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“exponential” as just “another word” to say “fast”: When colloquial discourse tells only part of a story

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As part of a research and development programme that examines intra- and extra-mathematical visibility of mathematics especially for students enrolled on a Bachelor's in Education, most of whom will soon train as primary teachers, I made “exponential growth” the focus of one part of the summative, portfolio-based assessment in a Mathematics Education course. Commognitive analyses of students' portfolio entries revealed a significant majority of student narratives about “exponential” as roughly synonymous to “very fast”. Here, I show evidence of how students navigate across literate and colloquial narratives about exponential growth, and I discuss how intra-mathematical deficits in these narratives may play out adversely in the students' capacity to recognise and engage with its manifestations in colloquial situations.

Keywords: Mathematics Education courses, social significance of Mathematics, commognition, exponential growth, literate and colloquial discourses

“If you look at the number of in-patients [...] on 7 September there were 536 cases. By the time you get to the beginning of October, it is over 2,500. As of today, it has breached 10,000 people in hospital. You do not need too much modelling to tell you that you are on an exponential upward curve of beds.”¹; “it's amazing how many clever people don't know what exponential growth means”² Chris Whitty, Chief Medical Officer for England

INTRODUCTION

Throughout the pandemic, public discourse about Covid-19 – in the UK, led primarily by daily, televised conferences of the Government's Chief Medical Officer and his Deputy as well as the Chief Scientific Adviser – brimmed with mathematical references. Amongst those was the abundantly used reference to “exponential growth”. As debates raged about whether and how to convince the public of the utter necessity for the personal, social and economic sacrifices that tackling the virus implied, a stark realisation started to emerge: that many of these references may not have the impact that the scientists who were making them were hoping to achieve. In tandem with findings from research that indicated how invisible mathematicians and mathematics often seem to be (Yeoman, Bowater & Nardi, 2017; Nardi, in press), I conjectured that relentless exposure of the public to said mathematical references may make some difference. To explore this conjecture, I made “exponential growth” one focus of the summative assessment in a Mathematics Education course I teach to final year BA Education students. Here, I report analyses of data I collected during this assessment.

¹ Prof. Whitty's evidence to Parliamentary Committee, 3 November 2020: transcript at <https://t.co/PjjRQBt3L>

² Plenary at [LGA/ADPH Annual Public Health Conference 2021: Rising to the challenge parts 1 and 2, 23 March 2021](#) | [Local Government Association](#),

In what follows, I discuss the background and rationale of the research and development work I am conducting in the context of the course's design, implementation and evaluation. I then outline the commognitive underpinnings of this work – with a focus particularly on pertinent constructs such as *literate* (as in, e.g., mathematical) and *colloquial* (as in everyday, public) *discourses* (Sfard, 2008; p.118). I then introduce the context, participants and data collection methods I use in this work. Subsequently, I present data and analysis relating to students' narratives about exponential growth as they emerged from aforementioned summative assessment. I conclude with reflections on what this analysis may imply for university mathematics pedagogy, especially for students on the cusp of entering the teaching profession.

INTRA- AND EXTRA- MATHEMATICAL VISIBILITY OF MATHEMATICS

Visibility of mathematics and mathematicians – whether, for example, in popular culture (Nardi, 2017) or in the narratives of secondary school students (Yeoman *et al.*, 2017) – is becoming a focus for mathematics education researchers as well as science communication scholars. “What does a mathematician do?” was one question that Yeoman *et al.* (2017) asked during focused group interviews with secondary students in a study that investigated student narratives about how research in various disciplines is conceived, conducted – and where its utility and significance lie. Evidence from the study (group interviews with 100 students aged 11-19, questionnaire responses from 2634 students) on whether “Mathematicians do a lot of research” – and examples of said research – was alarmingly scarce: examples of research in mathematics which participants considered worthwhile elicited *zero* responses (Nardi, in press).

Yeoman *et al.* (2017)'s evidence corroborates the claim that mathematics and mathematicians are relatively invisible and that public narratives about mathematics (including those of young people still in their schooling years) remain poor (Nardi, 2017). These findings also resonate with my experience as the faculty member in charge of *Children, teachers and mathematics: Changing public discourses about mathematics*, an introductory, optional course in Research in Mathematics Education (RME), to final-year BA Education students that I designed and have been delivering since 2012. The course aims to welcome Education undergraduates (of whom about three quarters will soon enter primary teacher preparation courses) into RME. Particularly, the course aims to trigger revisiting the students' own, often traumatic experiences of learning mathematics and to help them overcome their reticence about their mathematical ability. Over the years, from a relatively narrow exercise in tackling disaffection with the subject, the course has evolved into a platform on which student narratives about what mathematics is and what it is for are regularly challenged. The course is assessed through a portfolio of learning outcomes (see Course section of this paper): this invites responses to ten tasks that include biographical accounts of their mathematical experiences, responses to *mathtasks* (Nardi & Biza, in press) and “maths pitches”, snappy narratives which present an important piece of mathematics briefly and clearly to a member of the general public.

With Research Ethics Committee permission and student consent, student portfolio responses are analysed via a frame consisting of a typology of four characteristics informed by the Theory of Commognition (see next section of this paper).

As reported in (Nardi, in press), over the years, I observed that students' choices of topic for their "maths pitch" invariably focused on classical topics from mathematics created in the distant past. Consequently, I started requesting two pitches, one entitled "mathematics over time" and one entitled "mathematics today". In the dataset collected from the first student cohort with which I trialled the request for a dual pitch, the vast majority, over 80%, ignored the "today" in the brief and simply presented a bit of ancient mathematics, sometimes disguised as having contemporary relevance (say, Pythagoras' Theorem in the construction of buildings). This, I label a "fossil" narrative about mathematics. And, I label the few concretely described uses of mathematics in engineering, environmental science, economics, medicine and technology evidenced in a handful of portfolio entries a "fuel" narrative about mathematics.

What this, and other, portfolio evidence suggests resonates with Yeoman *et al.*'s (2017) conclusion that, to these undergraduates – now at the cusp of university graduation and entering the world of work, including the world of the classroom – mathematics remains largely invisible. I concur on this matter with (Herbst *et al.*, 2021) in wondering: should we not, as university mathematics educators, do better at enriching these professionals-soon-to-be narratives about what mathematics is, and what it is for?

Spurred on by the aforementioned omni-presence of references to "exponential growth" in pandemic-related public announcements, I invited students' "pitches" on this quintessential mathematical object in one recent portfolio task. I did so in awareness of research findings that alert us to documented student difficulties with this topic. For example, Ellis *et al.* (2016) recap findings about: university students' struggle with rules of exponentiation and with connecting these to rules for logarithms; and, about secondary students' struggle with the transition from linear to exponential representations and with identifying what makes data exponential. They also report that teachers find instruction about exponential growth challenging, particularly of how exponential growth as repeated multiplication connects with the closed-form equation.

Crucially to the focus of this paper, Ellis *et al.* (2016) also include in these challenges the recognition of growth as exponential in nature and the generalization of rules such as the multiplication and power properties of exponents. Overall, however, the bulk of prior research into the challenges posed by the learning and teaching about exponentiation has an intra-mathematical focus. The focus of the discussion I propose in this paper is on the boundaries of intra/extra-mathematical consideration of the mathematical object "exponential growth": how may manifestations of exponential growth feature in colloquial situations? How may students / teachers / members of the public navigate across literate and colloquial narratives about exponential growth? And, how may intra-mathematical deficits in narratives about exponential growth play out adversely in our capacity to recognise and engage with its manifestations in said colloquial situations?

COMMIGNITION, MathTASK AND FOUR CHARACTERISTICS

The theoretical perspective of this work is discursive and draws on the commognitive framework (Sfard, 2008) that sees Mathematics, Education and Mathematics Education as distinctive discourses and learning as communication acts within these discourses. In resonance with this discursive lens, the design of the Mathematics Education course that is the focus of this paper aims, *inter alia*: to attend to discursive differences, and potential conflicts, between Mathematics and colloquial discourses that students may experience; and, to assist students towards smooth navigation across these discourses.

A further, also commognitively-informed, element of the theoretical perspective of this work comes from the pre- and in- service mathematics teachers focussed [MathTASK](#) programme. In MathTASK, we engage teachers with fictional but realistic classroom situations, which we call *mathtasks* (Biza et al., 2018). Mathtasks are presented to teachers as short narratives that comprise a classroom situation where a teacher and students deal with a mathematical problem and a conundrum that may arise from the different responses to the problem put forward by different students.

Towards the analysis of student data collected during the delivery of the course, I deploy a typology of four interrelated characteristics that emerged from themes identified in prior MathTASK research (see detailed rationale, definitions and examples in Biza et al., 2018; Nardi & Biza, in press) and is tailored to aforementioned commognitive underpinnings: *consistency*; *specificity*; *reification of RME discourse*; *reification of mathematical discourse*. This typology is the basis for the assessment frame deployed towards the formative and summative assessment of the students' work during the course. Each one of the typology's four characteristics encapsulates features that can be traced firmly and concretely in the students' writing. So, for example, for reification of mathematics discourse, I scrutinize student portfolio entries in terms of how specific, relevant, reliable and accurate the use of mathematical terminology is. I ask questions such as: are mathematical terms used accurately? Is there direct relevance of a mathematical utterance to the claim being made? Is the utterance consistent with mathematical theory? Is the link between the utterance and the overall claim made explicit? Are any credible sources for the mathematical utterance quoted? Etc.

Seeking such evidence in the students' responses not only secures a verifiable (by course moderators and external examiners) route to a student's course mark; it also paves the way for identifying which student narratives about the teaching and learning of mathematics (and RME) subsequent versions of the course (and research thereof) need to challenge, and, hopefully, change. As part of my larger investigation into the visibility of mathematics in extra-mathematical situations, the portfolio item (the mathtask in Figure 1) and the analysis of corresponding portfolio entries that are the focus of this paper aimed to trace how students navigate colloquial and literate narratives on one mathematical object, "exponential growth", and whether, and how, students recognise mathematics in colloquial situations.

COURSE, PARTICIPANTS AND PORTFOLIO-BASED DATA COLLECTION

The course is part of the BA Education (BEd) programme's suite of optional courses. Contact time is four hours per week (two for lectures and two for seminars) for a period of twelve weeks. Lectures are teacher-led and partly interactive. Seminars are student-led. As about three quarters of the programme's graduates continue into training to become primary teachers, the course is designed to address directly the widely reported reticence of those students towards mathematics and their generally low self-esteem in mathematics. Its aim is to equip these students with the means to tackle the disaffection that often tantalises the relationship with mathematics experienced by themselves as well as the young people many of them will soon be preparing to teach (Nardi, 2017).

The inception of the course stems from acknowledging that the preparation of teachers rarely equips them for this complex task – and its twelve weeks of lectures and seminars are organized to address in-school (curriculum, classroom) and out-of-school (media, popular culture, arts) discourses on mathematics. In the Portfolio of Learning Outcomes, the course's single item of summative assessment, students are asked to: return to ten activities they prepared for in the weekly seminars; study the materials accumulated during the twelve weeks of the course; and, compose a revised contribution to each one of the ten activities, written in the light of what they learnt during those twelve weeks. The headings of the ten activities are:

1. *Mathematics and I*: A biographical account of your relationship with mathematics
2. *Mathematics in the media*: A brief analysis of a mathematics-related media excerpt (paper press or online)
3. *School mathematics and I*: Reflections on one aspect of the primary or secondary mathematics curriculum
4. *Mathematics over time*: A 2-minute Maths Pitch from the history of mathematics
5. *Mathematics today*: A 2-minute Maths Pitch on a contemporary application of mathematics
6. *Mathematics in the classroom*: A brief analysis of a classroom incident (with mathematical, social, affective, meta-mathematical elements)
7. *Mathematics in art and popular culture*: A brief analysis of a mathematics-related art or popular culture excerpt (film, TV, theatre, literature, arts, music)
8. *Mathematical ability on film*: A brief analysis of the portrayal of a mathematically able character on film
9. *Myths about maths*: A brief essay, with evidence, debunking myths about maths (such as Innate, Male, Introvert, Burn Out, Uncreative)
10. *Mathematics lesson plan*: A plan for a mathematics lesson on a topic (of each student's own choice).

Students are asked to deploy RME theoretical constructs introduced during the course: see (Nardi & Biza, in press) for examples of these across developmental, sociocultural, anthropological, embodied and discursive theories. They are also expected to refer to a small number of research papers (and, where needed, other publications such as policy documents, reports or media excerpts) in each part. An example of a mathtask students were asked to respond to in Part 6 of a recent portfolio is in Figure 1. In this paper, I focus on student responses to question 1, mainly the “exponential growth” part.

Ms Jones is about to start a mathematics lesson in a Year 6 class (student age 10-11). As she walks into class, she notices two students arguing about whether wearing a mask during the pandemic makes sense. One of the students, Neil, complains as follows:

Neil: Ms, I hate these! Why do we even care? Only one percent of people will end up in hospital anyway. Let's do the maths! One percent is almost nothing!

Ms Jones: Ok, Neil, yes, let's do the maths. [*to the class*] Any ideas anyone?

Anna: Well, I never liked percentages. They are so vague. Ok, here is my bag of crisps [*she takes a Kettle© chips 30 gr bag out of her backpack*]. And here is the one my dad asked me to bring him on my way back from school for tonight's game on TV [*takes Kettle© chips bag of 150gr out of her backpack*]. I will take one percent of this bag [*rattles 150gr bag*], any day, Neil!

Barack: Yeah, ok, unless the big one is almost empty and the little one is sealed and full [*laughs*]. And, Neil said let's do the maths. He didn't say let's talk about crisps ... Neil, you know what? In her silly crisps and the like kind of way, Anna has a point. One percent of what? If one hundred people get infected, then one person will go to hospital. We can sort of deal with that, right? [*writes 1 on his whiteboard*]. If one hundred thousand people get infected, then one thousand people go to hospital [*writes 100,000 and 1,000 on his whiteboard*]. Hm... and if one million people get infected [*writes 1,000,000 on his whiteboard*] ... You see where I am going? Oh, plus infections are doubling every day... I heard my big sister say something about a thing called expo. ... something growth, sounded like a tumour ... Scary. I am telling you. If we don't do something, we are doomed to be stuck with these masks and all for ever!

Clive: OMG, Barack! You and your depressing speeches! Too many zeros are giving me a headache by the way. I just want to ask Anna: do you *really* think this big bag of crisps in your backpack will still be there by lunchtime!?

[*The class erupts with laughter.*]

You are the teacher and you just heard what Neil, Anna, Barack and Clive said....

1. Here is a sentence that sounds like ones that we have been hearing in the news in the last twenty months or so: "*About 1% of infected people will need hospitalisation and the growth of infections is exponential*". How do you explain this sentence to someone who does not know what "percentage" and "exponential growth" mean?
2. How would you respond to Anna?
3. How would you respond to Barack?
4. How would you respond to Clive?
5. How would you respond to the whole class – also in the light of Neil's initial comment – and conclude the lesson?

Figure 4. The *Percentages, exponential growth and masks* mathtask from a Portfolio of Learning Outcomes. The focus of this paper is on Question 1.

“PLAIN” AND “FASTER” NARRATIVES ON EXPONENTIAL GROWTH

The focus of this paper is on 25 student entries (2 were non-responses). I grouped the students' narratives on exponential growth in the remaining 23 as follows (Table 1).

“plain growth” narratives (5 entries). In these entries, exponential growth is described as “a quantity increasing as time goes by” or as “something [that] constantly increases over time”. 3 entries contextualise the response in the case of a virus, noting that exponential growth indicates “the growing rates in infections” (with 2 specifying “over time”). Entries in this group tend to be low in specificity: none, bar one, includes reference to a particular numerical example, dataset or graph. The one that does (“...it is a *growth rate that becomes larger in relation to the total growing number* (a), [my emphasis] [...] 1% become infected and this percentage is *growing every day* (b) *as the rate is exponential* (c)”) sets out with two promising utterances – a and b – but is let down by a circular warrant, c, where the term “exponential” is used to define itself.

“faster and faster growth” narratives were discerned in the remaining 18 entries: 15 were solely verbal; and, 3 were reliant on closely connected graphical and verbal realisations of “exponential growth”. Amongst the 15, there were 5 consisting of exclusively general statements and 10 included specific examples. Entries in each category (of 5 and 10) were differentiated as per type of growth each described.

“faster and faster growth” narratives (verbal, general: 5 entries) sometimes stand on an ambiguous boundary with “plain growth” narratives. For example, in “[t]he number of infected people will *increase* (a), as they will likely pass the infection to *several people* (a), leading to a faster and faster increase (b) in case numbers”, I see (a) as alluding to a “plain” and (b) to a “faster and faster” narrative. Another student quotes the *Organisation for Economic Co-operation and Development* (OECD)'s [glossary](#) (“a circumstance in which growth compounds continuously at every instant of time”) but concludes their sentence with the “plain growth” utterance “meaning rates of infection are continuously growing”. Another student, even though in principle she would set out from asking her interlocutors for examples of “when we see [exponential growth] in everyday life” and she would “provide a definition and examples”, she then simply quotes the *Centre for Evidence-Based Medicine*'s [glossary](#) (“when the speed of growth is proportional to the size of the population”) and qualifies no further. A similar, if convoluted, attempt is in another student's “...a *specific way* (a) that numbers can increase over time. Described as a function, a quantity that undergoes *exponential growth is an exponential function of time* (b), that is, the variable representing time is exponential (*in contrast to other types of growth, such as quadratic growth*) (a).”: I see (a) as worthy attempts to highlight that exponential growth is not just any growth let down by (b)'s circular reasoning. All, bar one of these five entries, make no reference to any other realisations of “exponential growth”. The one that does (“... when the *rate of increase becomes faster as the population increases* (a). The *more people with covid* (b) the faster the rate of infection therefore a sharp curve would be *seen on a graph* (c).”) includes: a verbal, but of low specificity, reference to a graph (c); an attempt to capture the speed of increase (a); and, an attempt at contextualising (b).

‘faster and faster growth’ narratives (verbal, example-based: 10 entries) were a significant majority. Most of these 10 entries evidence a narrative that exponential growth is a particular type of growth (“another word which can be used to describe something that is fast. More so, it describes how something can grow more quickly over time”). I saw variable attempts to locate this particularity. In “...when an event increases more and more so the increase gets faster over time”, the student captures the difference of speed in the increase as “increasing at a faster rate than time is passing” and “the speed it will infect people will get faster” (two other students wrote similarly). These efforts are however followed by examples such as: “that on day 1 one person is in hospital day 2, two more people are in hospital and on day 3 ten more people are in hospital”; or, “if 5 people were reported as infected today and tomorrow 10 people were infected”. Such responses are quite far from identifying the exponential by which the growth takes place (noted in 4 of the 10 entries). What kind of increase exponential growth means is captured a bit more in another student’s entry who notes that “exponential growth refers to the power of the epidemic” and specifies with the example “[w]hen there are two people infected, then the number of infected people is no longer 2 but 4 people”. Other attempts at capturing “exponential” include: the syntactically challenged – and potentially conflating quadratic and exponential growth – “if the factor were n^2 starting at 2, value 1 would be 4 (2^2), value 2 would be 16 (4^2), value 3 would be 256 (16^2), etc.”; and, the elegant “say at the beginning of the pandemic we had an increase of 2 cases per day for the first week, but then the second week it is 4 per day, the third week it is 8, and so on”. Explicitly differentiating from other types of growth, another student writes: “if 1 person was infected, the next day that one person could infect maybe around 5 more. The next day those 5 people could infect 25 more, then 125 then 625 then 3,125 then 15,625 so you see that the more people get infected the more it spreads. It does not just go from 1 to 2, it has larger gaps between each jump”.

“plain” (5)	“faster and faster” (18)		
general (4)	general (5)	verbal only (4)	
		verbal with reference to graph (1)	
specific (1)	specific (13)	verbal only (10)	imprecise (5)
			ambiguous (3)
			precise (2)
		verbal with graph (3)	graph illustrative yet generic (1)
			graph illustrative, informative (2)

Table 1: “plain” and “faster” narratives on exponential growth in portfolio entries

Finally, 3 entries evidenced ‘*faster and faster growth*’ narratives (verbal, with graph) in which a graphical realisation of exponential growth has significance that varies from minimal (“...growing or increasing really fast and is often shown in a graph as a really steep gradient or line. Like in the graph below”, Figure 1a) to quite substantial (Figures 1b, 1c). The student entry that corresponds to Figure 1b

“[...] speed of growth proportional to the population. I would first explain linear growth (1 more person is infected each day, so 1 on day1, 2 on day2, 3 on day3, 4 on day4 etc). Then use doubling and compare to exponential growth (1 person is infected on day1, then 2 on day2, 4 on day3, 8 on day4 etc. A simple graph would help highlight the difference between exponential growth and linear growth.”

is purposefully embedded into a “doubling” example of exponential growth ($f(n)=2^{n-1}$) as distinct from linear growth ($f(n)=n$). Figure 1b captures the difference between the two with the annotation “disruption”. The student entry that corresponds to Figure 1c (“Something doubling each day [...] something becoming more and more rapid [...] a process that increases quantity over time [...] a relatively small number can become very large very quickly [...]”) evokes a different but also helpfully transparent realisation of “exponential growth”.

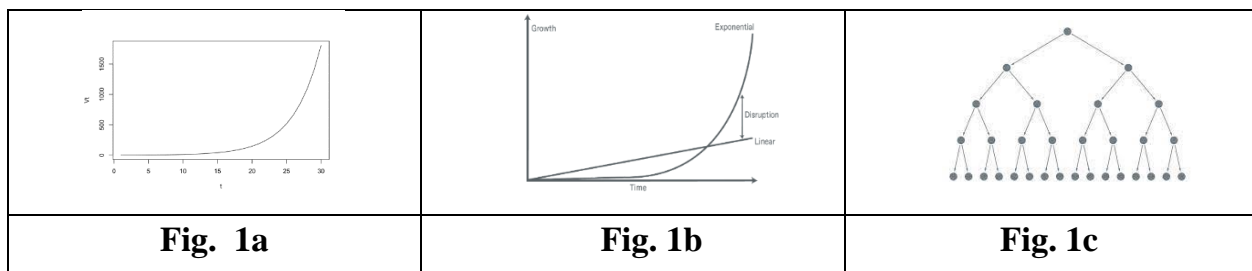


Figure 1: “exponential growth” images in 3 portfolio entries.

LOSING URGENCY OF “EXPONENTIAL” IN COLLOQUIAL DISCOURSE

Missing the exponential nature of the virus’ transmission proved to be a key casualty of governmental policy – and its endorsement by the public – during the pandemic in the UK. The bulk of the student portfolio entries discussed in this paper equate “exponential” more or less with “very fast”: this equating obfuscates the urgency of the exact sort of “fast” that exponentiality carries (as Professor Whitty’s quotations at the start of this paper amply demonstrate). While mathematics education research into the learning and teaching of exponentials has mainly focused on its intra-mathematical significance and precision, its extra-mathematical relevance and importance has been left as a matter to be dealt with outside the classroom: see, for example, Chapter 1 in Yates (2021) for a lucid demonstration of exponential growth in the context of algal bloom and nuclear reactions; or, the [Centre for Evidence-Based Medicine](#)’s nutshell account in the context of viral transmission. In “picturing a crisis” mathematics is here to help with “formatting solutions” (Skovsmose, 2021; p. 371) to it – crucially though, this can only happen in tandem with civic appreciation for mathematics. In preparing citizens for the workplace, especially long-term influencers such as teachers, our role in fostering this appreciation as university mathematics educators is key.

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