

# Critical velocity for superfluidity in reduced dimensions

From matter to light quantum fluids

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February 7, 2023

JH, Mathias Albert, and Pierre-Élie Larré. (2022), Phys. Rev. A 105, 023305



## Critical velocity for superfluidify in 1D & 2D



JH



Frédéric Hébert



Pierre-Élie Larré

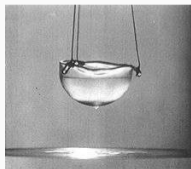
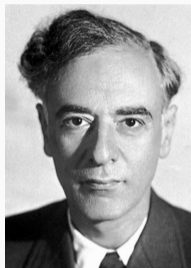


Mathias Albert

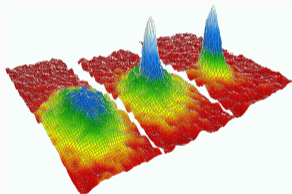
# Introduction

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# A BRIEF HISTORY OF SUPERFLUIDITY



1<sup>st</sup> evidence of BEC

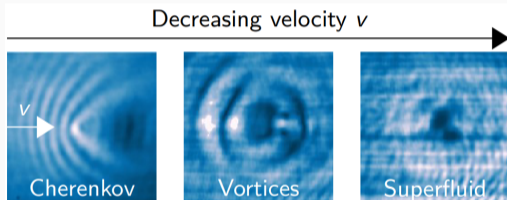


JILA, MIT 1995

SF observed in many systems:

- **BEC** (Ketterle, Cornell, Dalibard...)
- **Polaritons condensates** (Richard, Bramati, Bloch, Amo...)
- **Paraxial fluids of light** (Fleischer, Faccio, Michel, Bellec, Glorieux...)

## CRITICAL VELOCITY FOR SUPERFLUIDITY



Michel, Nat. Commun (2018)

Eloy, EPL (2021)

- Flow past an impurity without any loss of kinetic energy when  $v < v_c$
- Landau criterion: small obstacles only

### State of the art

1D: wide or narrow gaussian, cubic NL (Hakim, Leboeuf, Pavloff)

2D: wide impenetrable cylinder, cubic NL (Frisch, Rica, Swerger, Berloff, Dalibard)

## Our contribution

Arbitrary obstacle

General nonlinearity  $g(n)$

Treatment of the losses  
(1D)

Analytical results in 1D &  
2D

Dilute ultracold atomic gases ( $n = |\psi|^2$ )

$$i\hbar\partial_t\psi = \left[ -\frac{\hbar^2}{2m}\nabla^2 + U(\mathbf{r}) + gn^\nu \right] \psi$$

Paraxial fluids of light ( $I = |E|^2$ )

$$i\partial_z E = \left[ -\frac{\nabla^2}{2k} + U(\mathbf{r}) - \frac{\omega n_2}{c} \frac{I}{1 + I/I_{\text{sat}}} - \frac{i\alpha}{2} \right] E$$

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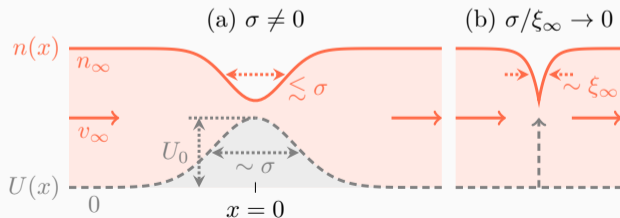
# Superfluid hydrodynamics

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2D mean-field regime, generalized nonlinear Schrödinger equation  
(Gross-Pitaevskii)

$$i\partial_t\psi = \left[ -\frac{1}{2}\nabla^2 + U(\mathbf{r}) + g(|\psi|^2) \right] \psi \quad (1)$$



Natural units

$$x \rightarrow \xi_\infty$$

$$v \rightarrow c_\infty$$

$$E \rightarrow \mu_\infty$$

Madelung transformation: hydrodynamic picture  $\psi = \sqrt{n} \exp\{i \int d\mathbf{r} v\}$

$$\partial_t \mathbf{v} = -\nabla \left( \frac{v^2}{2} + U(\mathbf{r}) + g(n) - \frac{1}{2} \frac{\nabla^2 \sqrt{n}}{\sqrt{n}} \right)$$

$$\partial_t n + \nabla \cdot (n\mathbf{v}) = 0$$

$$\cancel{\partial_t \mathbf{v}} = -\nabla \left( \frac{v^2}{2} + U(\mathbf{r}) + g(n) - \frac{1}{2} \frac{\nabla^2 \sqrt{n}}{\sqrt{n}} \right)$$

$$\cancel{\partial_t n} + \nabla \cdot (n\mathbf{v}) = 0$$

Without losses

**Stationary solutions**

Far away from the obstacle:

- density  $n \rightarrow n_\infty$
- velocity  $\mathbf{v} \rightarrow \mathbf{v}_\infty$

$$\begin{aligned}
 \cancel{\partial_t v} &= -\nabla \left( \underbrace{\frac{v^2}{2} + U(\mathbf{r}) + g(n) - \frac{1}{2} \frac{\nabla^2 \sqrt{n}}{\sqrt{n}}}_{= C^{te} = \frac{v_\infty^2}{2} + g(n_\infty)} \right) \\
 \cancel{\partial_t n} + \nabla \cdot \underbrace{(nv)}_{= C^{te} = ?} &= 0
 \end{aligned}$$

Without losses

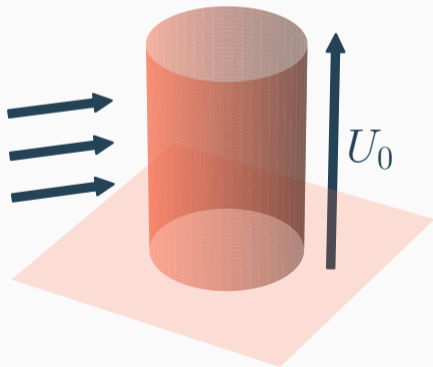
Stationary solutions

Far away from the obstacle:

- density  $n \rightarrow n_\infty$
- velocity  $v \rightarrow v_\infty$

# CRITICAL VELOCITY FOR SUPERFLUIDITY IN 2D

$$\frac{v^2}{2} + U(\mathbf{r}) + g(n) - \frac{1}{2} \frac{\nabla^2 \sqrt{n}}{\sqrt{n}} = \text{Cte} \quad \text{and} \quad \nabla \cdot (n\mathbf{v}) = 0 \quad \rightarrow \quad \text{nonintegrable}$$



## Our contribution

### State of the art

- $R \gg \xi_\infty$
- $U_0 = +\infty$
- Cubic NL

### Here

- $R \gg \xi_\infty$
- Arbitrary  $U_0$
- Arbitrary NL

Frisch, Rica, Zwerger, Berloff, Dalibard

# THE CONDITION TO OBTAIN THE CRITICAL VELOCITY

$$\underline{R \gg \xi_\infty:}$$

$$\frac{v^2}{2} + U(\mathbf{r}) + g(n) - \frac{1}{2} \frac{\nabla^2 \sqrt{n}}{\sqrt{n}} = \text{Cte}$$



$$g(n) = g(n(v)) = \text{Cte} - U(\mathbf{r}) - \frac{v^2}{2}$$

$n$  only depends on the modulus of  $v$

# THE CONDITION TO OBTAIN THE CRITICAL VELOCITY

$\nabla \cdot (n(v)\mathbf{v}) = 0 \rightarrow$  Make that **highly nonlinear** equation **linear**

Hodograph plane: Legendre transformation  $\Phi(\mathbf{v}) = \frac{m\mathbf{v}}{\hbar} \cdot \mathbf{r} - \phi(\mathbf{r})$

$$nv \frac{\partial^2 \Phi}{\partial v^2} + \frac{d(nv)}{dv} \frac{\partial \Phi}{\partial v} + v \frac{d(nv)}{dv} \frac{\partial^2 \Phi}{\partial \theta^2} = 0 \quad (5)$$

Elliptic solutions: complex-valued characteristic equation of (5)

$$\frac{d(nv)}{dv} > 0 \quad (6)$$

Condition relating to the maximum velocity  $v_{\max}$

# TREATMENT OF THE COMPRESSIBILITY

Solve  $\nabla \cdot (n\mathbf{v}) = 0$  to find  $v_{\max}$

First approximation:  $n = \text{Cte} \rightarrow$  **Incompressible fluid**  $\times$

Solution: include the **compressibility**  $\chi \propto \frac{1}{c_{\infty}^2}$  with a perturbative Janzen-Rayleigh expansion of the potential velocity

$$\phi = \phi_0 + \chi\phi_1 + \chi^2\phi_2 + \dots \text{ with } \mathbf{v} = \frac{\hbar}{m}\nabla\phi$$



# CRITICAL VELOCITY FOR SUPERFLUIDITY IN 2D

We want  $v_c(U_0)$ :

2 DIFFERENT CASES

Impenetrable obstacle

$$U_0 \gg \mu_\infty$$

The fluid goes around  
the obstacle

e.g. Rica, Phys. D (2001)

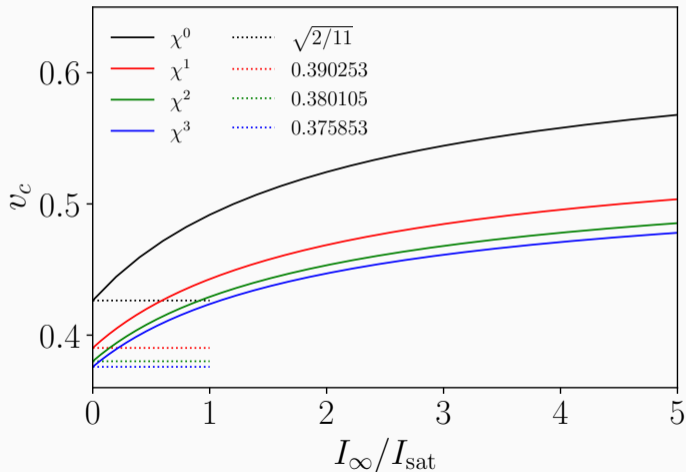
Penetrable obstacle

$$U_0 < \mu_\infty$$

The fluid goes around  
or crosses the obstacle

Theory for this?

# THE IMPENETRABLE OBSTACLE: $U_0 \gg \mu_\infty$

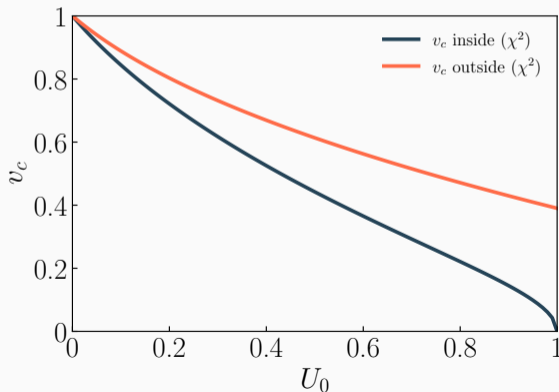


- $v_c$  indep. of  $U_0$
- **Superfluids of light**  
$$g(I) = \frac{\alpha I}{1+I/I_{\text{sat}}}$$
- $I_\infty/I_{\text{sat}} = 0$   
Cubic NL  $\rightarrow$  Rica

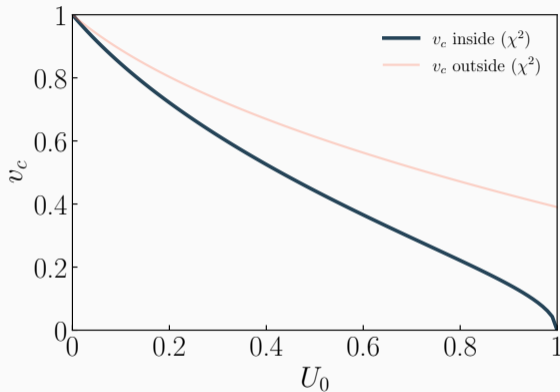
# THE PENETRABLE OBSTACLE: $U_0 < \mu_\infty$

$v_c$  does depend on  $U_0$

2 possible breakdowns of SF  $\rightarrow$  **2 velocities**

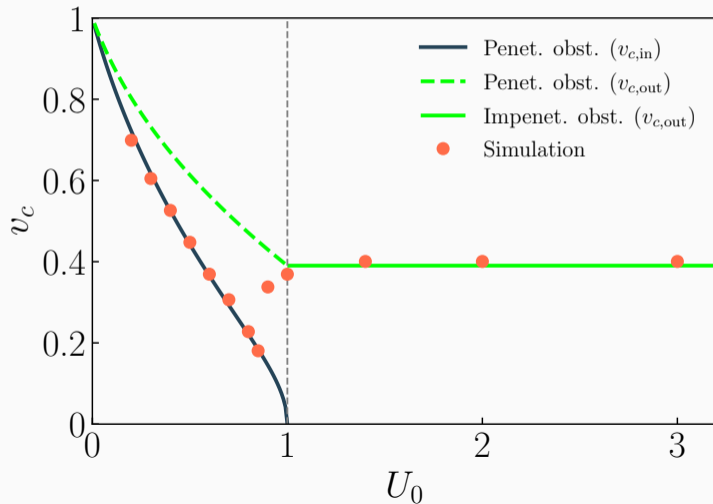


## THE PENETRABLE OBSTACLE: $U_0 < \mu_\infty$



The **real** critical velocity for SF is the **lowest** of the 2

# COMPARISON ANALYTICAL/NUMERICAL RESULTS



Similar results in  
Shin's group (exp.  
BEC)  
2015, 2022

# Conclusion

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# CONCLUSION

1D & 2D critical velocity beyond the Landau criterion

- General local nonlinearity** & **arbitrary obstacle**
- Treatment of **particle losses** (1D)
- Analytical results** in the mean-field regime

Method: search for the existence of stationary unperturbed solutions conditioned by the velocity

Perspectives: How do **quantum fluctuations** affect the mean-field SF transition? And what about **disordered systems** ?

**THANK YOU FOR YOUR ATTENTION!**