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Teaching mathematics to non-specialists: a praxeological approach

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The study presented here concerns teaching practices at university for non-specialist students. Referring to the Anthropological Theory of the Didactic, our aim is to investigate the personal didactical praxeologies of university teachers (at the School, Discipline and Content levels) and to observe what can be considered as specific for non-specialists in these practices. We interviewed three experienced teachers with different profiles and collected their teaching resources. Analysing this data, we identified several didactical praxeologies specific for non-specialists at the Discipline level. At the Content level we give the example of a specific didactical praxeology and claim that many more exist, due to the mathematics-didactic codetermination.

Keywords: Anthropological approach to didactic, Teachers' and students' practices at university level, Teaching and learning of mathematics in other fields, Teaching and learning of mathematics for engineers

INTRODUCTION AND BACKGROUND

In their survey of research in University Mathematics Education, Biza et al. (2016) mention 'mathematics teaching at tertiary level' and 'the role of mathematics in other disciplines' as two emergent themes. Indeed, research about each of these two themes has developed significantly in the last few years. However, studies combining these two themes are scarce. González-Martín et al. (2021) note in their synthesis of research concerning teaching mathematics to non-specialists that many authors evidence ruptures between the mathematics taught in mathematics courses and in courses of other disciplines. Nevertheless, Pepin et al. (2021), focusing on mathematics in engineering education, observe that while several studies address the issues of instructors' expectations and their views about the mathematics that should be taught to future engineers, only a few authors investigate the ordinary practices of teachers in their mathematics courses for future engineers. Interviews with teachers having different backgrounds (studies in mathematics, in engineering, in physics, professional experience as engineer) evidence that they declare having different practices, regarding e.g. the links between mathematics and other disciplines, or the level of rigour expected. These differences can be a consequence of their different backgrounds (Hernandes-Gomes & González-Martín, 2016; Sabra, 2019).

González-Martín (2021) uses the Anthropological Theory of the Didactic (ATD, Chevallard, 1999) for analysing textbooks and teaching practices in two different engineering courses using the concept of integrals. Analysing interviews with two

teachers, he observes that their practices seem to be strongly influenced by the textbooks they use; and that the mathematical content (integrals) finally plays a limited role in their courses. Following this work of González-Martín, we use ATD and the concept of didactical praxeologies to investigate teaching practices in the context of courses for first year non-specialists students. Our aim is to deepen our understanding of these practices, and elucidate the issues identified by the teachers and the strategies they develop to address these issues. Our work belongs to the DEMIPS [1] network in France (Theme 5: teachers' practices at tertiary level).

THEORETICAL FRAMEWORK

The theoretical framework guiding our study is the Anthropological Theory of the Didactic (ATD, Chevallard, 1999). Chevallard considers that the knowledge taught is shaped by the institutions. In our study, secondary school and university are institutions; a mathematics course or a chemistry course for first year students are also institutions. How the knowledge is shaped is described in ATD by the concept of praxeology. A praxeology comprises four elements: a type of tasks T , a technique τ to perform this type of task, a technology θ which is a discourse explaining the technique, and a theory Θ grounding the technology. In mathematical praxeologies, the type of task concerns mathematics, e.g. T_{nsv} : "Compute the norm of the sum of two vectors". This type of task can be present both in mathematics and in chemistry courses, and will be associated with different praxeologies in each course. In didactical praxeologies, the type of task concerns the teaching of mathematical praxeologies in a given institution: "Teach the mathematical praxeology associated with T_{nsv} ". A didactical type of tasks is associated with didactical techniques and technologies; the didactical theory usually remains implicit. The didactical and mathematical praxeologies mutually influence each other (Bosch & Gascón, 2001). The conditions and constraints underpinning any teaching or learning process (e.g. questions from the teacher to the students) can be located and analysed at different levels, classified in a scale extending from the more general to the more precise point of view. Florensa et al. (2018) separate this scale in an "Upper scale" (Humanity $\langle \rangle$ Civilisation $\langle \rangle$ Society $\langle \rangle$ School $\langle \rangle$ Pedagogy $\langle \rangle$) and a "Lower scale" ($\langle \rangle$ Discipline $\langle \rangle$ Domain $\langle \rangle$ Sector $\langle \rangle$ Theme $\langle \rangle$ Question).

While most studies referring to didactical praxeologies focus on six predetermined 'moments' (see e.g., González-Martín, 2021), in our study we consider the personal praxeologies developed by teachers (Bosch & Gascón, 2001). We try to identify didactical types of tasks T , techniques τ and technologies θ . These didactical praxeologies of the teacher are empirical (Bosch & Gascón, 2001), developed by the teachers along their work in different institutions. The levels presented above also concern didactical praxeologies, and identifying to which level a didactical praxeology belongs can enlighten teachers' practices (Florensa et al. 2018). Nevertheless as acknowledged by these authors this identification is complex; for this reason we have chosen here a simplified version of the codetermination scale: School $\langle \rangle$ Discipline (here mathematics) $\langle \rangle$ Content (e.g. vectors). What we call "School" includes the

whole “Upper scale”. For example, "Ensure that students do personal work" is a didactical type of task at the School level; "Teach students how to present the solution of a mathematics exercise" is at the Discipline level, while "Teach the mathematical praxeology associated with T_{nsv} " is at the Content level. Drawing on these theoretical elements, the research questions we study here are:

What didactical praxeologies are developed by teachers teaching mathematics to non-specialists students? What is specific for non-specialists students in these praxeologies?

METHODS

The DEMIPS theme 5 group designed interview guidelines in order to investigate university teachers’ practices (in mathematics, physics or chemistry). During the semi-structured interview, after general questions about their teaching experience and the courses they deliver, the teachers were asked to focus on a particular course. They were informed ahead of the interview, and were asked to bring with them the material they used for this course. Concerning this course, they were firstly asked to present it and the resources offered by the institution to the students (e.g. digital platform). Then they were asked about their views on the students’ needs and potential difficulties, about their own practices (including the resources they design, how they design them, their collective work with colleagues) for the tutorials and for the students’ assessment. Interviews were conducted during the academic year 2021-2022 (9 interviews when we write this paper). The material brought for the interview (e.g. exercises sheets for students, exam texts) was collected, and the interviews were transcribed.

For the study presented here, we selected 3 of the 9 teachers. We chose teachers who focused in the interview on courses for first-year non-specialist students; and experienced teachers, who might have a rich repertoire of didactical praxeologies. The profiles of the three teachers chosen are presented in table 1 below.

Teacher	TC	TM	TP
PhD in	Chemistry	Mathematics	Theoretical Physics
Personal information	Female, 28 years teaching exp.	Male, 28 years teaching exp.	Male, 16 years teaching exp.
Course	Chemistry for biology students, including "maths reminders"	Mathematics for future chemistry engineers	Mathematics remedial course for students who gave up in another course

Table 1. Profiles of the three teachers interviewed and courses.

For analysing the data collected, we started by searching in the interviews the type of tasks mentioned by each teacher. The three authors of this paper confronted their initial analyses, and the level (S, D, C) of the corresponding task. While the difference between the School and the Content level is clear, sometimes the Discipline level and the other two can overlap. Then we searched the interviews for the didactical techniques used by the teachers, and possible explanations/justifications of these techniques (interpreted as technologies). We confronted the teachers' declarations with the material collected, and also analysed this material (in particular the solutions of the exercises) to identify mathematical and didactical praxeologies at the Content level.

RESULTS

Firstly we present our results concerning the School and the Discipline levels; then we give one example at the Content level. Analysing our data, we observed ten types of didactical tasks that were shared by at least two of our three interviewees; five for each of these two levels (Table 2).

School level	<i>Ensure that students complete personal work</i> (T_{cpw})	Ensure that students work during the tutorial (T_{wt})	Ensure that students work autonomously during the tutorial (T_{awt})	Ensure that students take responsibility for their learning (T_{rl})	Assess students (T_a)
Discipline level (in maths)	<i>Foster students' interest and engagement</i> (T_{ie})	<i>Teach basic maths tools</i> (T_{bmt})	Teach reasoning, justification and proof (T_{rjp})	Foster students' ability to tackle a new problem (T_{inp})	<i>Restore students' self-confidence</i> (T_{sc}) (<i>not for TM</i>)

Table 2. - Didactical types of tasks shared by the teachers at the School and Discipline level. In italics: types of tasks potentially specific for non-specialists.

For the sake of brevity, we develop in what follows examples of praxeologies potentially specific for non-specialist students.

Example of a didactical praxeology at the School level

The didactical type of task "Ensure that students complete personal work" (T_{cpw}) was present in the three interviews but the techniques were different for the three teachers.

TC aims to make her students work *after* the classroom session by distributing a booklet containing many exercises for one chapter (collectively designed with other chemistry colleagues). Solutions are provided online *via* the institutional web environment and an online quiz - which mark counts for the global assessment - has to be filled by all the students. She also sends a reminder 2 days before the quiz is due. Before moving on to a new chapter, she spends 30 minutes during the tutorial to check that the work on the exercises has been done. Moreover in the final assessment all the

skills are evaluated. She explains that since the booklet has many exercises, and the assessment covers all topics, at least some students will need to do the exercises as homework since they do not have enough time during the tutorial. Another technique used by TC for performing T_{cpw} is the organisation of group work during the tutorial. She explains indeed that students pursue the group work after the tutorial and that helps them to achieve their personal work.

For TM, the techniques for T_{cpw} aim more to foster the students' preparation for the tutorials. He asks the students to partly read the course in advance in the handout he has edited for them. If he finds on the Internet valuable videos about the contents to be taught, he posts them *via* the institutional web environment. Before the tutorial sessions, he emails every week or every two weeks the list of exercises to prepare. At the end of a chapter, he provides some solutions of the exercises (hand notes or software computations) *via* the institutional web environment. The techniques used are often justified by TM by the will to save time during the class. The videos also allow him to provide a visualisation of some mathematical phenomena.

TP declares in his interview that he thinks that the students do not work out of the tutorial sessions. Nevertheless the students have exercises sheets; the work during the tutorial concerns only a part of these sheets; a complete correction is provided and can support students' personal work. We consider this as a technique for this type of task, and note at the same time that TP does not trust this technique.

Examples of didactical praxeologies at the Discipline level

To foster students' interest and engagement (T_{ie}), TP chooses contextualised exercises (referring to physics, but also to day life contexts). TP justifies this choice by explaining that it is likely to foster students' engagement, but also that students are used to contextualised exercises at secondary school. Another technique used by TP for the didactical task T_{ie} is to explain to students that they will need these mathematical tools. Some mathematical exercises are inserted in TC's booklet for chemistry. In her interview she declares that she observed during the first semester that starting with these exercises was a mistake. For the second semester she plans to begin the lesson with chemistry exercises that motivate the mathematical exercises that will follow: this is a new technique that she will use for T_{ie} . TM does not make the connection with other disciplines and declares "I would think that we are not really here to foster interest... We are here to make them learn things." Nevertheless, he declares that he uses videos that he appreciates to foster students' engagement; we consider this as a technique for T_{ie} .

To restore students' self-confidence (T_{sc}), the technique used by TC is based on the exercise booklet. It contains many exercises, classified by difficulty level. Students can start with a more difficult exercise but go back to easier exercises if they fail. She justifies the use of this booklet by explaining that students are not confident in their abilities, depending on the options they took in high school (this concerns both mathematics and chemistry, the students are biology majors). TP aims to restore the

self-confidence of his students by filling in their gaps: he attempts to restore their confidence by making them feel comfortable with the basic tools of calculation. He also wants the students to regain confidence in their reasoning, and urges them to check each step of this reasoning to be sure of its correctness. MP did not mention in his interview the need to restore its students' self-confidence.

Concerning the type of tasks "Teach basic maths tools" (T_{bmt}), in the booklet proposed by TC, some "boxes" are entitled "Mathematics reminder". They concern for example 2x2 linear systems, vectors or unit conversions: concepts taught at secondary school. Nevertheless the properties presented and the perspective on the concept is sometimes unfamiliar for the students, we discuss this with an example at the Content level. These "reminders" are sometimes followed by mathematics exercises; they were written by another chemistry teacher. This teacher had a long experience of teaching chemistry to selected students preparing to enter engineering schools, and TC trusts his experience concerning the students' needs in mathematics. TM considers that teaching basic mathematics tools is necessary but does not teach them himself since his engineering school dedicates one week at the beginning of the school year to an autonomous work of students on these basic mathematical tools for students who have just obtained their Baccalaureate (end of secondary school national exam in France). The technique used by TP is to make the students practise many exercises to develop their procedural fluency with basic contents: developing, factoring or solving first-degree equations. TP says in his interview: "the most important is to practice, practice, practice to develop their fluency [...]. It is important that they have the solutions, so that they can try, try, repeat exercises and check if their solution is correct."

Didactical and mathematical praxeologies at the Content level: norm of vectors

In this section we focus on the didactical praxeology used by TC for teaching the mathematical praxeology associated with "compute the norm of the sum of two vectors" (T_{nsv}). This praxeology is needed in chemistry within the theme entitled "polarity of molecules", for computing a "dipole moment". The students' booklet on this theme starts with a "mathematics reminder" about vectors. It includes in particular the general formula for the norm of a sum of two vectors, then it introduces the property presented in Figure 1.

The norm of a sum of two vectors that have the same norm but different directions can be written as:


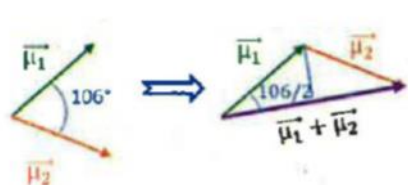
$$\|\vec{u} + \vec{v}\| = 2\|\vec{u}\|\cos\left(\frac{\theta}{2}\right)$$


Figure 1. Extract of the "mathematics reminder" about vectors

This property provides a technique for T_{nsv} , when the two vectors have the same norm. We note that this technique is not taught in mathematics courses, at university or at secondary school. The figure (see figure 1) next to it can be considered as a technology;

but there is no associated text. Further in the booklet, exercise 3 asks for a justification of the formula, in the case of two given vectors (forming a 106° angle). The justification expected is presented in the corrected booklet (figure 2).



$\| \vec{\mu}_1 + \vec{\mu}_2 \|$ is two times the projection of $\vec{\mu}_1$ on $\vec{\mu}_1 + \vec{\mu}_2$

$$\text{With } \cos\left(\frac{\theta}{2}\right) = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{\frac{\| \vec{\mu}_1 + \vec{\mu}_2 \|}{2}}{\| \vec{\mu}_1 \|}$$

$$\text{Thus } \| \vec{\mu}_1 + \vec{\mu}_2 \| = 2 \| \vec{\mu}_1 \| \cos\left(\frac{\theta}{2}\right).$$

Figure 2. Justification expected for the formula of the norm of the sum

This justification implicitly uses several mathematical properties of the triangle formed by $\vec{\mu}_1$, $\vec{\mu}_2$ and $\vec{\mu}_1 + \vec{\mu}_2$. The triangle is isosceles. Its perpendicular bisector is also its height (justification for the projection). Its vertex angle is $180^\circ - 106^\circ$, thus the basis angles are $(180^\circ - (180^\circ - 106^\circ))/2 = 106^\circ/2$. The triangle formed by $\vec{\mu}_1$, $(\vec{\mu}_1 + \vec{\mu}_2)/2$ and the height is right-angled (justification for the cosine formula). For this mathematical task in the chemistry course, the praxeology is not the praxeology that would be expected in the mathematics course. TC confirmed in the interview that the students have not been able to solve this exercise.

We observe in this example several issues associated with the didactical praxeology for teaching T_{nsv} (which belongs to the praxeology at the discipline level T_{bmt}). Firstly, the "reminder" can in fact correspond to new knowledge. Here the property can be proven with secondary school knowledge, but it requires a complicated proof. Moreover students are not familiar with vector projections at secondary school. Second, for what TC identifies as a mathematical exercise, the kind of justification expected is very different from what would be expected in a mathematics course.

DISCUSSION

Are the didactical types of task (table 2) and the associated praxeologies specific for the target public of non-specialist students? In this section we discuss our results in order to answer this question. Our aim was not to compare the three teachers; nevertheless we also present some hypotheses about the differences between the didactical praxeologies they developed for the same types of tasks.

At the School level, we observed five types of tasks shared by the three teachers for their (non-specialists) first year students. During the first year at university, whatever the subject taught, the teachers have to ensure that their students complete personal work (T_{cpw}) out-of-class; that they work -autonomously- during the tutorial (T_{wt} or T_{awt}); that they take responsibility for their learning (T_{rl}). The teachers also need to assess all the students (T_{a}). We hypothesised that the praxeology associated with (T_{cpw}) could be specific for non-specialists students, since these students probably dedicate only a limited amount of time to mathematics. TM asks to prepare exercises before the

tutorial. TC proposes a very elaborated booklet with many exercises whose solutions are available on the course's platform. She also fosters group work. TP proposes long lists of technical exercises with their solutions. This technique does not directly concern the amount of personal work, but its nature: TP wants that the students practice to develop their procedural fluency. It is both a technique for T_{cpw} (Institution) and T_{bmt} (Discipline). Thus we do not claim that we identified at the School level didactical praxeologies specific for non-specialists.

At the Discipline level, we also observed five didactical types of tasks shared by at least two teachers. In our analyses of the interviews, we did not find for the praxeologies associated with "Teach reasoning, justification and proof" and "Foster students' ability to tackle a new problem" elements that could be specific for non-specialists (the analyses are not presented here, due to space limitations). We contend that for the three other types of tasks, the praxeologies are specific.

The type of tasks "Foster students' interest and engagement" (T_{ie}) is specific because many non-specialist students are not motivated by mathematics. Two of the teachers (TC and TP) used as a technique the proposition of mathematics exercises in the context of another discipline, and we consider this technique as specific. Interestingly, TM made no links with other disciplines and chose to propose videos that he appreciated - he also declared that raising students' interest was not his role. This can be a consequence of his mathematical background (it is less natural for him to make links with other disciplines), but also of the type of engineering school and the role of mathematics in it. Indeed his students prepare for a competition; they are obliged to learn mathematics to succeed. The types of tasks "Teach basic maths tools" (T_{bmt}) and "Restore students' self-confidence" (T_{sc}) are specific as some of these students only have a limited mathematics background. Some of them have difficulties in mathematics (in particular TP's students who follow a remediation course); according to TP and TC, most of their students consider themselves as low-achievers in mathematics. As TC says: "they have prejudices about their level in maths [...] they did not take the maths specialty in grade 12, so they feel suck at maths and they don't like maths". TM does not mention the T_{sc} type of task. This is linked with his teaching context: his students were high-achievers at secondary school and are self-confident.

At the Content level, we observed that the mathematical praxeology for "Compute the norm of the sum of two vectors" (T_{nsv}) was different from what would be expected in a mathematics course. Thus the didactical praxeology for the types of tasks "Teach T_{nsv} " (belonging to "Teach basic mathematics tools") is specific for non-specialists. The didactical technique used by TC (and her chemistry colleagues) is "present a brief summary of the mathematical properties needed and propose a few exercises"; the technology seems to include "the mathematical concepts and properties are not new for the students". Nevertheless this raises an issue, since some of the properties are in fact new, and some of the concepts like projections of vectors are not familiar for the students. Sometimes mathematics teachers at the beginning of university are not aware

of what students precisely learned at secondary school; it is even more difficult for a chemistry teacher.

CONCLUSION

We observed in our analyses ten didactical types of tasks at the School or the Discipline level, shared by at least two of the three teachers we interviewed. Three of the didactical praxeologies at the Discipline level were specific for non-specialists students. At the Content level, we only presented one example of didactical praxeology, which was also specific. Drawing on previous works discussing mathematical praxeologies in courses for non-specialists (e.g., González-Martín, 2021), we hypothesise that most didactical praxeologies at this level are specific; indeed the mathematical and didactical praxeologies are co-determined (Bosch & Gascón, 2001). In order to validate this hypothesis, but also to improve our understanding of the didactic stakes in the teaching of mathematics to non-specialists, we plan to pursue our analyses with regard to the links between mathematical praxeologies and didactical praxeologies, to understand better the epistemological dimension of didactical praxeologies for non-specialists.

The theoretical approach we have chosen in terms of didactical praxeologies at three different levels allowed us to analyse the practices described by the teachers and the teaching resources they designed. This first step was needed to examine the specificity of their practices. We contend that this praxeological approach of the teachers' practices can contribute to our understanding of teaching at university level, for non-specialists or for other students. We plan to continue our study with more interviews with teachers intervening in diverse courses for non-specialists, and to observe their courses to confront these observations with the teachers' declarations.

The existence of specific didactical praxeologies for non-specialist students (developed by experienced teachers, in this study), suggest that novice teachers could benefit from a specific training. Research in mathematics education could contribute to formulating propositions for such a training.

NOTES

1. Didactic and Epistemology of Mathematics, interactions with Computer Science and Physics at Tertiary level, <https://demips.math.cnrs.fr/recherches/pratiques-des-enseignants/>

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