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Teacher learning from team teaching an advanced course in mathematics and didactics for prospective upper secondary teachers

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We report on a course in advanced mathematics and didactics aimed at prospective upper secondary teachers, taught jointly by a mathematician and a mathematics educator using a team teaching format. The course is novel in that it covers mathematical and didactical content concurrently and jointly. The present paper focuses on the team teaching aspect, with the aim of investigating what opportunities for learning, in particular for the teachers, arise from such a teaching practice. Drawing on commognitive theory, we analyse a teaching episode that displays how the team teaching might contribute to creating opportunities for meta-level learning, but also how implicit assumptions made by the teachers might create challenges for some students.

Keywords: Teachers' and students' practices at university level, Novel approaches to teaching, Upper secondary mathematics teacher education, Team teaching, Meta-level learning.

INTRODUCTION

There is some consensus among mathematicians and teacher educators (e.g., Dreher et al., 2018; Leikin et al., 2018) that prospective upper secondary mathematics teachers need a solid knowledge of central topics of upper secondary mathematics, such as calculus and algebra, including some more advanced topics. However, research (e.g., Wasserman et al., 2018; Zazkis & Leikin, 2010) has shown that both prospective and practicing upper secondary mathematics teachers have difficulty seeing the relevance of more advanced mathematics courses to their teaching practice. In 2021, the authors of this paper received funding from Uppsala University for developing a course partly aimed at addressing this problem. The design of the course is novel in that it covers mathematical and didactical content concurrently and jointly, not as two separate parts of the course. To this end, the course is taught through *team teaching* (Friend et al., 2010), where two teachers are present in the classroom conducting the teaching together. In this paper we will focus on this aspect of the course, with the aim of examining what opportunities for learning that might arise from team teaching. Although opportunities for student learning will be discussed, we will concentrate on opportunities for teacher learning afforded by the team teaching. We will also discuss some challenges that arose in the teaching of the course.

CROSS-DISCIPLINARY COLLABORATIVE UNIVERSITY TEACHING

There are several models for collaborative teaching (Friend et al., 2010, p. 12), ranging from “one teacher, one assistant” models, via parallel and alternative teaching, through to team teaching, “in which both teachers lead large-group instruction by both

lecturing, representing opposing views in a debate, illustrating two ways to solve a problem, and so on” (ibid., p. 12). The research literature on co-teaching of mathematics at the university level mostly consists of case studies in the context of elementary teacher education. For instance, Ford and Strawhecker (2011) developed a blended mathematics content/method course, co-taught by a mathematician and a mathematics educator in a format that included team teaching to a limited extent.

Studies involving co-teaching of university-level mathematics are rare. Lehmann and Gillman (1998) report on a semester of collaborative teaching. However, in that case both teachers were mathematicians. More directly relevant to the present study is the work of Zaslavsky and colleagues concerning a course in Mathematical Proof and Proving, taken primarily by prospective secondary mathematics teachers and co-taught by a mathematician and a mathematics educator. Sabouri et al. (2013) focus on the professional reflection of the teachers, building on one specific classroom situation to discuss “ways in which the collaboration between these two experts made them conscious of each other’s considerations and of the importance of questioning their assumptions and negotiating them” (p. 312). Meanwhile, Cooper and Zaslavsky (2017) conducted an in-depth analysis of a teaching episode to investigate the two teachers’ views on proof and proving as they come across in their teaching, and the affordances of these views on the students as future teachers. Perhaps unsurprisingly, the analysis revealed the mathematician to be more concerned with the content and presentation of the proof, while the mathematics educator paid more attention to the thinking of the prover, what Cooper and Zaslavsky label the “human element” of the proof. They also note that the teaching mostly took the “one teach, one observe” approach, and that a teaming approach might have created greater opportunities to discuss and reflect upon differences in views. In conducting their analyses, Cooper and Zaslavsky drew on a discursive approach to teaching and learning, the commognitive framework (Sfard, 2008). This framework also forms the theoretical backdrop of the present study.

UNIVERSITY MATHEMATICS TEACHING AS A DISCURSIVE PRACTICE

From a commognitive perspective, mathematics and mathematics teaching are seen as discursive practices, characterized by the *words* and *visual mediators* used, the *narratives* told about mathematical objects and their relations, and the repetitive patterns, *routines*, of the discourse (Sfard, 2008, p. 131-133). As a patterned activity, mathematical discourse is governed by two types of rules. *Object-level rules* are “narratives about regularities in the behaviour of objects of the discourse” (ibid., p. 201). Most of what we think of as mathematical rules or facts belong to this category, for instance, differentiation rules or the distributive law. *Meta-level rules*, on the other hand, govern the actions of the discursants, that is, they regulate “the activity of the discursants trying to produce and substantiate object-level narratives” (ibid., p. 201). An example could be rules for what counts as an acceptable mathematical proof. Meta-level rules (metarules for short) behave differently from object-level rules. They may *evolve* over time and vary between contexts. Moreover, where the rules governing the mathematical objects are explicitly articulated as mathematical narratives, metarules

can be *tacit*. Indeed, Sfard distinguishes between *endorsed* and *enacted* metarules, where the former “are explicitly recognized as a person’s own” (ibid., p. 204), while the latter are inferred by an observer of the discourse. Furthermore, metarules are typically *normative* and value-laden; *constraining* rather than deterministic; and *contingent* rather than necessary (ibid., p. 202). It should be pointed out, however, that not all metalevel rules satisfy all these characteristics. They should therefore not be taken as defining the notion of meta-level rule; the defining property of a meta-level rule is rather that it governs the actions of the discursants rather than the objects of the discourse. For more detail on this, see Viirman (2021, p. 469-470).

In commognitive terms, learning is defined as change in the learner’s mathematical discourse. Such change can be of two types: *object-level learning*, which involves expanding one’s discursive repertoire of objects and narratives about them; and *meta-level learning*, which involves changes in the meta-level rules of the discourse (Pinto, 2019, p. 4). Typically, meta-level learning involves discursive change that is not initiated by the learners themselves, and that evolves through repeated engagement with the new discourse. For instance, in the move from school to university, the meaning of, and rules governing, many familiar mathematical objects and narratives change in ways that are not always made explicit. A few studies have examined university mathematics teaching from the perspective of meta-level rules and learning. Viirman (2021) investigated how the discursive practices of seven university mathematics lecturers teaching first-semester courses served to model mathematical discourse, and Pinto (2019) compared the opportunities for object- and meta-level learning offered by two teaching assistants through their tutorial lessons in a Real Analysis course. As Pinto points out, “while instructors do not typically engage in open and explicit discussions about the meta-rules of their discourse in the course of their lectures, their recurrent actions, and their comments about their actions, provide glimpses into their mathematical discourse and its underlying meta-rules” (p. 4).

Using this terminology, we address the question of how and to what extent team teaching, with its element of teacher dialogue, can create opportunities for meta-level learning. Although we are interested also in opportunities for student learning, here our main focus is on the learning opportunities created for the teachers. For instance, we hypothesize that the dialog inherent to the team teaching might help make explicit enacted metarules both concerning mathematics and mathematics teaching. Before addressing the research question, however, some information on the course and the teaching, as well as on data collection and analysis, is needed.

METHODS

Setting – the course, the teachers, the teaching

The course is positioned late in the upper secondary teacher education program and runs with 1-2 two-hour sessions a week, a total of 30 sessions, throughout the spring semester, in parallel with the students’ final thesis work. The students have taken three semesters of mathematics prior to enrolling in this course, including, for instance,

linear algebra and calculus of one and several variables. For most students, these semesters are placed early in the program. Thus, their prerequisite knowledge is not always up-to-date.

For assessment purposes, the course consists of a mathematics part, 10 ECTS [1] credits, and a didactics part, 5 ECTS credits. The mathematical content of the course is taken from abstract algebra and real analysis, with a focus on topics relevant to upper secondary mathematics. For algebra, this means mainly ring and field theory, for instance Euclidean domains and field extensions, but only some rudiments of group theory. Applications include a treatment of the classical problems of geometric constructions (proving, for instance, that squaring the circle is impossible) and a discussion of the insolvability of the general fifth-degree polynomial equation. In analysis, we begin by detailing the construction of the real numbers through Cauchy sequences and then give a rigorous treatment of concepts familiar from upper secondary school calculus, such as limit, derivative and integral. Didactical topics covered include, for instance, the epistemological structure of calculus and well-known student difficulties with calculus topics, but also didactical perspectives on notions such as definitions, examples, proof, generalisation, representation and classification. The course is assessed through a variety of means, including two closed-book exams on mathematical content; an open-book exam on didactical content; and group and individual presentations on mathematical and didactical topics.

As already mentioned, the course is taught concurrently by the authors of this paper. The first author, ME, is a researcher in mathematics education, but also has a solid background in advanced mathematics (a masters degree in mathematics plus a number of additional courses at the doctoral level), whereas the second author, M, is a research mathematician, but also an award-winning teacher of university mathematics, professionally involved in didactical networks within the science faculty at Uppsala University. In the teaching of the course, the second author takes main responsibility for the mathematical aspects, and the first author for the didactical ones, but both are knowledgeable enough in the other's domain of expertise to be able to engage in meaningful dialogue. In planning and conducting the course, we wanted to establish connections between university and school mathematics teaching. Wasserman (2018, p. 6) describes a spectrum of such connections, on the level of content, disciplinary practice, classroom teaching and modelled instruction. We aim at achieving connections of all these types, for instance through explicitly pointing out content connections and ways in which the content can inform the students' future teaching practice, but also through encouraging student meta-reflection and through viewing our own teaching as a source of examples of practice, to be used for didactical reflection. We also try to achieve a high level of interaction with the students, by engaging them in conversation around the mathematics rather than just lecturing.

Data and analysis

So far, the course has been given once, with nine students enrolled. All sessions (except the first four, due to technical difficulties) were video-recorded. In addition, the first

author has kept notes from the joint planning of the course, and we also conducted some informal interviews with the students. So far, we have conducted preliminary analyses of a small part of the material, focusing on teaching episodes where there was much interaction between the two teachers, or between the teachers and the students. In particular, we were looking for exchanges where the focus was on meta-level rather than object-level discourse, that is, where discussion revolved around, for instance, more general aspects of mathematical discourse, on implications for teaching practice, or on didactical aspects of the teaching currently taking place. From these exchanges we then selected instances which we deemed particularly representative or enlightening. We then analysed these further, with the aim of investigating how the dialogue between the teachers, or between the teachers and the students, influenced the opportunities for learning, both for students and teachers, arising from the teaching. The final decision to make opportunities for teacher learning the focus of this paper led us to select the particular episode presented below. All dialogue was originally in Swedish, and has been translated by the first author.

RESULTS

A first, general, observation is that at the very onset of the course we realised that we had unrealistic expectations of the mathematical maturity of the students. We knew that it was some time since they had studied mathematics, and in the information distributed to the students prior to the course we emphasised the need to review material from earlier courses in algebra and calculus in preparation for this course. In particular, we designed the first session around the algebra of integers and polynomials, and asked the students to review the statements and proofs of the fundamental theorem of arithmetic, and of Euclidean division of integers and polynomials. We intended to have the session revolve around a discussion of the structure of these proofs, and of abstract properties of integers and polynomials more generally, to prepare the students for the introduction of the concept of a ring in the second session. However, it soon became clear that most of the students had struggled with the proofs, and we instead had to spend much of the session explaining them in detail. When we reflected on the session afterwards, M first attributed the student difficulty to particularities of the proofs. However, ME, who had been less directly involved in the teaching during the session, and thus had greater opportunity to reflect on the discourse as it unfolded, observed that some of the difficulties seemed to originate in lack of experience with proving more generally. Further reflection and observation during the following session led us to agree on ME's point of view.

Hence, we deemed it necessary to devote more teaching time to presenting and explaining the mathematical content. This placed more of the responsibility of the teaching on M, and made it more difficult for ME to interact, since there was less time available for didactical and mathematical reflection. Moreover, it was decided that ME should concentrate much of his teaching contributions on issues of proof and proving, and this is also the topic of the example episode that is presented and analysed below.

A teaching episode – proving an equivalence

This episode is taken from Session 6, and appeared in the context of proving a property of the valuation v in a Euclidean domain, namely that $v(a) = v(1) \Leftrightarrow a$ is a unit (has a multiplicative inverse). The form of the statement prompted some meta-level reflection from M, prior to the presentation of the proof, concerning the two implication claims implicit in an equivalence:

- M Often, you cannot prove these two claims together, that is, show this [points at the equivalence written on the board] directly, rather you show one claim at a time, first this one [points from left to right in the equivalence] and then this one [points from right to left], or the other way around, depending upon what you feel like. I think that we'll go this way first [points from right to left], because it is easier.
- ME That's a bit interesting, because in many contexts, when you first encounter the idea of proving things, in number theory and things like that, then you often work with expressions that you reformulate in various ways, and then these equivalences will somehow hold all the way through.
- M Unless you do something like extracting roots or something like that.
- ME Yes, exactly, so if that is, like, your entry point to proving things, then you run the risk of forgetting that a claim like this typically needs to be split up and treated in two separate parts, because you're somehow used to being able to do everything at once.
- M A typical problem with the presentation of proofs like that often is that the student starts with what they are supposed to prove. You write down the equality you want and then you manipulate it until you get the equality you had to begin with or the other way around or whatever, and sometimes it's like, if you interpret it kindly, those are the calculations that you need to do, but the structure of the proof is unclear if you do it like that, OK? It's always better if you need to prove an equality (...) even if you figure out what the process is by fiddling with the equality yourself, at least when you present it start with one side and rewrite it until you get the other side 'cause then you'll have a clear direction of your argument. OK? Don't start with the claim, the equality, and start working on both sides at once, because it is not systematic.
- ME No, because we don't know that the equality holds to begin with.
- M For instance that, yes [laughs].
- S So, then, one direction is a is a unit and then that [referring to the claim $v(a) = v(1)$] holds, or if that holds then a is a unit?
- M Exactly, it's those two. So that, an equivalence [writes on the board: "An equivalence arrow is two implications"] an equivalence arrow is two

implications, OK? So, we have A is equivalent with B [$A \Leftrightarrow B$] means that A implies B [$A \Rightarrow B$] and that B implies A [$B \Rightarrow A$].

In this dialogue, the respective statements by M and ME prompted responses that elaborated on what was just said, adding further levels of reflection. From the point of view of an insider to the discourse, what was happening could be described as follows: M began by stating a general principle of proof, which prompted ME to reflect on potential didactical problems with the simple types of claims that typically are secondary school students' first contact with proof, which in turn caused M to highlight a typical student mistake with proof and to formulate an explicit metarule for this kind of proof. Prompted by a student comment about the particular proof under consideration, he also put the initial observation about equivalences and implications into symbolic form on the board.

However, given the students' difficulties during the first sessions and their lack of recent engagement with mathematics, it was likely too optimistic to assume this degree of insidership to mathematical discourse from many of them. A closer analysis of the dialogue reveals a number of places where M and ME made implicit assumptions on the students' familiarity with mathematical discourse, and in particular on their ability to move between object-level and meta level discourse.

The example given, in quite abstract and imprecise terms, by ME in his first statement assumed both that the students understood what kind of proof situations he was referring to (proof through algebraic manipulation) and that they were capable of reflecting on these at the meta-level, as examples of ways of mathematical reasoning. In his response, M aimed to complete the argument with details (not all algebraic manipulations result in equivalences), but again it was implicitly assumed that the students could follow this meta-level argument. For M, as an insider, it was clear what situation ME was referring to, but possibly it was less clear to the students.

In his next statement, ME reached his intended conclusion by connecting the two strands of the argument; if your main encounters with proof are through proving algebraic identities, often as a sequence of equivalences, then you can easily be led to think that this is how you typically prove equivalences. However, this again assumed that the students were able to follow this quite abstract meta-level reasoning, reflecting on what unites and separates two forms of mathematical argumentation, one of which had only been described too abstractly and with lack of precision. When M then connected this to a well-known type of student mistake, the fact that ME's argument concerned proofs of algebraic identities was clear to him as an insider, but in fact this is the first place where the term 'equality' (or 'identity' – the word 'likhet' in Swedish can mean both) was explicitly mentioned. The point was possibly further obscured by the fact that M was also unable to resist making a joke in the process (the manipulations resulting in getting back where you started). Still, M did formulate a useful explicit meta-rule for proving equalities, and at least one student (S) had apparently been able to keep track of the original narrative, prompting him to also explicitly state the connection between equivalences and implications.

The deepened analysis presented above indicates how ME and M made implicit assumptions on familiarity with mathematical discourse that possibly posed a challenge for the students. The analysis suggests at least two possible sources of problems. First, ME's comment was highly abstract, lacking a concrete example to specify the kind of situation he was talking about. Second, the language he was using was not sufficiently precise. For M, with access to a large and well-organised "library" of mathematical, as well as didactical, objects and examples, ME's statements had clear referents, despite their vague formulations. However, even if the students might very well be aware of the relevant examples, it is unlikely that such vague prompting would suffice for them to recall them in this context. Still, there are tensions here. If ME had taken the time to support his remark with a carefully presented example given with precise details, it would have ceased being just a remark, thus disrupting the ongoing mathematical argument. Moreover, ME's comment was prompted by an observation made in the moment and had to be made before work on the actual proof started, meaning that the time available for coming up with a supporting example was limited.

Lest we paint too bleak a picture here, despite these misgivings the dialogue format of the team teaching did support a deepened didactical reflection in the moment, in the process creating opportunities for meta-level learning concerning proofs and proving, although perhaps not for all students. In particular, here it led to an enacted metarule being made explicit for the students, in a way that would likely not have happened had only one of the teachers been doing the teaching.

DISCUSSION

The analysis presented above exemplifies how team teaching led both M and ME to realise the need for constant awareness of students' less developed mathematical discourse. Despite M and ME being didactically knowledgeable teachers, in their meta-level discussion they nevertheless lose track of this. Still, the episode also illustrates how team teaching can contribute to making metarules visible to students, thus creating opportunities for meta-level learning by providing insight into the "explicit discussions about the meta-rules of their discourse" (Pinto, 2019, p. 4) typically only glimpsed in teaching. But it also suggests ways in which team teaching can create opportunities for teachers' in-the-moment reflection, as one teacher is not solely responsible for managing the lesson. In the episode analysed, comments made by M prompted ME to reflect on the teaching taking place, a reflection that continued after the particular session. In this way, the team teaching also contributed to teacher learning. Only a small part of the data has been analysed so far and we expect further analysis to provide stronger support for the value of team teaching for meta-level learning.

Widening the perspective from this particular episode to the course as a whole, between the teaching sessions M and ME reflected on and discussed the teaching, providing analysis and critique. This process led to both teachers gaining new insight into their teaching practice, in a way that would have been more difficult, and possibly would not have happened, had either been the sole teacher of the course. This resonates with the observations made by Sabouri et al. (2013), where the collaboration between the

mathematician and the mathematics educator “made them conscious of each other’s considerations and of the importance of questioning their assumptions and negotiating them”. Examples of learning that led to changes in teaching practice include the realisation of the crucial importance of timing when making comments, in order not to disrupt an ongoing argument, and of keeping comments short and precise. Moreover, ME has come to realise that the (over-)abstraction evident in the episode above is a recurring tendency in his teaching that he needs to work on, while M has noted how impatience sometimes causes him to not allow students enough time to reflect and formulate their arguments when conducting group discussions. Here it is also worth noting that the process of researching your own practice in itself creates opportunities for teacher learning, which are enhanced if you are two people doing the analysis.

There are also a few general observations to be made in relation to previous research. We note that the modelling of mathematical discourse that Viirman (2021) found in his analyses of university mathematics lecturing occurred also in this context. Moreover, the different roles discerned by Cooper and Zaslavsky (2017), where the mathematician took main responsibility for content, while the mathematics educator focused more on the “human element”, were less present in this course. In the episode discussed above, both M and ME made didactical reflections about proving. Indeed, it was a conscious choice when planning the course to try to avoid this strict separation of roles, showing by example that mathematical and didactical reflection go hand in hand.

We also want to make some remarks concerning difficulties we have encountered when trying to implement team teaching. Similarly to what Cooper and Zaslavsky report, much of the teaching took the “one teach, one observe” approach, rather than the more interactive format we had aimed for. This was due partly to us not being able to do the detailed planning of the sessions together, and partly to the need for more presentation of content described earlier, which sometimes caused ME to refrain from, for instance, initiating discussion of the lecturing of M as a model of instruction (Wasserman, 2018) since he knew that we were pressed for time. Moreover, in the light of the analysis above, there is a risk that the kind of reflection needed to make sense of meta-level comments made in the moment might contribute to student cognitive overload. In conclusion, however, so far we feel that the team teaching has a definite potential to contribute positively to student and teacher learning, particularly at the meta-level. Moreover, it is a very enjoyable form of teaching, something also highlighted by, for instance, Lehmann and Gillman (1998), and we recommend others to try it if they get the opportunity.

NOTES

1. European Credit Transfer and Accumulation System. One academic year corresponds to 60 ECTS credits.

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