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# Resources to question mathematical knowledge: from teacher education to university mathematics

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*Our research is centred on the design, implementation and analysis of inquiry-based teaching proposals based on study and research paths within the anthropological theory of the didactic. We briefly present a study and research path for teacher education (SRP-TE) designed and implemented in a mathematics education course for pre-service preschool and primary school teachers. After describing the main elements of the SRP-TE, we explain how it provides students with tools to question and analyse the mathematical knowledge involved in activities related to numeral systems, randomness, and modelling. We conclude with the possibility to extend the instructional strategies and resources designed for SRPs-TE to more general SRPs intended to teach mathematics outside the teacher's educational setting.*

*Keywords: Teachers' and students' practices at the university level, study and research paths, teacher education, resources, anthropological theory of the didactic.*

## STUDY AND RESEARCH PATHS AND TEACHER EDUCATION

From the approach of the anthropological theory of the didactic (ATD), our education systems today are situated closer to the so-called pedagogical “paradigm of visiting works” (Chevallard, 2015). This paradigm is mainly characterised by the introduction of a set of knowledge works teachers present to the students for them to study and know – or visit – its main characteristics and usages. The counter-paradigm of “questioning the world” helps analyse the limitations of the visiting works paradigm and study its potential to evolve. The paradigm of questioning the world focuses teaching and learning processes on the research and study of open questions. The visiting of works does not disappear, but is subjected to another purpose: the study of questions. That is, knowledge works are important insofar as they help develop answers to the questions addressed: their value is not intrinsic, but functional.

Several investigations within the ATD focus on studying the conditions that can facilitate the transition from the paradigm of visiting works to that of questioning the world, especially in the case of university education (Barquero et al., 2021). In this perspective, the proposal of *study and research paths* (SRPs) (Bosch, 2018) has been implemented in the past decades at different school levels. SRPs are instructional proposals specific to the paradigm of questioning the world that start with the approach of an open question not initially associated with a particular answer or piece of knowledge. Implementing SRPs requires creating new conditions in current educational systems, especially at the level of the didactic contract. Running them helps

observe the institutional constraints that emerge during their implementation and hinder their development.

SRPs have also been proposed to support teacher education in what we call *study and research paths for teacher education* (SRP-TE). An SRP-TE also starts from a problematic question, in this case, related to the teaching profession. The initial proposal of (Ruiz-Olarría, 2015) starts by carrying out an SRP related to the teaching question addressed, and using the analysis of the SRP experienced as a tool for the design and eventual experimentation of an adapted SRP (Barquero et al., 2018). We consider SRPs-TE especially important to support collaborative work between researchers in didactics, educators, and teachers in the study of the conditions for the change of paradigm and the co-creation of instructional tools to design and implement new teaching proposals.

Despite the ill-defined position of teacher education in the context of university teaching and research, it seems appropriate to approach SRPs and SRPs-TE from a common perspective. On the one side, SRPs-TE can be considered as a specific kind of SRPs for degrees in teacher education, not different in essence from SRPs implemented in Engineering, Economics, Medicine or Administrative degrees. At the same time, the specificities of SRPs-TE and the fact that they include the design and analysis of an SRP can shed new light on the study of the institutional conditions needed by the paradigm of questioning the world.

In this paper, we start by presenting the specificities of the SRP-TE as a university training device. We then briefly introduce three cases of SRPs-TE that have been designed and implemented for pre-service primary school teachers at the University of Barcelona, and focus on the one concerning the teaching of inferential statistics in primary school. We are particularly interested in showing how, throughout the SRP-TE, educators transpose (from research to teacher education) some epistemological and didactic tools that help teachers manage didactic processes. We conclude with the possibility to extend the instructional strategies and resources designed for SRPs-TE to more general SRPs outside the teacher's educational setting.

## **AN SRP-TE ABOUT RANDOMNESS**

Five modules structure the generic proposal of SRPs-TE as described in (Ruiz-Olarría, 2015; see also Barquero et al., 2018). However, each case requires its own adaptations according to the conditions of the instructional practice: sometimes not developing all the modules or splitting some of them. They are defined as follows:

*Module 0:* Introduction of an initial question  $Q_{0-TE}$ , which is the starting point of the SRP-TE and which will progressively be addressed in all the modules. This initial generating question of TE is related to a teaching problem, and usually takes the form of *How to teach a content ♥?*

*Module 1:* Carrying out an SRP that appears as a possible answer to  $Q_{0-TE}$ . It is experienced with participants assuming the role of students, and the educator guiding its implementation.

*Module 2:* Analysing the SRP experienced from a didactic perspective.

*Module 3:* Adapting the design of the SRP and implementing it in a given school setting; observing and collecting data for further analysis.

*Module 4:* Analysing the SRP implementation and identifying its potential and limitations as a response to the initial question  $Q_{0-TE}$ .

Several SRPs-TE have been implemented for pre-school, primary, secondary and university teachers (Barquero et al., 2018). A common aspect of all these proposals is that they start by carrying out an SRP with pre-service teachers (module 1). In some cases, they are based on an SRP previously implemented with students; in other cases, the starting point is an SRP newly designed for the teacher education context.

In the case of the SRP-TE we are considering, the initial question  $Q_{0-TE}$  is related to the teaching and learning of inferential statistics in primary school. This SRP-TE has been implemented since academic year 2014/15 at the University of Barcelona in the last compulsory course of Didactics of Mathematics (first semester, 6 ECTS) with groups of about 50 pre-service teachers. The participants are fourth year students of the degree of primary school teacher education. They work in groups of 3-5 members during the whole course. This is not the only SRP-TE implemented in this same course or in previous courses. Table 1 summarises the SRPs-TE that have been implemented under similar conditions, including the generating question  $Q_{0-TE}$ , the modules implemented, if the SRP-TE is based on a previously experimented SRP or not, and the approximate hours of work in the classroom.

Table 1: SRPs-TE implemented in the Didactics of Mathematics course

University year (over 4)	Generating question $Q_{0-TE}$	Implemented modules	Available SRP previously implemented	Duration
2	How to teach the rationale and usefulness of positional numeral systems?	0, 1, 2	No (Sierra, 2006)	30 hours
4	How to teach randomness and statistics in primary school?	0, 1, 2	What is hidden inside the bottle? (Brousseau et al., 2002; Granell & Barquero, 2019)	10 hours
4	How to teach mathematical modelling in primary school?	All	The cake box (Chappaz & Michon, 2003)	40 hours

The first SRP-TE is based on the work of Sierra (2006) that proposes to approach additive, additive-multiplicative and positional numeral systems from the analysis of their properties to not only represent numbers or quantities, but also order them and operate with them. The last SRP-TE takes a situation regarding building boxes with paper based on Chappaz and Michon's proposal as the initial question. The need to build boxes of different sizes to answer a baker's demand motivates the consideration of empirical, geometrical, numerical, and algebraic models (Wozniak et al., in press). Its analysis leads to considering specific notions to describe modelling processes, and to recognise the role of experimental work in mathematics.

The second SRP-TE is the one we are considering in more detail. Figure 1 summarises the content of its modules.

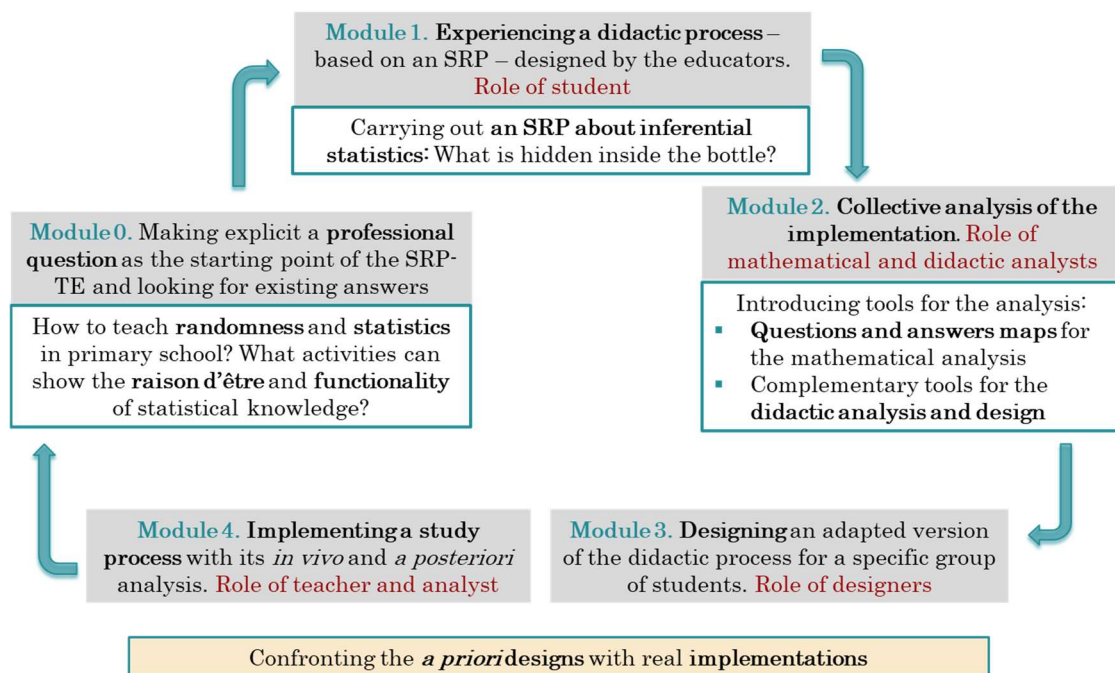


Figure 1: Modules of the SRP-TE about randomness and statistics.

In module 0, the following initial generating question  $Q_{0-TE}$  is proposed: *How to teach randomness and statistics in primary school? What activities can show the rationale and functionality of statistical knowledge? How to design and analyse them?* The teachers are invited to search for available answers in the different accessible media (books, textbooks, curricula guidelines, etc.), which eventually include some instructional proposals coming from educational research about the teaching and learning of statistics in primary school. From this analysis, the teachers often conclude that most of the proposals are focused on introducing techniques for manipulating data for their graphical representation and the calculation of numerical statistics. There are not many activities that propose building an appropriate experimental milieu to deal with statistical variation.

In *module 1*, the teachers are asked to act as students, and experience a teaching project based on the proposal of an SRP entitled “What is hidden inside the bottle?”. This project comes from the activity designed by Brousseau et al. (2011) consisting of a long process of didactic engineering design, planned to be experienced during 26 sessions with grade 5 students. In its adaptation for this SRP-TE, we only implemented three sessions of two hours with the prospective teachers as an introduction to this project. The situation is presented by some coloured balls hidden inside an opaque bottle with a small hole in the lid. The experiment consists in predicting how many balls of each colour there are in the bottle and of what colour. We know the total number of balls inside the bottle, but we do not have access to them. The only way to collect data is by shaking the bottle, and registering the colour of the ball appearing in the hole of the lid.

The initial questions are “What is the number of balls of each colour inside the bottle? What hypotheses can we formulate about its content and colour distribution? How reliable are they?”. These same questions are presented in three different situations. The first one consists of having a bottle with 5 balls of two colours and no restriction with regard to data registration. In the second, each working team designs a bottle with 5 balls and 2 colours,. They exchange bottles with another team, and they ask the questions to be answered by the team receiving their bottle. In the third case, the bottles have 25 balls of 4 different colours. When implementing this activity, the student teachers are asked to report about their work and to explicitly describe the tasks and questions they deal with, the kind of strategies they use, and the answers obtained. At the beginning of each session, we share these reports in a common forum to have information about all the questions addressed and the answers provided.

*Module 2* consists in the collective analysis of the implementation. The teachers are asked to change roles to become “mathematical analysts”. Some specific terms related to randomness and data analysis are agreed upon at this moment such as “event”, “sample”, “sampling universe”, “sample size”, “frequency”, “likely”, “unlikely”, “possible”, “impossible”, “hypothesis”, etc. The educator also introduces one of the main tools at this stage: the *question-answer maps* (Q-A maps) for the mathematical analysis of the process followed, consisting of a diagram of all the questions, answers and strategies appearing in the activity experienced and their connections (Figure 2).

Later, Q-A maps become a key tool for the analysis of the student teachers’ own experience of the SRP, as well as reference models to analyse the students’ work experienced in primary school. The strategy the educator adopts here is to share with students some classroom experiences with a similar SRP as the one developed in primary school to contrast their analysis with real students’ work. To do this, the educator counts on the case of “The COSMOS project: ordering Hama Beads for the school” (Granell & Barquero, 2019), which was implemented in grades 4, 5 and 6 at *Col·legi Sant Lluís*, a primary school in Barcelona. The teacher is a former student of the University of Barcelona who continues to collaborate with the group of researchers-educators. At this point, the pre-service teachers are introduced to the work the teacher

prepared in collaboration with our research team regarding the design of the project. It includes the following materials: the *a priori* designs of the SRP and the lesson plans guiding the implementation, the students' answers to some activities, and video recordings of some parts of the sessions with grade 4 students.

A2-Recorregut d'Estudi i Investigació per a Cicle Inicial

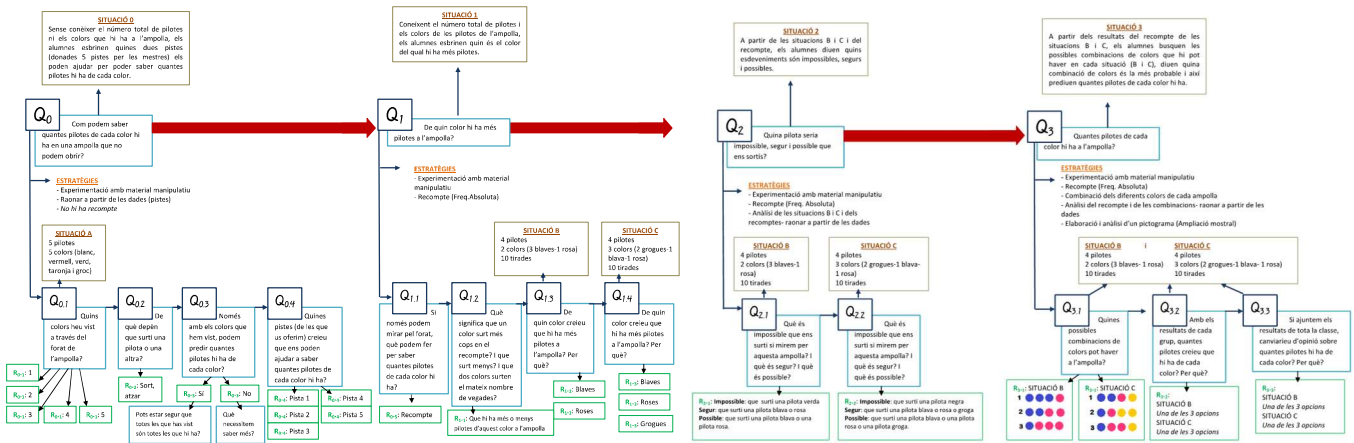


Figure 2: Example of a Q-A map proposed by a working team

## SRPs-TE AND THE PARADIGM OF QUESTIONING THE WORLD

The general structure of an SRP-TE relies on several key features that are inherent to the paradigm of questioning the world. The first one is to start the SRP-TE with a professional teaching question  $Q_{0-TE}$  the student teachers are asked to address together with the educators (who are, in this case, also researchers in didactics). The second one is that the final “product” is not supposed to be a previously known answer, as the questions addressed are open questions in mathematics education. Along with the modules of the SRP-TE, the aim is to initiate collaborative work between teachers and educators to search for already available answers for  $Q_{0-TE}$  (in the form of teaching proposals), and to analyse them as potential teaching proposals. This collaboration leads to sharing, studying and building *mathematical knowledge for teaching* about the topic at stake concerning the initial and derived questions, which might include some “visits” of mathematics or didactics works.

Despite the specificities of the teacher education context, some SRP-TE features are also important for other SRPs (without TE).  $Q_0$  might be an open question, of interest to the university context in which it should be posed and addressed. In the context of teacher education, as the profession behind is clear, searching initial generating questions legitimated by the profession does not seem difficult. On the contrary, what seems difficult to manage is their openness and the temporary and progressive character of the always partial and evolutive answers built. In other university contexts, the legitimacy and pertinence of possible  $Q_0$  is a crucial aspect to be discussed by researchers and teachers, as well as with students. What seems easier to manage is the openness of the initial questions and the convergency or delimitation towards the



answers produced. A strategy used on several occasions is to introduce an external demander  $Z$ , who is the person or group of people that comes up with the assignment (Barquero et al., 2021).  $Z$  can be also the recipient of the final answers, and the person or people responsible for evaluating and validating their pertinence. In the case of teacher education, the collaboration with an external validator  $Z$  could include agents from the teaching profession, or producers of teaching materials and resources.

Moreover, the collaboration between students (pre-service teachers) and teachers (educators) is productive as far as they jointly advance in establishing some *shared knowledge* about  $Q_0$  and producing responses to  $Q_0$  (or its derived questions). This is another important aspect of SRPs that collides with the traditional contract of the paradigm of visiting works. In an SRP, the teacher does not know the answer to  $Q_0$  in advance. As  $Q_0$  is an open and sometimes ill-formulated question, an answer does not always exist: it is defined and built during the enquiry process.

The other issue is how students and teachers can share, talk about, and refer to the knowledge they are constructing before getting the final answer to  $Q_0$ . In this regard, an important feature of SRPs-TE is that the *analysis of the SRP* is explicitly included as part of the enquiry. Therefore, educators introduce *epistemological* and *didactic tools* as far as they enable teachers to tackle the questions raised and elaborate answers to them. The didactic knowledge provided to pre-service teachers is not presented beforehand – which is the common strategy in the paradigm of visiting works – but introduced “on-demand” and motivated by its utility to address teaching questions. An important aspect in this regard is that educators are not supposed to teach future teachers “what and how to teach”, but to help them in their search, construction, experimentation, analysis, assessment, etc. of different teaching proposals.

Another characteristic of an SRP-TE is that its module 1 – “experiencing an SRP” – aims to create a rich *empirical milieu* shared by teachers and educators to collectively identify and analyse some of the school conditions and constraints affecting the didactic designs and teacher actions. The description of the SRP that has been carried out generates the necessity of new tools (outside the traditional content description) for the mathematical analysis of this open activity. To address this necessity, educators introduce Q-A maps as an epistemological tool to analyse the dynamics of knowledge production. Q-A maps are very well accepted by teachers who use them not only as an epistemological tool for the *a posteriori* analysis of study processes but also for their *a priori* analysis—when they design the lesson plans to anticipate and evaluate possible paths to be followed by students. They also employ them in the *in vivo* analysis—when they use them as a tool to analyse real-time study processes for primary school students.

Finally, a specificity of SRPs-TE is that they include carrying out an SRP and its subsequent analysis. To distinguish between the two different paths, the strategy followed in the SRP-TE here presented consists of asking the participants to assume different roles during the instructional process. They start with the role of the “student” addressing an open question, then the role of the “analyst” of the study process that has just been followed, and finally the “teacher” role, including design and analysis



responsibilities. This role-playing appears to be a successful strategy to facilitate the identification of different types of “mathematical” and “didactic” analyses, and to approach  $Q_0$  from different angles, which can seem complementary but is necessary. These different perspectives on a single SRP, a more executive one and a more reflexive or analytical one, can also be extrapolated outside SRPs-TE. In an inquiry process of an initial question  $Q_0$ , the first aim is to produce an answer to  $Q_0$ . However, there is also a secondary more reflexive or “methodological” aim about the process followed, and the by-products that are worth preserving for further inquiries.

## **CONCLUSIONS: LESSONS FROM SRPs-TE TO NOURISH UNIVERSITY SRPs**

As they are part of university education, teacher education degrees are faced with the same instructional problems as other degrees. Because researchers in didactics are usually involved in teacher education, they tend to consider these educational processes to be of a different nature. Being researchers, this was our spontaneous attitude in our research on SRPs, which were treated separately from SRPs-TE. At best, as mentioned before, some results of SRPs were introduced into SRPs-TE, like the use of Q-A-maps, and the exploitation of previously implemented SRPs within the SRP-TE process. What happens when we adopt the opposite perspective, and try to see which aspects of SRPs-TE can be passed on to SRPs? What can we learn from the teacher education experiences that could be useful—or at least worth trying—in other types of degrees?

The role-play strategy used in SRPs-TE can be taken as an example. It is motivated by the need to differentiate between carrying out a study process (the experimented SRP) and its description and analysis. This helps introduce an *in vivo* analysis during the SRP where the educator (acting as a teacher) comments on the strategies followed by the student teachers and by the teacher’s own strategy. In other words, SRPs-TE facilitate a type of analysis of the study process that reveals to be useful for managing an SRP: we can easily talk about the work that is carried out, which helps control it and better understand what we are doing and where we want to go. Therefore, the possibility of translating this *in vivo* analysis to university SRPs can be considered with the aim of improving their management and outcomes. At the end of the SRP, when the final question has been produced, a kind of complementary report about the process followed to produce the answer could be introduced. This is a very common type of professional process related to the need for accountability, assessment, and quality control that is rarely transposed in the classroom.

There is a more subtle issue related to the previous one. In SRPs-TE, the analysis of the SRP carried out is performed with didactic and knowledge resources that are sometimes elaborated for this purpose by research in mathematics education. A previously mentioned example are Q-A maps, but we can also think of less generic tools such as the description of data analysis or modelling processes, which require specific terms that are not always covered by the traditional mathematical terminology.

The institutionalisation of these tools that help describe and better understand (and control) the process followed is easier in the context of teacher education because the content at stake is not only “mathematics”, but also “didactics of mathematics”. It thus gives more flexibility to the educator to elaborate discourses about the mathematical activity “in process”, and also about the final answer produced. It is, however, more difficult to do so in another kind of degree when the teacher is supposed to teach “statistics” or “mathematical modelling” and not “a discourse about statistics” or “a discourse about modelling”. As if the difference between “mathematical” and “a discourse about mathematics” was so clear, or as if mathematics could be done without having a discourse about mathematics. In teacher education, it seems legitimate to teach Q-A maps and other epistemological resources elaborated by didacticists for the analysis of mathematical activities. It does not seem so legitimate in other kinds of degrees, although it is equally necessary.

Approaching university SRPs from the perspective of SRPs-TE helps us think about new strategies to develop. It also reveals new constraints linked to the nature of the knowledge to be taught and the aim of the instructional process: how the knowledge or activity at stake is conceived by the educational institution, what legitimates a given instructional process, etc. It also questions the boundaries between didactics and mathematics. This is a boundary that does not exist in teacher education and that may hinder rather than clarify what is done and what can be done in university education.

## **ADDITIONAL INFORMATION**

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