

CORRELATIONS AND SYMMETRIES OF 1D SPIN MIXTURES



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Correlations and symmetries of 1D spin mixtures

Why ?

Theoretical treatment

Example of result

Outcomes

Correlations and symmetries of 1D spin mixtures

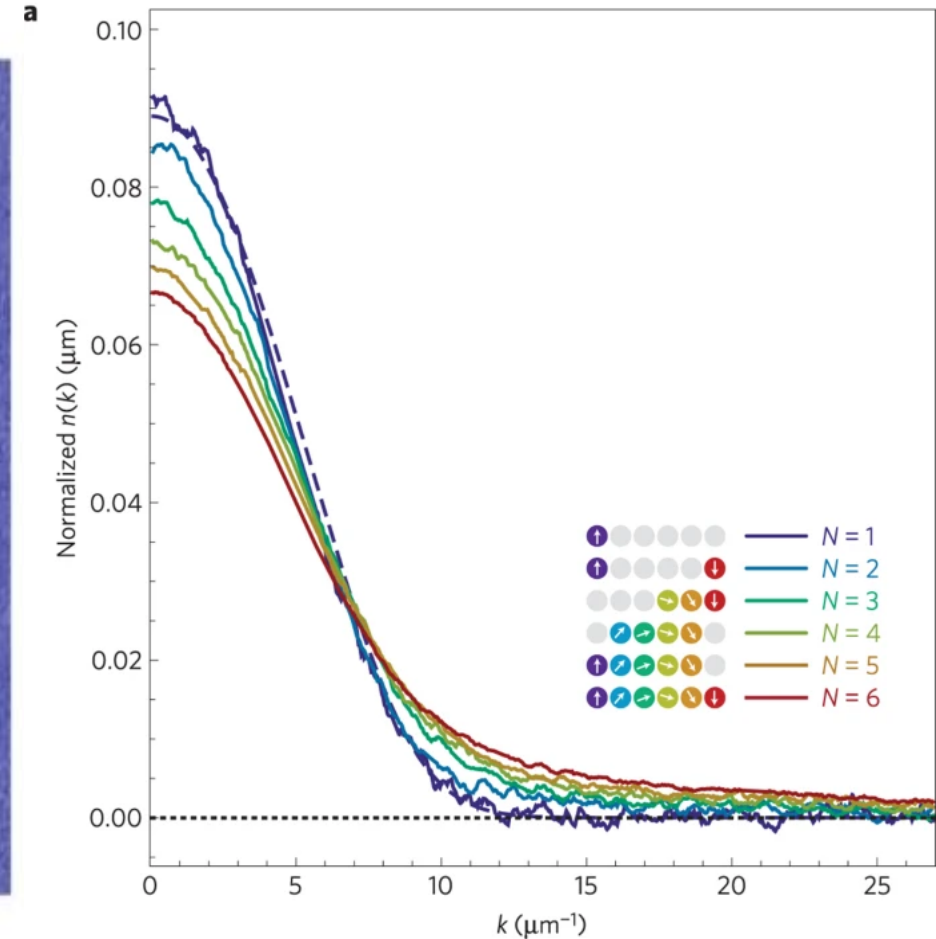
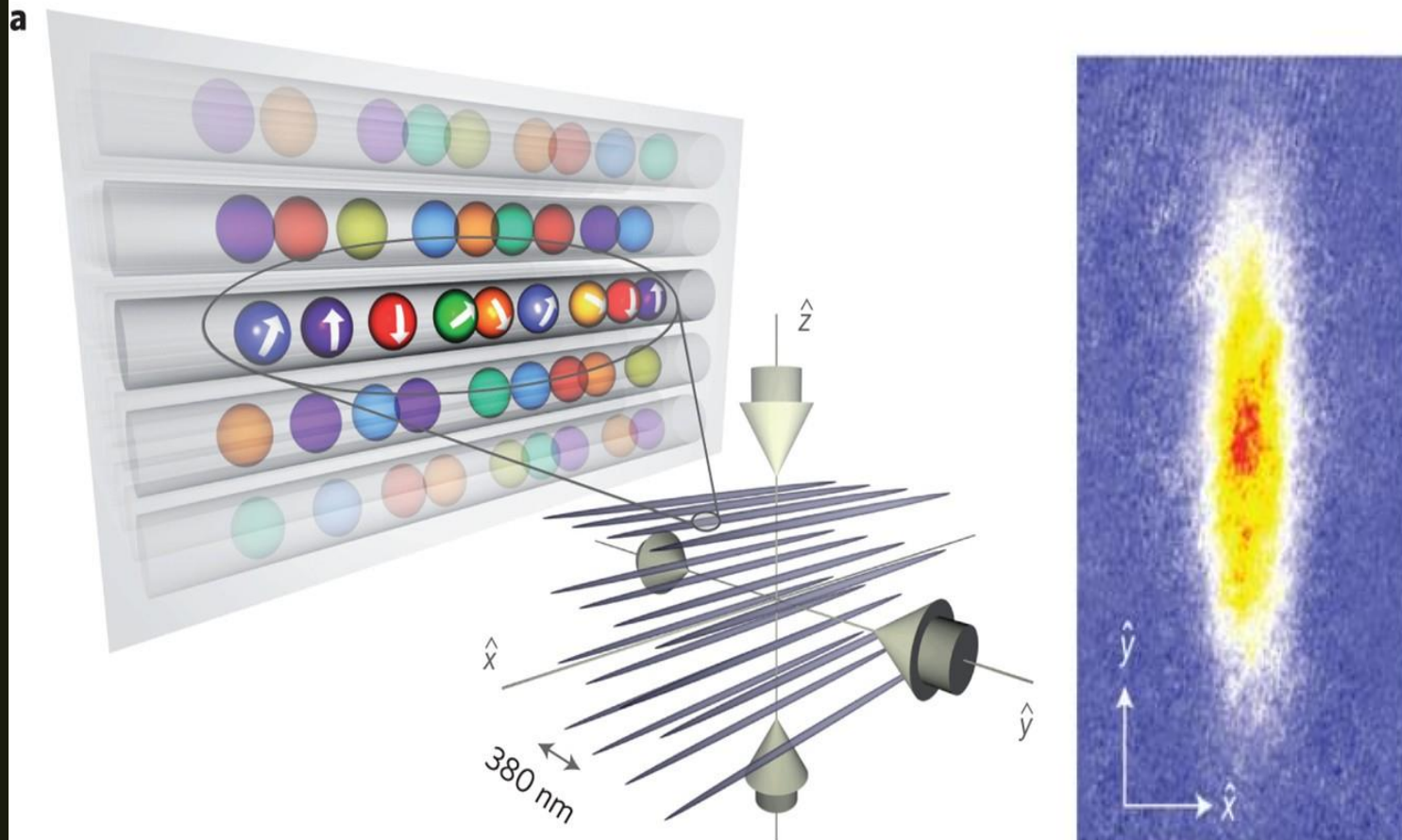
Why ?

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Correlations and symmetries of 1D spin mixtures



Why ?

1D cold atoms

Exactly
solvable

- Integrable systems
- Observables accessible
- Deep understanding

Close to
experiment

- ✓ Fermions 1 composante
- ✓ Bosons 1 composante
- ✓ Fermions Mixture
- × Bosons Mixtures
- × Fermions and bosons

Versatile
systems

- Number of particles
- Number of species
- Strength of interaction
- Type of trap
- Temperature
- ...

But incoming !!

Correlations and symmetries of 1D spin mixtures

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Theoretical treatment

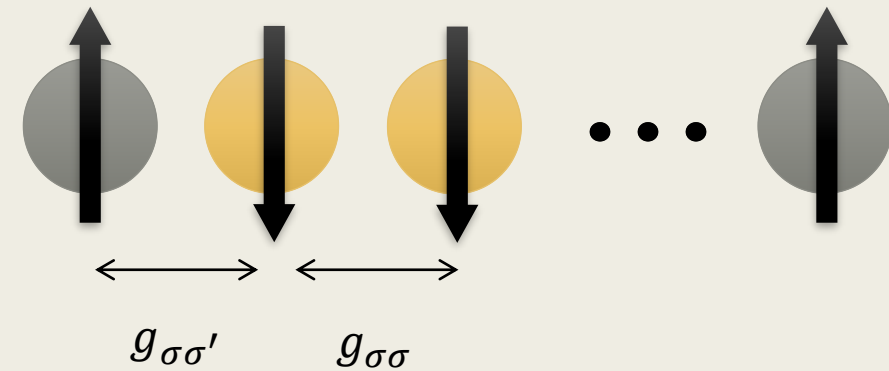
Example of result




Outcomes

Theoretical treatment

Our model

$$H = \underbrace{-\frac{\hbar^2}{2m} \sum_{i=1}^N \frac{\partial^2}{\partial x_i^2}}_{\text{Kinetic energy}} + \underbrace{\sum_{\sigma \leq \sigma'} \sum_{1 \leq i < j \leq N} g_{\sigma\sigma'} \delta(x_i - x_j)}_{\text{Interaction potential}}$$



- i, j : position label of particles 
- σ, σ' : spin label of particles 
- $g_{\sigma\sigma'}$: interaction strength 

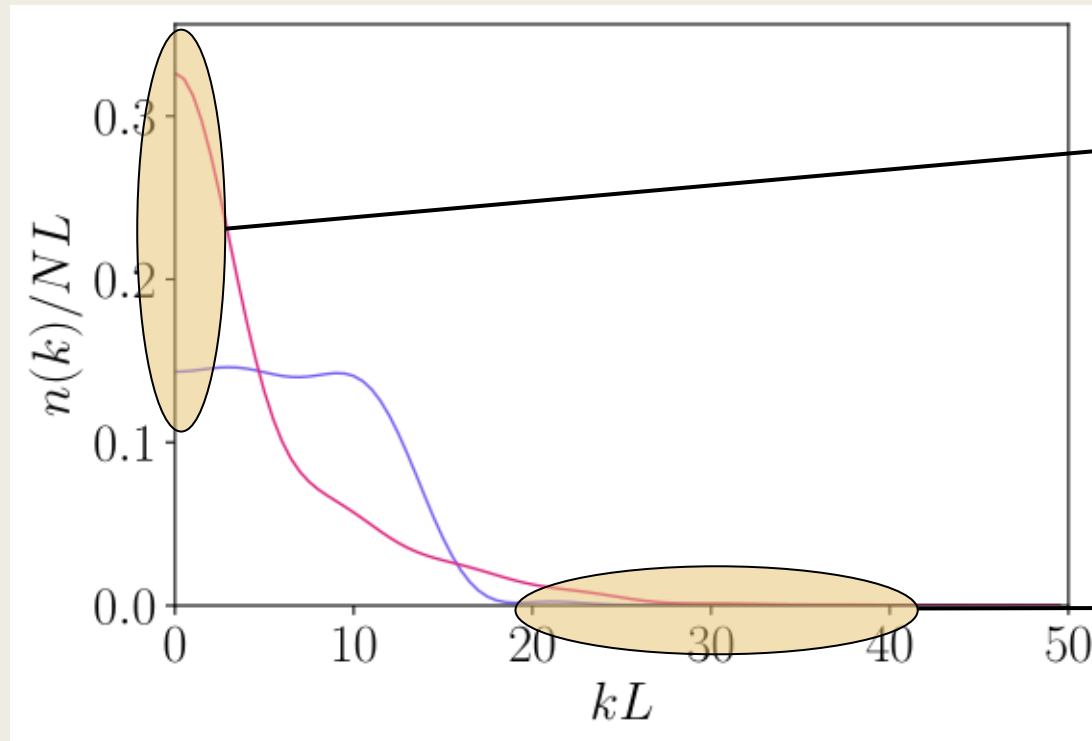
$$|\Psi_{Tot}\rangle = \text{[grey up, yellow down, grey up, yellow down]} + \text{[grey up, grey up, yellow down, yellow down]} + \dots$$

Theoretical treatment

Example of observable

Momentum distribution

$$n(k) = \int dx dy e^{ik(x-y)} \rho_1(x, y)$$



Zeroth-momentum distribution
(long-distance coherence)

$$n(0) = \int dx dy \rho_1(x, y)$$

Tan contact
(short-distance coherence)

$$n(k)k^4 \underset{k \rightarrow \infty}{=} \mathcal{C}$$

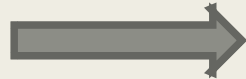
Theoretical treatment

Example of tool: Symmetry analysis

What is the symmetry?

How to discriminate?

Wave function



Young Tableau



Symmetry operator

$$\Psi_0(x_1, \dots, x_N)$$


$$\Psi_1(x_1, \dots, x_N)$$

⋮

$$\Psi_e(x_1, \dots, x_N)$$

⋮

$$\Psi_D(x_1, \dots, x_N)$$

Spinless Bosons : 

Spinless Fermions :



⋮



Other mixtures :



$$\hat{\Gamma}^{(2)} = \sum_{ij} \hat{P}_{ij}$$

$$\gamma_e^{(2)} = \langle \Psi_e | \hat{\Gamma}^{(2)} | \Psi_e \rangle$$

Correlations and symmetries of 1D spin mixtures

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Example of result

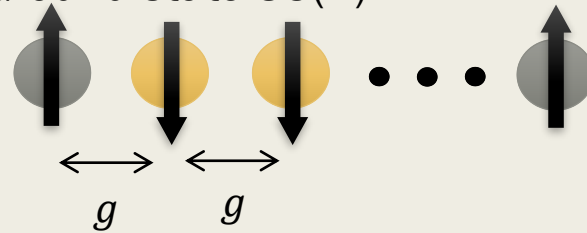
Outcomes

Example of result

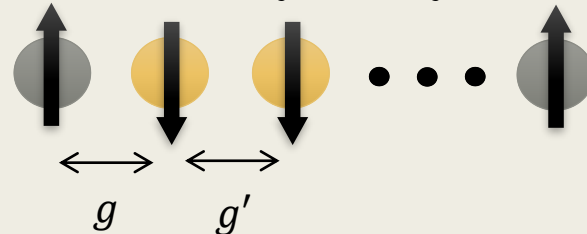
Long-distance coherence

Zeroth-momentum :

■ Ground state SU(2)

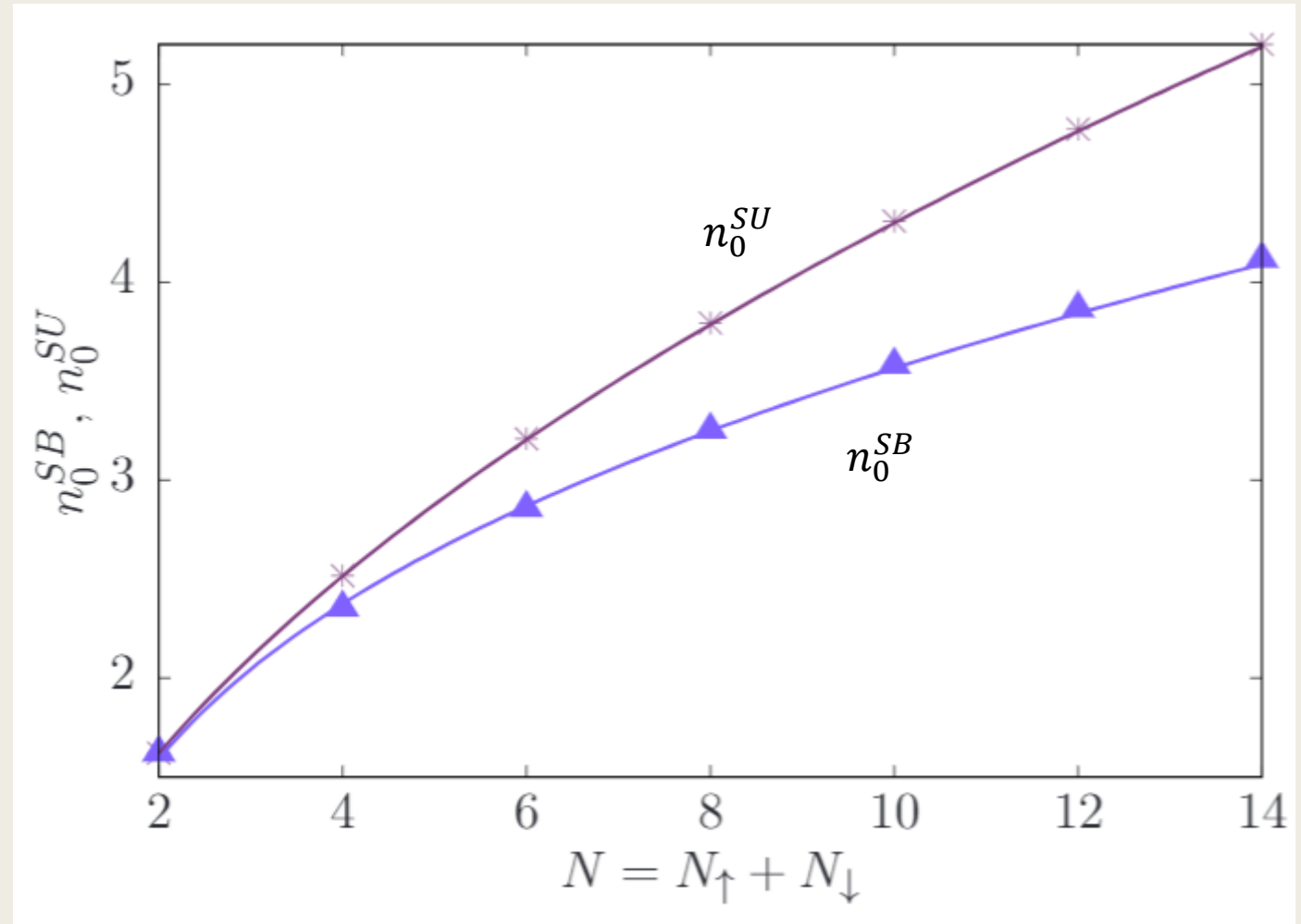


■ Ground state symmetry breaking



Thermodynamic limit :

$$\frac{n_0^{SB}}{n_0^{SU}} \xrightarrow{N \rightarrow \infty} \frac{1}{2}$$



Correlations and symmetries of 1D spin mixtures

Why ?

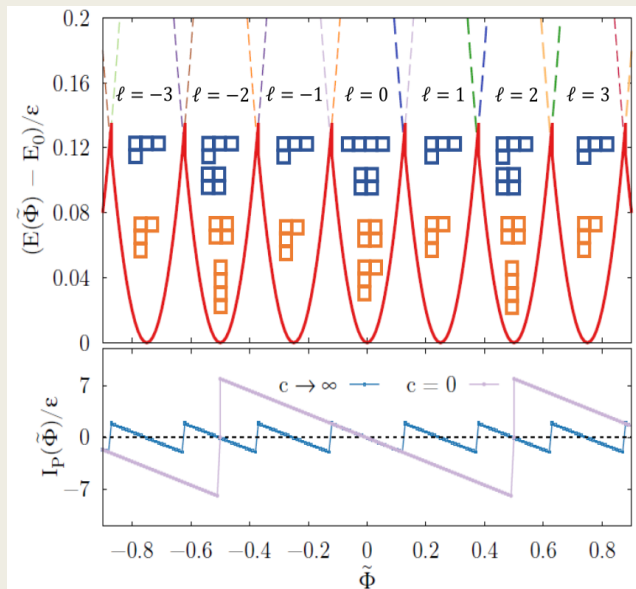
Theoretical treatment

Example of result

Outcomes

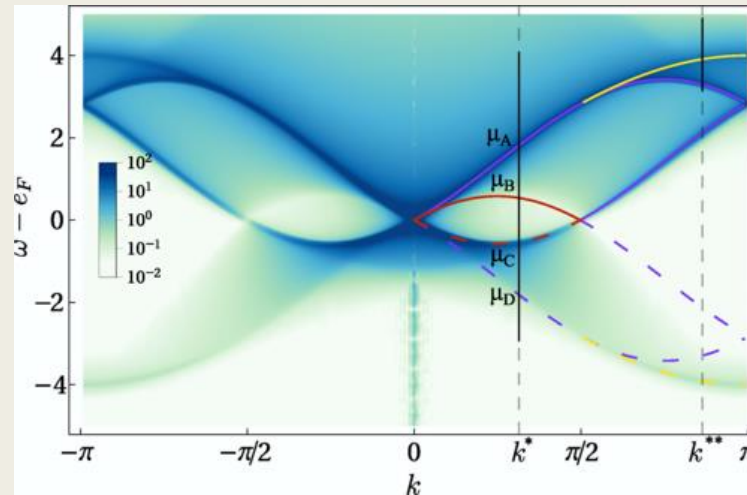
outcomes

Persistent current in the ring

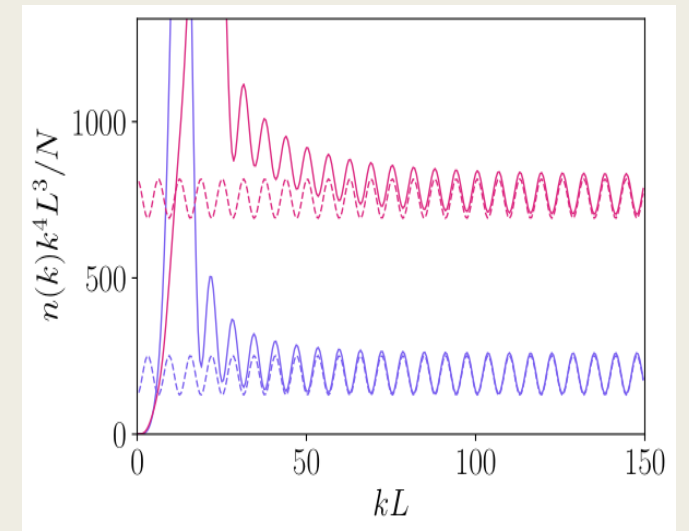


Submitted : [arXiv:2211.16194](https://arxiv.org/abs/2211.16194)

Correlations in the box



J. Settimo et. al. Phys. Rev. Lett. **126**, 065301



Spectral function or quench

$$c^{FF} = \langle C_{HW} \rangle + (-1)^N \langle C_{osc} \rangle \cos(kL)$$

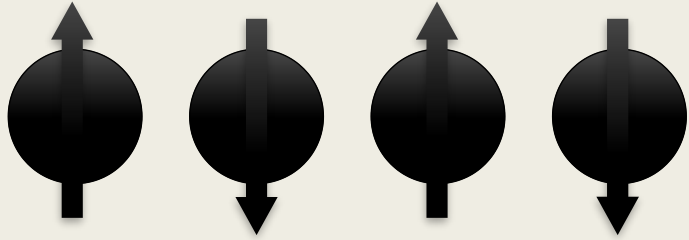
$$c^{TB} = N \langle C_{HW} \rangle + \langle C_{osc} \rangle \cos(kL)$$

Correlations and symmetries of 1D spin mixtures

Thank you
for your attention

The spectrum

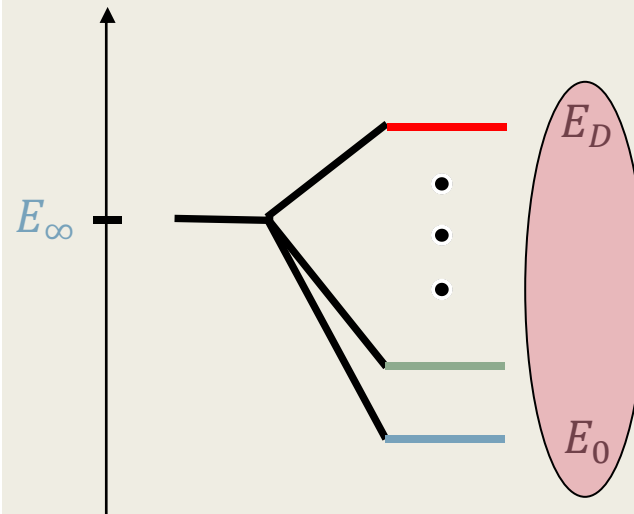
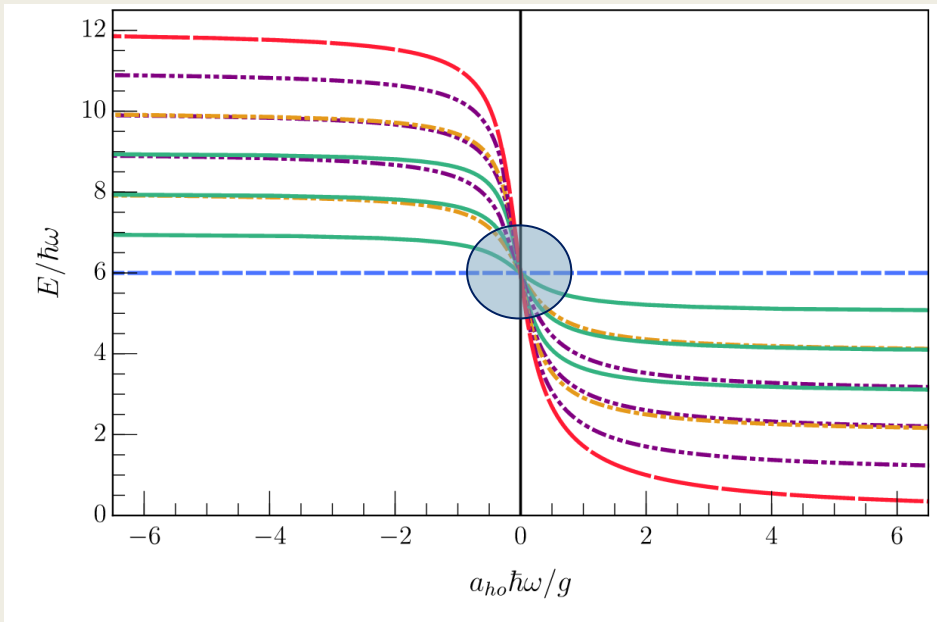
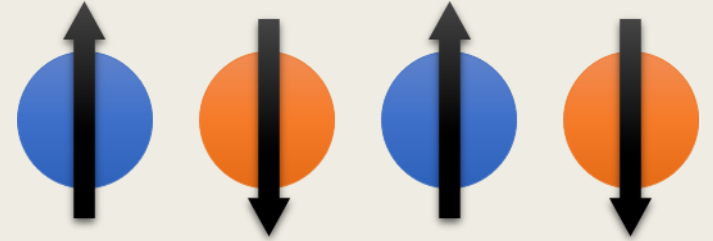
Infinite interaction



$$E_g \simeq E_\infty + \frac{1}{g} \left[\partial_{1/g} E \right]_{g \rightarrow \infty}$$



1/g expansion



Give the symmetry correction

- $\sigma = \sigma'$: Imposed symmetry
- $\sigma \neq \sigma'$: Degrees of freedom

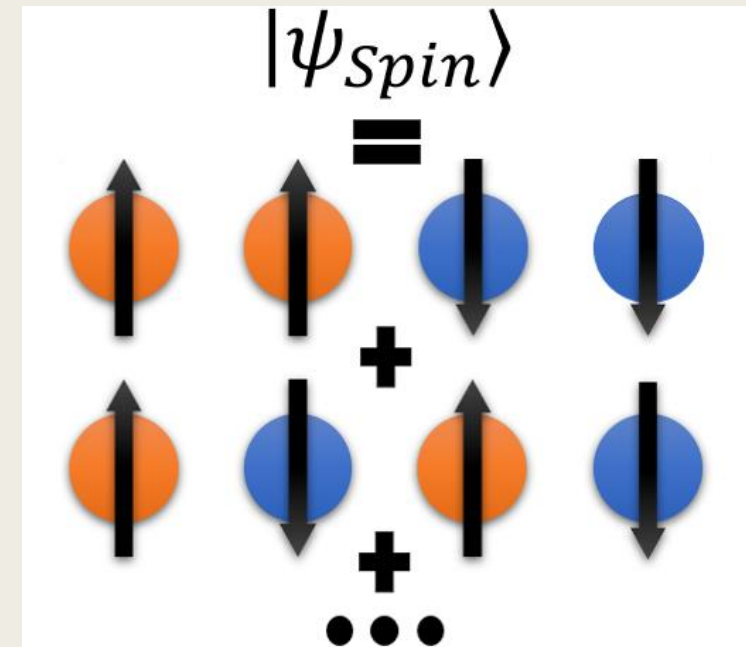
$$D = \frac{N!}{\prod_{\sigma} N_{\sigma}!}$$

The wave function

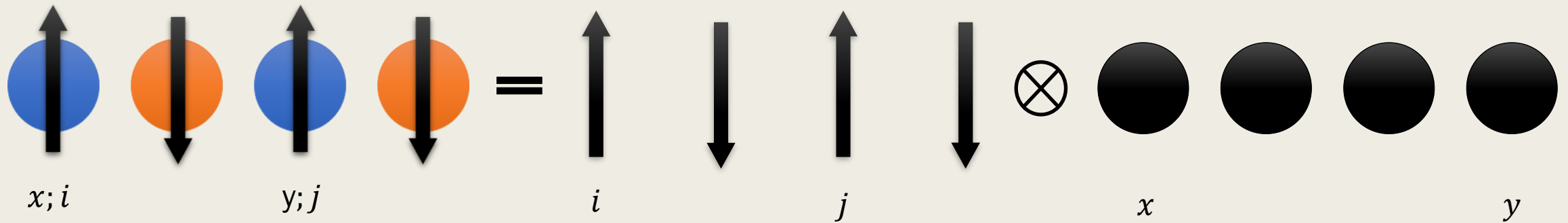
$$\begin{aligned}\Psi(x_1, \dots, x_N) &= |\Psi_{Spin}\rangle \otimes |\Psi_{Spatial}\rangle \\ &= \sum_{P=S_N} a_P \theta(x_1 < \dots < x_N) \Psi_{A,S}(x_1, \dots, x_N)\end{aligned}$$

$$E_g \simeq E_\infty + \frac{1}{g} [\partial_{1/g} E]_{g \rightarrow \infty}, \text{ with } [\partial_{1/g} E]_{g \rightarrow \infty} = -K(a_P)$$

1. $\Psi_{A,S}$: The natural one particle orbitals for the spatial part
2. a_P : Diagonalize a simple matrix for the spin part



Our tools: 1st order correlation function



1st order correlations

Magnetisation correlations

2 points correlations

$$\rho_1(x, y) = \langle \Psi^\dagger(x) \Psi(y) \rangle$$

$$= \rho_m^{(i,j)} \otimes \rho^{(i,j)}(x, y)$$

$$\rho_m^{(i,j)} = f(a_P)$$

“Probability” that the i -th and j -th particle have simultaneously the same magnetization m

$$\rho^{(i,j)}(x, y) = f(\phi_j(x_i))$$

“Probability” that the i -th particle is in x while the j -th particle is in y

Correlations and symmetries of 1D spin mixtures

Result

Symmetry breaking

[PhysRevA.106.033312](#)

Study of the symmetries

SU(2) Bosons $g \rightarrow \infty$:

Ground state



$\gamma_{GS}^{(2)}$

Most excited state

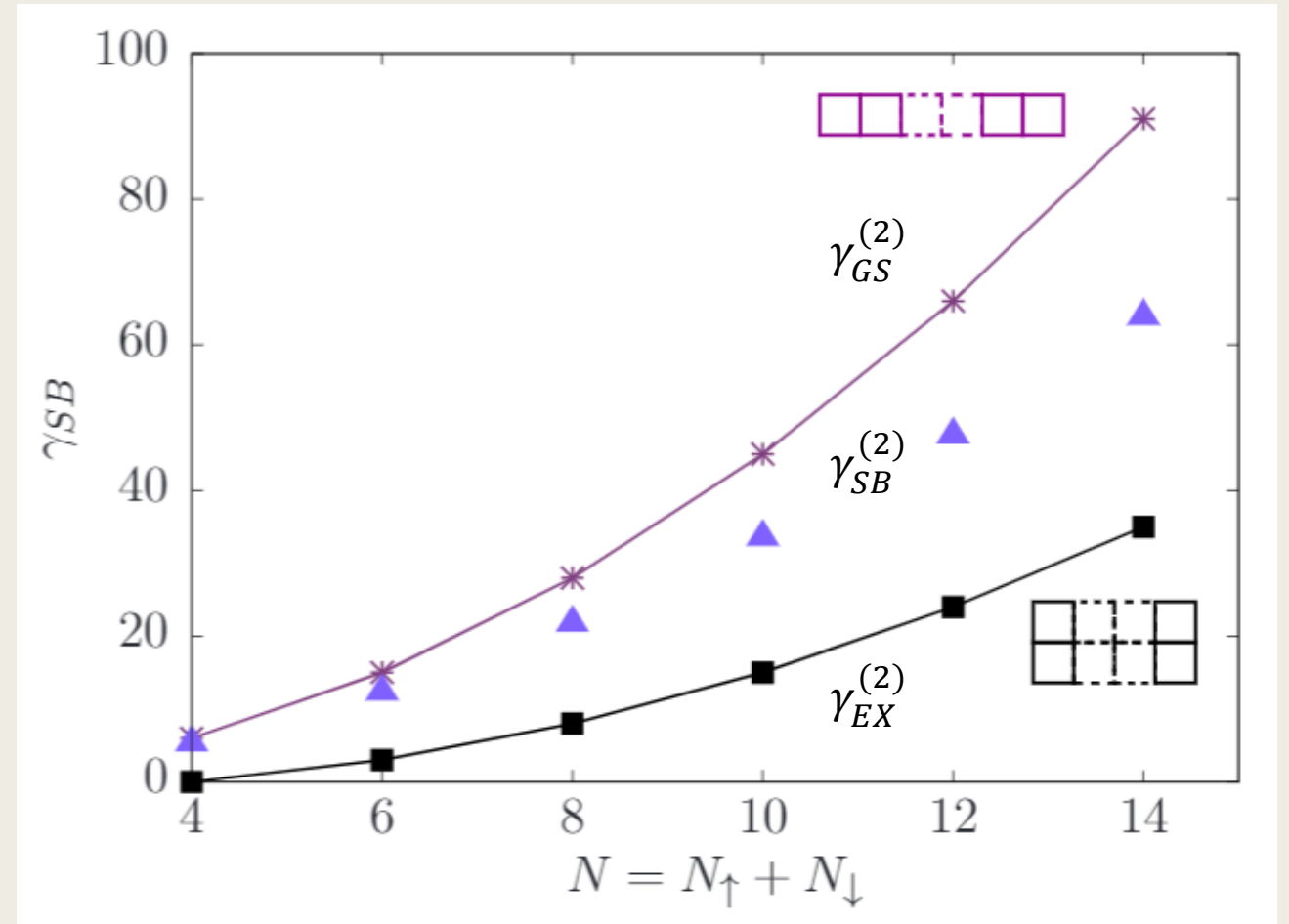


$\gamma_{EX}^{(2)}$

Symmetry breaking:

 $\gamma_{SB}^{(2)}$

- Undefined symmetry for the ground state
- Still mixture of Young tableau



Short-range correlations

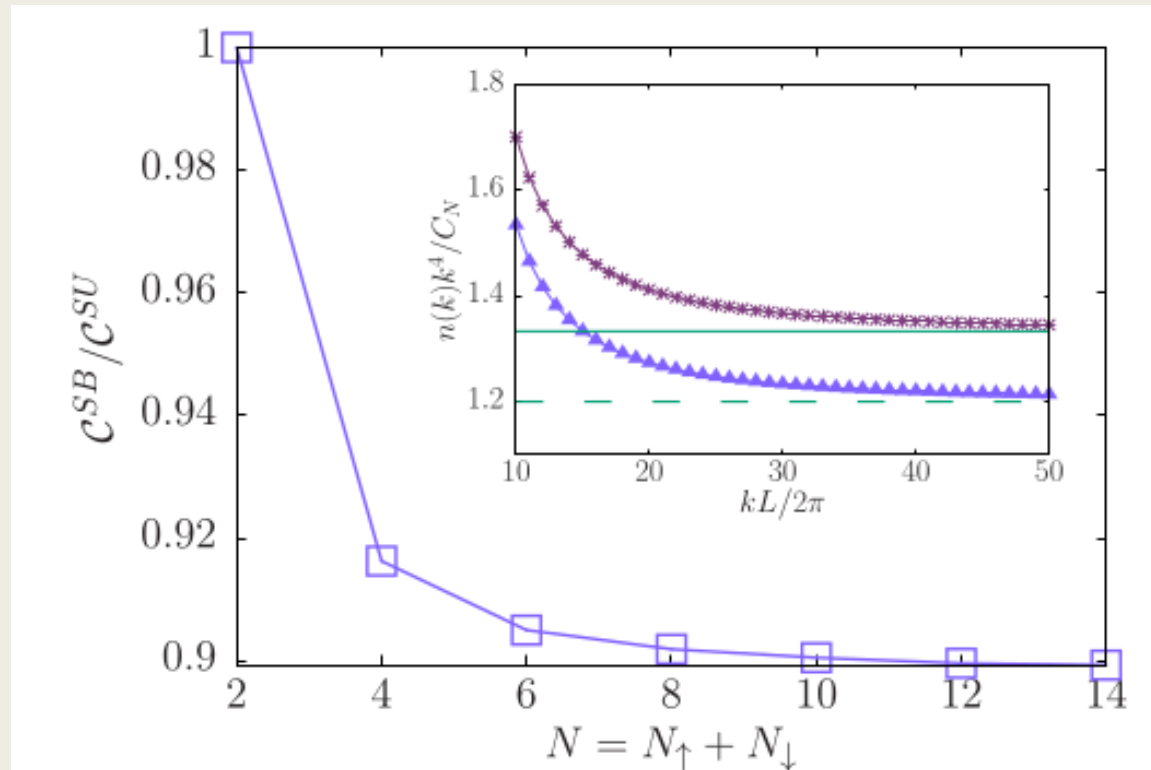
$$\mathcal{C} \underset{k \rightarrow \infty}{=} n(k)k^4$$

Scaling law :

$$\mathcal{C}^{Ring} \propto \mathcal{C}_N = \frac{(N\pi)^2 (N^2 - 1)}{L^3}$$

Thermodynamic limit:

$$\frac{\mathcal{C}^{SB}}{\mathcal{C}^{SU}} \underset{N \rightarrow \infty}{\simeq} 0,9$$



$$\text{---} \frac{\mathcal{C}_{SU}^{Ring}}{\mathcal{C}_N}$$

$$\text{- - -} \frac{\mathcal{C}_{SB}^{Ring}}{\mathcal{C}_N}$$

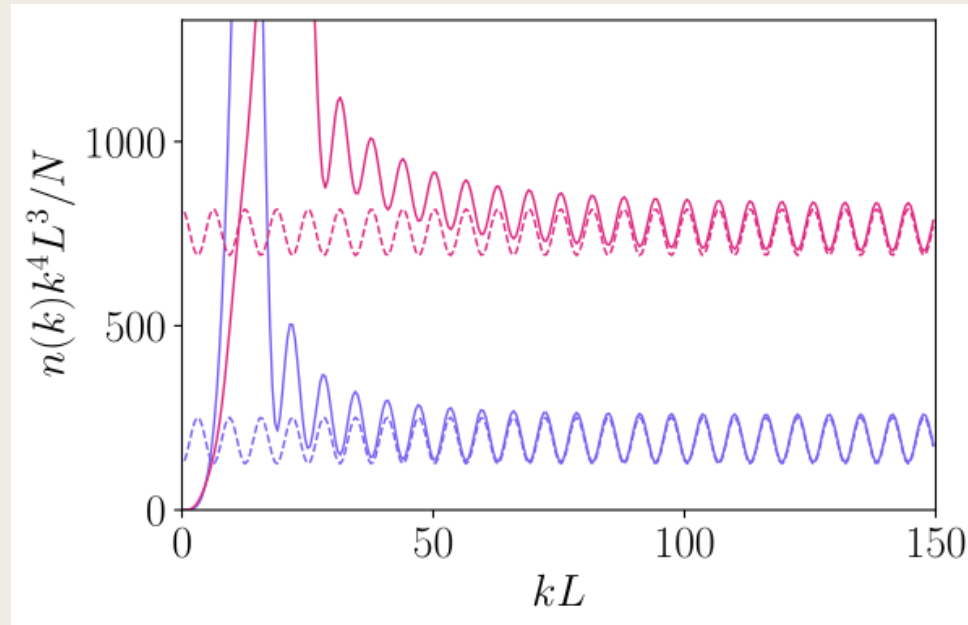
Correlations and symmetries of 1D spin mixtures

Results

Contact in the box

Exact form for the contact

Spinless particles



$$c^{FF} = \langle c^{FF} \rangle + (-1)^N \langle c_{osc} \rangle \cos(kL)$$

$$c^{TB} = N \langle c^{TB} \rangle + \langle c_{osc} \rangle \cos(kL)$$

Spin mixtures

$$c^{Mix} = \langle c_{\rho_2} \rangle + \langle c^{FF} \rangle + (-1)^N \langle c_{osc}^{Mix} \rangle \cos(kL)$$

$\langle c^{FF} \rangle$: Hard wall cusp contribution

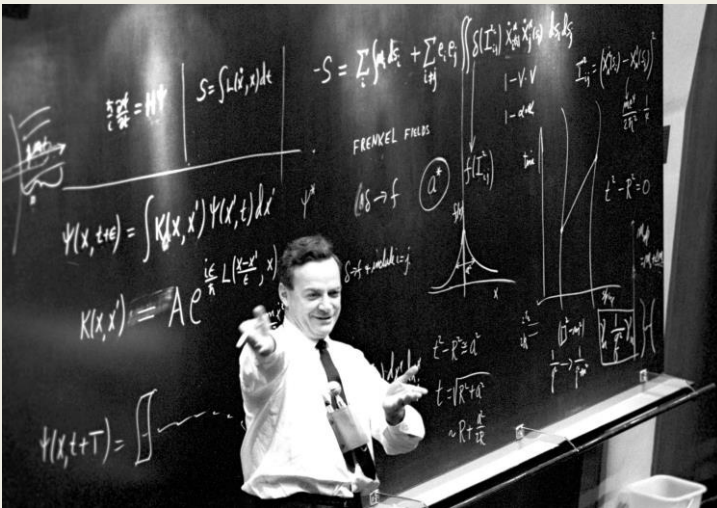
$\langle c_{osc}^{Mix} \rangle$: Depend only on long-range spin correlations

Why ?

Quantum simulation

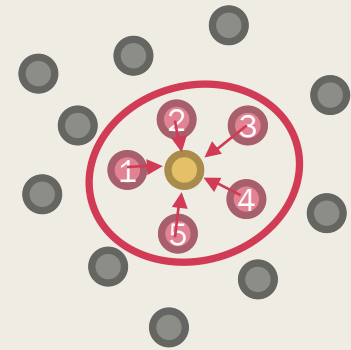
“Nature isn’t classical, dammit, and if you want to make a simulation of nature, you’d better make it quantum mechanical, and by golly it’s a wonderful problem, because it doesn’t look so easy.”

R. Feynman, 1982



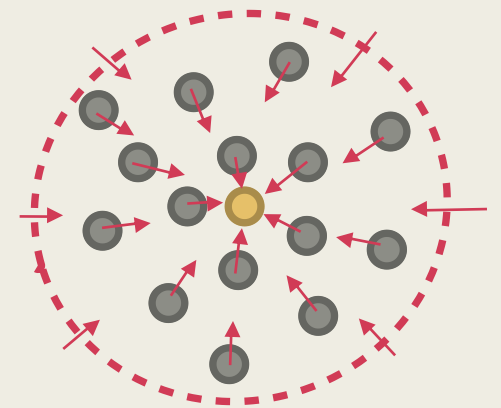
Weakly correlated systems

$$D_{\bullet} = f(I_{\bullet 1}, \dots, I_{\bullet 5})$$



Strongly correlated systems

$$D_{\bullet} = f(I_{\bullet 1}, \dots, I_{\bullet N})$$



Why ?

Benchmark

