

# The characteristics of proof teaching in a first-year university mathematics course

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## INTRODUCTION

Mathematical proofs have a central role in developing, establishing and communicating mathematical knowledge. Given the importance of mathematical proofs, introductory courses in mathematics bachelor's degrees are dedicated to introducing students to the proving process. The goal of this study is to characterise proof teaching in university mathematics lectures by using the framework of commognition (Sfard, 2008). The basic tenet of this framework is that thinking can be conceptualised as communication with oneself. This poster focuses directly on proof teaching in an introductory mathematical lecture. The research question of this study is: what are the characteristics of proof teaching in a first-year university mathematics course?

## METHODOLOGY

Six online lectures (out of 17) by an exemplary mid-career lecturer teaching a first-year university mathematics course on real analysis were analysed. Inductive thematic analysis was performed, using themes (characteristics of proof teaching) from Karavi et al. (2022). Thus, the following characteristics were used, flexibility (i.e., performing a proof in more than one way), bondedness (i.e., making connections between the different steps of the proof), applicability (i.e., discussing the application of a proving process in other situations), agentivity (i.e., making decisions for the proving processes, evaluating and showcasing how one can explore them), objectification (i.e., increasing the level of abstraction of a mathematical object) and substantiability (i.e., establishing the criteria to judge and reflecting on the essence and key ideas of the outcome of the proving process).

## RESULTS

All quotes in this section were taken from the proof of the characterisation of compact sets ( $K$  in  $\mathbb{R}$  is compact if and only if  $K$  is closed and bounded). We identified a structure for the teaching of proofs in the first-year university mathematics course under study that we discuss briefly in this section. When introducing a proof the lecturer first stated what exactly needed to be proved.

The first statement is  $K$  as a subset of real numbers is compact, and the theorem asserts that this is equivalent to saying that  $K$  is both closed and bounded.

He showed how this theorem relates to previous proofs or results (bondedness)

So, this connects back to two questions earlier on the chat, are all compact sets closed?

He explored what definitions and tools could be used and what proving approach will be taken (agentivity).

Well, let's do a proof by contradiction. Let's assume that a set  $K$  is not bounded, so I assume  $K$  is compact, and I assume  $K$  is not bounded. And then I want to force a contradiction.

During the proof itself, the lecturer related the (steps of the) proof to other steps, proofs and results (bondedness).

Okay, this is only half of the proof. If  $K$  is compact, then it is bounded. But I still need to show that  $K$  is closed as well. Okay, so that will be the next slide.

He applied previously known theorems and definitions (applicability).

So here we go. Since  $x$  is a limit point of  $K$ , we know that there has to be a sequence  $x_n$  in  $K$ , such that  $x$  equals the limit of this sequence  $x_n$ .

He showed that (steps of the) proof can be performed in multiple ways (flexibility), and finally repeated the statement of what exactly was proved.

So, here's one characterization of compact sets, a set is compact if and only if it is closed and bounded.

When closing the proof, the lecturer repeated the main ideas of the proof and explained them in other ways (substantiability).

So, if I assume that  $K$  is compact, then the assumption that  $K$  is not bounded gives me a contradiction. And therefore, the compactness of  $K$  implies boundedness of  $K$ .

In this poster, we explored the proof teaching through the identification of characteristics that originated from the literature. Each characteristic appeared to have a specific function given its place in the proving process. These identified characteristics can be a starting point for future researchers who aim to further investigate proof teaching.

## REFERENCES

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