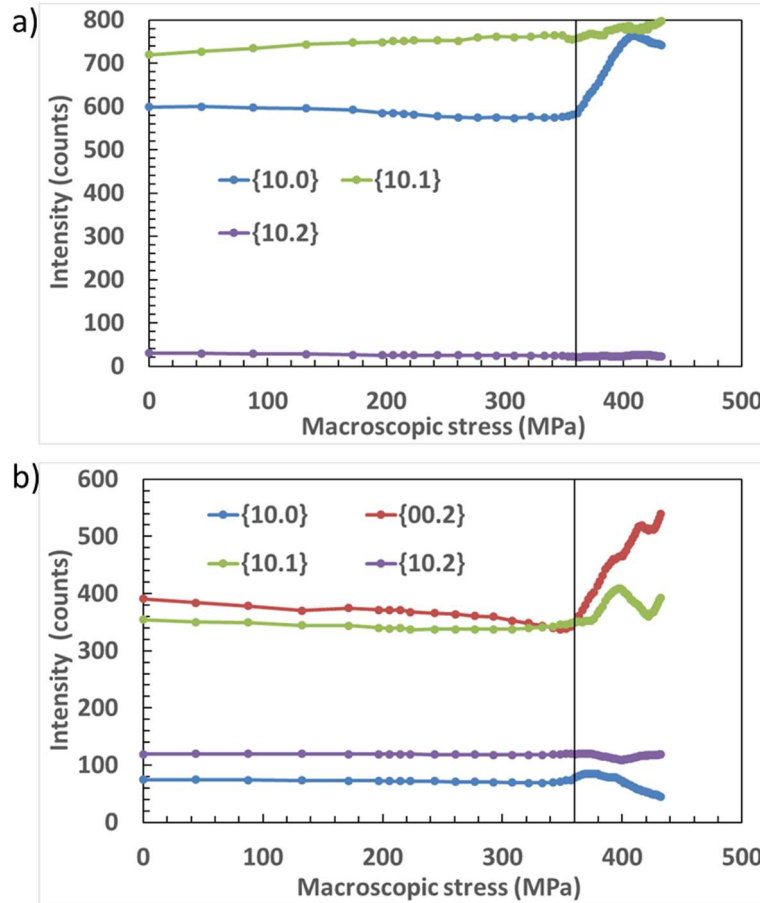


**Fig. 7:** Experimental  $\{10.0\}$  and  $\{00.2\}$  pole figures of the a) as-received N-sample and b) after the tensile loading up to 6 %. Pole figures (PFs) show the ND–LD plane, the center of the poles being the TD.

Tensile loading along the longitudinal (or extrusion) direction, with the  $\vec{c}$ -axis close to the transverse direction (**Fig. 7.a**), reorients the  $\vec{c}$ -axis of the grains undergoing twinning deformation. The twinned area fraction can be quantitatively calculated focusing on the difference between  $\{00.2\}$  pole figures of both as-received and deformed samples. Based on the work of Brown et al. [32] and integrating over the white framed area in **Fig. 7.a**, the total twin volume fraction after 6 % applied macroscopic strain is about 13 %.

Contrary to the previous analysis, crystallographic texture of the S-sample could not be measured in synchrotron experiments after the tensile test. Thus, analysis has focused on the evolution of the intensity measured during *in situ* straining as shown in **Fig. 8**. Gharghouri et al. [33] were the first to show that severe intensity variations during plastic deformation can be reliably attributed to twinning activity. Hence, Muránsky et al. [12], Brown et al. [32] and Gharghouri et al. [33] highlighted a direct correlation between the intensity evolution of  $\{10.0\}$  and  $\{00.2\}$  diffraction peaks, measured with detectors placed at  $90^\circ$  from each other and the tensile twinning  $\{10.2\}\langle 10.\bar{1} \rangle$  that reorients the twin grains by  $90^\circ$  [31]. Our study confirms findings of these authors: a dramatic increase in  $\{10.0\}$  intensity in the loading direction (LD) is observed above around 365 MPa macroscopic stress (**Fig. 8.a**). At the same time, in the transverse direction (TD), the intensity of the  $\{10.0\}$ -orientated grains decreases, while the  $\{00.2\}$  peak intensity increases significantly (**Fig. 8.b**). This reflects a change in the grain volume fraction that diffracts in the TD and LD. Owing to the texture of the as-received S-sample, the changes of  $\{00.2\}$  and  $\{10.0\}$  peak intensities in these two directions are due to tensile twinning  $\{10.2\}\langle 10.\bar{1} \rangle$  activity.

The evolution of the intensity of others reflections such as  $\{10.1\}$  in TD (Fig. 8.b) could naturally also be related to other twinning mode activity but it is difficult to discern which one only in the light of the information given by present synchrotron X-ray diffraction experiments.



**Fig. 8:** Evolution of the diffraction peak intensities (Miller indices indicated in the plot)

obtained from *in situ* synchrotron X-ray diffraction measurements in the a) loading (LD) and transverse (TD) directions

### 3. Twinning scheme for Elasto-Plastic Self-Consistent model

The one-site EPSC approach applied in this work incorporate the single crystal plasticity model derived by the pioneering works of Kröner [34,35] and Hill [36]. It adopts a weighted population of grains approach which exploits the experimentally measured crystallographic

textures as inputs. Each grain is assumed to have the form of an ellipsoidal inclusion embedded in a homogeneous effective medium with anisotropic properties representative of the textured polycrystalline aggregate. The model has been extensively described in [9,15,37], therefore only a brief description will be given here. More information concerning the approach used can also be found in [38–41]. This model enables to incorporate the measured experimental results as well as to determine the strain hardening parameters, the related Critical Resolved Shear Stress (CRSS) and the mechanical strain/stress state at the mesoscopic level, for the diffracting volume and during the mechanical tests.

The slip rate  $\dot{\gamma}^g$  on a slip system  $g$  is linked to the resolved shear stress rate  $\dot{\tau}^g$  [40,41] by:

$$\dot{\gamma}^g = T^g(\tau^g, \tau_c^g, \dot{\tau}^g) \dot{\tau}^g \quad \text{Eq. 2}$$

where  $\tau^g$  is the resolved shear stress and  $\tau_c^g$ , the critical resolved shear stress.  $T^g$  is the deformation mechanisms management function and is expressed by:

$$T^g = \frac{1}{H^{gg}} \left( \frac{1}{2} (1 + \tanh(\tau^g)) \right) \left( \frac{1}{2} \left( 1 + \tanh \left( k \left( \frac{\tau^g}{\tau_c^g} - 1 \right) \right) \right) \right) \left( \frac{1}{2} (1 + \tanh(\dot{\tau}^g)) \right) \quad \text{Eq. 3}$$

where  $H^{gg}$  is the self-hardening coefficient and  $k$ , a numerical parameter. The hyperbolic tangent function was used because it enables the mechanical and hardening behaviors, observed in practice, to be reproduced [40,41].

The hardening matrix  $H^{gr}$  is introduced to describe the CRSS rate evolution in a system  $g$ , as a function of the plastic slip (or twinning) on the other deformation systems  $r$ :

$$\dot{\tau}_c^g = \sum_r H^{gr} \dot{\gamma}^r \quad \text{Eq. 4}$$

where  $\dot{\tau}_c^g$  is the CRSS rate. Due to a lack of relevant experimental data on latent hardening nature, especially in titanium alloys, the interactions between the different deformation modes are described by a simple law. In the present work, the hardening matrix is described by the following equation [39]:

$$H^{gr} = q.H^{gs}$$

Eq. 5

$H^{gs}$  is the self-hardening coefficient and  $H^{gr}$  corresponds to the latent hardening when  $g$  slip system is different from  $r$ . The  $q$  factor defines the degree of latent hardening.

In the present study, twinning deformation is treated as a directional slip mechanism [42–44]. It is characterized by the normal to the twinning plane  $n^t$  and the twinning direction  $m^t$ . When a twinning system becomes active, a part of the parent grain is reoriented and stress relaxation is introduced. In order to take into account these phenomena, different algorithms have been proposed. Van Houtte [45] was the first to propose a “Monte Carlo” approach consisting of randomly sampling and reorienting full grains. A scheme of such an approach has been proposed by Tomé et al. [46]: reorienting the grain according to the most active twinning system, named Predominant Twin Reorientation (PTR) formulation. Lebensohn and Tomé [47] have proposed another one called a Volume Fraction Transfer (VFT) formulation. The Euler space is partitioned in cells and, instead of keeping the grain volume fraction constant while changing its orientation, as done in PTR, the volume fraction assigned to each cell evolves with deformation. This last approach enables to predict the crystallographic texture evolution but it has the disadvantage of losing the grain identity and it does not allow to implement a realistic hardening scheme.

The scheme proposed in this paper, called Modified Predominant Twin Reorientation (M-PTR), is based on Tomé et al. [46] and Clausen et al. [48] assumptions.

### 3.1 Modified Predominant Twin Reorientation (M-PTR)

The M-PTR scheme accounts for: (i) the stress relaxation associated with the twin formation and, (ii) the twinning fraction effect on texture evolution. At each imposed strain increment, the philosophy of the PTR scheme is adopted. The accumulated shear of each twin system in

each grain is recorded and the Predominant Twin System (PTS) is identified as the one that involves the maximum volume fraction  $f_{cum}^t$  (Eq. 6):

$$f_{cum}^t = \sum \frac{\dot{\gamma}^t}{S^t} \quad Eq. 6$$

Here,  $\dot{\gamma}^t$  represents the shear due to the twin system  $t$  and  $S^t$  is the characteristic twinning shear.

Once the predominant twin system is identified, the total fraction  $f_{cum}^{gr}$  accumulated in each grain is given by the sum over all  $N^t$  active twin systems:

$$f_{cum}^{gr} = \sum_{t=1}^{N^t} f_{cum}^t \quad Eq. 7$$

In the EPSC model, a threshold fraction  $f_{limit}$  is introduced in Eq. 8. The idea behind this parameter is to take into account the twinning nucleation mechanism and the initial growth process in the parent grain. More information about these phenomena can be found in [49–52]. When the volume fraction  $f_{cum}^{gr}$  in a parent grain reaches the threshold  $f_{limit}$ , that fulfills the following equation (Eq. 8), the PTS system is selected and the corresponding grain is fully reoriented.

$$f_{cum}^{gr} \geq f_{limit} \quad Eq. 8$$

$f_{limit}$  is the model parameter that will be adjusted. Based on Tomé et al. [46], Tomé and Lebensohn [53] and Wronski et al [54] studies, the threshold value  $f_{limit}$  is taken between 5 and 50 %. If  $\Omega^m$  is the parent grain orientation, the twin orientation (child)  $\Omega^{tw}$  will be determined by the relation (Eq. 9) [55,56]:

$$\Omega^{tw} = R^{tw} \Omega^m \quad Eq. 9$$

The lattice rotation tensor  $R^{tw}$  is given by:

$$R^{tw} = 2n^t \otimes n^t - I \quad Eq. 10$$

where  $n^t$  is the twinning plane normal and  $I$  is the unit tensor.

Due to the fact that the twinning deformation is a secondary deformation mechanism in our study (see [section 2](#)), the nucleated twin is treated as a new independent grain and the new orientation is kept constant until the end of the simulation.

### *3.1. 1 Twin variant selection criterion*

Experimental studies have shown that each active twin system has six possible variants. Therefore, it is important to define a criterion allowing for the selection of the most appropriate twinning variant. Electron Backscatter Diffraction (EBSD) statistical analyses on zirconium [\[50\]](#) showed that  $\{10.2\}$  twinning variants with the highest Schmid factors were the most active (about 56-60 % activation probability). Similar dependencies were observed for  $\{10.2\}$  and  $\{11.2\}$  twinning systems during tensile or compressive loading of commercially pure titanium in [\[54,57,58\]](#). However, it has been noticed that variants with lower Schmid factors may also contribute to plastic deformation. Thereby, it is clear that the selection process is crucial since it affects the microstructure and the texture evolution of the sample. For example, Wronski et al [\[54\]](#) have used a Visco-Plastic Self-Consistent (VPSC) model with a simple Monte Carlo twin variant selection criterion to predict with accuracy the crystallographic texture evolution in titanium during tensile and compression tests. On the basis of these analyses and in order to provide a reasonable description of the selection process, one variant is picked randomly among the three with the highest twin fractions. The goal behind is to allow for the relatively less predominant variants to accommodate plastic deformation.

### *3.1. 2 Estimation of the stress relaxation*

As mentioned previously, twinning activity induces not only the twinned zone reorientation but also a stress relaxation in the parent grain [\[48,59,60\]](#). Stress relaxation has been implemented on the assumptions made in [\[48,59,60\]](#). Elastic strain and stress, originally

present and developed in the new formed (child) grain are estimated by applying continuity equations for tractions and displacements across the twin boundary. The stress and strain at the grain level are assumed to be uniform in the present model and the elastic strains within the plane of the child twin are supposed to be compatible to the matrix on the other side of the twin boundary as described in [48]. Furthermore, stresses acting on the twinning plane are also assumed to be equal across the twin interface. When the newly child grain is created and grown to an accumulated twin volume fraction  $f_{cum}^t$  (given in Eq. 6) of the parent grain, a shear strain  $\gamma^t$  for the twin system  $t$  is calculated from the equation below:

$$\gamma^t = f_{cum}^t S^t \quad \text{Eq. 11}$$

The plastic shear  $\gamma^t$  generates necessarily a back-stress between the parent and child grains in order to accommodate the plastic deformation. It is determined thanks to the Eq. 12 assuming that the imposed plastic shear is equal and opposite to the elastic shear:

$$\varepsilon^{II\ 0} = -R^t \gamma^t \quad \text{Eq. 12}$$

Here,  $\varepsilon^{II\ 0}$  represents the elastic back-strain and  $R^t$ , the Schmid tensor for the twin system  $t$ .

According to Clausen et al. [48], parent crystal is assumed to be infinitely stiff in comparison with the newly formed twin. It is thus not accommodating any of the plastic shear, forcing the twin to develop an equal and opposite elastic strain. The back-stress is computed with the following relation (Eq. 12), using the twin elastic stiffness tensor:

$$\sigma^{II\ 0} = c^t \varepsilon^{II\ 0} \quad \text{Eq. 13}$$

In the formulation applied, the parent and the twin child single-crystal elastic coefficients are supposed to be the same. Then, the stress relaxation is introduced by correcting the current stress with the calculated back-stress  $\sigma^{II\ 0}$ . The sum is made over the twin systems  $t$  which become active in the current step [48,60]:

$$\sigma^{II} = \frac{\sigma^{II} + \sum_t \sigma^{II\ 0} f_{cum}^t}{1 - \sum_t f_{cum}^t} \quad \text{Eq. 14}$$

### 3.2 Model parameter determination

In order to perform the numerical simulations, the model parameters must be identified to describe simultaneously the macroscopic stress-strain curve and the lattice strains measured on the various  $\{hk.l\}$  reflections. The single crystal elastic constants used in these simulations are shown in Table 2.

**Table 2:** The single crystal elastic constants (GPa) used in M-PTR model

Reference	$c_{11}$	$c_{12}$	$c_{13}$	$c_{33}$	$c_{44}$	$c_{66}$
[61]	162.4	92	69	180.4	46.7	35.2

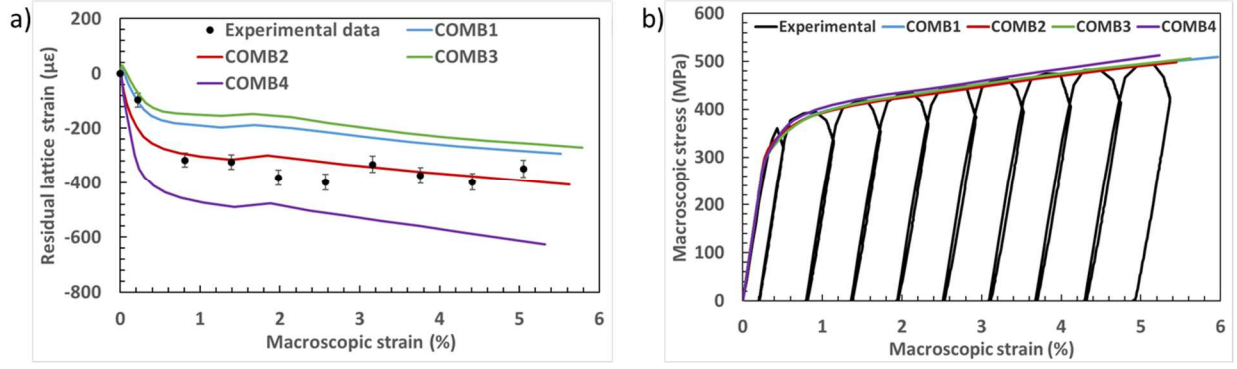
Experimental PFs were used to generate the Orientation Distribution Function (ODF). A set of 2000 grain orientations weighted by the measured volume fraction has been selected from the experimental ODF. The choice of this number is the result of a preliminary study that allows us to determine the sufficient numbers of grains which describe correctly the experimental data (i.e. neutron or synchrotron measurements) [9,11]. On the basis of experimental observations in [4,7,62,63], the deformation systems considered in the simulations are: prismatic slip (denoted  $P\langle a \rangle$ ), basal ( $B\langle a \rangle$ ), pyramidal ( $\Pi_1\langle a \rangle$ ), first-order pyramidal ( $\Pi_1\langle c + a \rangle$ ) and  $\{11.2\}$  twinning ( $ctw\{-11.2\}$ ) for neutron data simulations and, prismatic slip ( $P\langle a \rangle$ ), basal ( $B\langle a \rangle$ ), pyramidal ( $\Pi_1\langle a \rangle$ ), first-order pyramidal ( $\Pi_1\langle c + a \rangle$ ),  $\{10.2\}$  twinning ( $ttw\{-10.2\}$ ) and  $\{11.2\}$  twinning ( $ctw\{-11.2\}$ ) for X-ray synchrotron data simulations. In the two cases, the optimal value assigned to the parameter  $k$  (see Eq. 3) is 20. This is to ensure the stability and numerical convergence of the simulations [40,41].

As previously pointed out, owing to a lack of relevant data on the nature of latent hardening in HCP titanium alloys and to be more consistent with the theoretical and experimental information currently available, a simple and empirical law in Eq. 5 is applied to describe the interactions between different deformation modes. The  $q$  parameter (see Eq. 5), which

determines the degree of latent hardening has been adjusted to provide the best accordance between the experimental and predicted results (set to a value of 1.3 for the simulations). Concerning the twinning activity, the threshold fraction  $f_{limit}$  is set to 25 % what leads to a good description of the material behavior.

Due to the easy activation of prismatic deformation mode  $P\langle a \rangle$ , its CRSS value  $\tau^{P\langle a \rangle}$  is directly related to the macroscopic yield strength and the elastic-plastic transition during tensile tests. The self-hardening coefficient  $H^{P\langle a \rangle}$  was determined by fitting experimental macroscopic stress-strain curves at the plasticity activity outset. These parameters are then adjusted comparing neutron and synchrotron diffraction measurements. About basal ( $B\langle a \rangle$ ), pyramidal ( $\Pi_1\langle a \rangle$ ), first-order pyramidal ( $\Pi_1\langle c + a \rangle$ ),  $\{10.2\}$  twinning ( $ttw\{-10.2\}$ ) and  $\{11.2\}$  twinning ( $ctw\{-11.2\}$ ), there is less consensus on their relative role on the overall behavior. These deformation mechanisms activation sequence can be found in the literature and may differ from one type of alloy to another and / or from one author to another [64–68].

The main goal here is to identify a set of parameters able to explain the results obtained at both the macroscopic scale and diffracting volume level. Several numerical simulations are performed to determine the right set of parameters, in a first step, to successively favor in turn one or several mechanisms. Fig. 9.a shows an example of comparison of the simulated results for  $\{20.1\}$  reflection with different activation sequence combinations (denoted COMB1, COMB2, COMB3 and COMB4) and the measured residual lattice strain in neutron diffraction. A qualitative analysis shows that only COMB2 sequence reproduces correctly the behavior of  $\{20.1\}$  grain groups. This illustrates clearly the influence of the choice of the number of deformation modes and their hierarchy on the mechanical behavior at the diffracting volume level, even if all these combinations describe the observed macroscopic behavior (Fig. 9.b).



**Fig. 9:** a) Residual experimental (symbols) and predicted (lines) lattice strain evolution of {20.1} grains in loading direction versus applied stress and b) Experimental and simulated macroscopic stress–strain curves during neutron diffraction experiments. COMB1:  $P\langle a \rangle - B\langle a \rangle - \Pi_1\langle a \rangle - \Pi_1\langle c + a \rangle - \text{ctw}\{11.2\}$ , COMB2:  $P\langle a \rangle - \Pi_1\langle a \rangle - B\langle a \rangle - \Pi_1\langle c + a \rangle - \text{ctw}\{11.2\}$ , COMB3:  $P\langle a \rangle - B\langle a \rangle - \Pi_1\langle c + a \rangle - \text{ctw}\{11.2\}$  and COMB4:  $P\langle a \rangle - \Pi_1\langle a \rangle - \Pi_1\langle c + a \rangle - \text{ctw}\{11.2\}$

In order to obtain the final material parameters from neutron strain measurements, a least squares method is used, in a second step, to optimize the parameters by minimizing a function  $e^2$  defined as the sum of squares between deviations of the experimental and simulated quantities, over all  $p$  grain groups:

$$e^2 = \sum_{i=1}^p \left( \frac{\langle \varepsilon(hk.l, \varphi, \psi) \rangle_{V_{d\text{ meas}}}^i - \langle \varepsilon(hk.l, \varphi, \psi) \rangle_{V_{d\text{ simul}}}^i}{u(\langle \varepsilon(hk.l, \varphi, \psi) \rangle_{V_{d\text{ meas}}}^i)} \right)^2 \quad \text{Eq. 15}$$

where  $\langle \varepsilon(hk.l, \varphi, \psi) \rangle_{V_{d\text{ meas}}}^i$  is the measured value,  $\langle \varepsilon(hk.l, \varphi, \psi) \rangle_{V_{d\text{ simul}}}^i$  the simulated value, and  $u(\langle \varepsilon(hk.l, \varphi, \psi) \rangle_{V_{d\text{ meas}}}^i)$  the uncertainties of the measured lattice strains.

The final set of values that gives the optimum agreement between the measured and simulated results are summarized in [Table 3](#).

**Table 3:** Optimized set of CRSS and hardening coefficients (MPa) obtained with the model.

In blue, CRSS normalized values with respect to  $\text{CRSS}^{\text{P(a)}}$ , determined with the EPSC model

and neutron diffraction results.

	Neutron diffraction		Synchrotron diffraction	
	CRSS	$H^{\text{gg}}$	CRSS	$H^{\text{gg}}$
$\text{P}\langle\mathbf{a}\rangle$	115 (1)	120	110 (1)	100
$\text{B}\langle\mathbf{a}\rangle$	150 (1.30)	180	150 (1.36)	150
$\Pi_1\langle\mathbf{a}\rangle$	130 (1.13)	140	120 (1.09)	120
$\Pi_1\langle\mathbf{c} + \mathbf{a}\rangle$	240 (2.09)	250	180 (1.64)	200
$\text{ttw}\{-\mathbf{10.2}\}$	-	-	190 (1.73)	250
$\text{ctw}\{-\mathbf{11.2}\}$	250 (2.17)	300	220 (2)	300

#### 4. Model results and discussion

Before starting the simulations, residual stresses were quantified in the as-received N-sample and S-sample.

Lattice parameters  $a = 2.9509 \text{ \AA}$  and  $c = 4.6844 \text{ \AA}$  of the N-sample were found from a Rietveld refinement [10] of the initial entire diffraction ring for measurements along the LD.

From these, the ideal d-spacing  $\langle d_0^{\text{Rietveld}}(hk.l) \rangle_{V_d}$  for all peaks  $\{hk.l\}$  can be calculated from:

$$\frac{1}{\langle d_0^{\text{Rietveld}}(hk.l) \rangle_{V_d}} = \frac{4h^2 + hk + k^2}{3a^2} + \frac{l^2}{c^2} \quad \text{Eq. 16}$$

The difference between the predicted  $\langle d_0^{\text{Rietveld}}(hk.l) \rangle_{V_d}$  and measured d-spacing  $\langle d(hk.l, \varphi, \psi) \rangle_{V_d}$  by neutron diffraction is assumed to be due to initial residual lattice strain

$\varepsilon_{hk.l}^{\text{res}}$   $\left( \varepsilon_{hk.l}^{\text{res}} = \frac{\langle d_0^{\text{Rietveld}}(hk.l) \rangle_{V_d}}{\langle d(hk.l, \varphi, \psi) \rangle_{V_d}} - 1 \right)$  in the extrusion direction (see Table 4).

444

445 **Table 4:** Initial residual lattice strains in N-sample in the extrusion direction (LD)

{hk.l}	{10.0}	{00.2}	{10.1}	{10.2}
$\epsilon_{hk,l}^{res} (\times 10^{-6})$	$91 \pm 38$	$22 \pm 50$	$-57 \pm 40$	$-62 \pm 46$
{hk.l}	{11.0}	{10.3}	{11.2}	{20.1}
$\epsilon_{hk,l}^{res} (\times 10^{-6})$	$-40 \pm 38$	$64 \pm 48$	$-55 \pm 42$	$-116 \pm 39$

446

447 Similar order of magnitude is also found along TD. This indicates that the maximum  
 448 deviation is equivalent to a strain of  $120 \times 10^{-6}$ , and thus corresponds to initial residual  
 449 stresses below 10 MPa.

450 The as-received cold rolled sheet used in this study has already been studied in previous work  
 451 [15]. Analysis of the residual stress obtained from {30.2} and {21.3} reflections have then  
 452 shown values between  $-10 \pm 35$  MPa and  $+12 \pm 24$  MPa along the rolling direction. The stress  
 453 values along the TD are also in the same order of magnitude. Eventually, the two samples N  
 454 and S display rather low residual stress values and latter do not play, *a priori*, a significant  
 455 role on the elastoplastic behavior. For this reason, initial residual strains / stresses in the as-  
 456 received material were not accounted for in the model.

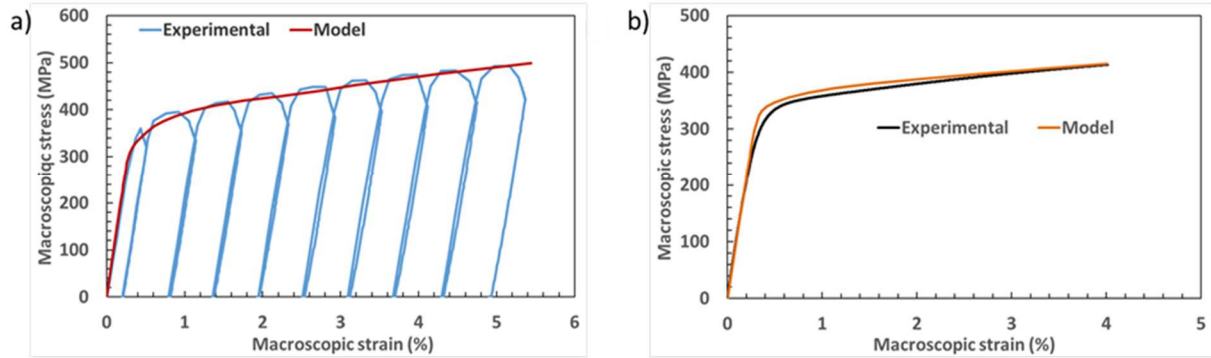
457 

#### 4.1 Neutron diffraction results

458 The EPSC model developed in the section 3 is used here to simulate the mechanical behavior  
 459 of the CP Ti- $\alpha$  alloy. Experimental and predicted macroscopic responses of the N-sample  
 460 under uniaxial tensile test along the extrusion direction (or LD) is shown Fig. 10.a.

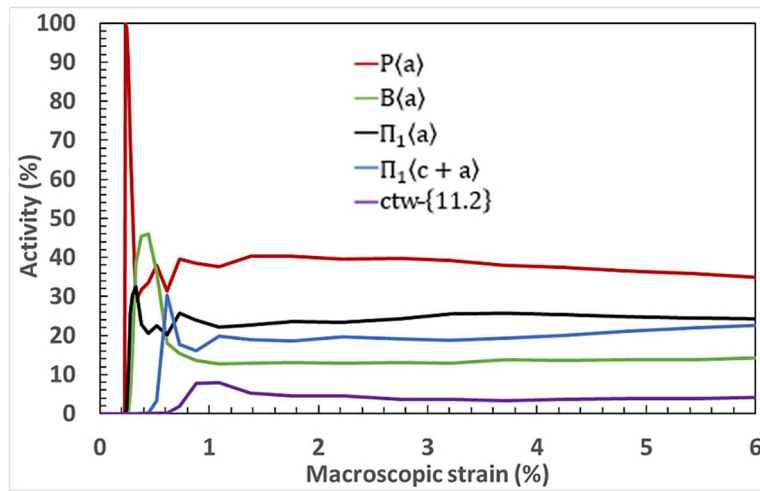
461 The plastic slope predicted by the model is in good agreement with that measured up to 5 %  
 462 macroscopic strain. The calculated value of the Young's modulus of 107 GPa is close to the

experimental one equal  $102 \pm 1$  GPa. This is consistent with the values commonly found in the literature [69].



**Fig. 10:** Experimental and simulated macroscopic stress–strain curves in a) neutron and b) synchrotron diffraction experiments

Fig. 11 shows the predicted relative activity of each deformation mode considered in the simulation. The experimental macroscopic yield stress is about 280 MPa corresponding to 0.29 % total strain. The adjusted CRSS values are consistent with the EBSD analyses of Barkia et al. [67] with the following order:  $P\langle a \rangle$  -  $\Pi_1\langle a \rangle$  -  $B\langle a \rangle$  -  $\Pi_1\langle c + a \rangle$  -  $ctw\{11.2\}$ .



**Fig. 11:** Relative contribution of each deformation mode to the total plastic deformation as a function of deformation during neutron diffraction experiments

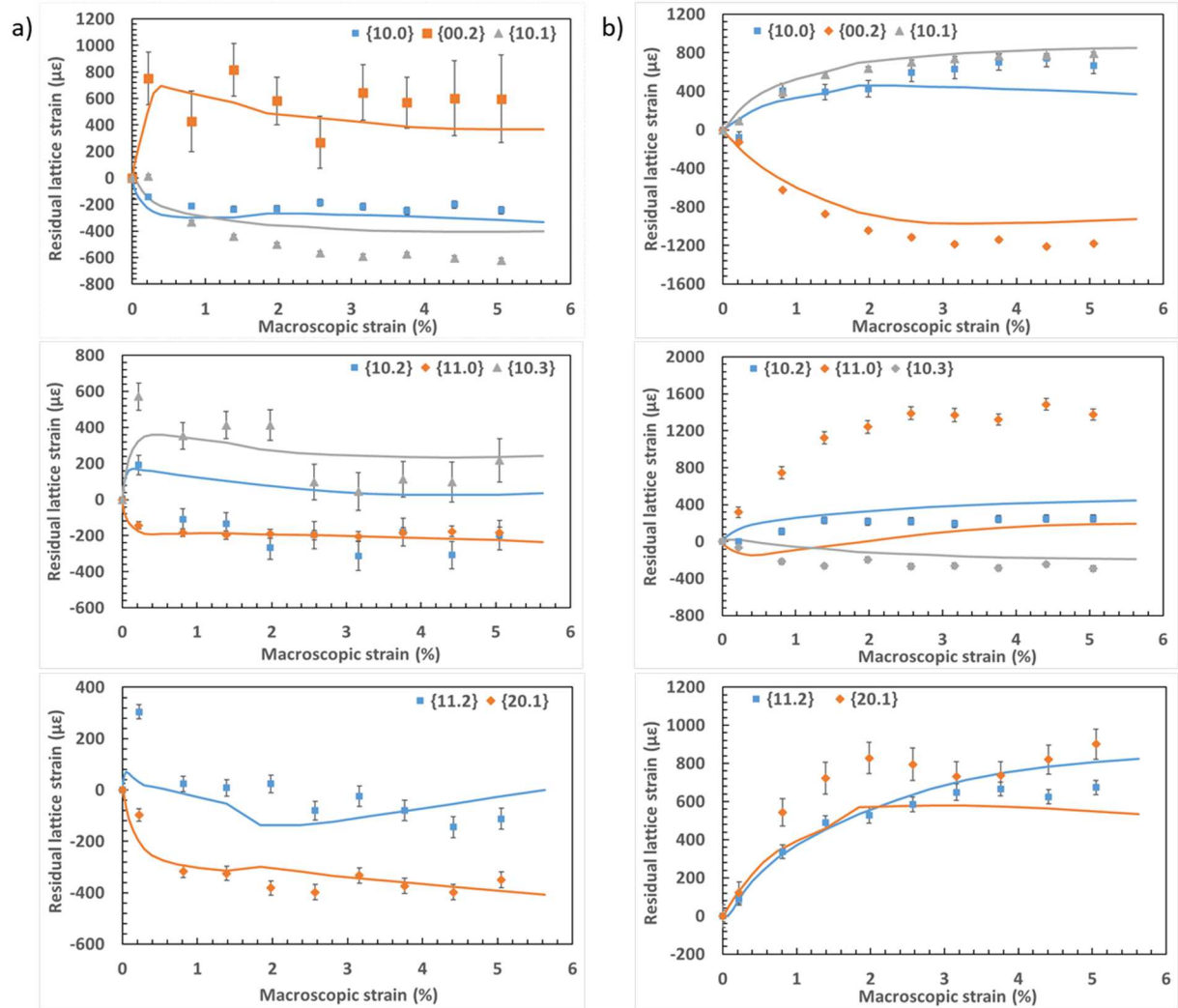
The main deformation mode is prismatic slip. For example, at 3 % macroscopic total strain, the proportions of activated systems are: 39 % for prismatic slip, 13 % for basal slip, 25 % for pyramidal  $\langle a \rangle$  slip, 18 % for pyramidal  $\langle c + a \rangle$  and 3.5 % for  $\{11.2\}$  twinning.

This activation sequence (Fig. 11) will influence the behavior of the polycrystal at the local level (i.e. at the diffracting volume level). Then, it is interesting to see its impact on the different mechanical states of  $\{hk.l\}$  grain families. To illustrate this, the evolution of residual intergranular strains is shown in Fig. 12 for both longitudinal (loading direction) and transversal directions. When a polycrystalline aggregate is subjected to a tensile test in the elastic region, the lattice strains develop linearly with the load up to the micro-yielding point. There is a significant difference in the elastic response for each  $\{hk.l\}$  grain family due to the elastic anisotropy and the range of possible crystallographic orientations. Slip activity is initiated in plasticity regime in some grains similarly orientated. One can notice the accommodation of the imposed plastic deformation and these grains undergo less loads.

The neutron experiments results shown in Fig. 12 clearly highlight an anisotropic mechanical behavior of  $\{hk.l\}$  groups of grains.

According to the simulations, plasticity initiation begins with the yielding of  $\{10.0\}_{//}$  grains at 260 MPa (corresponding to 0.24 % of total strain). These grains are favorably oriented for prismatic slip  $P\langle a \rangle$ , the lowest-CRSS mechanism, correlated with a favorable Schmid factor (i.e. 0.5) for these orientations [4,6,67]. Indeed, the grains that activate the prismatic slip first and most easily are those which have their lattice crystalline  $\vec{c}$ -axis perpendicular to the loading direction. These grains undergo plastic deformation below the macroscopic yield stress ( $R_E = 280$  MPa). It should be recalled that the outset of microplasticity does not cause

the macroscopic yield because slip activity is initiated in a small number of grains. As the applied stress increases,  $\{20.1\}_{//}$  orientation begins to shed load at 300 MPa. This can be explained by the activation of pyramidal slip  $\Pi_1\langle a \rangle$ . The  $\{10.1\}_{//}$  and  $\{10.2\}_{//}$  orientated grains yield at 330 MPa, reflecting the high activity of the basal slip ( $\sim 45\%$  of activity proportion).



**Fig. 12:** Residual experimental (symbols) and predicted (lines) lattice strain evolution in both (a) longitudinal and (b) transversal directions versus applied stress during neutron diffraction experiments.

One can notice the linear deviation of the hardest  $\{11.2\}_{//}$  grains (Fig. 12) with a change in relative activity between prismatic  $P\langle a \rangle$  and basal  $B\langle a \rangle$  slips (Fig. 11). Other orientations such

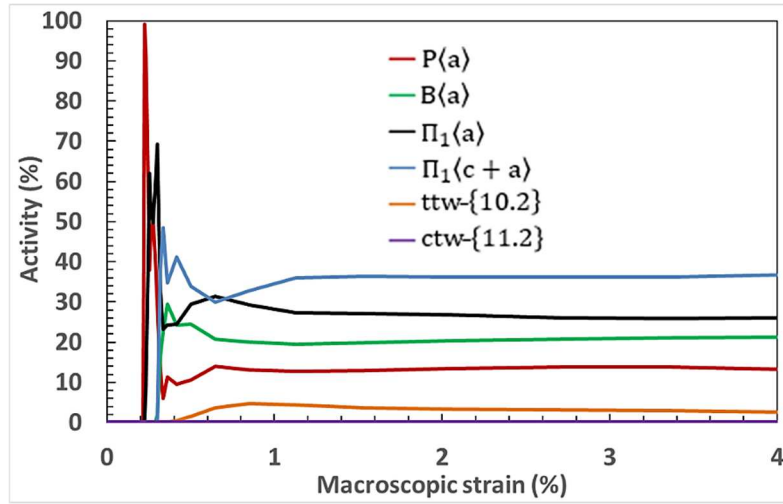
509 as  $\{10.3\}_{//}$  and  $\{00.2\}_{//}$  yield only at 355 MPa with the activation of the pyramidal mode  
 510  $\Pi_1\langle c + a \rangle$  or  $\{11.2\}$  twinning deformation. These  $\{hk.l\}_{//}$  grains have the  $\vec{c}$ -axis nearly  
 511 aligned along the LD and they are poorly orientated for the soft  $\langle a \rangle$  slip mechanism to be  
 512 activated. In the LD, the largest compressive residual strains are observed for  $\{10.1\}_{//}$  and  
 513  $\{20.1\}_{//}$  grains ( $\sim -620 \mu\epsilon$  and  $\sim -399 \mu\epsilon$  at 5 % of total strain, respectively). Similar  
 514 behaviors can be seen along  $\{10.0\}_{//}$ ,  $\{10.2\}_{//}$  and  $\{11.0\}_{//}$  orientations, although the lattice  
 515 strains are relatively smaller:  $\sim -246 \mu\epsilon$ ,  $\sim -312 \mu\epsilon$  and  $\sim -183 \mu\epsilon$ , respectively. These  
 516 behaviors are well captured by the model except for  $\{10.2\}_{//}$  orientation where a shift of 335  
 517  $\mu\epsilon$  can be noticed between the experimental and simulated results. As shown in Fig. 12.a, the  
 518  $\{00.2\}_{//}$  grain orientation develops significant tensile residual lattice strains ( $\sim 760 \mu\epsilon$ ) during  
 519 the first 1 % of total strain. This indicates that the load is redistributed towards the plastically  
 520 harder grains like the  $\{00.2\}_{//}$  ones after the initiation of plasticity. This phenomenon is  
 521 expected since all these grains are poorly orientated for the soft  $\langle a \rangle$  slip mechanisms. A  
 522 similar behavior is observed for the  $\{10.3\}_{//}$  grains orientation relatively close to the  $\{00.2\}_{//}$   
 523 one, although the experimental lattice strains are relatively smaller: 570  $\mu\epsilon$ . This set of grains  
 524 is also unfavorable to  $\langle a \rangle$  slips. Their residual strain evolution is well described by the  
 525 numerical simulations.

526 In the TD (Fig. 12.b), the model predicts the evolution of intergranular strains in the majority  
 527 of the  $\{hk.l\}$  orientations studied even if some discrepancies are observed. All  $\{hk.l\}_{\perp}$  grains  
 528 present an opposite behavior to that observed in the longitudinal direction, highlighting the  
 529 Poisson effect. The residual lattice strains evolution of  $\{11.0\}_{\perp}$  reflection is under-predicted.  
 530 A large gap of 450  $\mu\epsilon$  is observed between the experimental and simulated results, beyond 2.3  
 531 % of total strain. For  $\{20.1\}_{\perp}$  grains, the model captures the tensile state, although the state is  
 532 underestimated by  $\sim 250 \mu\epsilon$  at 3 % of total strain. Nevertheless, for the majority of

orientations, in particular:  $\{10.0\}_\perp$ ,  $\{00.2\}_\perp$ ,  $\{10.1\}_\perp$ ,  $\{10.2\}_\perp$ ,  $\{10.3\}_\perp$  and  $\{11.2\}_\perp$ , the predictions are fairly accurate with the exception of the  $\{11.0\}_\perp$  reflection. Therefore, one can only to conclude that the difference could not be attributed to a lack of description of the hardening. In our opinion, this difference could be explained by the presence of heterogeneities in the set of grains participating to this orientation or by the reverse plasticity when the material is subjected to reverse loading. The differences between model calculations and measurements are more pronounced than in the  $\{hk.l\}_{//}$  grains. Similar observation has been made in previous studies [13,48].

#### 4. 2    *Synchrotron diffraction results*

In contrast to neutron diffraction experiments, *in situ* synchrotron measurements have been performed during a continuous tensile test. Fig. 10.b shows the experimental and predicted bulk response. The simulated results describe with accuracy the elastic and plastic slopes up to 4 % macroscopic strain. The calculated Young's modulus of 107 GPa is very close to the experimental one of  $109 \pm 2$  GPa. These values are consistent with those found in the neutron diffraction analysis. This proves that the model parameters allow to describe the elastic behavior of the material, suggesting that the elastic constants used in the modeling are quite reasonable with a well-described crystallographic texture. The yield stress is 240 MPa with the prismatic slip  $P\langle a \rangle$  as the main deformation mode (Fig. 13).



**Fig. 13:** Relative activities of the different active deformation modes for uniaxial tensile loading during synchrotron diffraction experiments

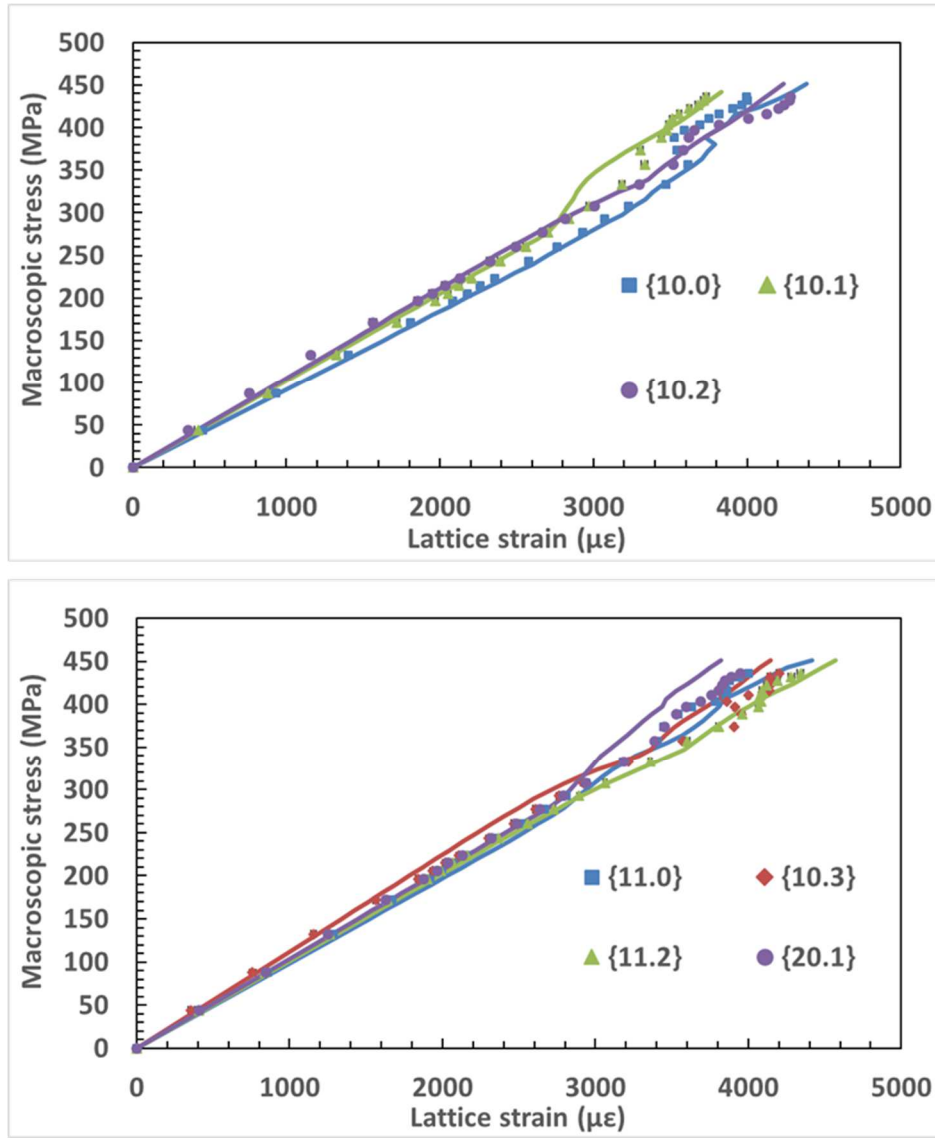
Its initial CRSS value (110 MPa, see Table 3) is consistent with that found for N-sample and more generally with  $\alpha$ -titanium [64–68].

Concerning the prediction of lattice strain evolution, only the results obtained in the longitudinal direction were retained. Indeed, for a given  $\{\mathbf{hk.l}\}_{//}$  reflection, all diffracting grains have the same orientation of the  $\vec{c}$ -axis with respect to the tensile direction. They have more-or-less the same deformation history and the Schmid factors for each deformation system are the same for these grains. The grains which participate in the  $\{\mathbf{hk.l}\}_{\perp}$  reflection have a wide range of crystallographic orientations with respect to the loading axis. In other words, a rotation of a grain around an axis perpendicular to the loading direction highly changes the stiffness in the tensile direction and, consequently, the strain and stress state in the grain. The lattice strain of the diffracting grains, for the considered  $\{\mathbf{hk.l}\}$  reflection, shows large variation.

The mechanical response in the TD is very sensitive to the texture and the elastic strain measured is thus the average over a set of diffracting grains with different strain history. Many authors have already pointed out this problem [11,13,70,71]. This can explain the more

pronounced discrepancies noticed between calculations and measurements in the direction perpendicular to the tensile axis in the [section 4. 1](#). Moreover, the number of diffracting grains in X-ray synchrotron experiments is weak as compared to neutron diffraction experiments due to the reduced gauge volume. Therefore, the lattice strains determined with synchrotron X-ray radiation are less representative of the polycrystal due to a smaller number of crystallites contributing to the diffracting volume.

The comparison with the homogenization approach is then more problematic especially in the TD. In the LD, this problem remains present but, for a given  $\{\mathbf{hk.l}\}_{//}$  reflection, all the grains have an identical lattice plane normal in a direction parallel to the tensile axis with a very similar mechanical state for these grains in plasticity. The range of lattice strains for all grains in the diffracting volume along the longitudinal direction is lower than the range of the strains for the transverse direction within a given  $\{\mathbf{hk.l}\}$  family. This compensates somehow the lack of statistics met with the gauge volume. Therefore, the focus is made on the internal strain evolution in the LD. The lattice strain evolution is reasonably predicted along LD ([Fig. 14](#)). In the elastic regime, a linear evolution is observed whose slope depends on the grain family due to the elastic anisotropy, each group of grains shows a more or less stiff behavior.



**Fig. 14:** Experimental (symbols) and predicted (lines) lattice strain evolution along LD for tensile test during synchrotron diffraction experiments

For example,  $\{10.0\}_{//}$  grains family, which exhibit the most compliant behavior with the lowest elasticity modulus  $E_{hk,l}$  (Table 5), accumulate the largest elastic lattice strains as compared to those met for  $\{1.01\}_{//}$  and  $\{10.2\}_{//}$  reflections. The elastic modulus  $E_{hk,l}$  are very well predicted by the model.

**Table 5:** Measured and predicted elastic modulus  $E_{hk,l}$  (GPa) for different reflections  $\{hk, l\}_{//}$

$\{10.0\}$	$\{10.1\}$	$\{10.2\}$	$\{11.0\}$
------------	------------	------------	------------

<b>Experimental</b>	$93 \pm 0.8$	$103.2 \pm 1$	$104.2 \pm 1.8$	$97.8 \pm 1.4$
<b>Model</b>	92.2	102.5	105.8	98.6
	<b>{10.3}</b>	<b>{11.2}</b>	<b>{20.1}</b>	–
<b>Experimental</b>	$113.6 \pm 2$	$104 \pm 1$	$100 \pm 1.4$	–
<b>Model</b>	112.3	103.8	101.6	–

593

594 In general, the simulations exhibit a reasonable agreement with the experiment in terms of  
595 both trends and magnitudes. The relative activities of the deformation modes are plotted in  
596 **Fig. 13**. The  $\{10.0\}_{//}$  and  $\{11.0\}_{//}$  grain families yield first at 240 MPa. Such micro-yielding  
597 is correctly reproduced by the model, and it is clear that this first inflection is induced by the  
598 activation of prismatic slip  $P\langle a \rangle$ . At 260 MPa, the  $\{20.1\}_{//}$  and  $\{10.1\}_{//}$  grain families begins  
599 to shed load due to the activation of pyramid slip  $\Pi_1\langle a \rangle$ . A third inflection taking place at 306  
600 MPa is observed for the  $\{10.2\}_{//}$  and  $\{11.2\}_{//}$  orientations. This inflection is a result of the  
601 activity of basal slip  $B\langle a \rangle$ . A significant load partitioning takes place later in  $\{10.3\}_{//}$  grains,  
602 and is associated with the noteworthy activity of pyramidal  $\Pi_1\langle c + a \rangle$ . This slip mode  
603 becomes very quickly the major slip system governing the material plasticity until 4 % of the  
604 total strain (**Fig. 13**).

605 The effect of crystallographic texture difference between N- and S-samples is clearly visible  
606 in the evolution of relative activities of the different active deformation systems. In the N-  
607 sample, the majority of the grains have the  $\vec{c}$ -axis nearly perpendicular to the loading axis and  
608 they are favorably oriented for the prismatic slip  $P\langle a \rangle$  while in the S-sample, the rolling  
609 texture induces the significant activity of pyramidal  $\Pi_1\langle c + a \rangle$  (see **Fig. 11** and **Fig. 13**). At ~  
610 365 MPa, the lattice strain relief in the  $\{10.0\}_{//}$  grains is caused by the  $\{10.2\}$  tensile  
611 twinning inducing a stress relaxation. This observation has been made in previous works [71–

73], and is well captured by the simulations. It is noted that {11.2} compressive twinning during the deformation is almost negligible, which is consistent with the initial crystallographic texture of the S-sample (Fig. 7).

For diffraction measurements, the residual strains usually show poorer agreement between measurements and simulation results than the *in situ* strains measured under a mechanical load. This can be explained by the fact that the model does not take into account slip activity [70] or twinning-detwinning [74] one during unloading (i.e the reverse plasticity, when the material is subjected to reverse loading, is not predicted by the model). The experimental macroscopic stress-strain curves show actually a slight reverse plasticity during the unloads, especially for large macroscopic strain values (see Fig. 10), for stress values close to zero. Some discrepancies between the model predictions and the measurements could therefore be expected. Despite this limitation, residual strain evolution is well described by the numerical simulations for the majority of {hk.l} orientations along the LD and TD (see Fig. 12). The observed discrepancies ( $\{10.2\}_{//}$ ,  $\{11.0\}_{\perp}$  for example) between the model predictions and the measurements could be partially explained by this lack of description. On the other hand, the main limitation of synchrotron is the small gauge volume resulting in a limited number of diffracting grains for a given reflection and an exacerbated sensitivity of the results to spatial heterogeneities. The comparison with a homogenization approach becomes then more problematic. In our study, in order to overcome this problem, only longitudinal strains have been analyzed and intensity of each diffraction ring has been integrated within "cake shape" sectors defined by  $\beta \pm 10^\circ$ . The arithmetic mean values of the d-spacings were subsequently calculated from the averaging of the symmetrically equivalent  $\gamma$  and  $\gamma + 180^\circ$  angles (see section 2.3). Moreover, the ratios between the CRSS of the different deformation modes with respect to the one of the P<a> family are in agreement with the ranges identified with neutron diffraction. These results are in accordance with those obtained, for example, by Barkia et al.

[67] by TEM (slip trace analysis) or Amouzou et al. [68] with an EVPSC model. The same activation sequence combination has been used for these two experiments. Therefore, we can deduce that the longitudinal lattice strains determined with synchrotron X-ray radiation are representative due to a sufficient number of crystallites contributing to the diffracting volume for the LD.

Nevertheless, due to the crystallographic texture, the activity rates for the different deformation modes are not the same according to the samples of this study.

## 5. Conclusion

In this work, the evolution of lattice strains and texture under mechanical loading in  $\alpha$ -Ti alloy has been examined using two different techniques: neutron and X-ray synchrotron diffractions. These results have been compared with the prediction of EPSC model and the elastoplastic deformation has been studied at the different scales. The major conclusions are summarized as below:

- An extended EPSC model taking into account twin domain reorientation and associated stress relaxation has been developed. On the basis of this study, the relative activity of slip and twinning deformation modes occurring in the samples have been determined and analyzed. By tuning the hardening coefficient and the initial CRSS, it is shown that the model predictions, in terms of lattice strain evolution (8 and 7 different reflections for neutron and synchrotron diffraction, respectively) and macroscopic stress-strain curves are in good agreement with the experimental data. The lattice strain relief caused by the twinning is well captured by the simulations.
- Our work highlights the strong crystallographic texture effect onto the deformations mechanisms activation sequence of N- and S-samples. In the N-sample, the majority of the grains have the  $\vec{c}$ -axis nearly perpendicular to the loading axis and they are

favorably oriented for the prismatic slip  $P\langle a \rangle$  (and for the tensile twinning  $\{10.2\}\{10.\bar{1}\}$ ) while in the S-sample, the rolling texture induces the significant activity of pyramidal  $\Pi_1\langle c + a \rangle$ .

➤ X-ray synchrotron source offers fast rates of data acquisition (few seconds for X-rays and about 40 minutes for neutron diffraction in this work) enabling real time and *in situ* strain distribution measurements during continuous mechanical loading. Despite a small irradiated volume for X-rays, the inflections of  $\{hk.l\}$  orientations due to the slip or twinning deformation modes are relatively well reproduced by the model. This agreement is only obtained in the LD with intensity of each diffraction ring has been integrated within "cake shape" sectors defined by  $\beta \pm 10^\circ$ . The arithmetic mean values of the d-spacings were calculated from the averaging of the symmetrically equivalent  $\gamma$  and  $\gamma + 180^\circ$  angles.

➤ The advantage of neutron diffraction techniques is notably related to a substantial and representative scattering volume of  $4 \times 4 \times 4 \text{ mm}^3$ . Despite the fact that the model does not take into account slip activity or twinning-detwinning one during unloading, residual strain evolution is well described by the numerical simulations for the majority of  $\{hk.l\}$  orientations along LD and TD.

➤ The comparison between the extended EPSC model with *in situ* or *post mortem* characterization give the same trends: the  $\langle a \rangle$  slip on prismatic planes is the easiest deformation mechanism. Pyramidal  $\langle a \rangle$  slip is the second easiest and basal  $\langle a \rangle$  slip the third.  $\langle c + a \rangle$  slip is regarded as more difficult than  $\langle a \rangle$  slip mechanisms which result in twinning to accommodate strain along the  $\vec{c}$ - axis.

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